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Adaptive mesh refinement in a parallel space-time solver for Navier–Stokes equations

Proposal for a master thesis

Introduction

Recently, in [1, 5, 8], we considered the shallow water equation as a model for the behavior of a fluid in a rectangular basin Ω which is excited periodically. The excitation is caused by periodic swayings of the ground of the basin with a frequency ω, imposing a periodic behavior of the fluid with the period $T = 2\pi/\omega$ [7].
More recently, in [2, 6], we investigated a two-dimensional, viscous, incompressible channel flow with an oscillating disk, see Fig. 1. The oscillating disk introduces a fundamental frequency ω to the system.

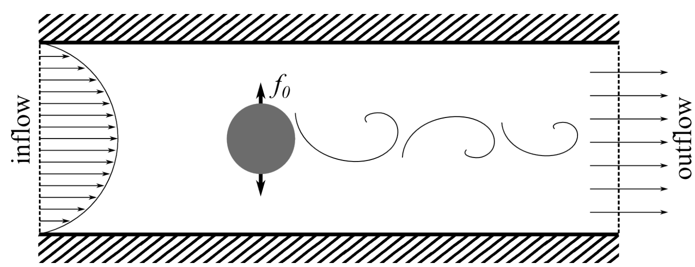

Figure 1: Example problem: Oscillating disk in a viscous channel flow. The disk oscillates with the frequency ω and creates an unsteady wake.

With a classical Navier–Stokes solver, the simulation is started with a given initial flow field. For sufficiently small Reynolds numbers, the flow will go through a transient phase and will eventually lock onto a periodic solution with the period $1/\omega$.
In [2], similarly as in [1] we modeled the fluid in space-time $\Omega \times [0, T)$. We imposed periodic boundary conditions in time. The discretization of the Navier–Stokes equations by finite differences in space *and* time leads to a very large nonlinear system of equations that requires parallel solution. This nonlinear system entails a high potential for parallelization, not only in space but also in time.
We solved this large nonlinear system by a Newton–Krylov method. That is, the nonlinear system is solved by Newton iteration. In each Newton step a linear system is to be solved for the Newton correction. To that end a Krylov space method is used, in our case the preconditioned GMRES method. The complete solver is implemented by means of the Trilinos framework [4, 9].

Scope of work

In order to be able to solve large problems we would like to introduce AMR, i.e., adaptive mesh refinement (and coarsening) into our code. The idea is to have fine meshes only where it is needed, that is, where the flow changes rapidly (in space or time). To the best of our knowledge, AMR has so far not been applied in space-time approaches. However, Weinzierl and Köppl [10] introduced a multigrid solver for such types of problems.
The goal of this master thesis is to

1. Introduce an AMR approach into our code.

   This involves an efficient estimator of the local error, as well as the implementation of a grid manager. We favor a tool like Boxlib [3].

2. Apply the approach to the periodic channel flow.

3. Compare the new enhanced code with the original plain structured approach in terms of execution time and memory consumption.

Deliverables

The work is to be documented in a short and concise thesis (LaTeX, PDF). It must be written such that it is intelligible to a fellow-student.
The code should be written as clean as possible. It must be complemented by a short user's guide.

Presentation

After 3–4 weeks a 10 min presentation and a timetable is due. After about 2 months a 15 min progress report is requested. At the end of the thesis, the work is to be presented in a 30 minutes' talk. Details will be determined later.

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