Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Computer Science Department
Prof. Dr. Peter Arbenz

Fast solution of axisymmetric time-harmonic Maxwell equations

Proposal for a master thesis

Introduction

The design of Radio Frequency (RF) structures is a complex optimization process. RF structures are elements in which charged particle are accelerated by an oscillating electric field which is obtained by exciting the proper eigenmode of the accelerator cavity. The shape of the eigenmodes as well as their frequencies are determined by the shape of the cavity.

Let Ω be a 3-dimensional (3D) domain that represents the cavity. Then the eigenfrequencies and eigenmodes are determined by the time-harmonic Maxwell equation, an eigenvalue problem (here in weak form),

$$
\begin{array}{ll}
\multicolumn{2}{l}{\textit{Find } (\lambda, \boldsymbol{e}, p) \in \mathbb{R} \times H_0(\mathbf{curl}\,;\Omega) \times H_0^1(\Omega) \textit{ such that } \boldsymbol{e} \neq \mathbf{0} \textit{ and}} \\
\text{(a)} \quad \int\limits_\Omega \mu\, \mathbf{curl}\, \boldsymbol{e} \cdot \mathbf{curl}\, \boldsymbol{v} d\boldsymbol{x} + \int\limits_\Omega \mathbf{grad}\, p \cdot \boldsymbol{v} d\boldsymbol{x} = \int\limits_\Omega \varepsilon\, \boldsymbol{e} \cdot \boldsymbol{v} d\boldsymbol{x}, & \forall \boldsymbol{v} \in H_0(\mathbf{curl}\,;\Omega), \\
\text{(b)} \quad \int\limits_\Omega \boldsymbol{e} \cdot \mathbf{grad}\, q = 0, & \forall q \in H_0^1(\Omega).
\end{array}
\tag{1}
$$

For the definition of the function spaces used in (1) we refer to [6]. We assume the permittivity ε and the permeability μ to be constant scalars. Discretizing (1) by the finite element method yields a matrix eigenvalue problem

$$\boldsymbol{A}\boldsymbol{x} = \lambda \boldsymbol{M}\boldsymbol{x}, \qquad \boldsymbol{C}^T \boldsymbol{x} = \mathbf{0}. \tag{2}$$

We have developed an efficient parallel Jacobi–Davidson eigensolver Femaxx to solve this particular eigenvalue problem [1, 3, 5, 7, 8].

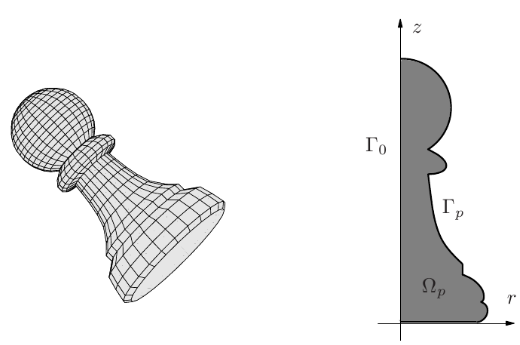

Figure 1: The cross section Ω_p (right) of the axisymmetric domain Ω (left), represented by the shaded region, is confined by two boundary sets. The first set, called Γ_0, comprises all the points of $\partial\Omega_p$ that lie on the rotation axis. The second one consists of the remaining boundary points, i.e. $\Gamma_p = \partial\Omega_p \setminus \Gamma_0$ [4].

It may be desirable that the shape of Ω is such that (1) has, e.g., some prescribed eigenvalues. If we want to determine such a shape by some optimization procedure[1] then we have to solve very many eigenvalue

[1] Compare proposal for the master thesis on "Multi-objective shape optimization".

problems (2). Full 3D problems are too expensive to this end. Therefore we want to simplify the task by assuming that Ω is *axisymmetric*, see Figure 1. Then the 3D problem (1) can be replaced by *a few* 2D eigenvalue problems of similar structure. Chinellato [2, 4] has implemented a solver for the axisymmetric time-harmonic Maxwell equation, however for the more difficult case of vertical-cavity surface-emitting lasers (VCSELs). Matrix eigenvalue problems of the form

$$\boldsymbol{A}\boldsymbol{z} \equiv \begin{bmatrix} \boldsymbol{A}_1 + m^2\boldsymbol{A}_2 & m\boldsymbol{A}_3 \\ m\boldsymbol{A}_3^T & \boldsymbol{A}_4 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \lambda \begin{bmatrix} \boldsymbol{M}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} \equiv \lambda\, \boldsymbol{M}\boldsymbol{z}, \tag{3}$$

have to be solved for $m = 0, 1, \ldots$, in practice just for a very small number of m's. In (3) the diagonal blocks $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, and $\boldsymbol{A}_4$ are symmetric and $\boldsymbol{M}_1$, $\boldsymbol{M}_2$ are symmetric positive definite.

Description of the task

The solution of the axisymmetric eigenvalue problem (3) shall be implemented as efficiently as possible to enable the solution of large numbers of such problems as is necessary in optimization.

The complex-symmetric VCSEL solver of Chinellato shall be adapted to the real-symmetric case. Femaxx can be used as a solver, together with the Trilinos packages it employs. The matrices can be generated according to the VCSEL solver. The multilevel preconditioner used in Femaxx has to be modified for the two dimensional case. Note that this preconditioner did not work with the complex-symmetric problem.

The implementation should exploit the fact that there are many similar problems (varying m, varying Ω) to be solved. This may incorporate the reuse or clever modification of the preconditioner. Reusing parts of the search space is another option.

The target machine is the Brutus cluster at ETH[2].

Requirements

- Good knowledge in C++.
- Good knowledge of finite elements.
- The attendance of a parallel computing course is very useful.
- Willingness to work in an interdisciplinary environment.

Deliverables

The work is to be documented in a short and concise thesis (LaTeX, PDF). It must be written such that it is intelligible to a fellow-student.

The code should be written as clearly as possible. It must be complemented by a short user's guide for the software that has been developed.

Presentation

After about one month a presentation of the project is to be given with a schedule of the work to be done.

At the end of the thesis, the work is to be presented in a talk at a seminar of the Chair of Computational Science. The date of the talk will be determined later.

Contact

- Prof. Dr. Peter Arbenz, arbenz@inf.ethz.ch, Tel: 044 632 7432
- Dr. Andreas Adelmann, andreas.adelmann@psi.ch, Tel: 056 310 42 33

[2] http://en.wikipedia.org/wiki/Brutus_cluster

References

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