# Assignment 6

1. **Rayleigh quotient iteration:**

   **(a)** Compute the smallest eigenvalue and corresponding eigenvector of the symmetric tridiagonal matrix from assignment sheet 5 question 2.

$$A := \frac{20^2}{\pi^2}\begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & \ddots & \ddots & \\ & & \ddots & \ddots & -1 \\ & & & -1 & 2 \end{pmatrix} \in \mathbb{R}^{19\times 19}.$$

   Use the *Rayleigh quotient iteration* (RQI) with the starting vector of $(1,1,\ldots,1)^T$. What convergence do you observe?

   **(b)** Now we consider the modified matrix $B$.

$$B := \frac{20^2}{\pi^2}\begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & \ddots & \ddots & \\ & & \ddots & 2 & -1 \\ & & & -1 & 1 \end{pmatrix} \in \mathbb{R}^{19\times 19}.$$

   This matrix is singular, so the smallest eigenvalue is zero. Modify RQI to find the second smallest eigenvalue. Is the starting vector $(1,1,\ldots,1)^T$ still a good choice?

   **(c)** Show a situation in which RQI does not converge.

2. **Subspace iteration:**

   **(a)** Implement the *subspace iteration* for solving $A\boldsymbol{x} = \lambda B\boldsymbol{x}$ in a MATLAB function. Suppose that $A$ and $B$ are *real, symmetric* $n \times n$ matrices and $B$ is *positive definite.* In addition, the matrices are not so big that we cannot factor them. The algorithm makes a Rayleigh-Ritz step in every fifth iteration.

   Input:

   - Matrices $A$ and $B$.
   - Number of eigenpairs to be calculated $p$.
   - $n \times q$ initial matrix $X$, $q \geq p$.
   - Shift $\sigma$.
   - Tolerance $\tau$ for the stopping criterion $\|(I - X_k X_k^T B)X_{k-1}\|_B \leq \tau$.
     (Here $X_k^T B X_k = I_q$)

Please submit your solution via e-mail to Peter Arbenz (`arbenz@inf.ethz.ch`) by April 17, 2018. (12:00). Please specify the tag **EWP18-6** in the subject field.

Output:

- vector $\boldsymbol{d}$ contains the eigenvalues in ascending order.
- $n \times p$ matrix $V$ contains the corresponding eigenvectors.

Transform the general eigenvalue problem into a special eigenvalue problem by using the Cholesky decomposition of $B$. As a starting point, use the MATLAB function `svit.m`.

As a bonus, you can incorporate deflation in the algorithm. After convergence of an individual eigenvector, it will no longer be iterated. You can exploit that the small eigenvalues converge faster than the large eigenvalues.

**(b)** Calculate the ten smallest eigenvalues and corresponding eigenvectors of the eigenvalue problem “vibrations in the car interior”, which was discussed in the lecture notes. Choose $\sigma = -0.01$ and $\tau = 1 \cdot 10^{-6}$.

The matrices are constructed by the following MATLAB script:

```
[p,e,t]=initmesh('auto');
[p,e,t]=refinemesh('auto',p,e,t);
[p,e,t]=refinemesh('auto',p,e,t);
p=jigglemesh(p,e,t);
[A,M]=assema(p,t,1,1,0);
```

Here `auto` defines the geometry of the car.

The eigenfunctions are stored in a $n \times 10$ matrix `U`, and they can be plotted, e.g., the 4th eigenfunction is plotted as follows

```
pdeplot(p,e,t,'xydata',U(:,4),'mesh','on','colorbar','off')
axis([-1 26 -1 15])
axis equal
axis off
colormap(hsv)
```

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