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Adaptive mesh refinement for a spectral time-periodic Navier–Stokes solver

Proposal for a master thesis

Introduction

The continued increase in computational power nowadays is mainly due to the increasing number of processing units rather than due to the increasing processor speed. Therefore flow solvers have to be able to use an ever increasing number of processing units. Most modern solvers for the Navier–Stokes equations are parallelized by decomposing the spatial flow domain into small sub-domains allowing them to simulate larger and larger problems. However, bigger spatial domains typically also entail bigger physical timespans which have to be covered in a simulation. This problem occurs especially in flow problems that are concerned with the spatial evolution of time-harmonic perturbations (e.g. vibrating-ribbon problem). The numerical simulation of such problems with time-stepping methods requires a long transient phase at the beginning of each simulation which is of no particular physical interest. Only after this transient, the physically relevant steady-state solution is established which is often periodic in time (e.g. Karman vortex street) or chaotic as in fully developed turbulent flows.
We are using an alternative approach which circumvents this transient by solving directly for a time-periodic steady-state solution by introducing periodic boundary conditions in time.
We write the set of partial differential equations as

$$\begin{aligned}
\alpha^2\partial t\mathbf{u} + \mathrm{Re}(\mathbf{u}\cdot\nabla)\mathbf{u} &= -\mathrm{Re}\nabla p + \Delta\mathbf{u}, && \mathbf{x}\in\Omega\,, \quad t\in[0,2\pi]\,,\\
\nabla\cdot\mathbf{u} &= 0, && \mathbf{x}\in\Omega\,, \quad t\in[0,2\pi]\,,\\
\mathbf{u}(\mathbf{x},t) &= \mathbf{u}_{\mathrm{bc}}(\mathbf{x},t), && \mathbf{x}\in\partial\Omega\,,\\
\mathbf{u}(\mathbf{x},0) &= \mathbf{u}(\mathbf{x},2\pi), && \mathbf{x}\in\Omega\,.
\end{aligned}$$

Here, α and Re denote the Womersley and the Reynolds number, respectively. $\mathbf{u}(\mathbf{x},t)$, $p(\mathbf{x},t)$ are functions in space and time, denoting velocity and pressure, respectively. Ω is the spatial domain. Our implementation employs the spatial finite-difference discretization and the parallelization schemes of the well-established IMPACT code [3]. The temporal discretization is done using a truncated Fourier series $\mathbf{u}(\mathbf{x},t)=\sum_{k=-N_f}^{N_f}\hat{\mathbf{u}}_k(\mathbf{x})\exp(ikt)$, $p(\mathbf{x},t)=\sum_{k=-N_f}^{N_f}\hat{p}_k(\mathbf{x})\exp(ikt)$. So, we get

$$\left.\begin{aligned}
ik\alpha^2\hat{\mathbf{u}}_k + \sum_{\substack{l=-N_f\\|k-l|\le N_f}}^{N_f}\mathrm{Re}(\hat{\mathbf{u}}_{k-l}\cdot\nabla)\hat{\mathbf{u}}_k - \boldsymbol{\Delta}\hat{\mathbf{u}}_k + \mathrm{Re}\nabla\hat{p}_k &= \mathbf{0},\\
\nabla\cdot\hat{\mathbf{u}}_k &= 0,
\end{aligned}\right\}\quad \text{for } \mathbf{x}\in\Omega\,, -N_f\le k\le N_f. \tag{1}$$

where N_f is the number modes.
The target of this project is to investigate adaptive mesh refinement in a parallel space-time set-up. Fourier coefficients belonging to higher Fourier numbers k naturally oscillate with higher frequency in space and therefore need a finer resolution in space to be resolved. The benefit of finer meshes for higher Fourier numbers should be investigated. Furthermore the truncation of the Fourier series N_f can be chosen differently for different spatial subdomains, which can be exploited to gain efficiency.

Scope of work

The student will study the implementation of our parallel space-time framework (PIMPACT), and delve into the respective literature in order to develop adaptive mesh refinement strategies in space and time. The student will implement this solver using the PIMPACT framework and will investigate the performance and scalability of its implementation by numerical experiments.

Description of the task

The goal of this master thesis is to

1. Formulate mesh refinement strategy for the time-periodic Navier-Stokes solver.
2. Implement this strategy using PIMPACT.
3. Investigate performance and speedup of solver.

Procedure and deliverables

- After one month: short 15 minute talk on the project, including a schedule with tasks & milestones.
- There will be frequent meetings to check the progress of the work.
- The work is to be documented in a short and concise thesis (LaTeX, PDF). It must be written such that it is intelligible to a fellow-student.
- The thesis is to be presented in a 30 minutes talk.
- The code should be written as clean as possible.

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References

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