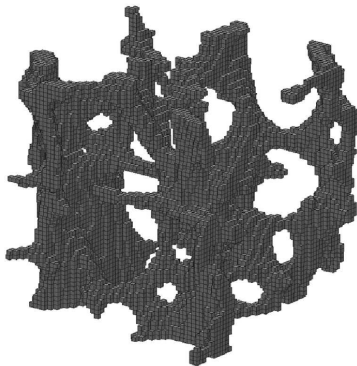


# Marching cubes in bone structure analysis

## Proposal for a master thesis

### Introduction

The aging of the human population made the number of bone fractures due to osteoporosis increase considerably. Worldwide, lifetime risk of osteoporotic fractures in women is estimated close to 40%; in men risk is 13%. Not surprisingly, research of osteoporosis plays a prominent role in health science. Often, large scale computer simulations are conducted to improve the understanding of the bone structure and bone strength [1, 2].



These typically symmetric positive definite systems are solved by the preconditioned conjugate gradient algorithm on large parallel computers. These simulations are based on linear or non-linear elasticity and lead to very large linear or non-linear systems of equations with hundreds of millions of unknowns [1, 2, 5, 6]. The equations of elasticity are discretized by finite elements. In bone structure analysis the domains are obtained by CT scans and are composed by millions of tiny cubes (voxels), see the image on the left.

In recent years we have developed a computer program called ParFE that solves these problems very efficiently, in terms of computing time and in terms of memory space [3, 5–7]. The program essentially executes the conjugate gradient algorithm, combined with the “smoothed aggregation multilevel preconditioner” ML [10]. The program exploits the Trilinos framework [12].

This is a cooperation with Prof. Ralph Müller of the Institute for Biomechanics, ETH Zürich.

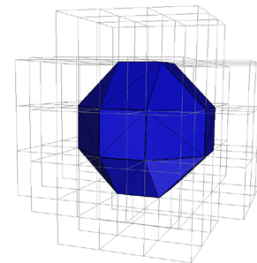
### Description

It turned out that the restriction to cube-shaped elements sometimes leads to inaccurate results. This happens in particular if additional materials (e.g. of artificial implants) are considered in the computations. In a recent master thesis we investigated smoothing of bones surfaces and material interfaces by admitting distorted cubes [3]. However, this often worsens the condition of the system matrices.

Therefore, in this project, ParFE shall be extended by so-called *marching cubes*<sup>1</sup>. Marching cubes is a computer graphics algorithm for reconstructing (among other things) a surface from medical volumetric datasets [4, 8]. The surface computed is polygonal. The marching cubes combines simplicity with high speed since it works almost entirely on lookup tables.

The problem is to form a facet approximation to an isosurface through a scalar function sampled on a rectangular 3D grid. In our situation this function is one at vertices in the bone structure, and zero elsewhere.

Given one grid cell defined by its vertices and scalar values at each vertex, it is necessary to create planar facets that best represent the isosurface through that grid cell.



<sup>1</sup>See [http://en.wikipedia.org/wiki/Marching\\_cubes](http://en.wikipedia.org/wiki/Marching_cubes) and <http://local.wasp.uwa.edu.au/~pbourke/geometry/polygonise/>.

Computations will be done on the Brutus-Cluster and later on the Cray XT5 at CSCS (Swiss Supercomputing Centre). Both machines are clusters of AMD Opterons, though their interconnect differs substantially.

## Requirements

- Good knowledge in C++.
- Good knowledge of finite elements.
- The attendance of a parallel computing course is very useful.
- Willingness to work in an interdisciplinary environment.

## Deliverables

The work is to be documented in a short and concise thesis (L<sup>A</sup>T<sub>E</sub>X, PDF). It must be written such that it is intelligible to a fellow-student.

The code should be written as clean as possible. It must be complemented by a short user's guide.

## Presentation

At the end of the thesis the work is to be presented in a talk at a seminar of the Chair of Computational Science. The date of the talk will be determined later.

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