Compact finite difference formulations for the Poisson equation with application to particle accelerators

Proposal for a bachelor/master thesis

Introduction

The 7-point stencil \( \nabla^2 \) to approximate the Laplacian \(-\Delta u(x)\) on a regular grid with spacing \( h \) is well known,

\[
-\nabla^2 u(x) = \frac{1}{h^2} \left( 6u(x) - \sum_{j=1,2,3} u(x \pm he_j) \right) + O(h^2) \equiv -\nabla^2_7 u(x) + O(h^2).
\]

This stencil is used almost always if the Laplacian is approximated by finite differences for solving, e.g., the Poisson equation, the Navier–Stokes equations, or other equations [2].

The purpose of this thesis is to investigate so-called compact formulations for the Laplacian [4]. One of them is the 19-point stencil

\[
-\nabla^2_{19} u(x) \equiv \frac{1}{6h^2} \left( 24u(x) - 2 \sum_{j=1,2,3} u(x \pm he_j) - \sum_{j,k=1,2,3, j\neq k} u(x \pm he_j \pm he_k) \right).
\]

Provided \( f(x) \) is sufficiently smooth, a forth order finite difference approximation for the Poisson equation \(-\Delta u(x) = f(x)\) is obtained by

\[
\nabla^2_{19} u(x) = f(x) + \frac{h^2}{12} \nabla^2_7 f(x) + O(h^4).
\]

Scope of work

If the work is to become a bachelor thesis the compact formulation is to be incorporated into a (MPI-parallelized) Poisson solver that employs the 7-point stencil so far. If the work is to become a master thesis then additionally the solver is to be integrated into a full-fledged particle solver where a sequence of Poisson problems has to be solved.

Since the compact finite difference formulation is much more accurate than the 7-point stencil, a coarser grid suffices to get the same accuracy in the solution of the Poisson equation. The question is thus: how much coarser can we choose the grid without losing accuracy in the overall solver. If time permits the compact finite difference formulation is to be incorporated into a solver for non-square domains [1].
Requirements

- Student in computational science or related fields.
- Very good knowledge in numerical mathematics.
- Fluent in C++.
- For the master thesis, attendance in the lecture on “Particle Acceleration Methods” by Dr. Adelmann is advantageous.

Deliverables

- The work is to be documented in a short and concise thesis (\LaTeX, PDF). It must be written such that it is intelligible to a fellow-student.
- The code should be written as clean as possible. It must be properly documented.
- At the end of the thesis, the work is to be presented in a 30 minutes’ talk.

Contact

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References


