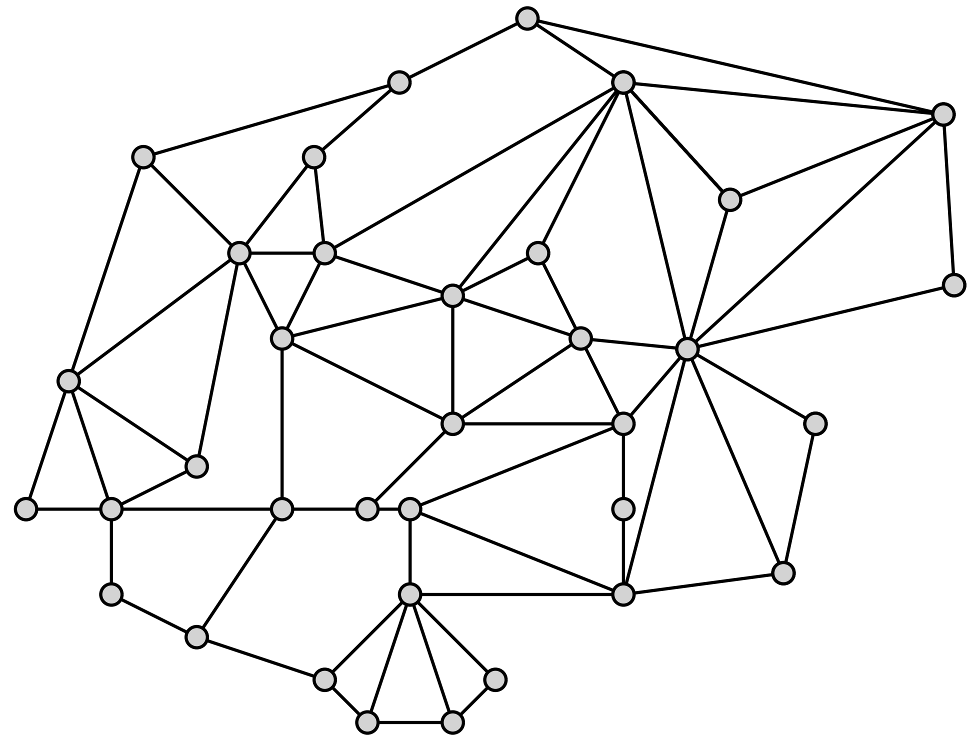
Deterministic Distributed Matching via Rounding

Manuela Fischer

joint work with Mohsen Ghaffari

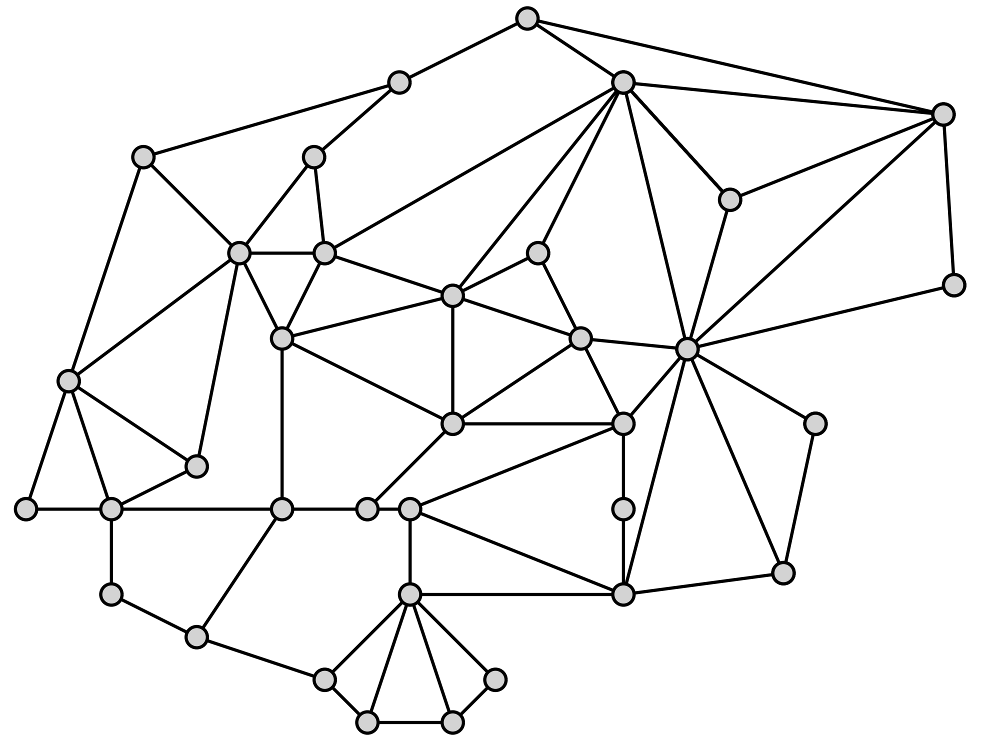
16.05.2017

Distributed LOCAL Model



Distributed LOCAL Model

n nodes, maximum degree Δ
unique IDs



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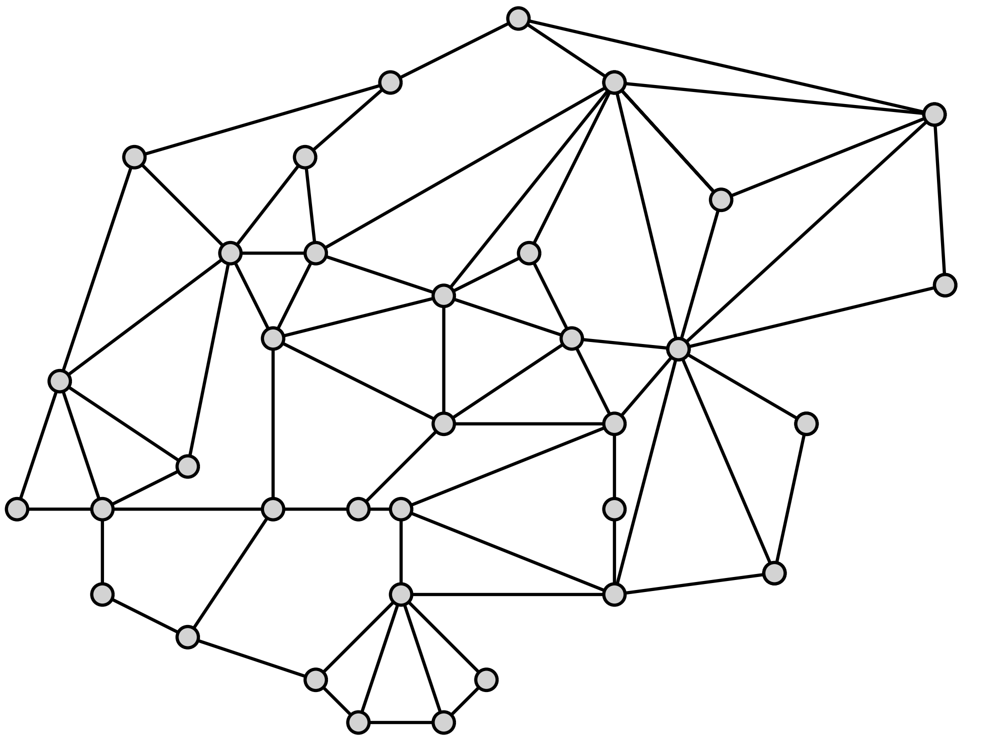

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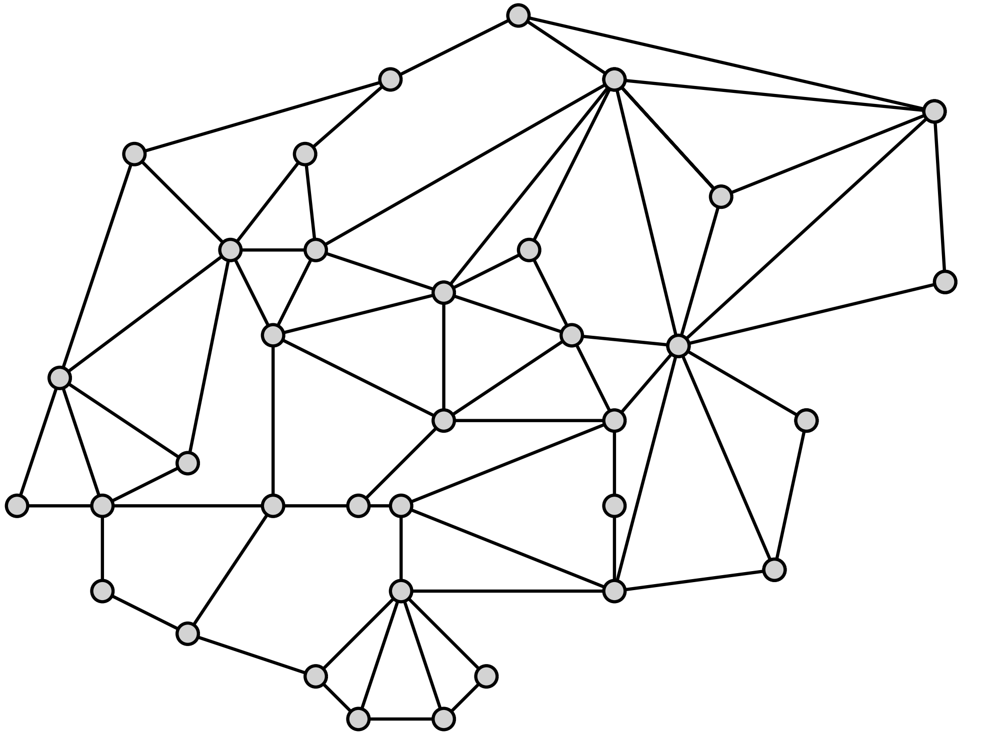

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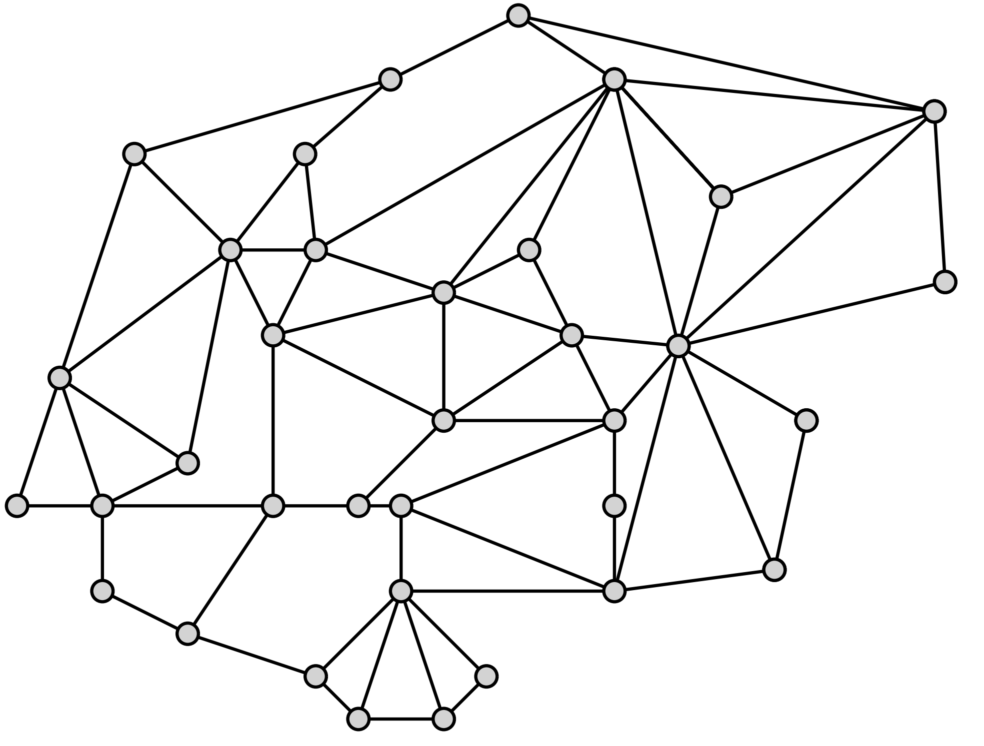

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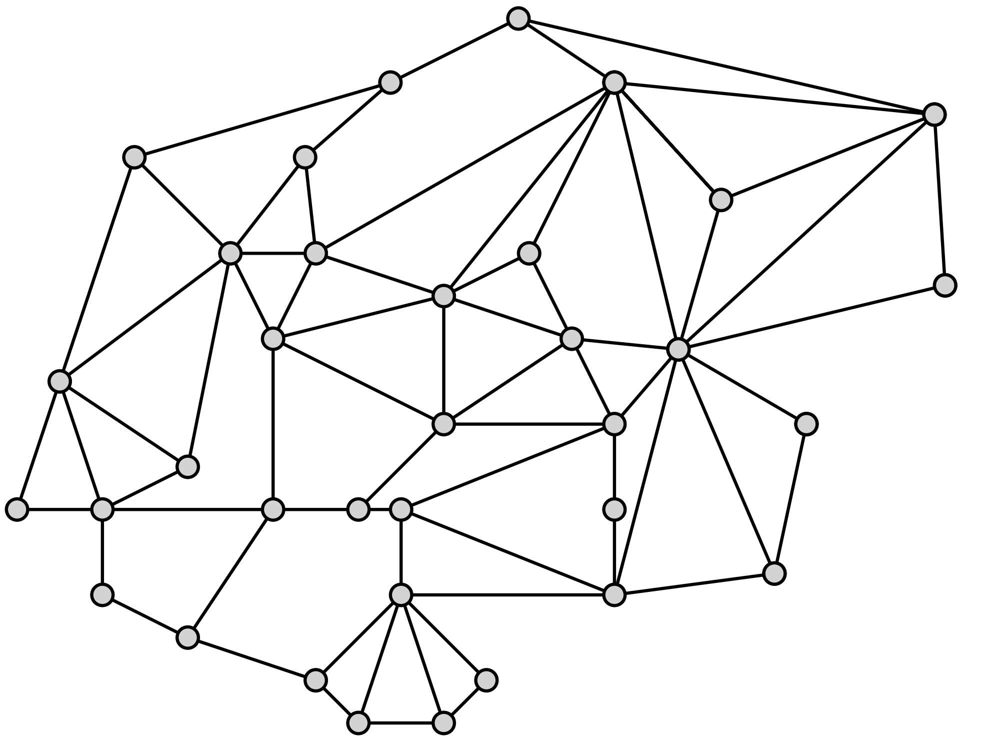

complexity = number of rounds = dependency radius

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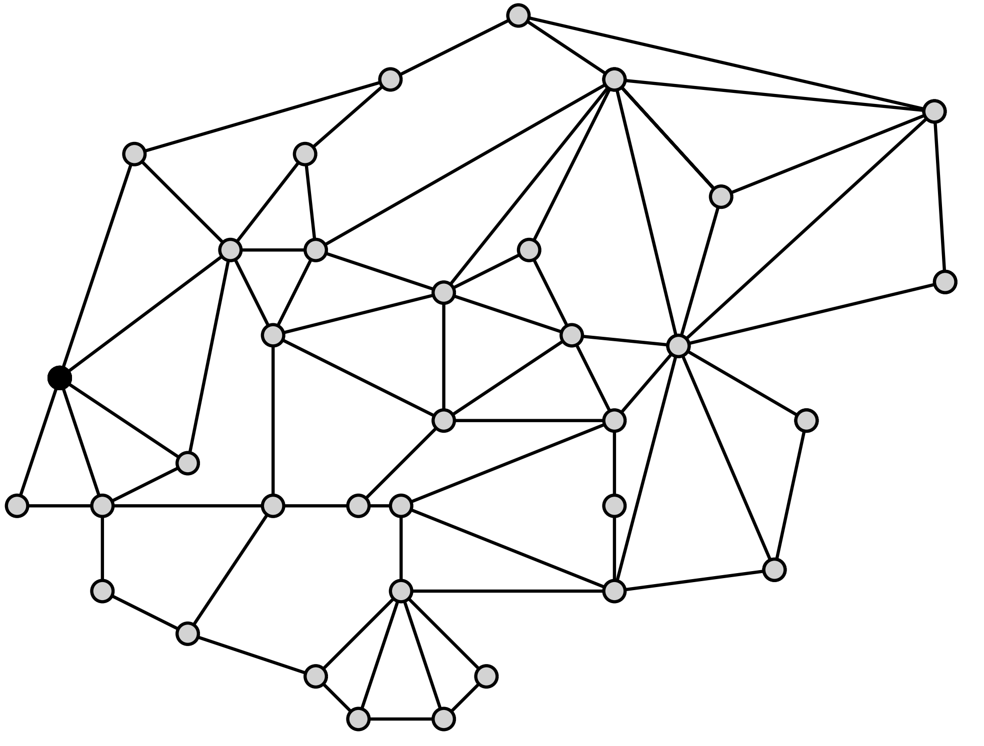

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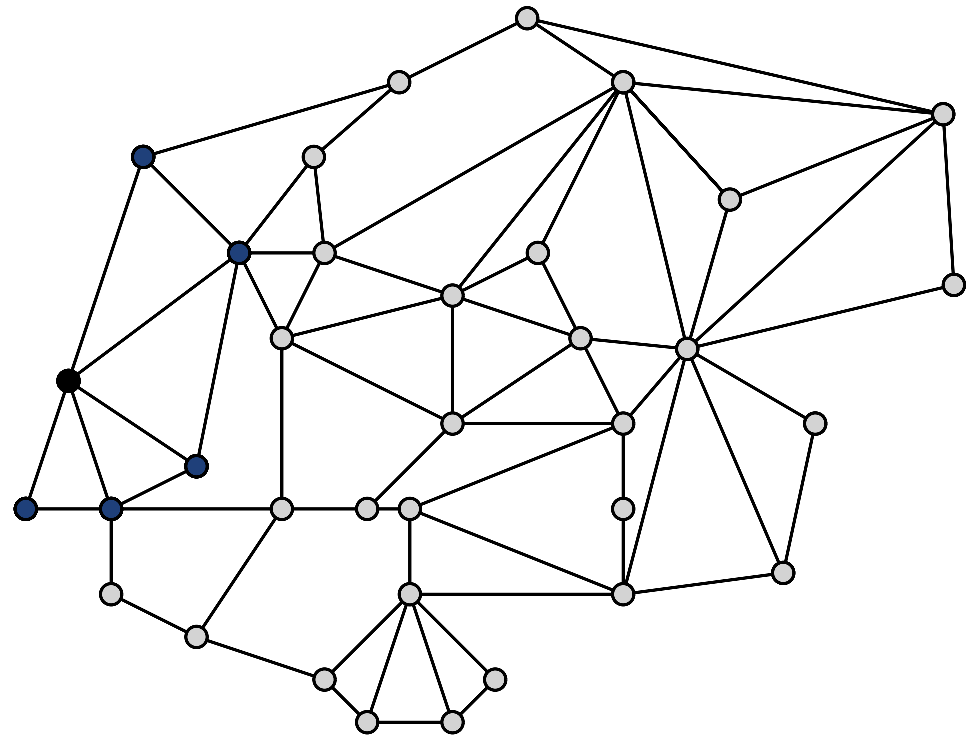

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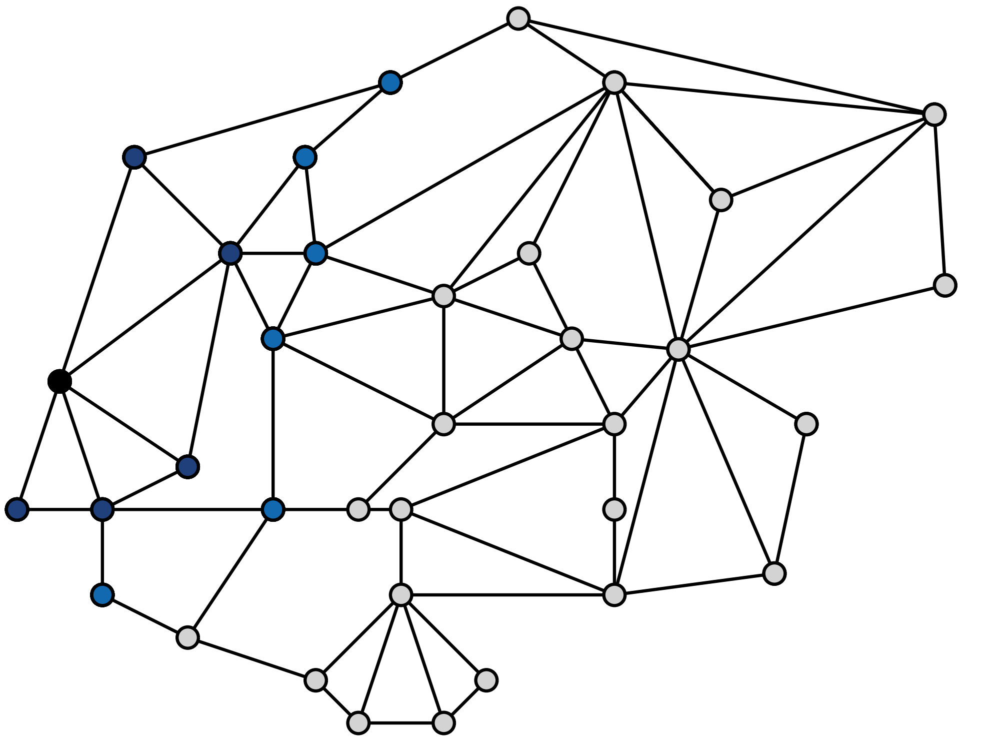

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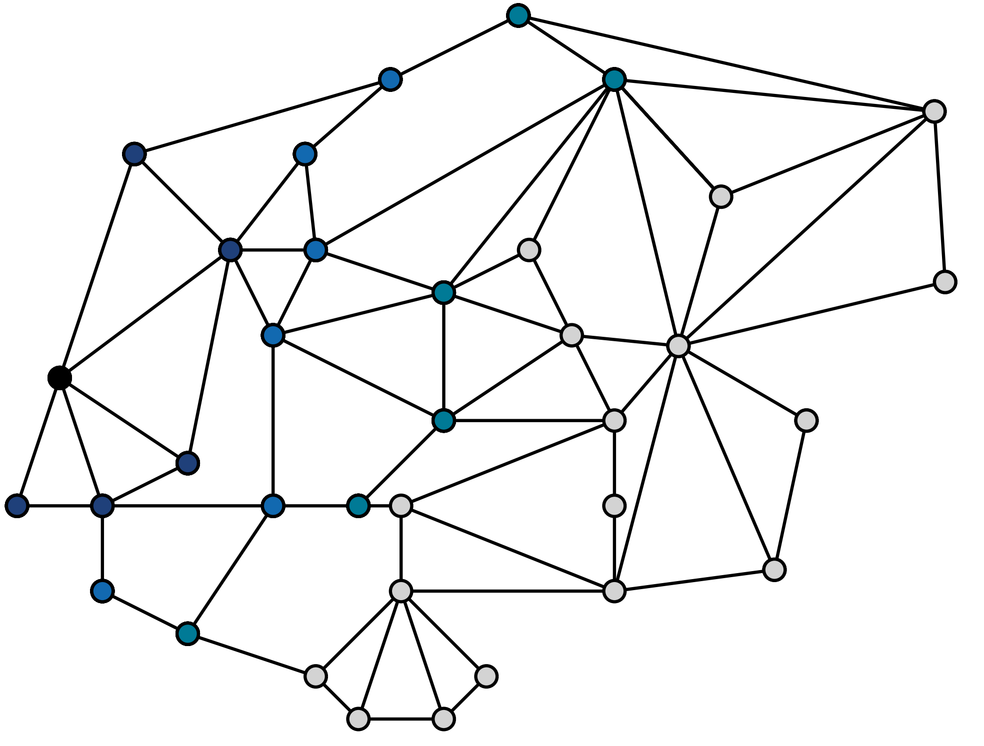

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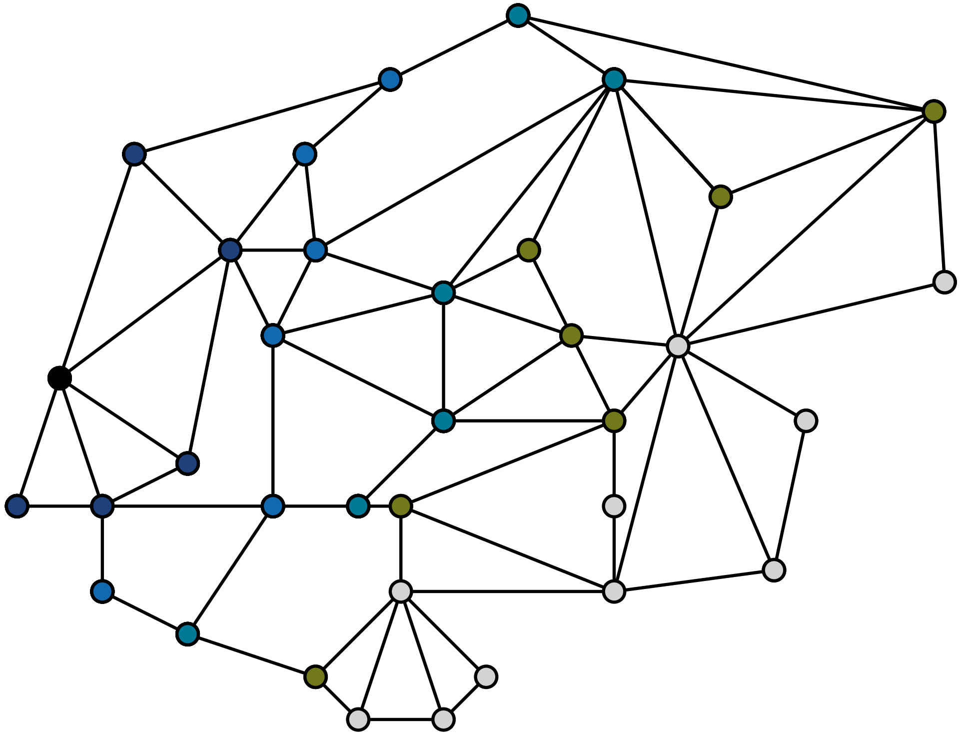

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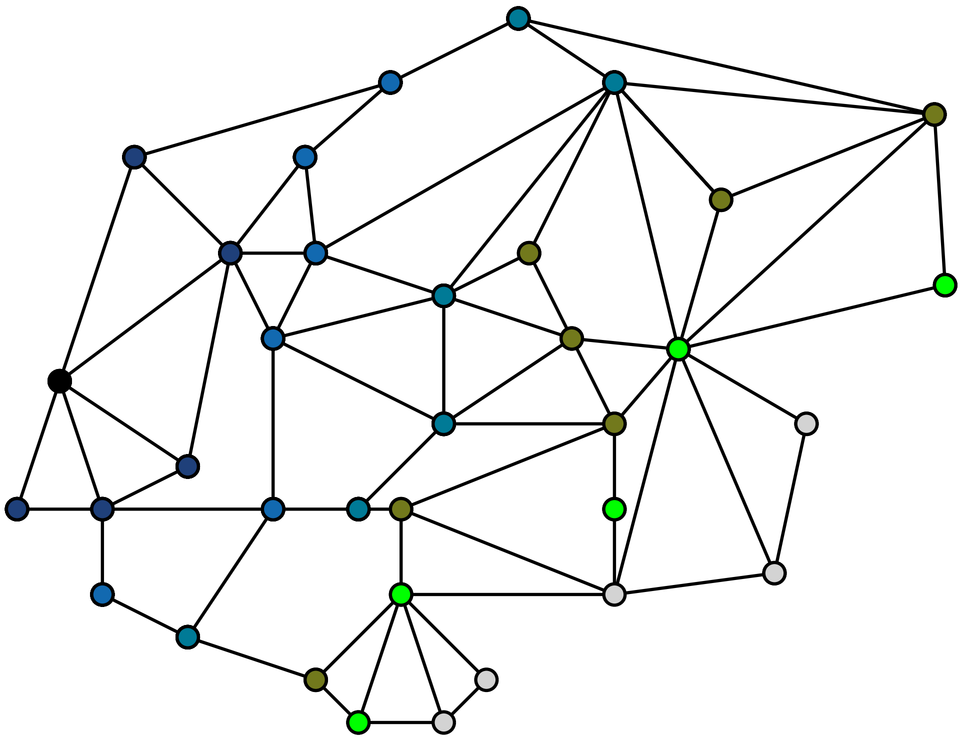

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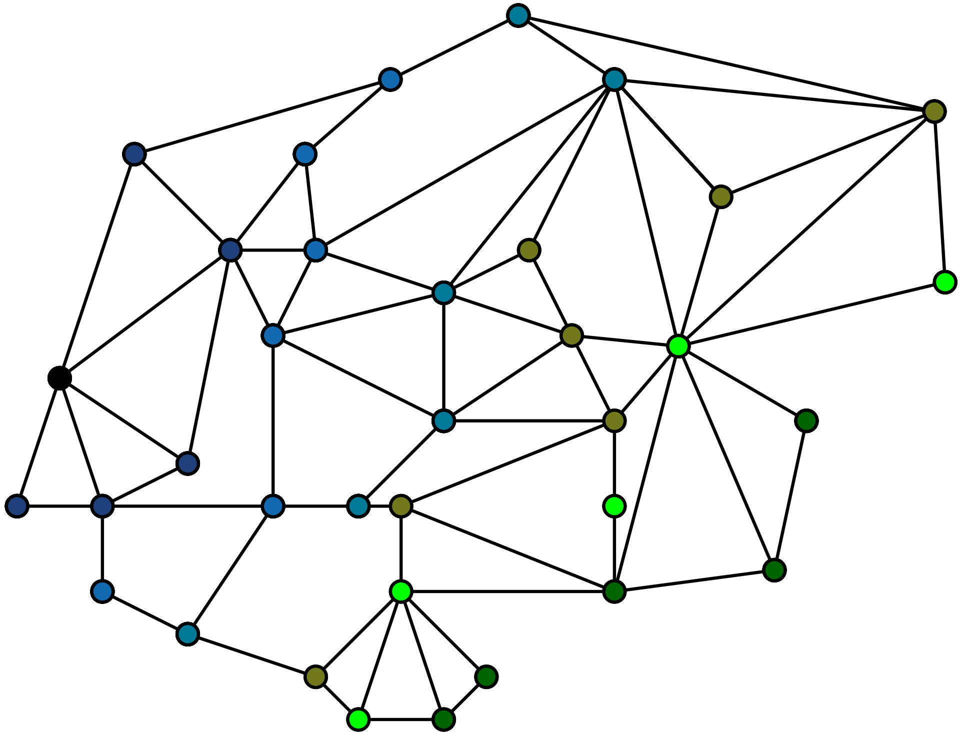

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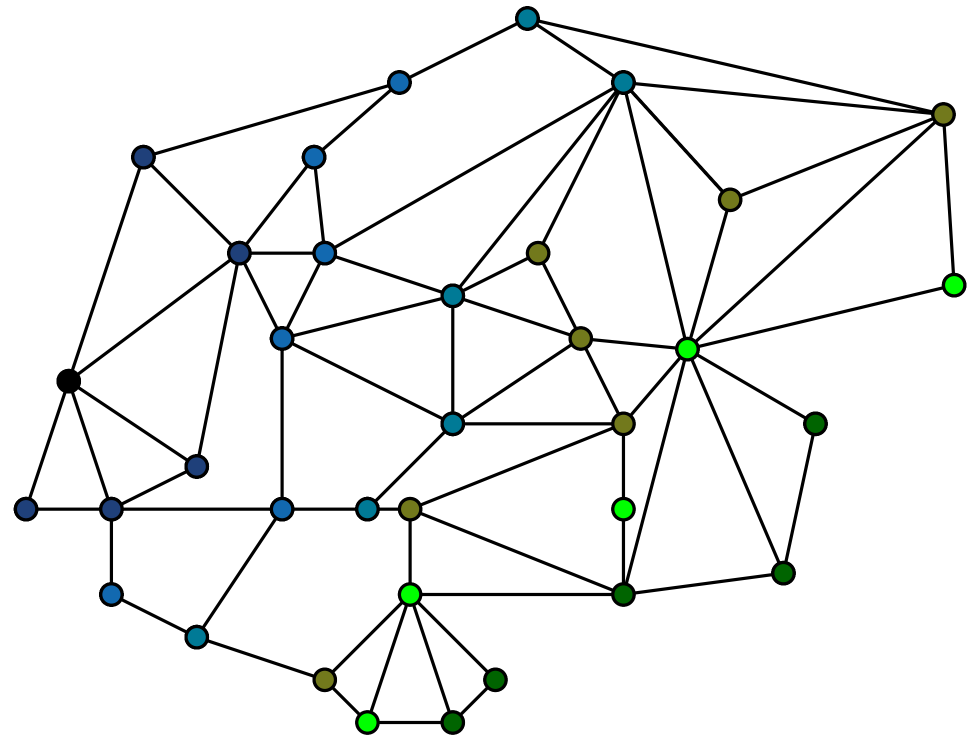

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local problems: $o(D)$ complexity

Distributed LOCAL Model: Bigger Picture

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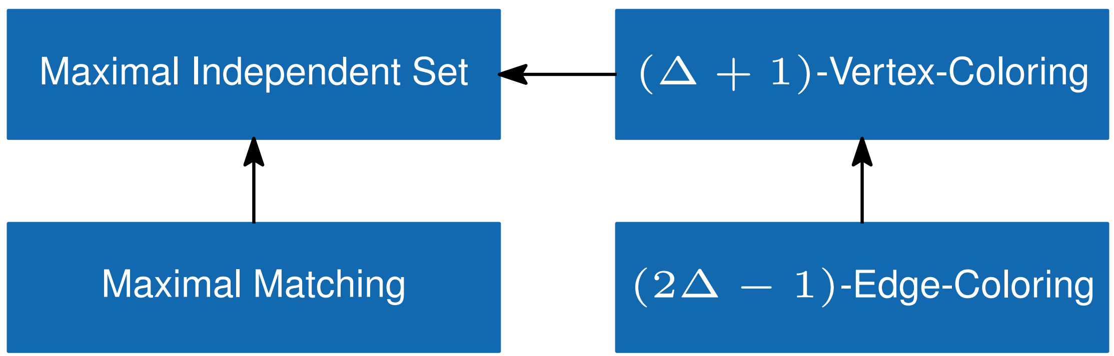
Four Classic LOCAL Problems (studied since 1980s)



Distributed LOCAL Model: Bigger Picture

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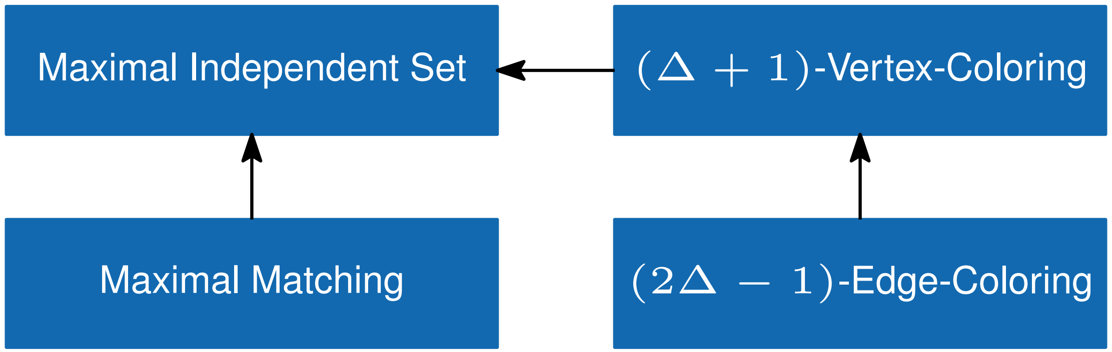

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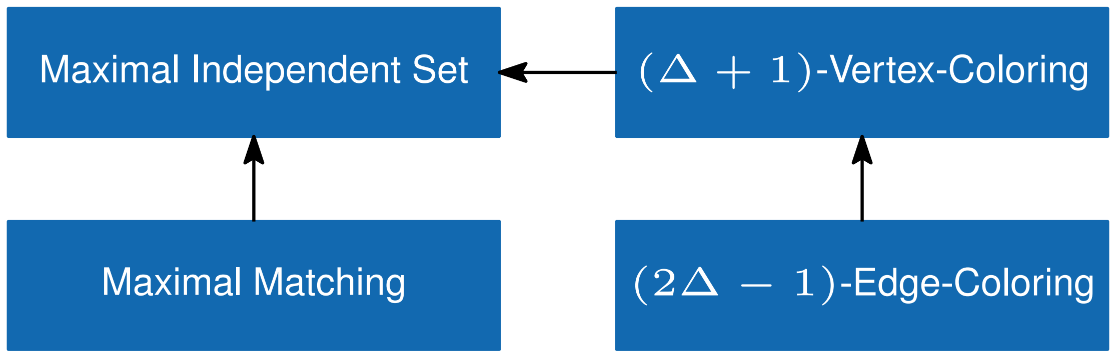

goal: efficient deterministic algorithms for these problems

completeness of rounding (by Ghaffari, Kuhn, Maus [STOC'17]):

only obstacle for finding efficient deterministic distributed algorithms is

efficient deterministic distributed rounding method

Matching Approximation:
State of the Art & Our Result

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Fast deterministic $O(1)$-approximation

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Our Result [F., Ghaffari 2017]

$O(\log^2 \Delta + \log^* n)$	for constant approximation

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$O(\log^2 \Delta \cdot \log \frac{1}{\varepsilon} + \log^* n)$	for $(2+\varepsilon)$-approximation
$O(\log^2 \Delta \cdot \log n)$	for maximal matching (2-approximation)

Further Extensions & Corollaries of Our Result

$O(\log^2 \Delta \cdot \log \frac{1}{\varepsilon})$	for $(2+\varepsilon)$-approximate weighted matching
$O(\log^2 \Delta \cdot \log \frac{1}{\varepsilon})$	for $(2+\varepsilon)$-approximate (weighted) b-matching
$O(\log^2 \Delta \cdot \log \frac{1}{\varepsilon})$	for an ε-almost maximal matching
$O(\log^2 \Delta \cdot \log \frac{\Delta}{\varepsilon})$	for $(2+\varepsilon)$-approximate minimum edge dominating set

Intuitive Idea

(1) What is simple case?

(2) How to reduce to it?

(1) Simple Case: Bipartite Bounded-Degree Graphs

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Lemma:

In bipartite graphs,
a maximal matching can be found in $O(\Delta)$ rounds.

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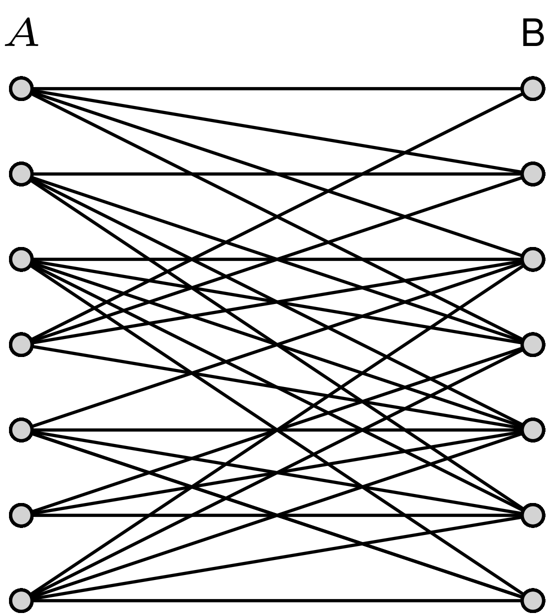

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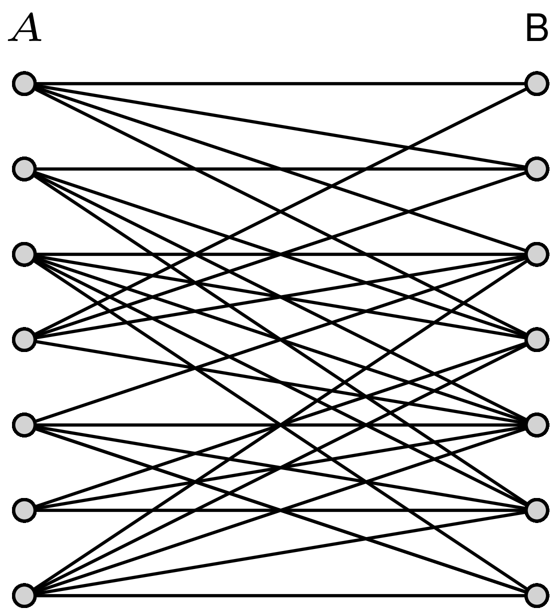

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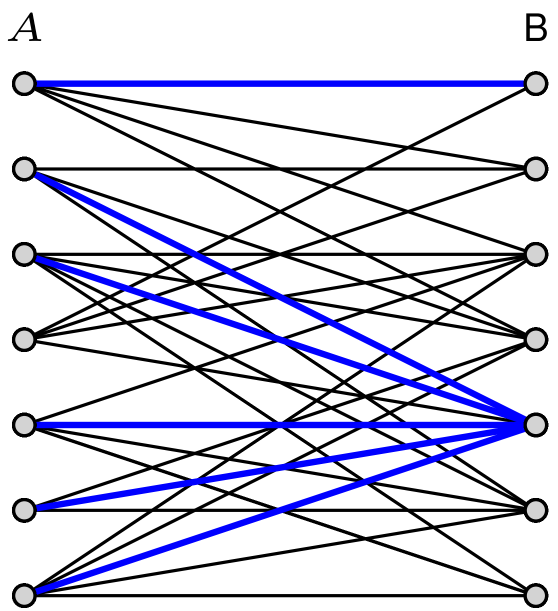

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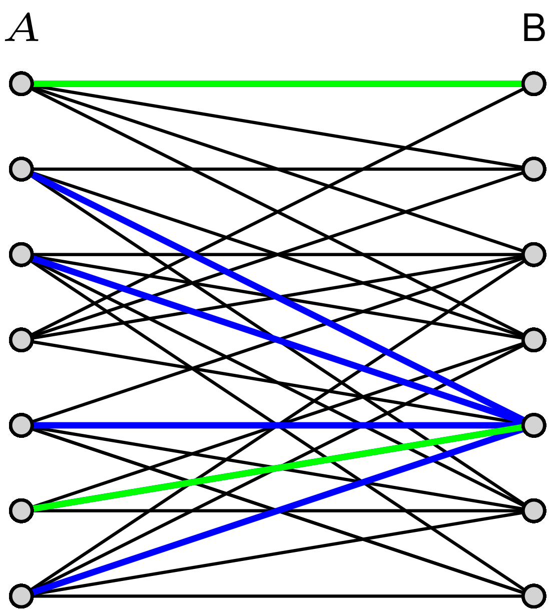

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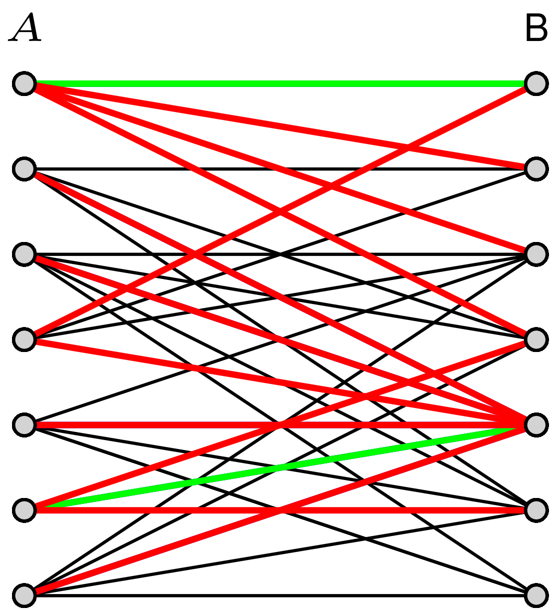

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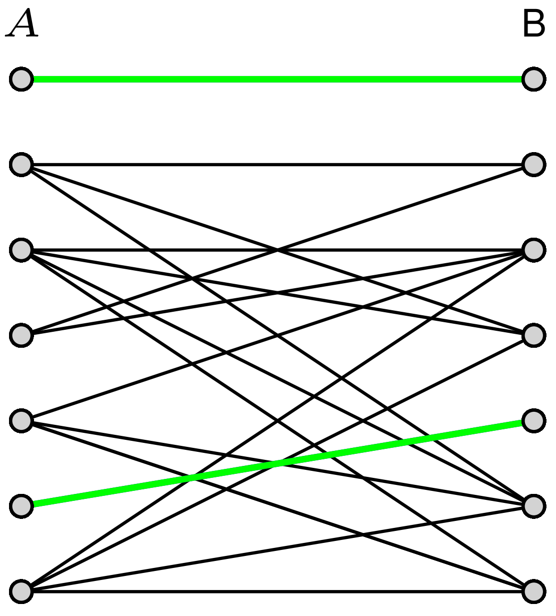

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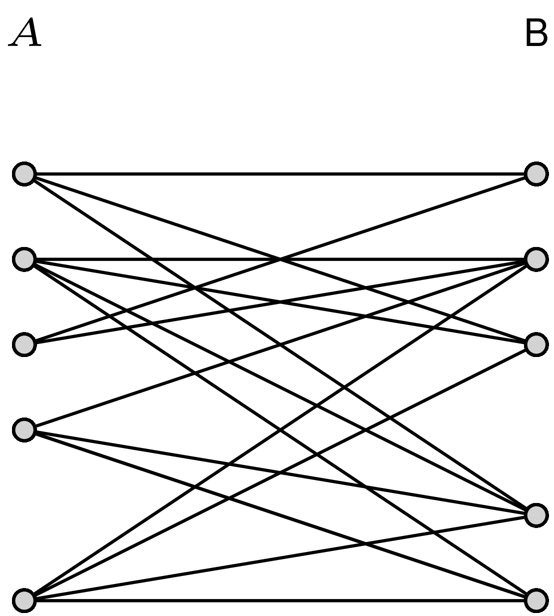

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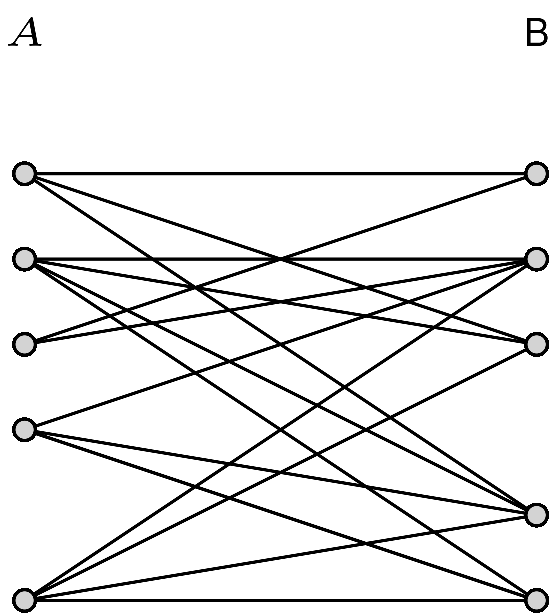

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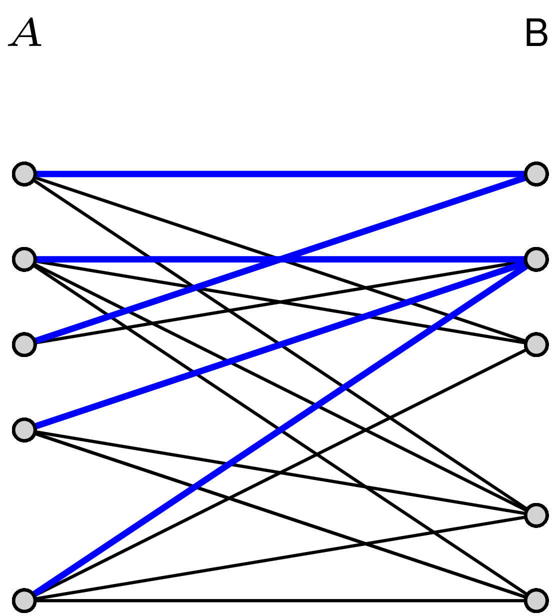

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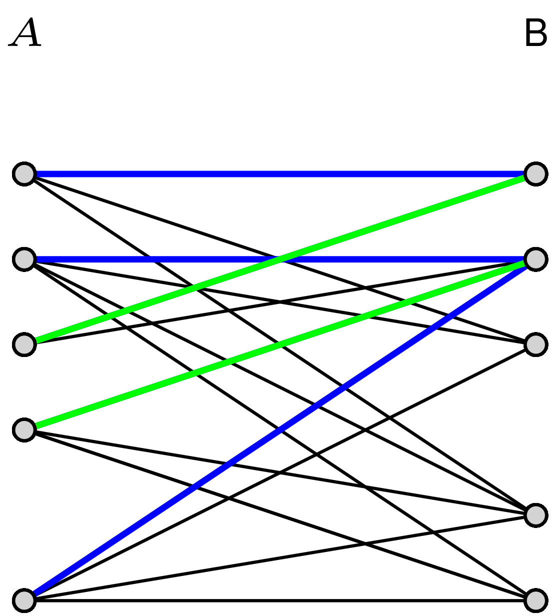

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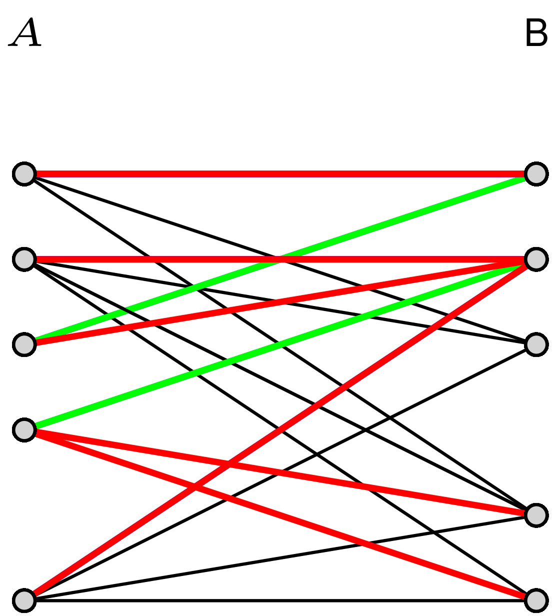

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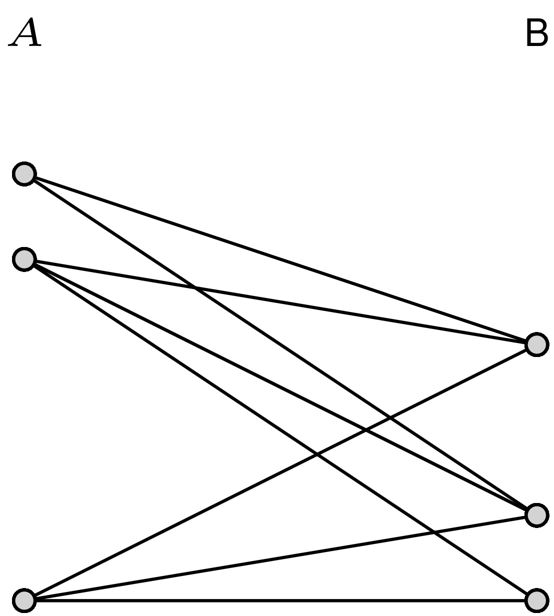

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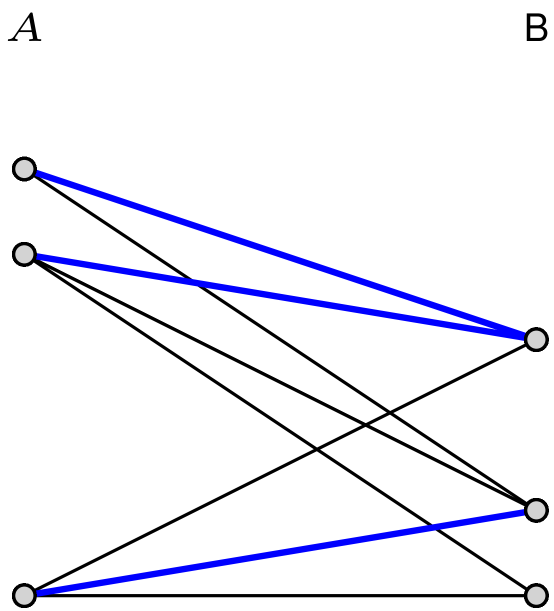

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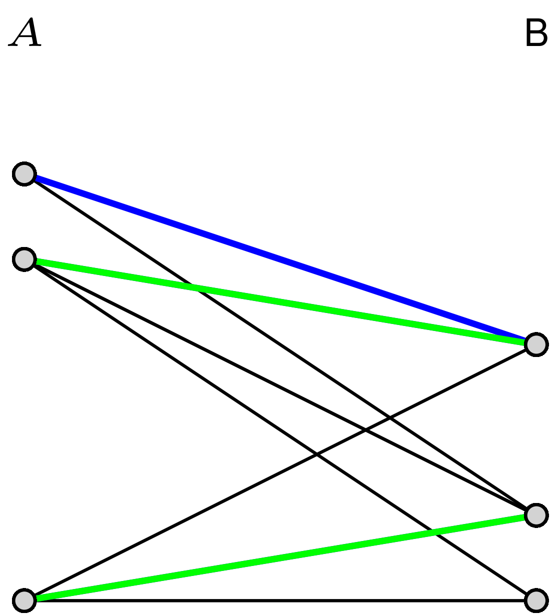

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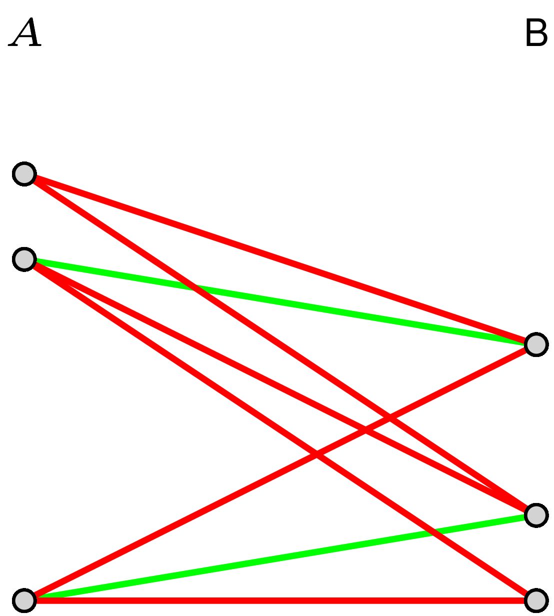

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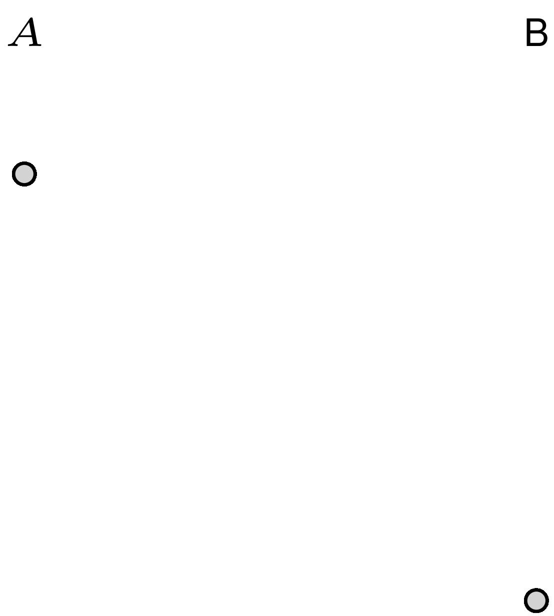

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$O(\log \Delta)$-round $O(\prod_i r_i)$-approximation

Our Method:

Matching Approximation via Rounding LPs

Fractional Matching

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(Integral) Matching as Integer Program

Fractional Matching

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$x_e \in \{0, 1\}$ for all $e \in E$

Fractional Matching

(Integral) Matching as Integer Program

$\sum_{e \in E(v)} x_e \leq 1$ for all $v \in V$

$x_e \in \{0, 1\}$ for all $e \in E$

Fractional Matching

(Integral) Matching as Integer Program

$\max \sum_{e \in E} x_e$

s.t. $\sum_{e \in E(v)} x_e \leq 1$ for all $v \in V$

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Fractional Matching

Fractional Matching as Linear Program

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Fractional Matching as Linear Program

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why useful?

instead of just setting half of edges to 0 and half to 1,

pursue a more gradual approach with fractional matchings

Fractional Matching

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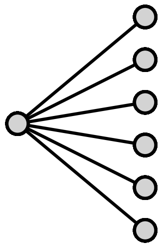

Fractional Matching

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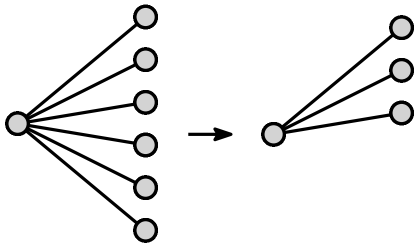

Fractional Matching

Fractional Matching as Linear Program

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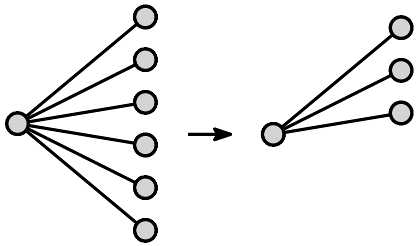

lose $\approx$ 2-factor in value

Fractional Matching

Fractional Matching as Linear Program

$$\max \sum_{e \in E} x_e$$

s.t. $\sum_{e \in E(v)} x_e \leq 1$ for all $v \in V$

$x_e \in [0,1]$ for all $e \in E$

why useful?

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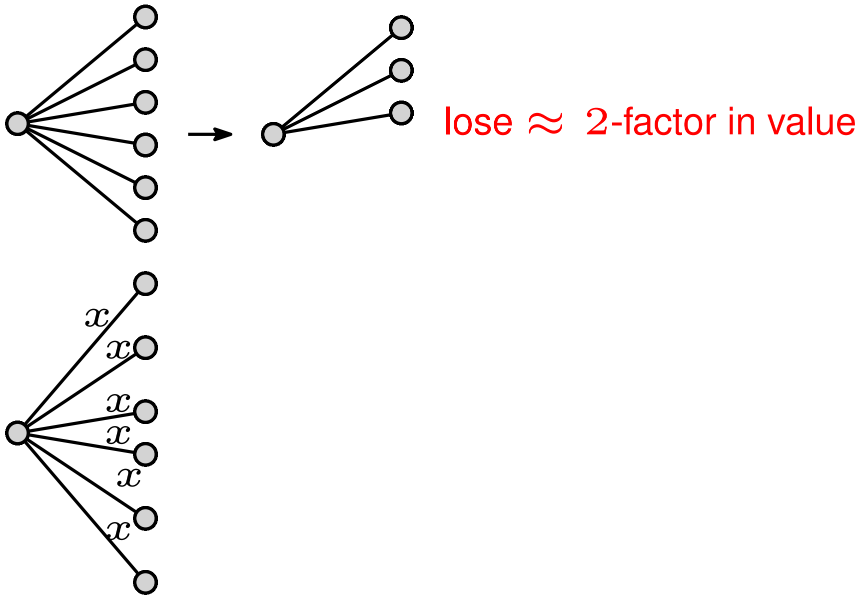

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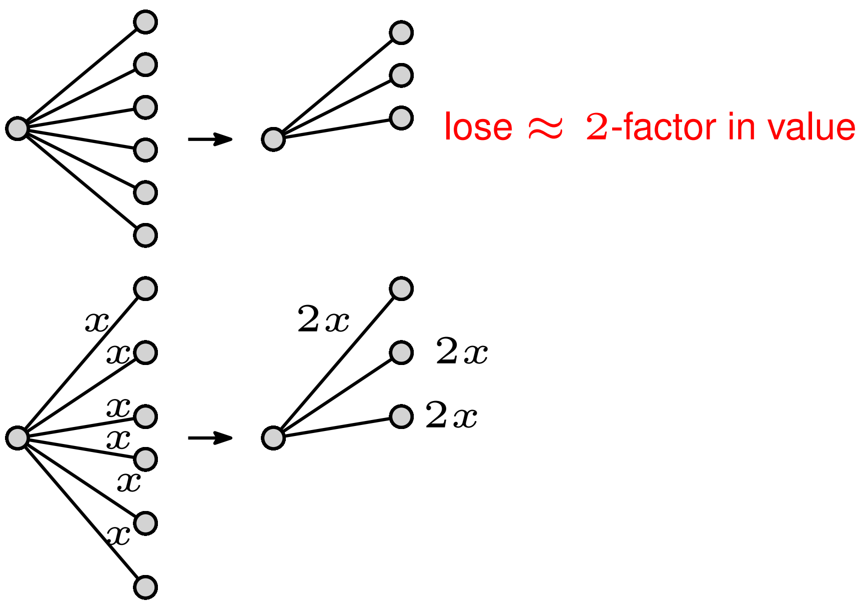

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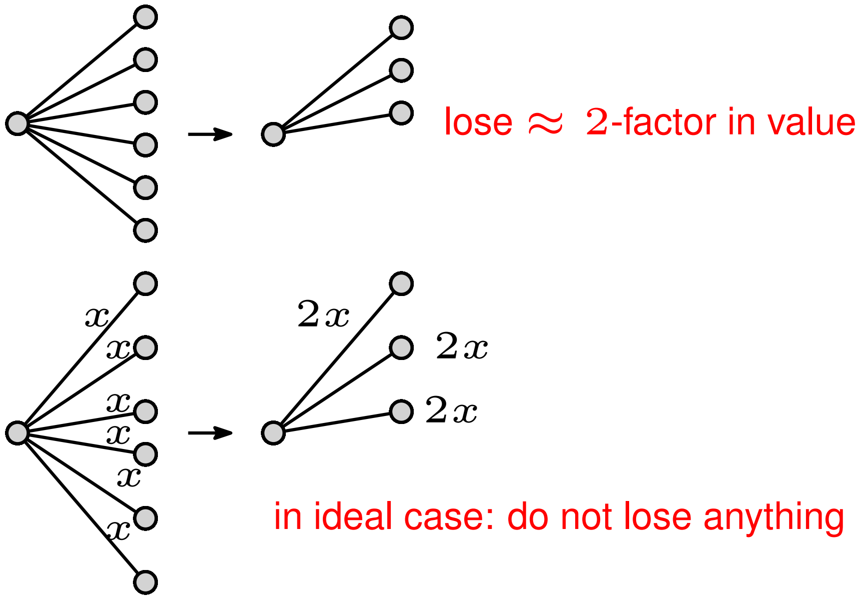

Our Matching Approximation Algorithm: Outline

Our Matching Approximation Algorithm: Outline

(1) Fractional matching

Our Matching Approximation Algorithm: Outline

(1) Fractional matching

(2) Iterative rounding

Our Matching Approximation Algorithm: Outline

(1) Fractional matching

(2) Iterative rounding

(3) Final Rounding

(1) Find Fractional Matching

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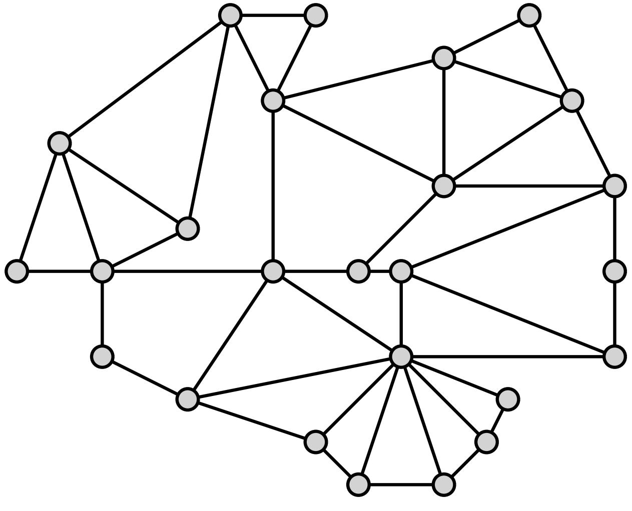
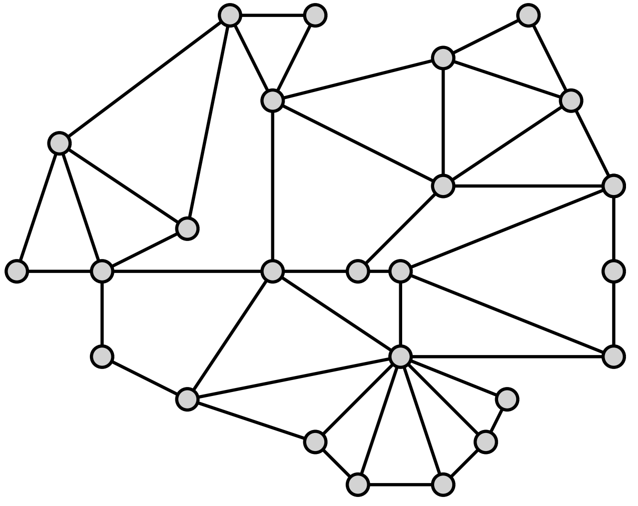


(1) Find Fractional Matching



start with $x_e = 2^{-\lceil \log \Delta \rceil}$

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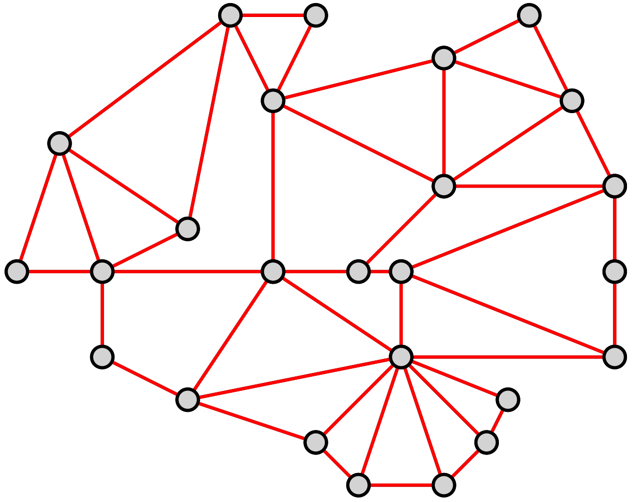

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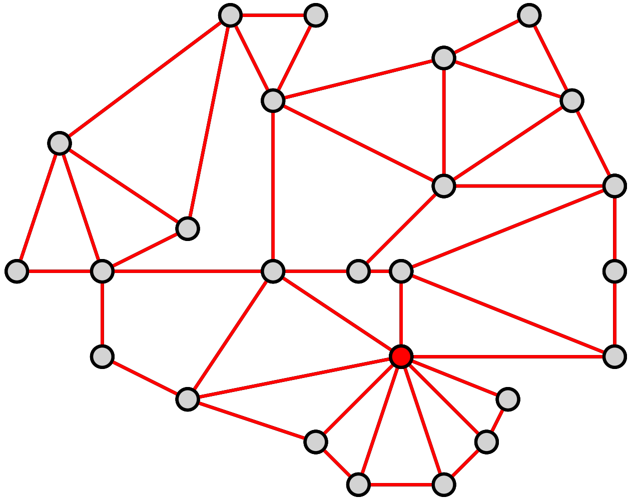
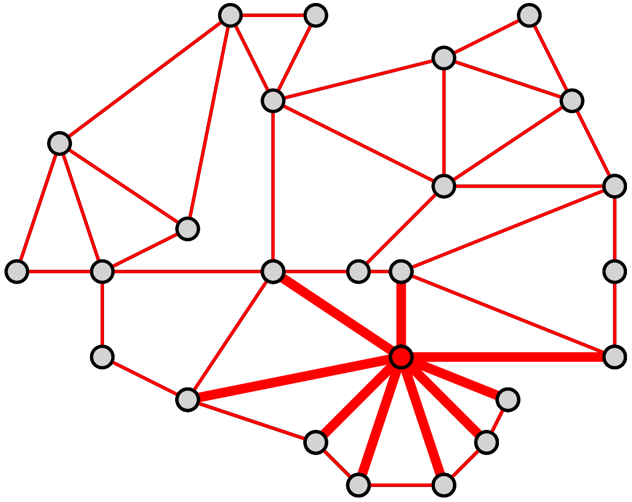


start with $x_e = 2^{-\lceil \log \Delta \rceil}$

mark $\frac{1}{2}$-tight nodes

(1) Find Fractional Matching

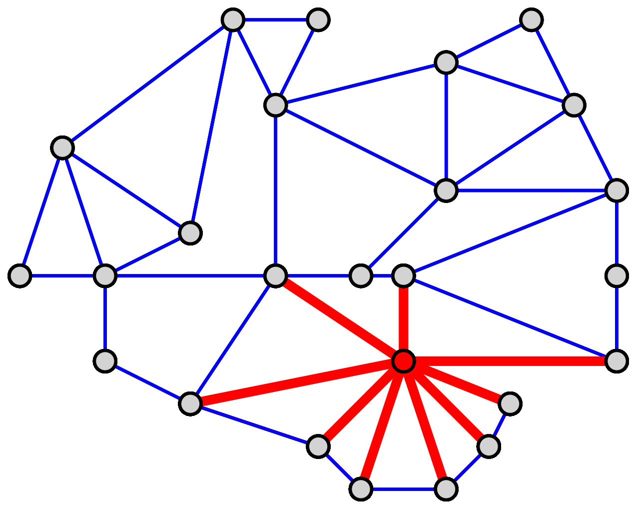


start with $x_e = 2^{-\lceil \log \Delta \rceil}$

mark $\frac{1}{2}$-tight nodes

block its edges

(1) Find Fractional Matching



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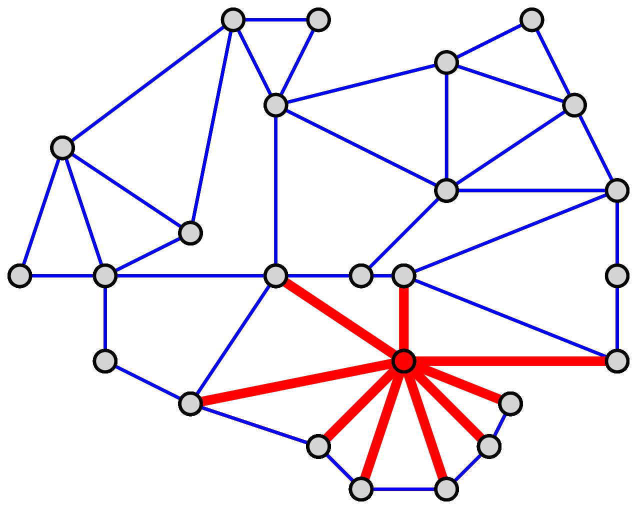
double value of all other edges

(1) Find Fractional Matching



start with $x_e = 2^{-\lceil \log \Delta \rceil}$

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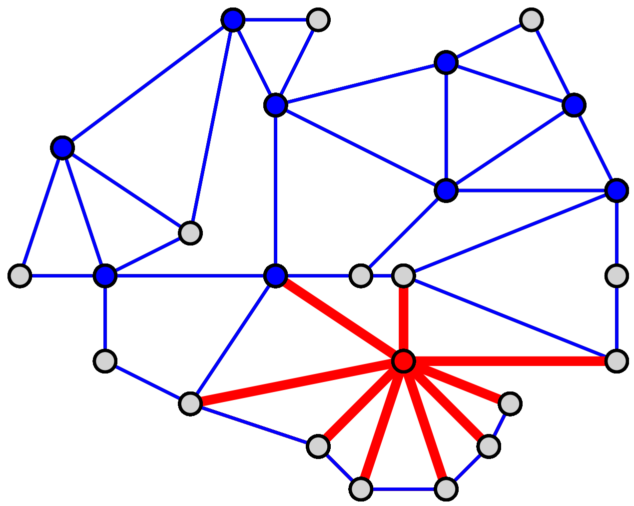

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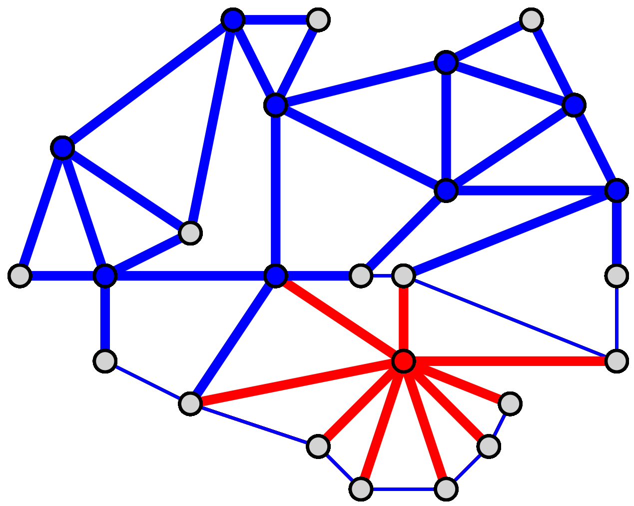

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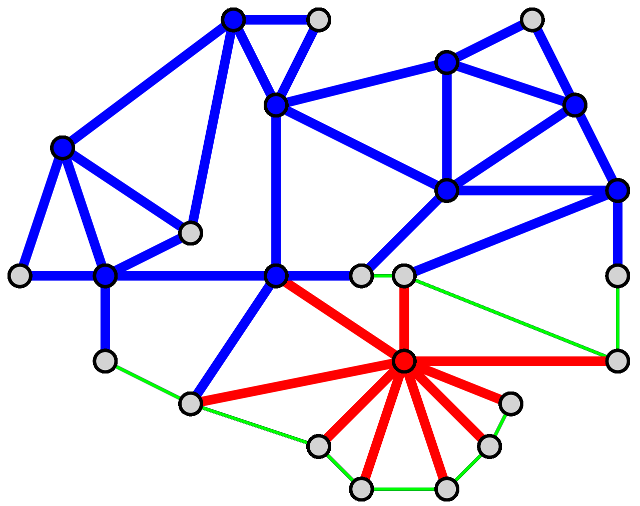

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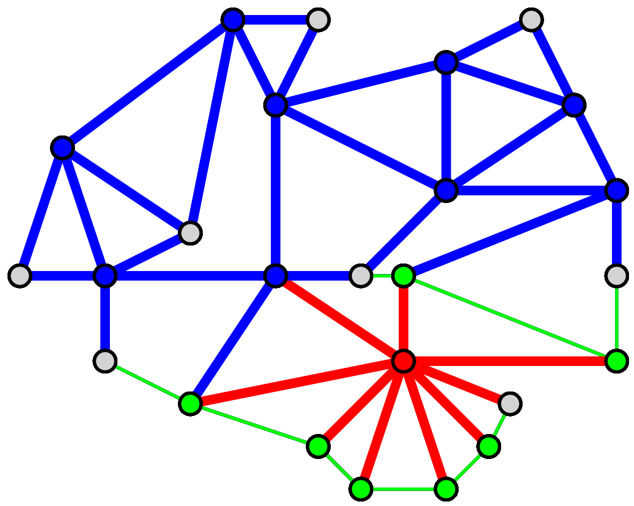

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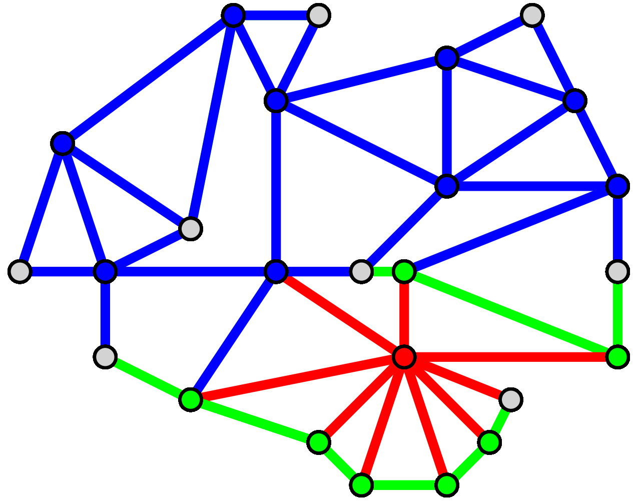

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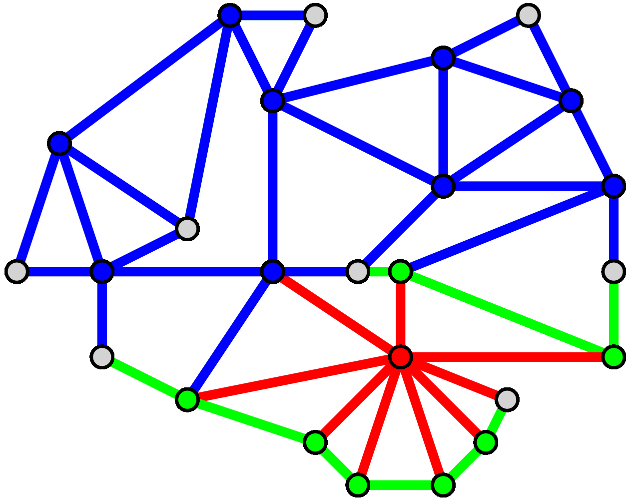

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until all edges are blocked

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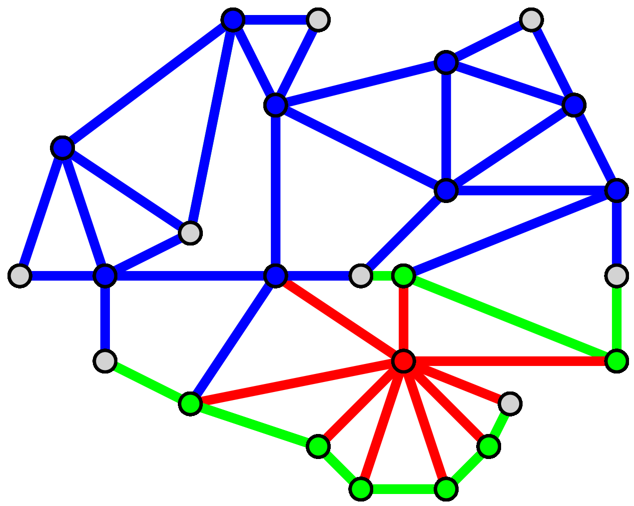

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4-approximation

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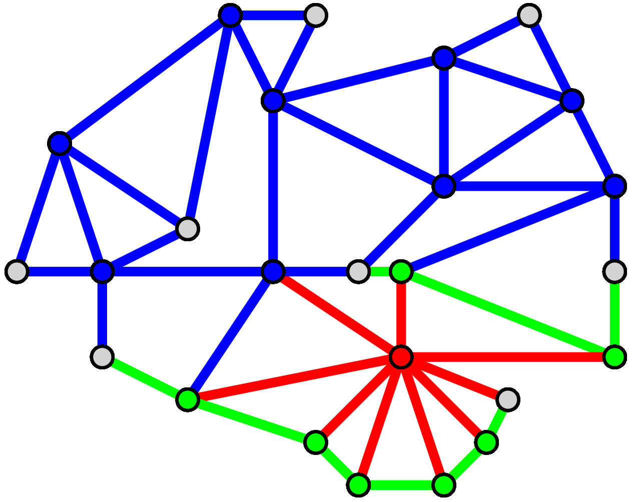
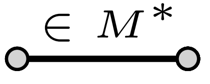


4-approximation



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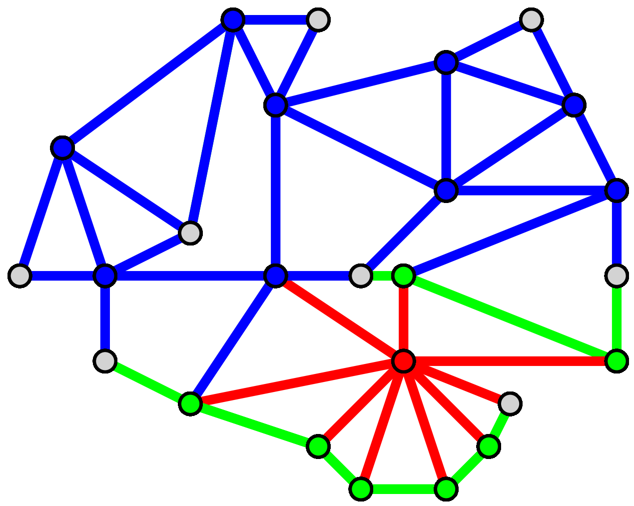

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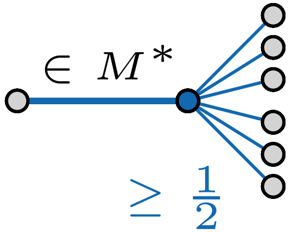

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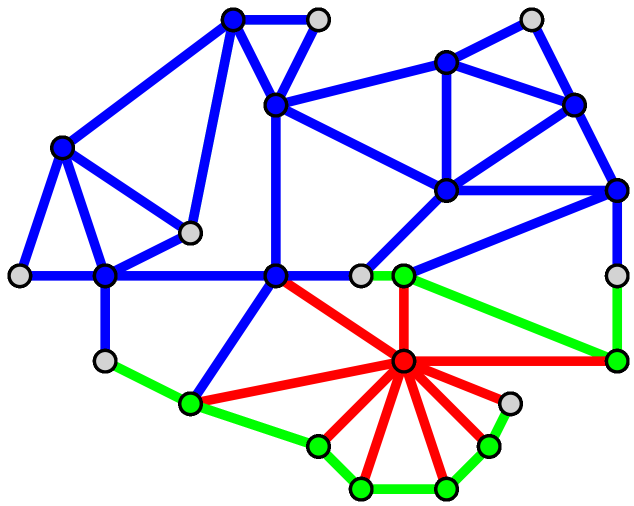

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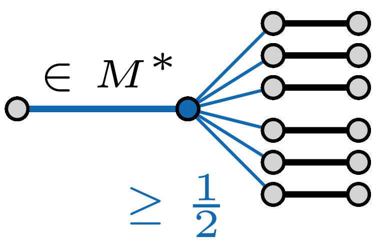

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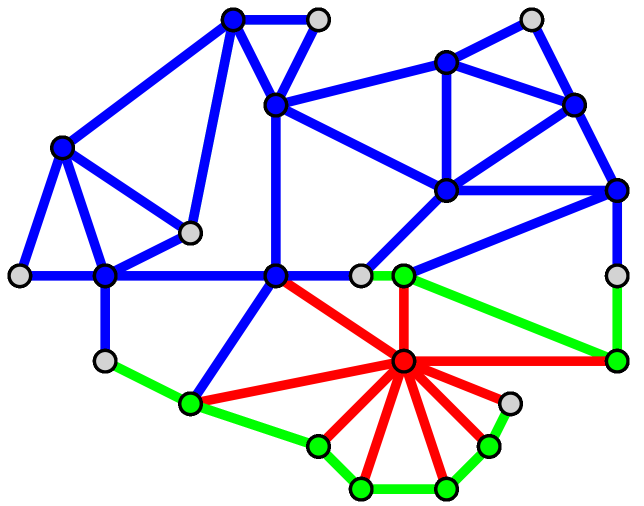

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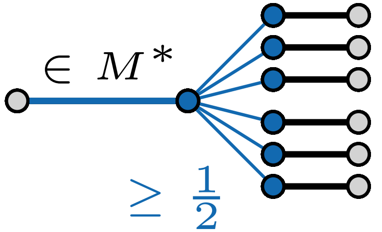
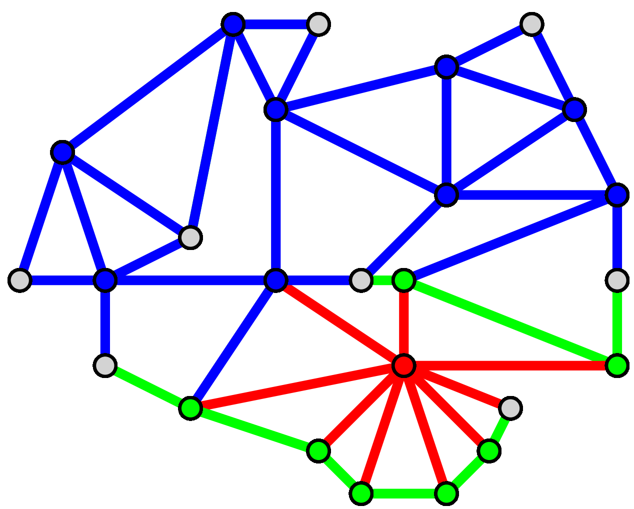


possible overcounting by 2-factor

(1) Find Fractional Matching



4-approximation in $O(\log \Delta)$ rounds

start with $x_e = 2^{-\lceil \log \Delta \rceil}$

repeat

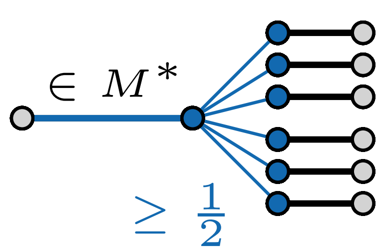
mark $\frac{1}{2}$-tight nodes

block its edges

double value of all other edges

until all edges are blocked

4-approximation



possible overcounting by 2-factor

(2) Iterative Rounding

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for $\lceil \log \Delta \rceil \geq i \geq 5$

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round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}

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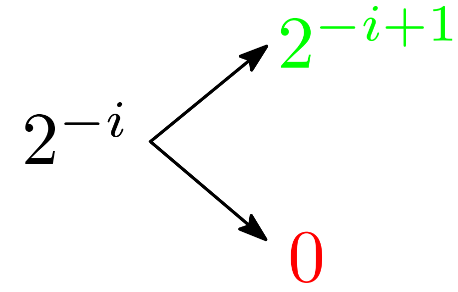

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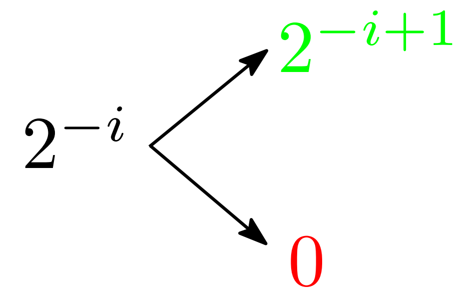

graph induced by $x_e = 2^{-i}$

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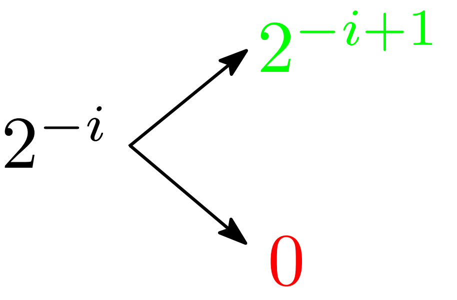

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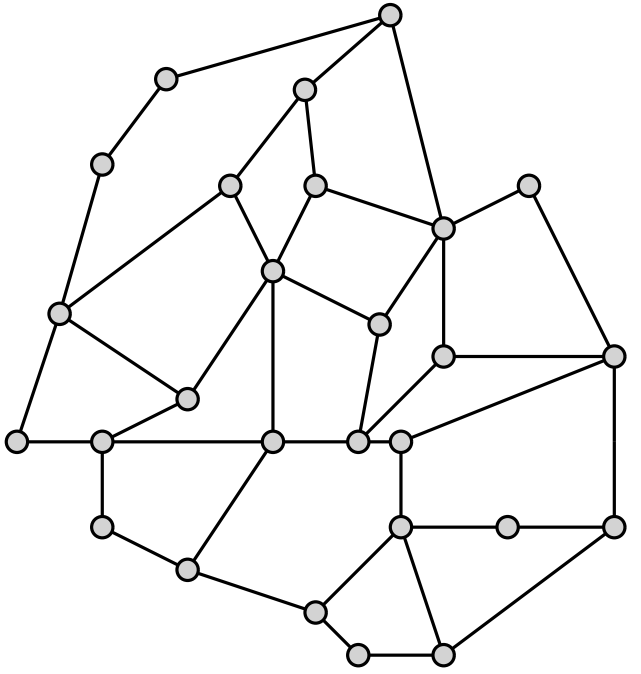
decompose into paths/cycles



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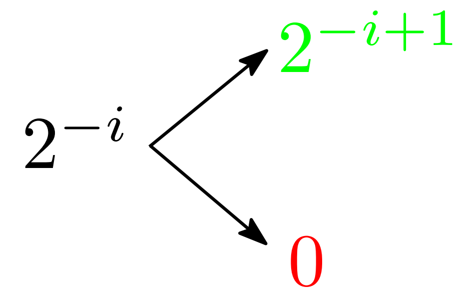

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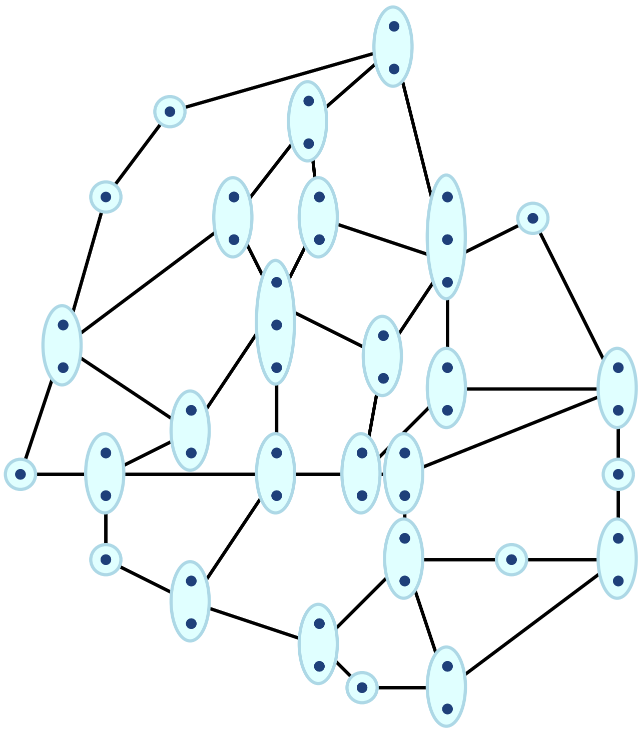

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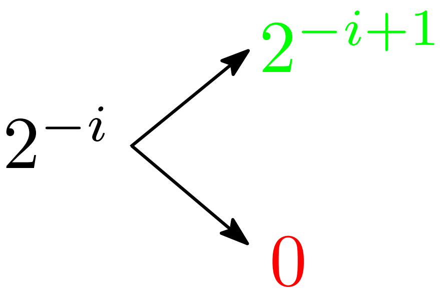

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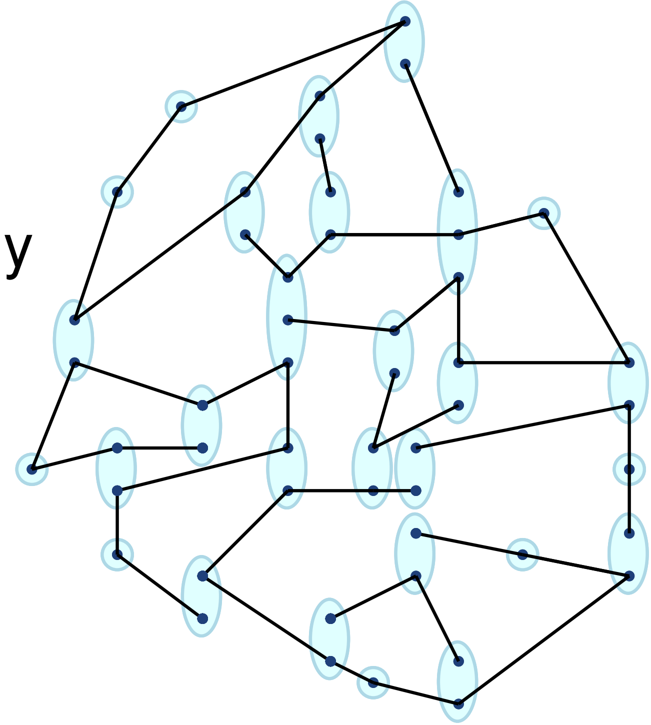
decompose into paths/cycles

deal with each path/cycle separately



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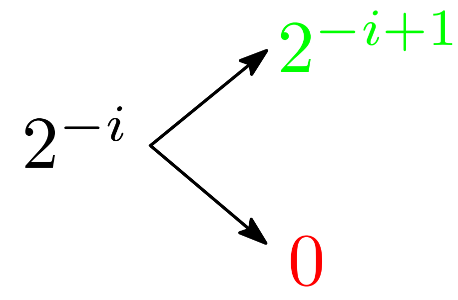

Short Cycles

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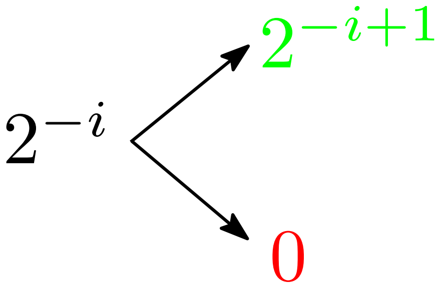

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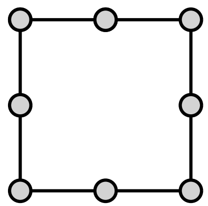

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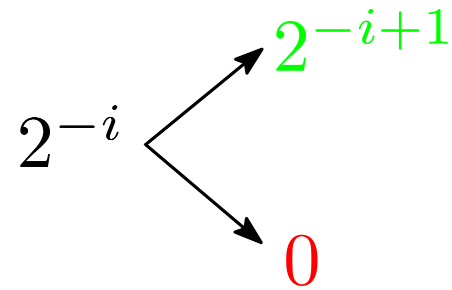

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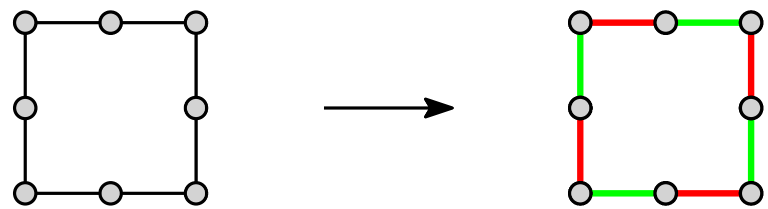

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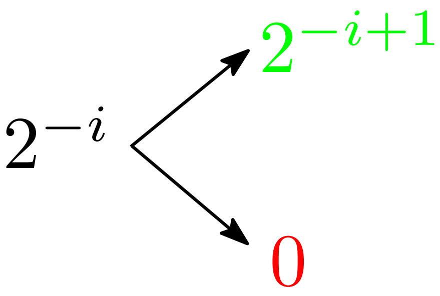

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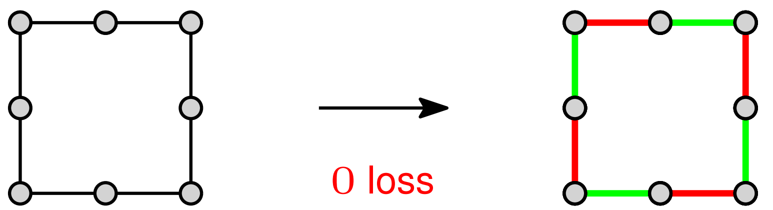

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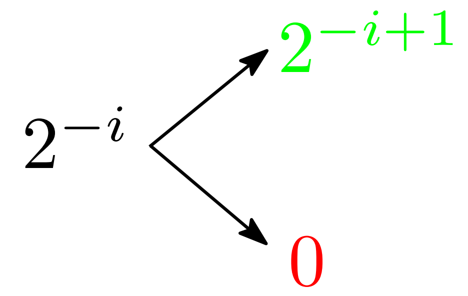

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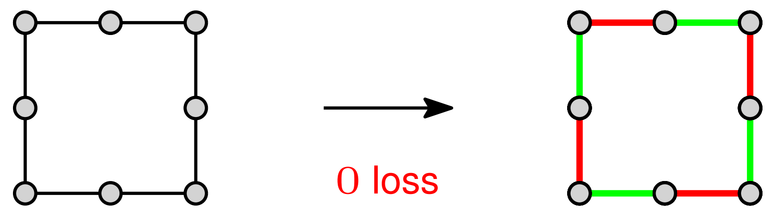

bipartite only!

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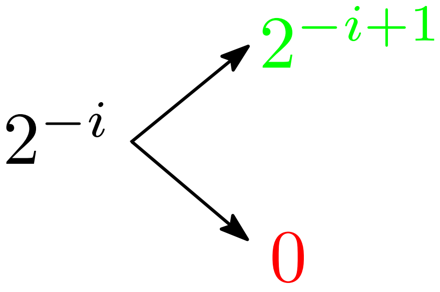

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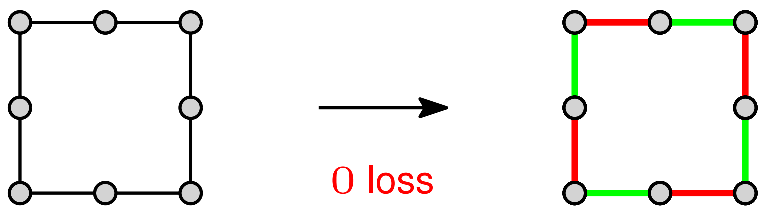

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otherwise can lose $O(1)$

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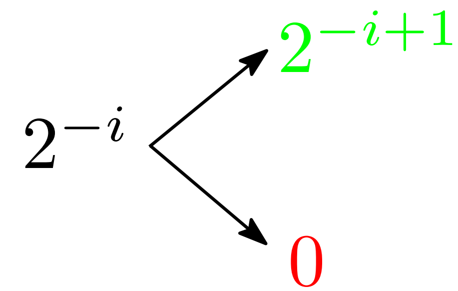

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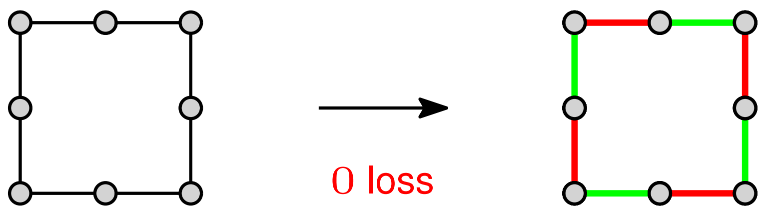

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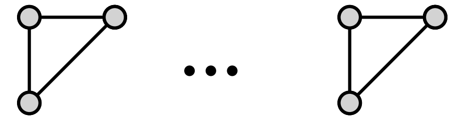

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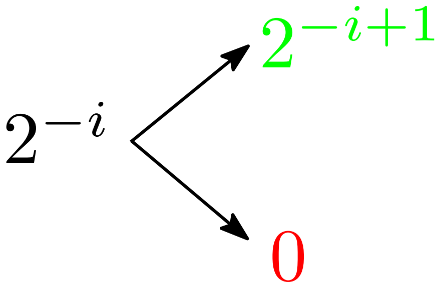

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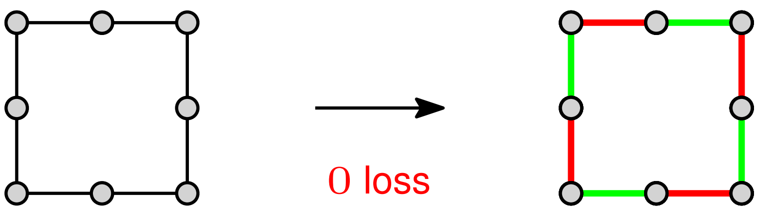

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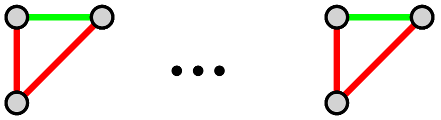

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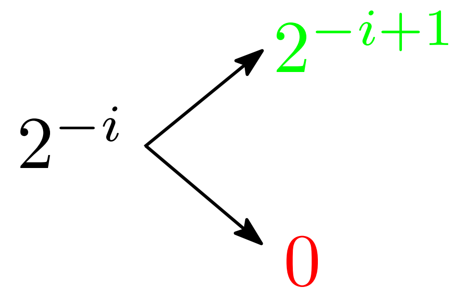
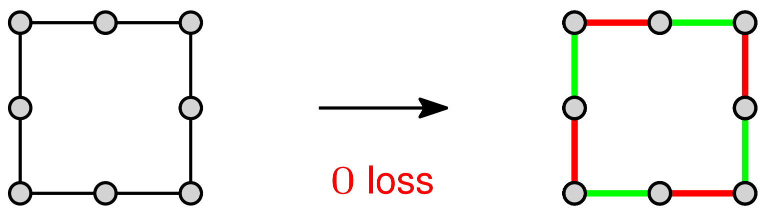
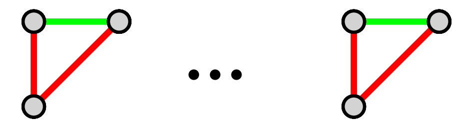


Short Cycles



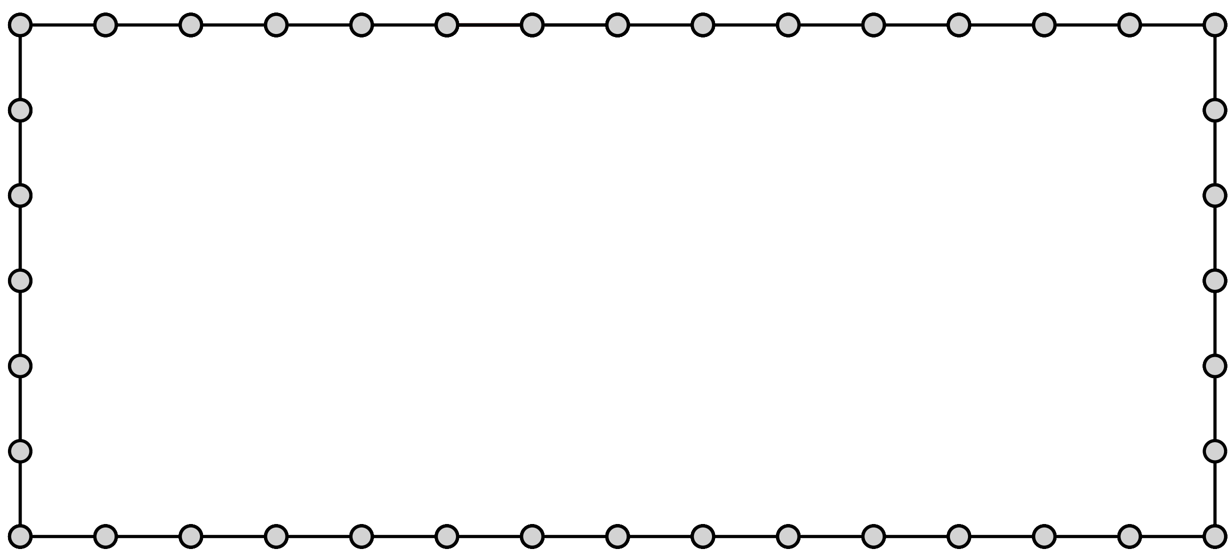
bipartite only!

otherwise can lose $O(1)$



Long Cycles

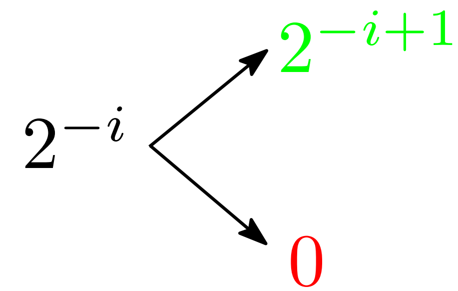
& Long Paths



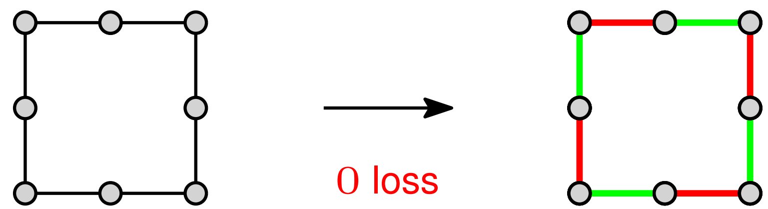
(2) Iterative Rounding

for $\lceil \log \Delta \rceil \geq i \geq 5$

round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}



Short Cycles



bipartite only!

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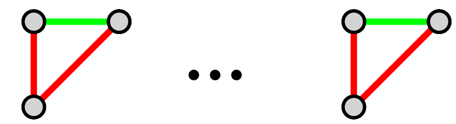
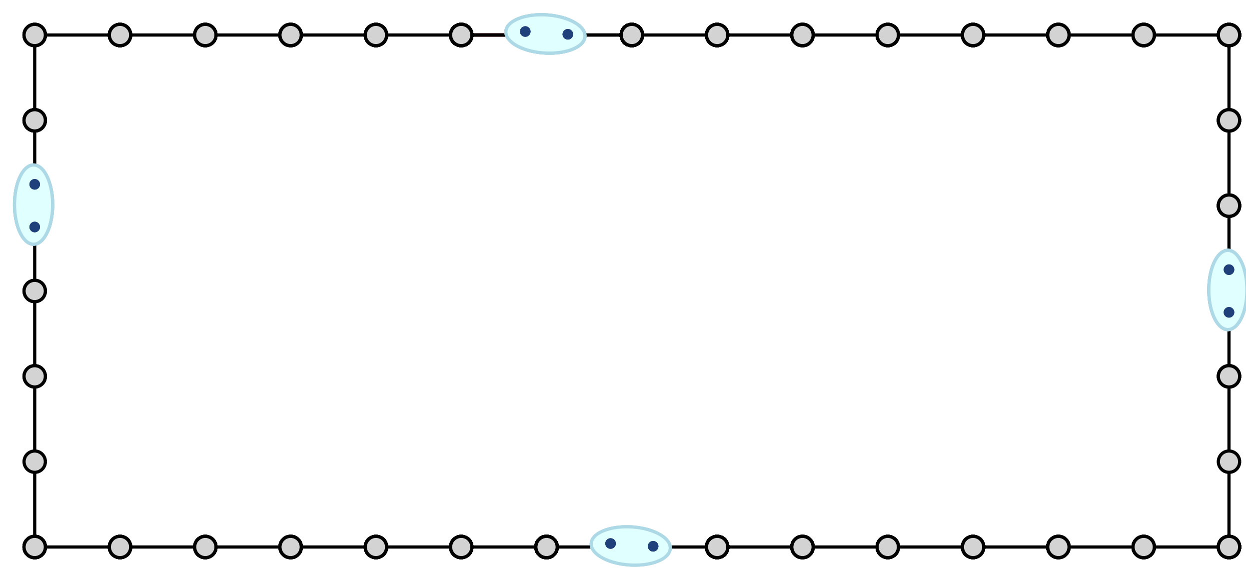


Long Cycles & Long Paths



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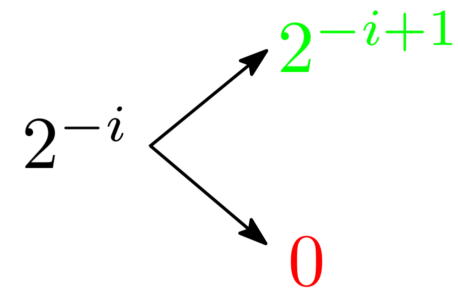
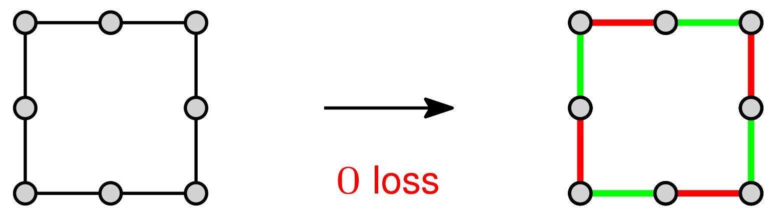
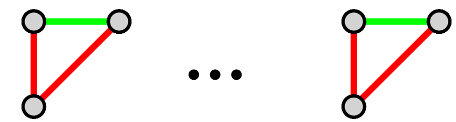


Short Cycles



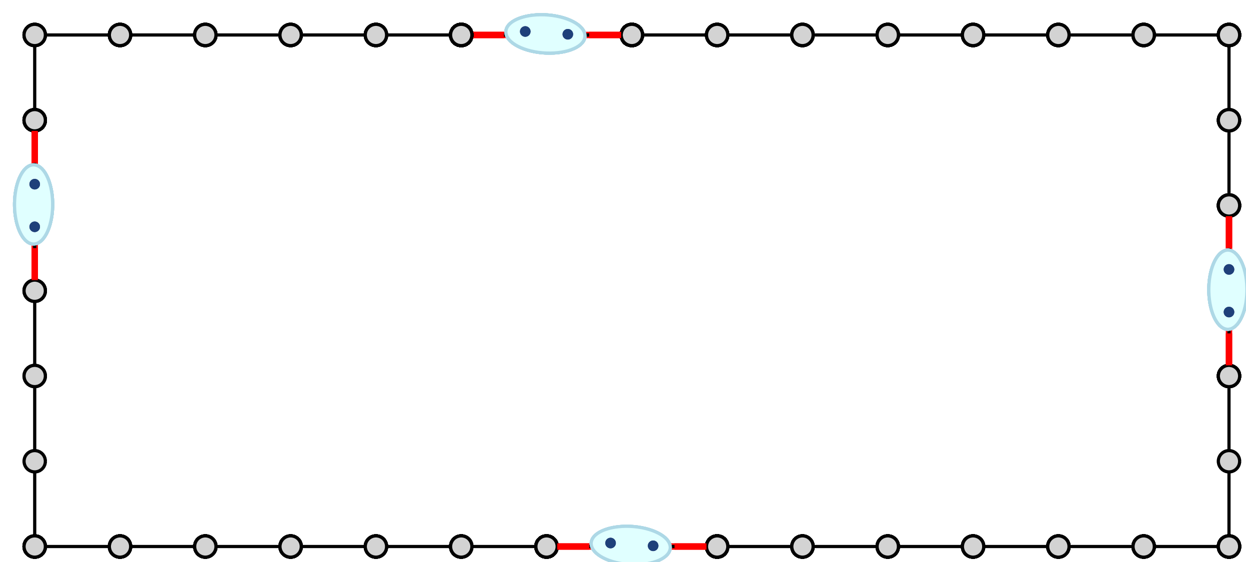
bipartite only!

otherwise can lose $O(1)$



Long Cycles

& Long Paths



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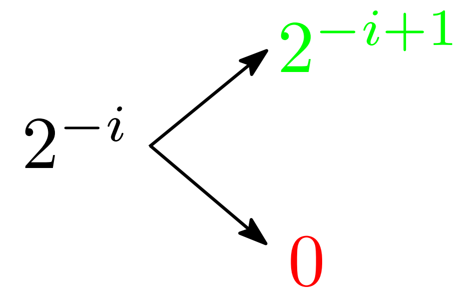
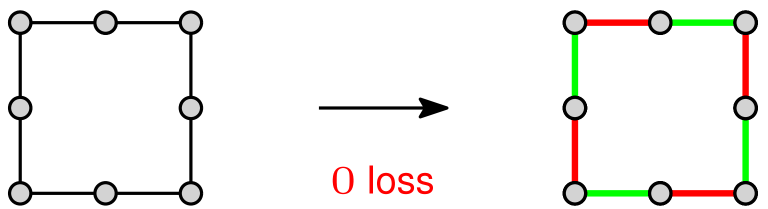


Short Cycles



bipartite only!

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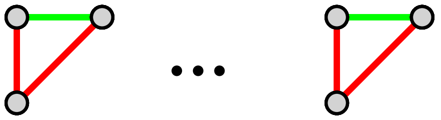
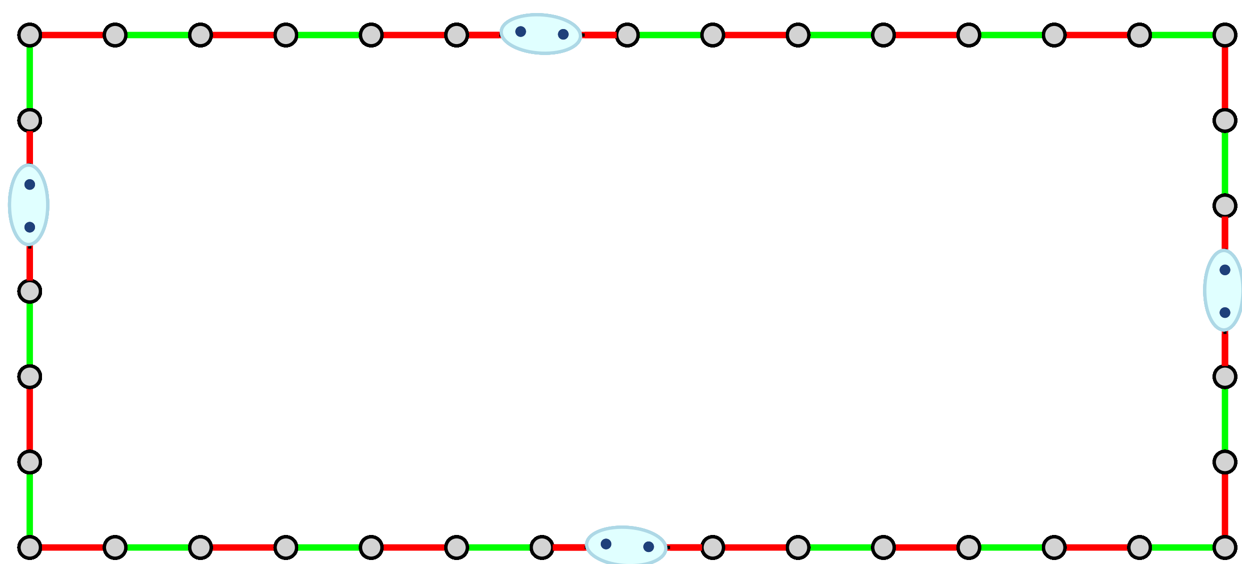


Long Cycles

& Long Paths



(2) Iterative Rounding

for $\lceil \log \Delta \rceil \geq i \geq 5$

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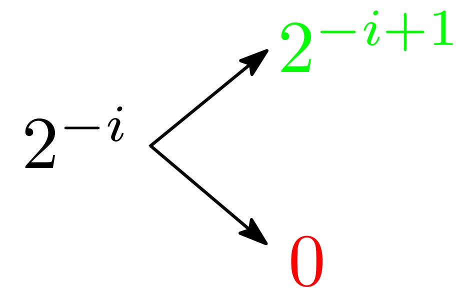
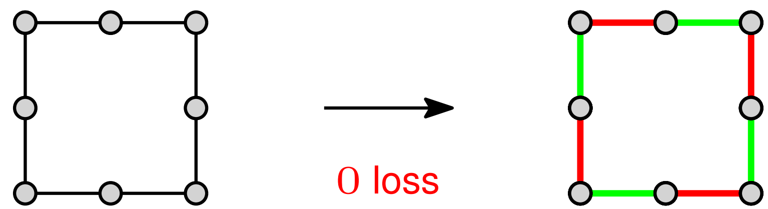
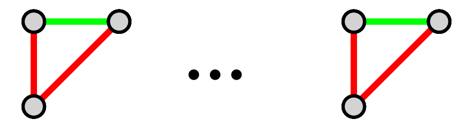


Short Cycles

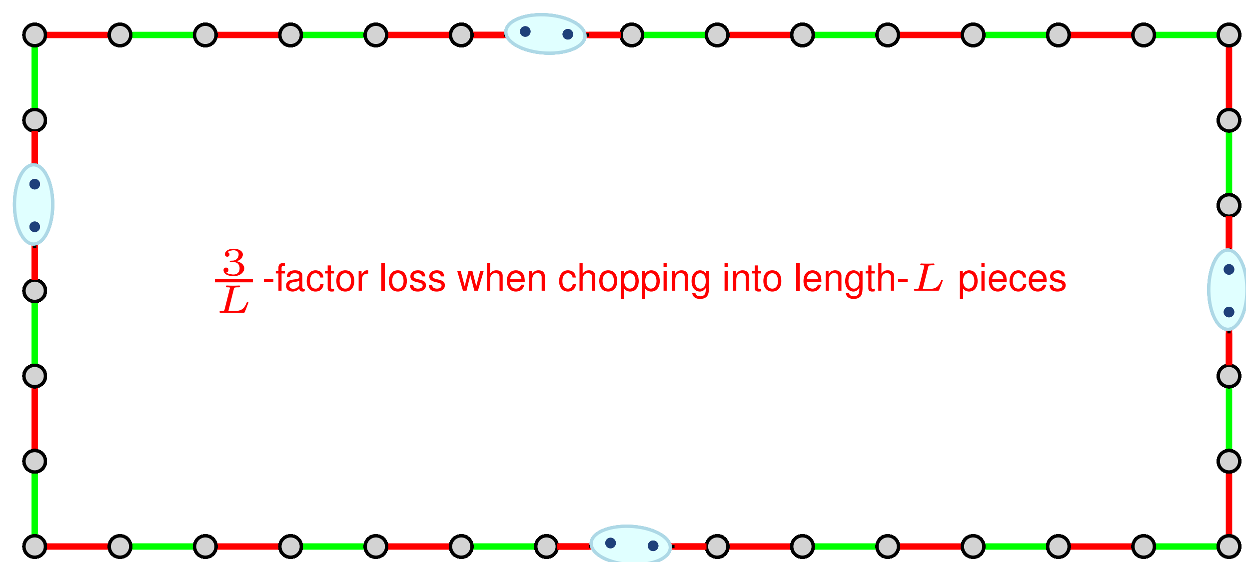


bipartite only!

otherwise can lose $O(1)$



Long Cycles & Long Paths



(2) Iterative Rounding

for $\lceil \log \Delta \rceil \geq i \geq 5$

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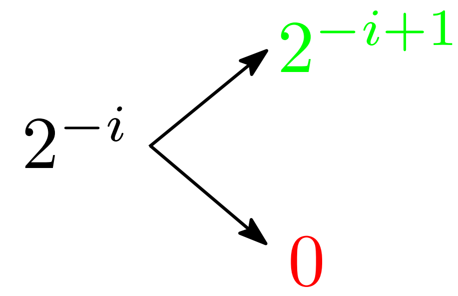
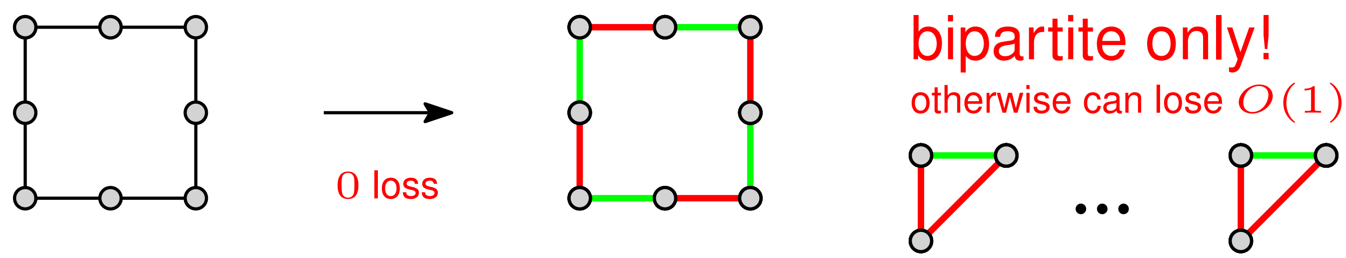
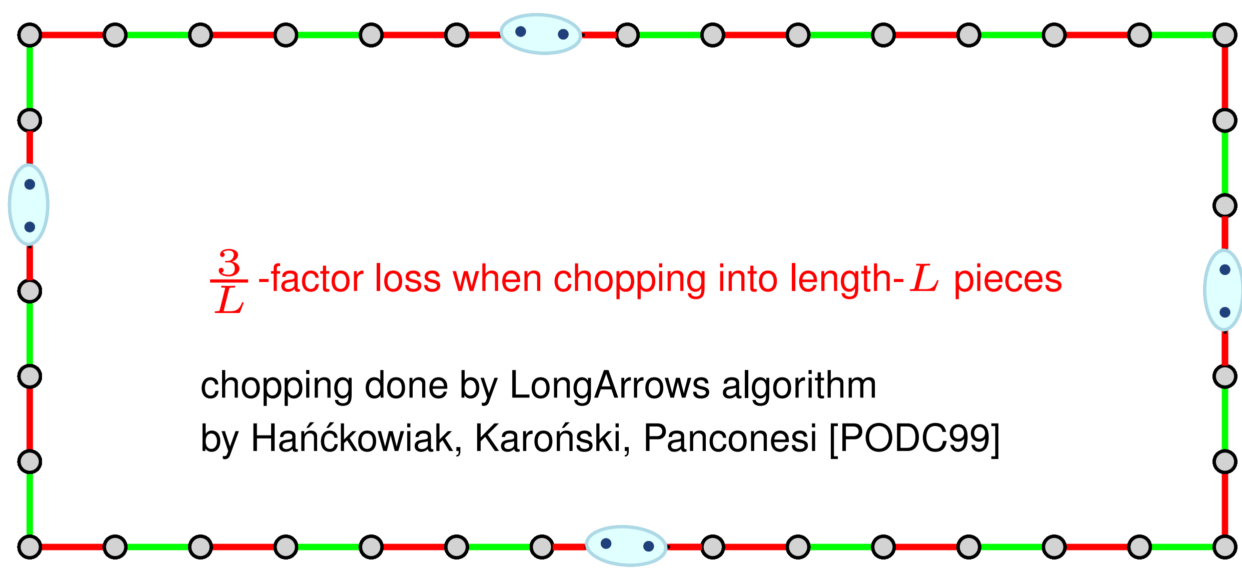


Short Cycles



Long Cycles

& Long Paths



(2) Iterative Rounding

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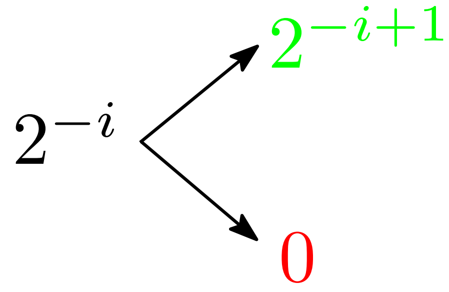
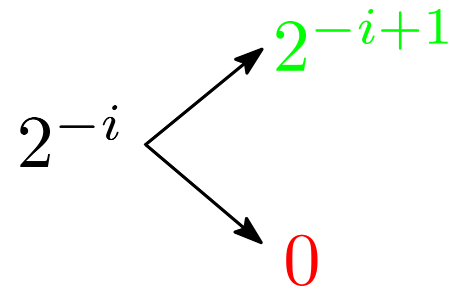
Short Paths



(2) Iterative Rounding



for $\lceil \log \Delta \rceil \geq i \geq 5$

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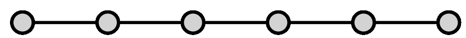
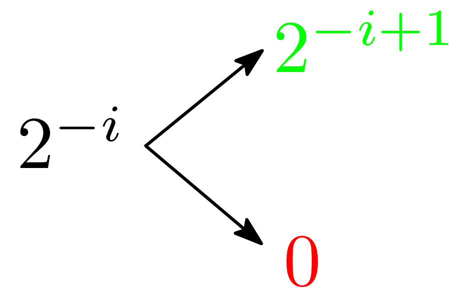
Short Paths



(2) Iterative Rounding



for $\lceil \log \Delta \rceil \geq i \geq 5$

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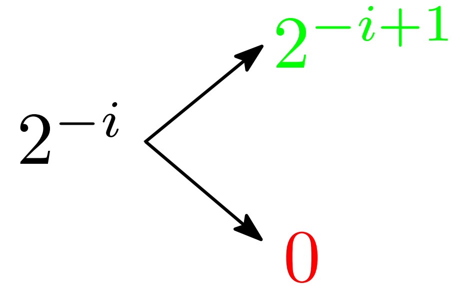
Short Paths 

(2) Iterative Rounding



for $\lceil \log \Delta \rceil \geq i \geq 5$

round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}

Short Paths

(2) Iterative Rounding

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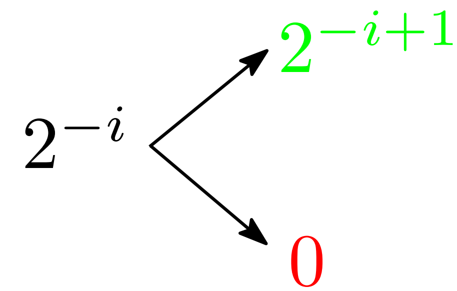


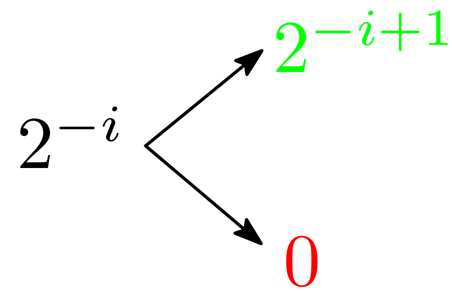
Short Paths

lose constant factor!

(2) Iterative Rounding

for $\lceil \log \Delta \rceil \geq i \geq 5$

round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}



Short Paths

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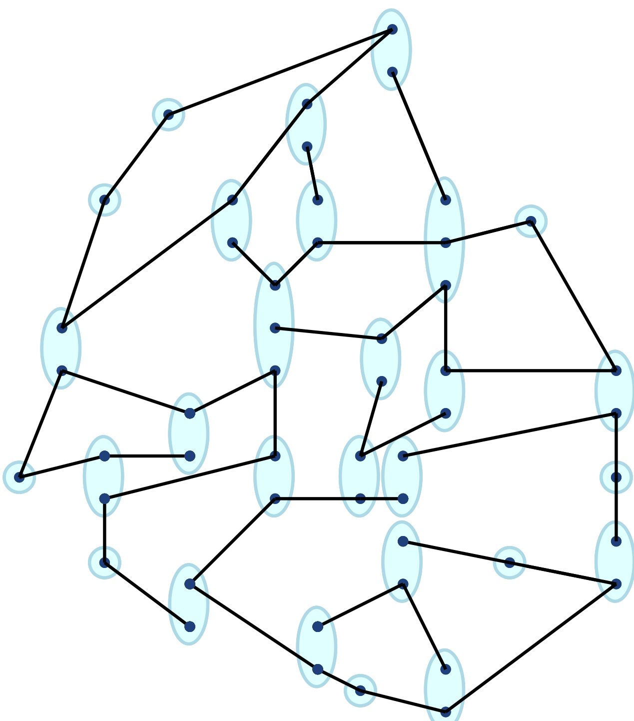

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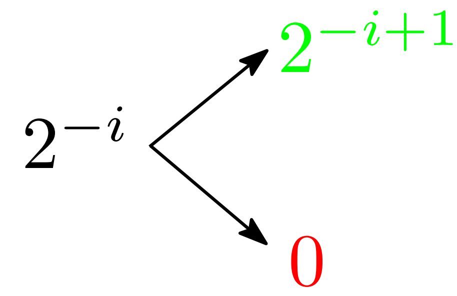
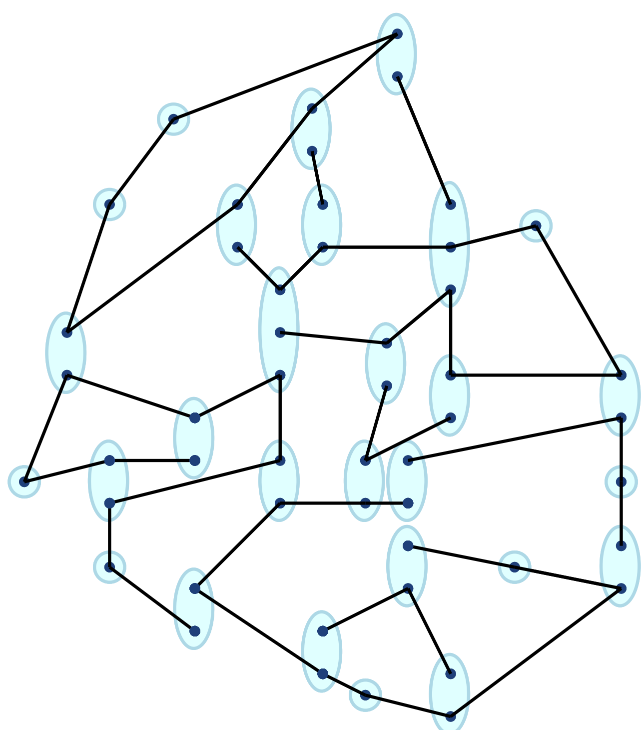


Short Paths

lose constant factor!



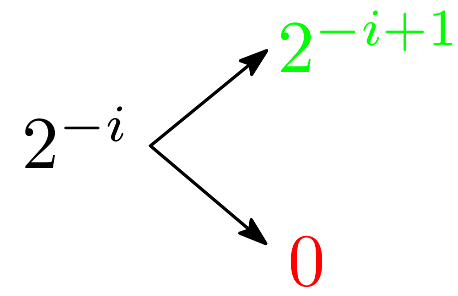
for $\frac{1}{2}$-loose nodes
& even-length paths

Manuela Fischer

(2) Iterative Rounding

for $\lceil \log \Delta \rceil \geq i \geq 5$

round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}



Short Paths

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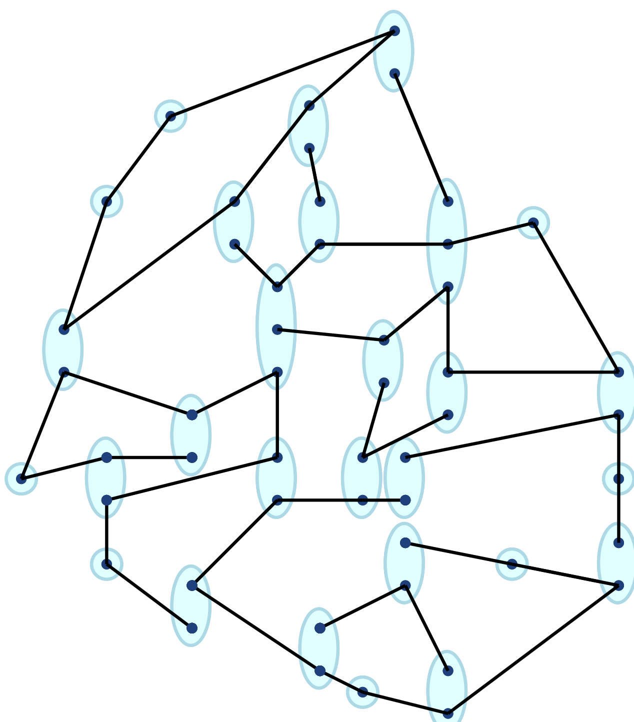
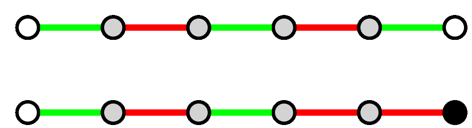


for $\frac{1}{2}$-loose nodes
& even-length paths

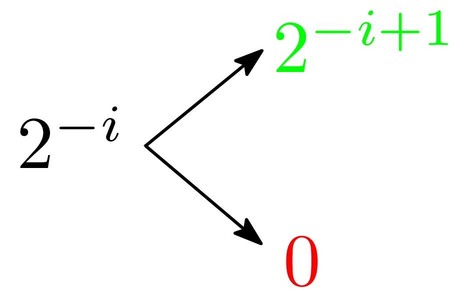


for $\frac{1}{2}$-loose nodes
& odd-length paths

(2) Iterative Rounding

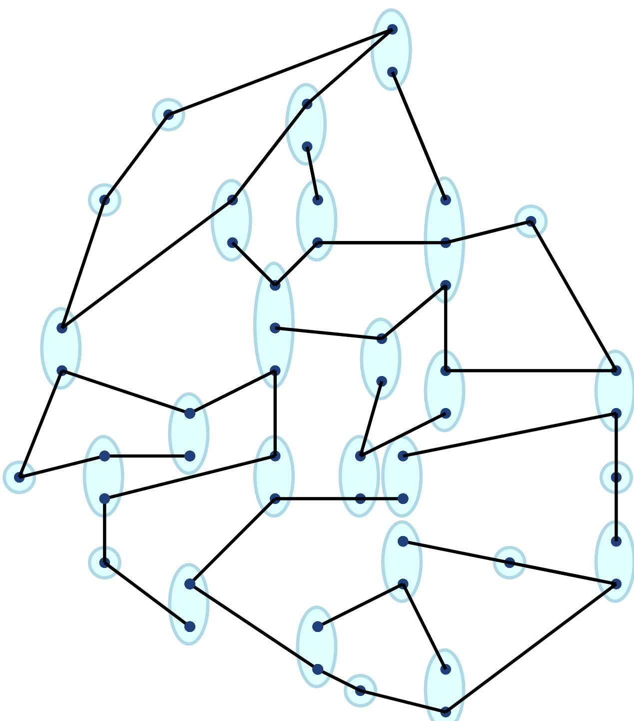
for $\lceil \log \Delta \rceil \geq i \geq 5$

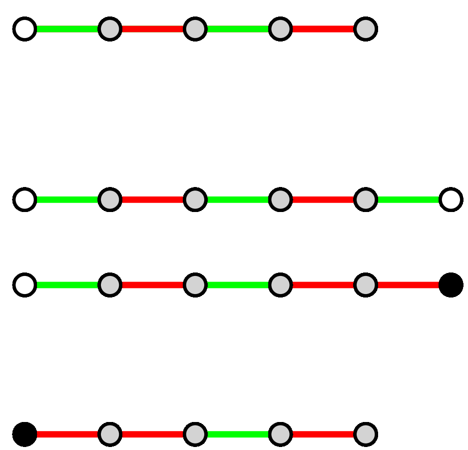
round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}



Short Paths

lose constant factor!





for $\frac{1}{2}$-loose nodes
& even-length paths

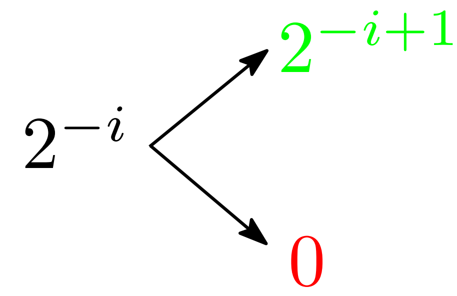
for $\frac{1}{2}$-loose nodes
& odd-length paths

for $\frac{1}{2}$-loose nodes
& even-length paths

(2) Iterative Rounding

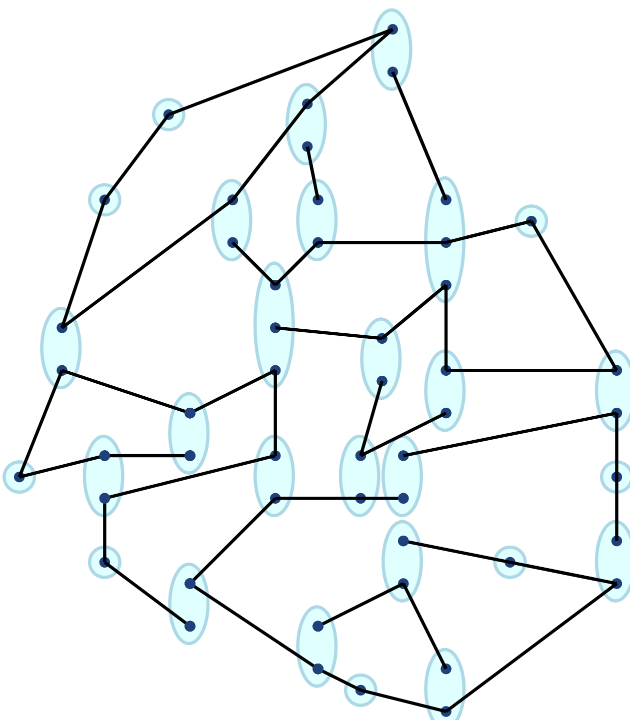
for $\lceil \log \Delta \rceil \geq i \geq 5$

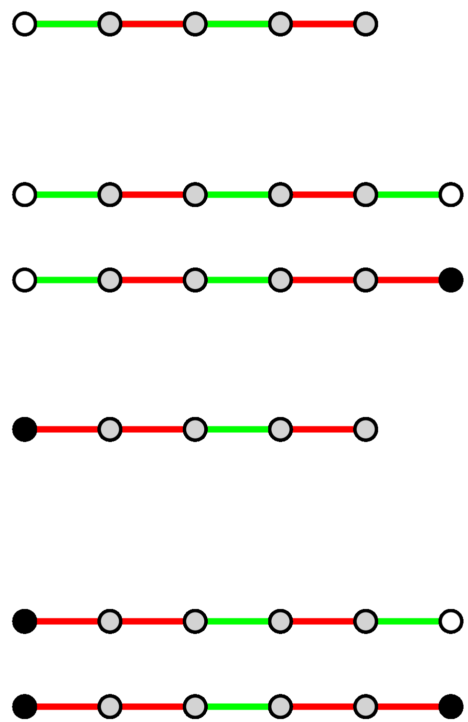
round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}



Short Paths

lose constant factor!





for $\frac{1}{2}$-loose nodes & even-length paths

for $\frac{1}{2}$-loose nodes & odd-length paths

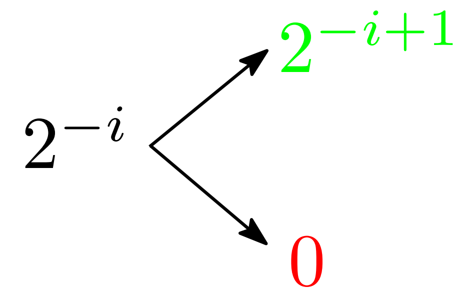
for $\frac{1}{2}$-loose nodes & even-length paths

for $\frac{1}{2}$-tight nodes & odd-length paths

(2) Iterative Rounding

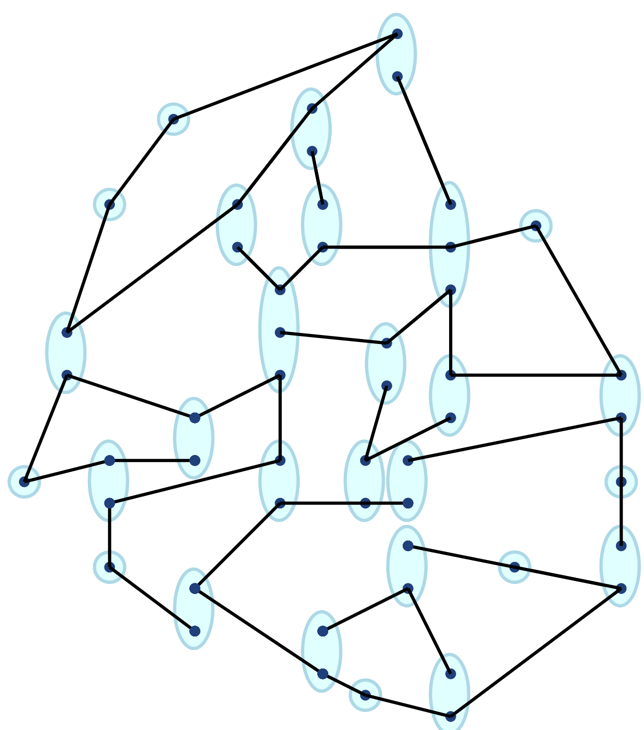
for $\lceil \log \Delta \rceil \geq i \geq 5$

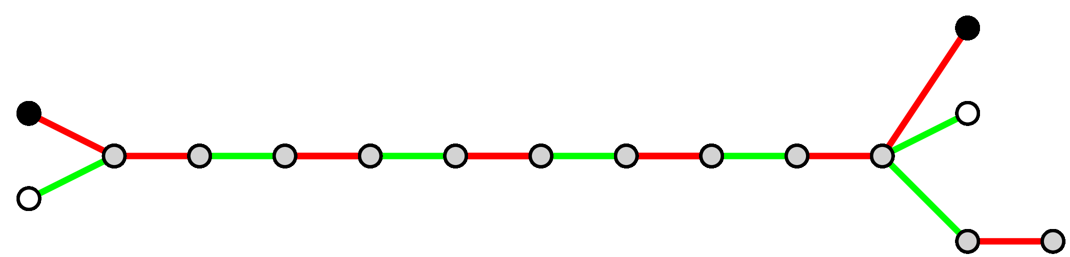
round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}



Short Paths

lose constant factor!





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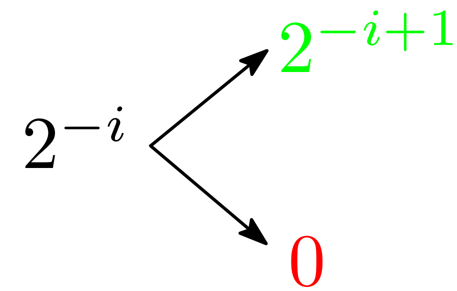
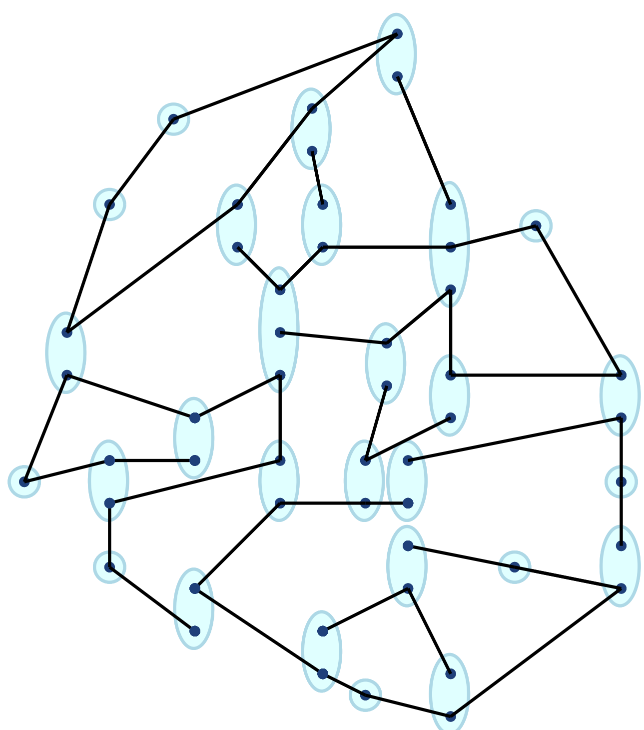
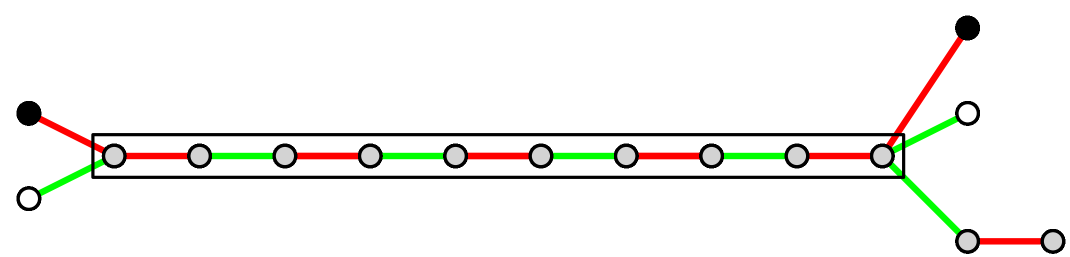


Short Paths

lose constant factor!





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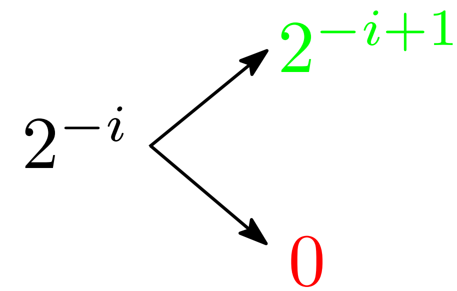
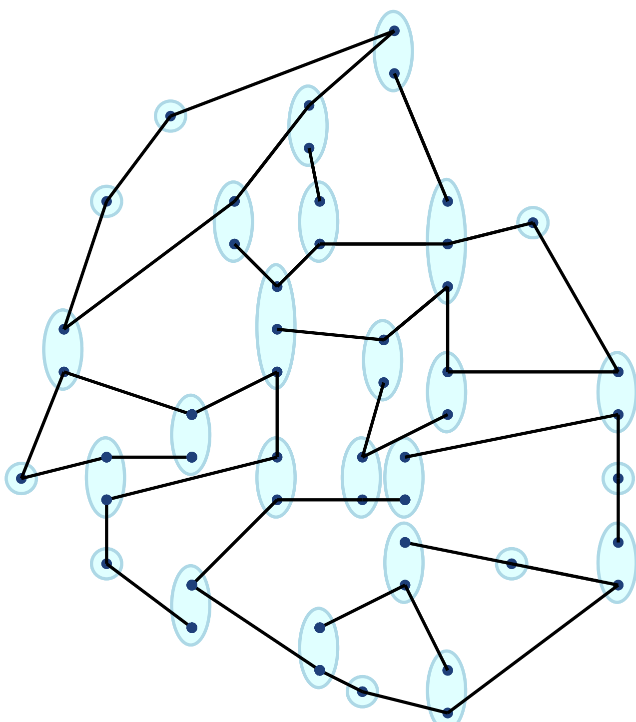
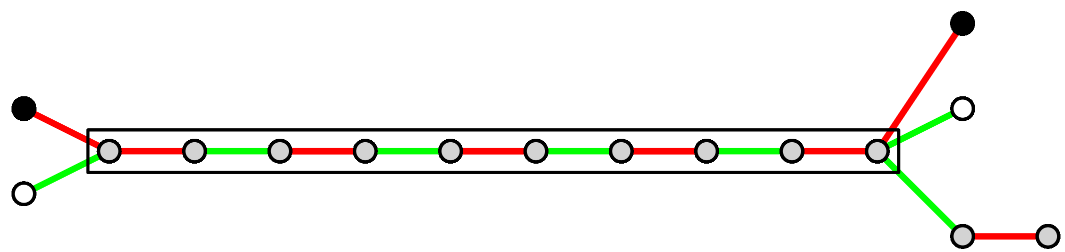


Short Paths

lose constant factor!



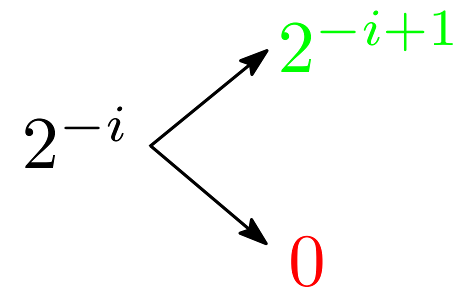


lose at most 2^{-i+1} per $\frac{1}{2}$-tight vertex

(2) Iterative Rounding

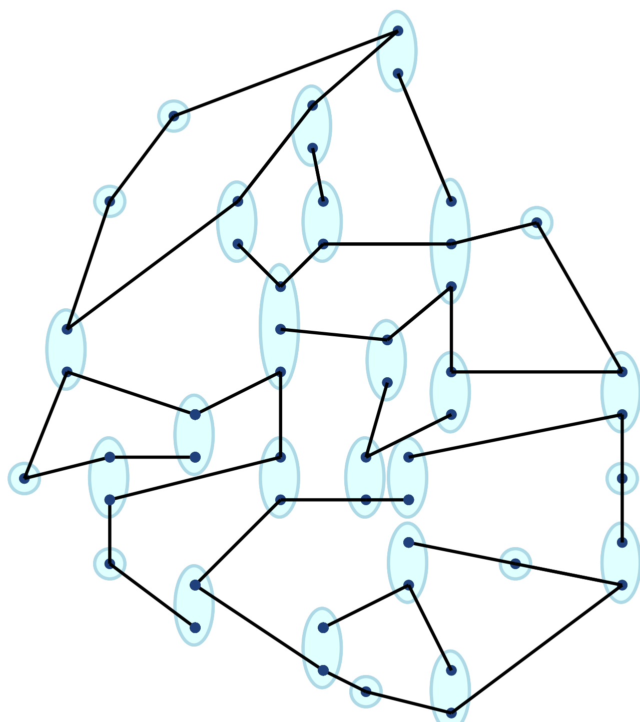
for $\lceil \log \Delta \rceil \geq i \geq 5$

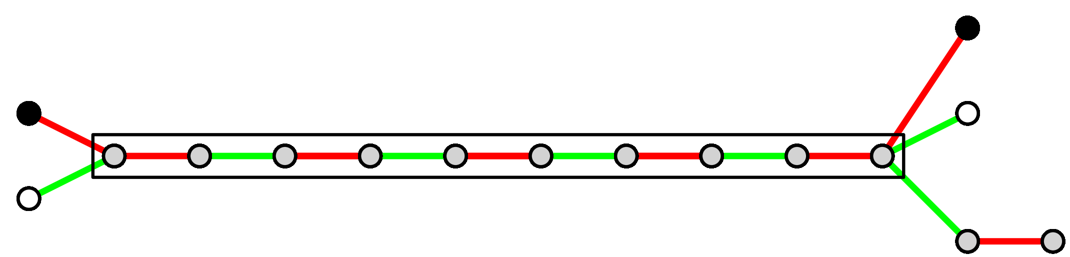
round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}



Short Paths

lose constant factor!





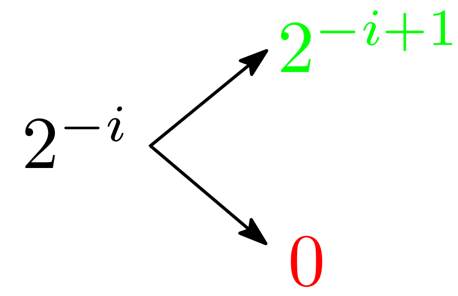
lose at most 2^{-i+1} per $\frac{1}{2}$-tight vertex

thus at most 2^{-i+2}-factor of its value

(2) Iterative Rounding

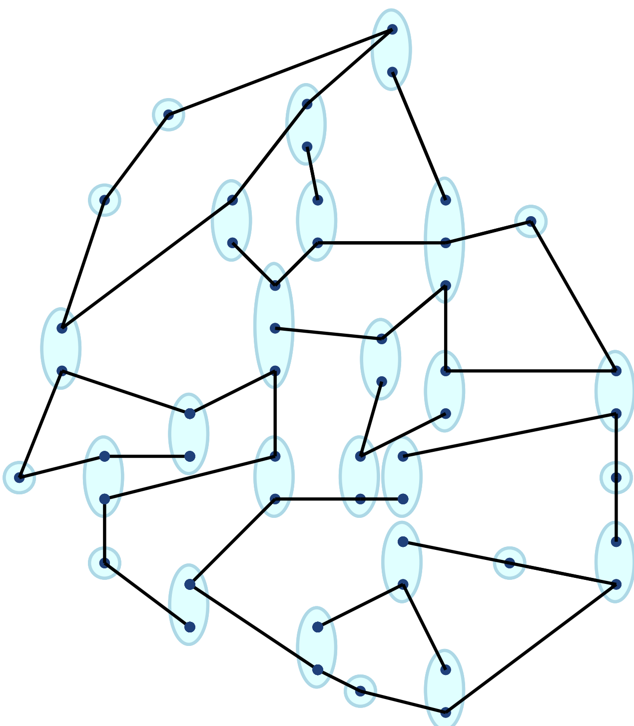
for $\lceil \log \Delta \rceil \geq i \geq 5$

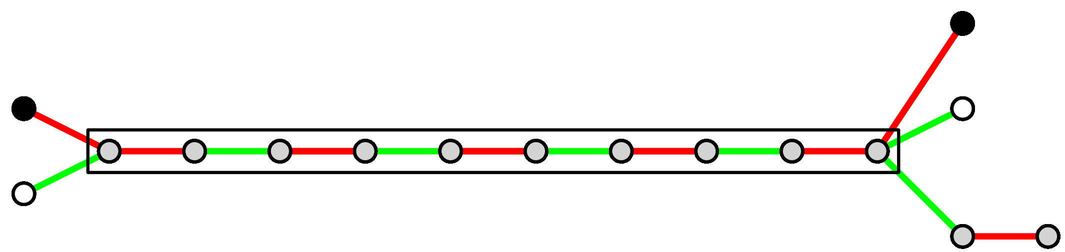
round values $x_e = 2^{-i}$ to 0 or to 2^{-i+1}



Short Paths

lose constant factor!





lose at most 2^{-i+1} per $\frac{1}{2}$-tight vertex

thus at most 2^{-i+2}-factor of its value

overall, lose at most 2^{-i+3}-factor

(2) Iterative Rounding

Short Cycles — no loss at all

Long Cycles & Long Paths — $\frac{3}{L}$-factor loss when chopping into length-L pieces

Short Paths — 2^{-i+3}-factor loss

(2) Iterative Rounding

Short Cycles — no loss at all

Long Cycles & Long Paths — $\frac{3}{L}$-factor loss when chopping into length-L pieces

Short Paths — 2^{-i+3}-factor loss

reduce value to $(1 - \frac{3}{L} - 2^{-i+3})$-factor of previous value

(2) Iterative Rounding

Short Cycles — no loss at all

Long Cycles & Long Paths — $\frac{3}{L}$-factor loss when chopping into length-L pieces

$L = \lceil \log \Delta \rceil$

Short Paths — 2^{-i+3}-factor loss

reduce value to $(1 - \frac{3}{L} - 2^{-i+3})$-factor of previous value

$$\sum_{e \in E} x'_e \geq \prod_{i=5}^{\lceil \log \Delta \rceil} (1 - \frac{3}{L} - 2^{-i+3}) \sum_{e \in E} x_e \geq \frac{1}{5} \sum_{e \in E} x_e$$

(2) Iterative Rounding

5-approximation in $O(\log^2 \Delta)$ rounds

Short Cycles — no loss at all

Long Cycles & Long Paths — $\frac{3}{L}$-factor loss when chopping into length-L pieces

$L = \lceil \log \Delta \rceil$

Short Paths — 2^{-i+3}-factor loss

reduce value to $(1 - \frac{3}{L} - 2^{-i+3})$-factor of previous value

$$\sum_{e \in E} x'_e \geq \prod_{i=5}^{\lceil \log \Delta \rceil} (1 - \tfrac{3}{L} - 2^{-i+3}) \sum_{e \in E} x_e \geq \tfrac{1}{5} \sum_{e \in E} x_e$$

(3) Final Rounding

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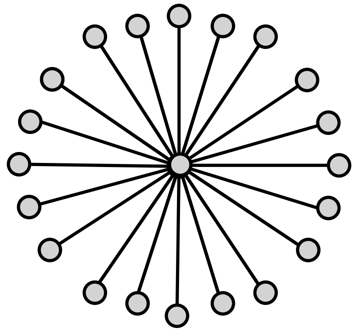
values in $\{2^{-i} \mid 4 \geq i \geq 1\} \cup \{0\}$

(3) Final Rounding

values in $\{2^{-i} \mid 4 \geq i \geq 1\} \cup \{0\}$



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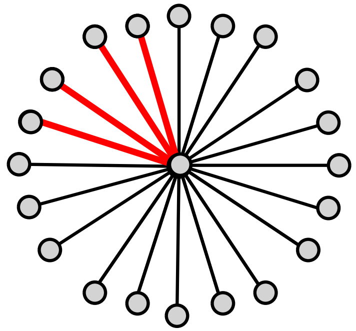

maximum degree in induced graph ≤ 16

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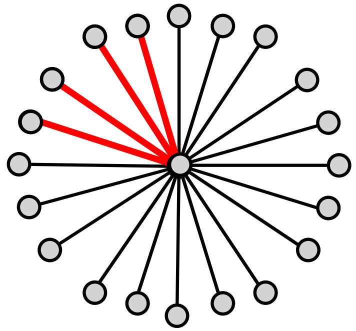

maximum degree in induced graph ≤ 16

find maximal matching in $O(1)$ by Proposing Algorithm

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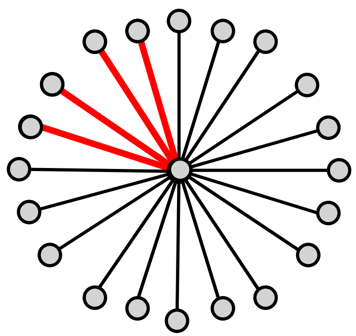

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matches constant fraction of the remaining edges

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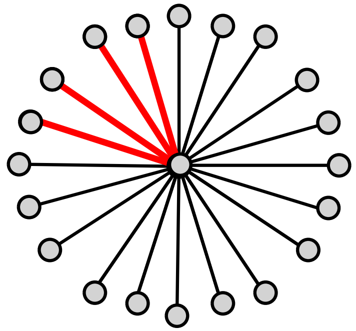

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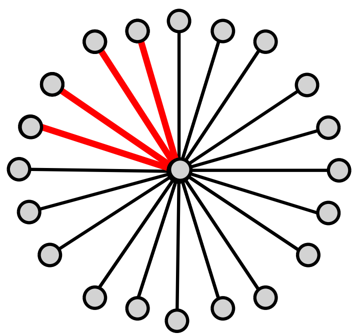
(any maximal matching has size $\geq \frac{1}{2\Delta - 1} |E|$)

(3) Final Rounding

$O(1)$-approximation in $O(1)$ rounds

values in $\{2^{-i} \mid 4 \geq i \geq 1\} \cup \{0\}$



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Our Matching Approximation Algorithm: Recap

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(1) Fractional matching

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4-approximation

$O(\log \Delta)$ rounds

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5-approximation of fractional matching of (1)

$O(\log \Delta)$ iterations $O(\log \Delta)$ rounds: $O(\log^2 \Delta)$ rounds

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values in $\{1, 0\}$: integral matching

$O(1)$-approximation of fractional matching of (2)

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Our Matching Approximation Algorithm: Recap

$O(1)$-approximation in $O(\log^2 \Delta)$ rounds

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4-approximation

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Our Matching Approximation Algorithm: Recap

$O(1)$-approximation in $O(\log^2 \Delta)$ rounds in bipartite graphs

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4-approximation

$O(\log \Delta)$ rounds

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From Bipartite to General Graphs

From Bipartite to General Graphs

(1) Turn G into auxiliary bipartite graph B

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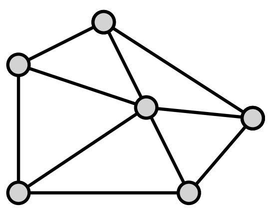

From Bipartite to General Graphs

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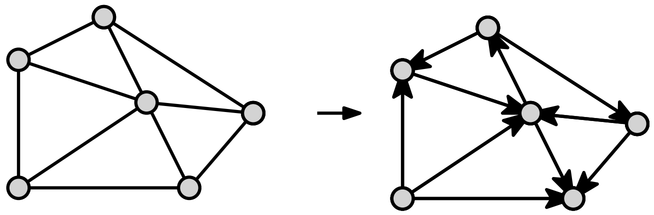

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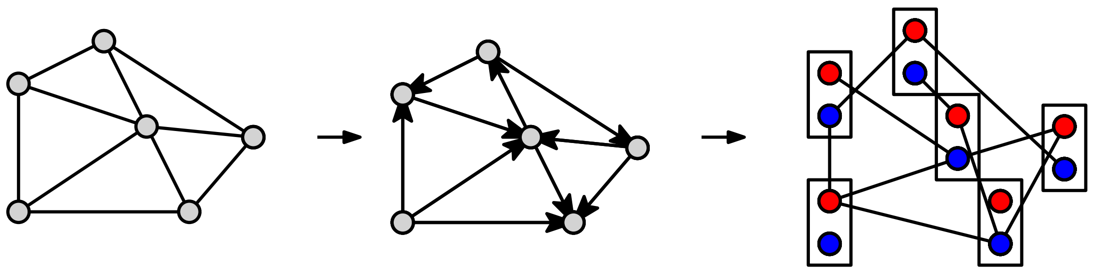

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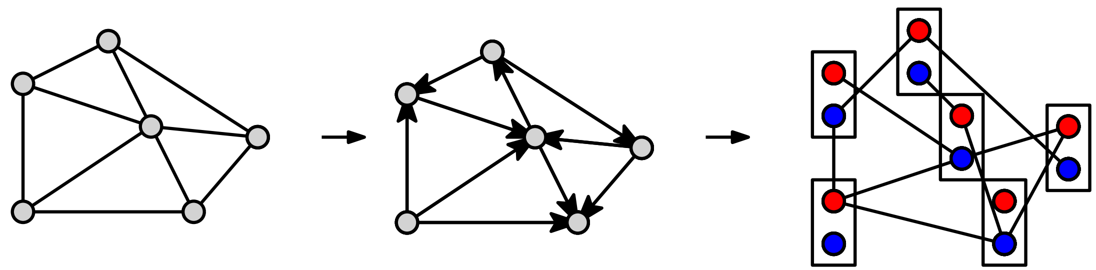

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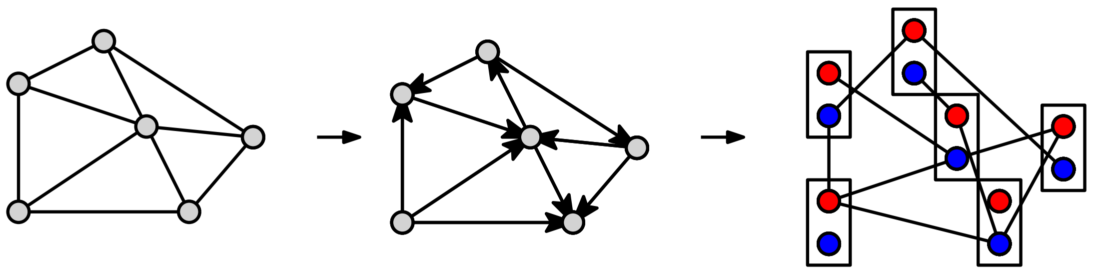

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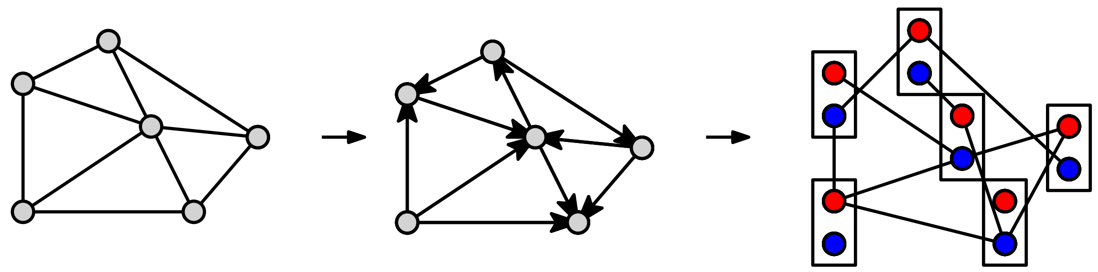

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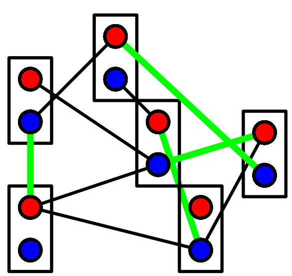

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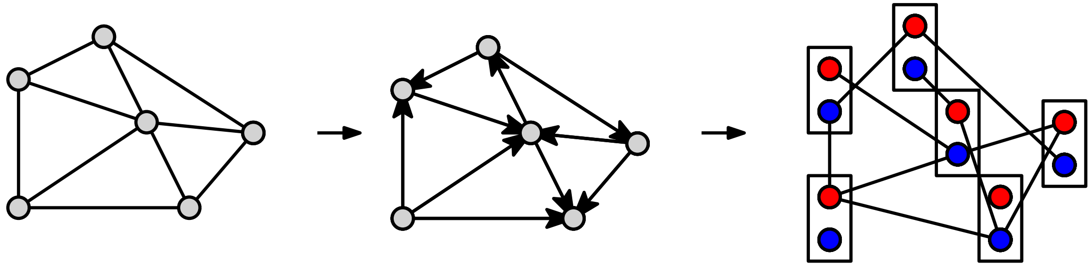

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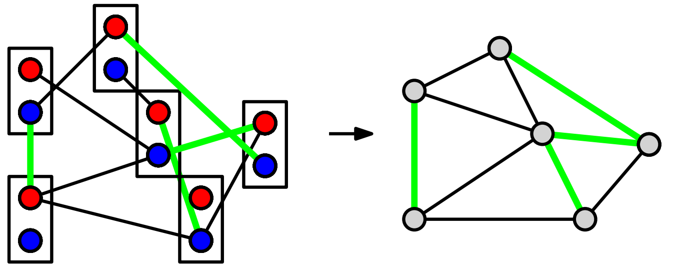

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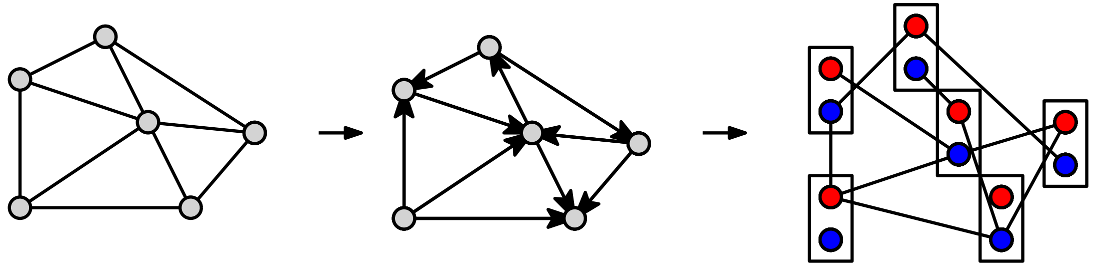

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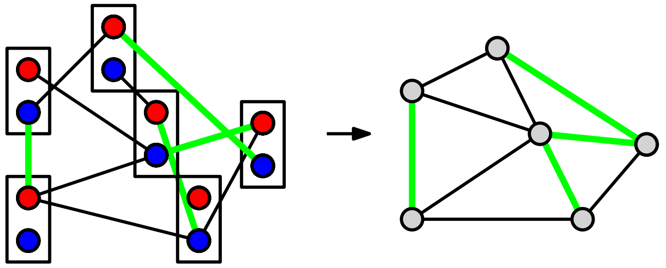

degree-2 graph

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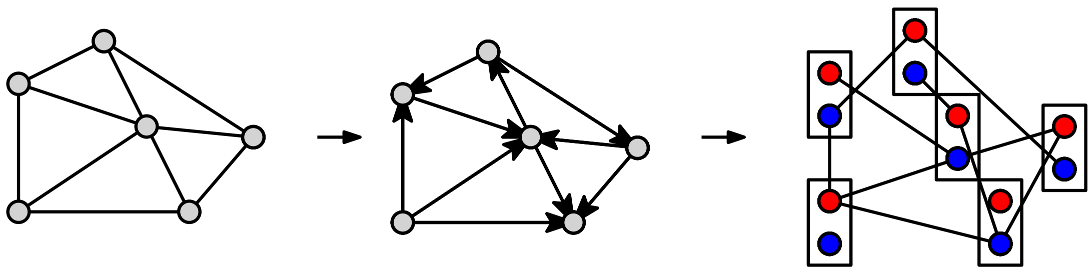

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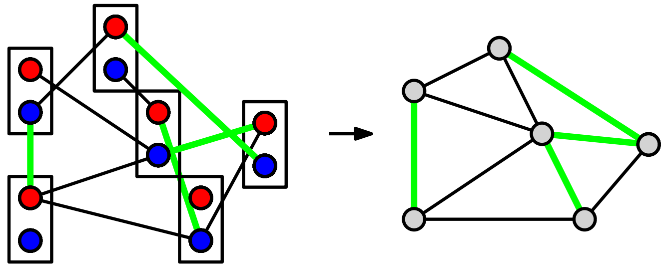

degree-2 graph

but not bipartite

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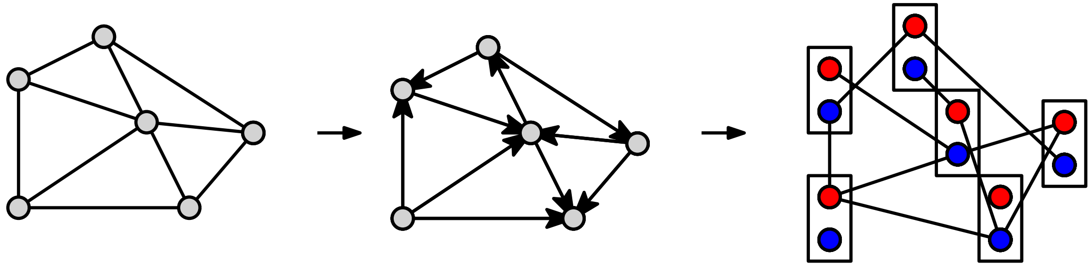

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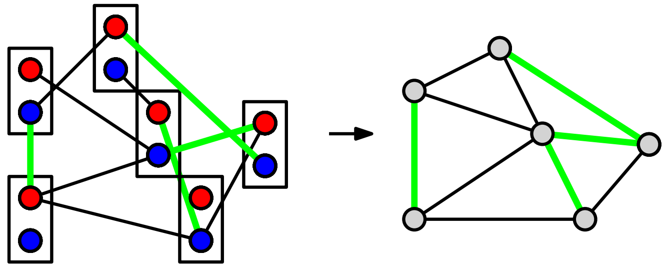

degree-2 graph

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maximal matching in $O(\log^* n)$

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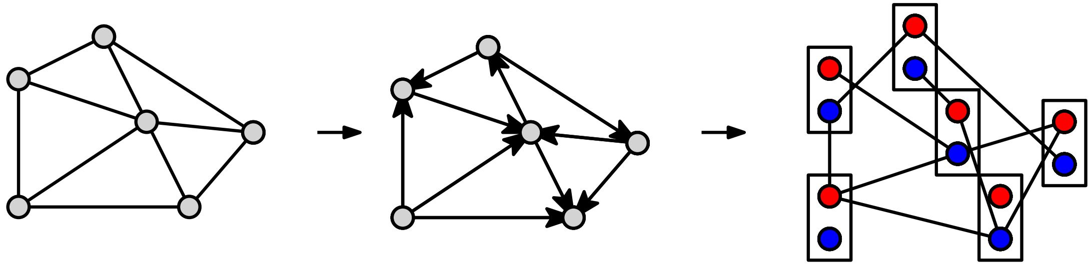

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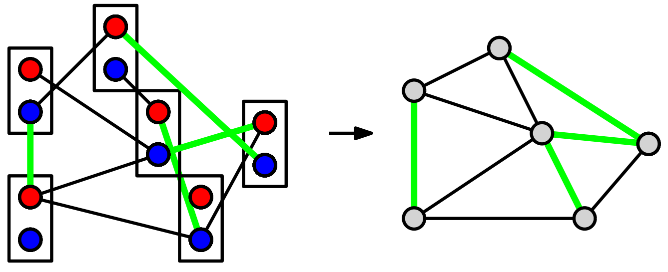

degree-2 graph

but not bipartite

maximal matching in $O(\log^* n)$

matching at least $\frac{1}{3}$-fraction of the edges

From Bipartite to General Graphs

$O(1)$-approximation in $O(\log^2 \Delta + \log^* n)$ rounds

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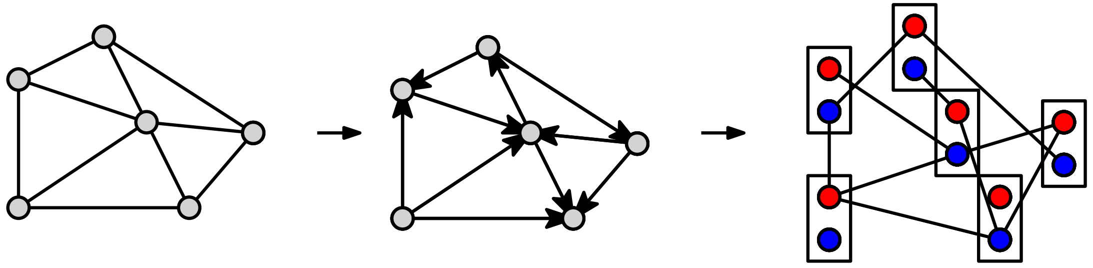

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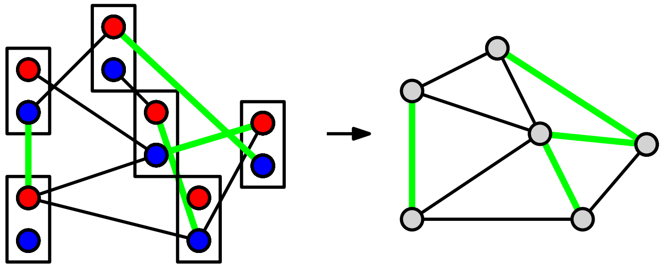

degree-2 graph

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maximal matching in $O(\log^* n)$

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From Bipartite to General Graphs

$O(1)$-approximation in $O(\log^2 \Delta + \log^* n)$ rounds in general graphs

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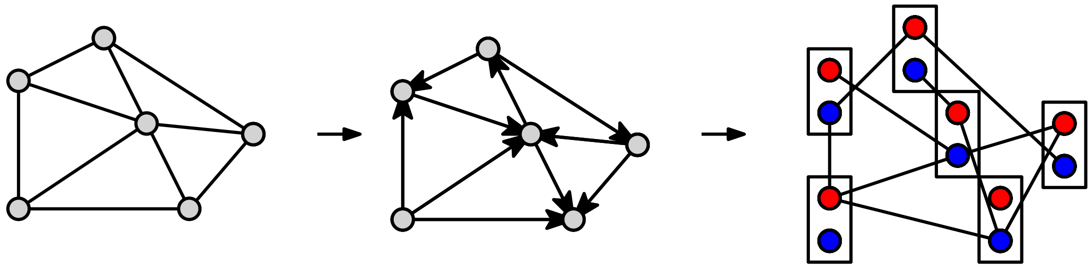

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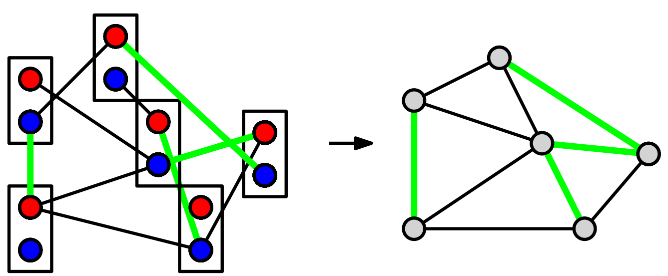

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Improving Approximation Ratio

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Observation: matching size in remainder graph reduces by $O(1)$-factor

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M^*

$|M| \geq \frac{1}{c}|M^*|$

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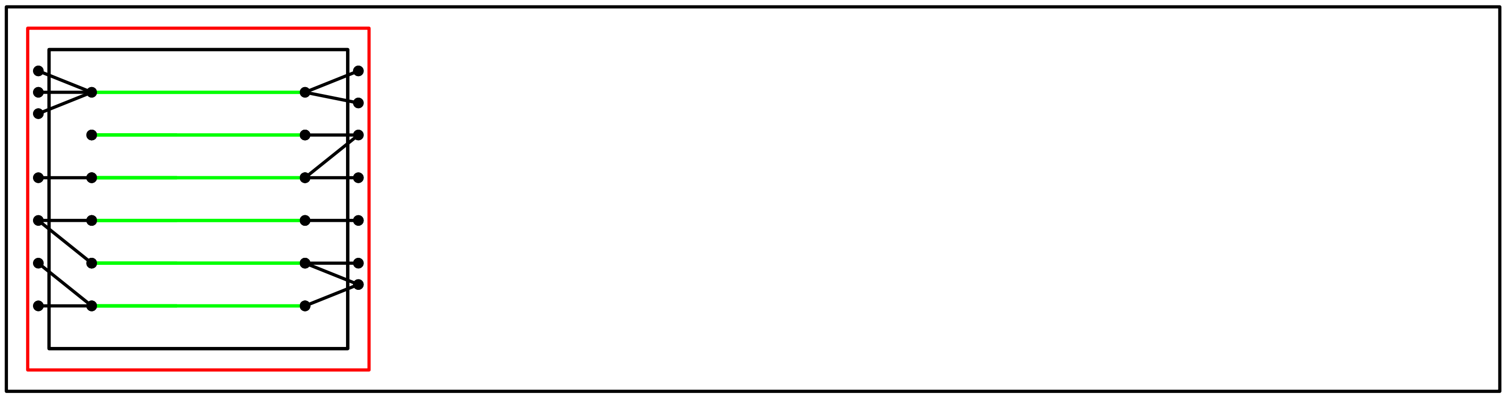


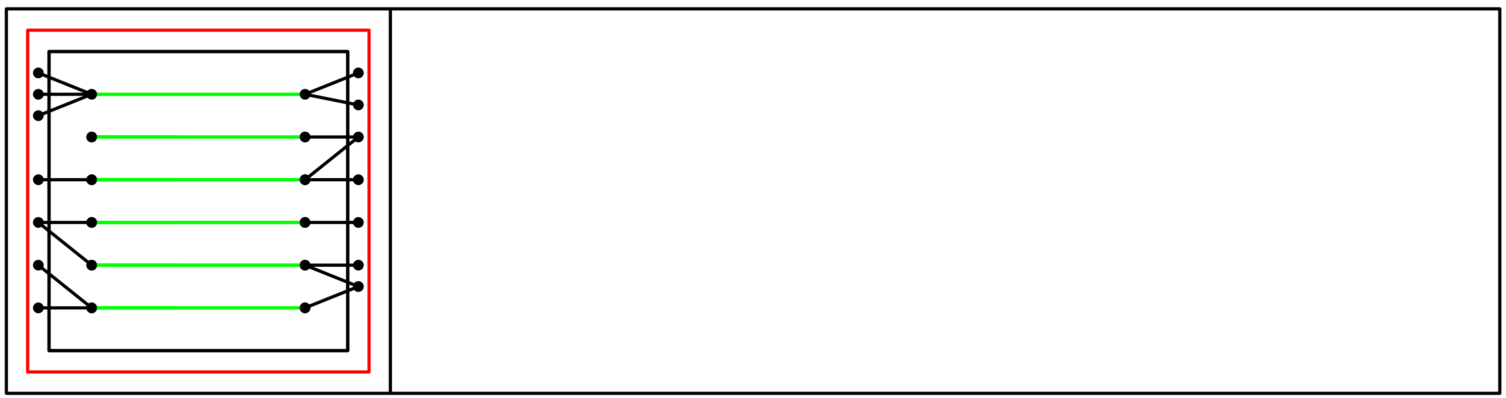
G

M^*

$|M| \geq \frac{1}{c}|M^*|$

Improving Approximation Ratio

Observation: matching size in remainder graph reduces by $O(1)$-factor



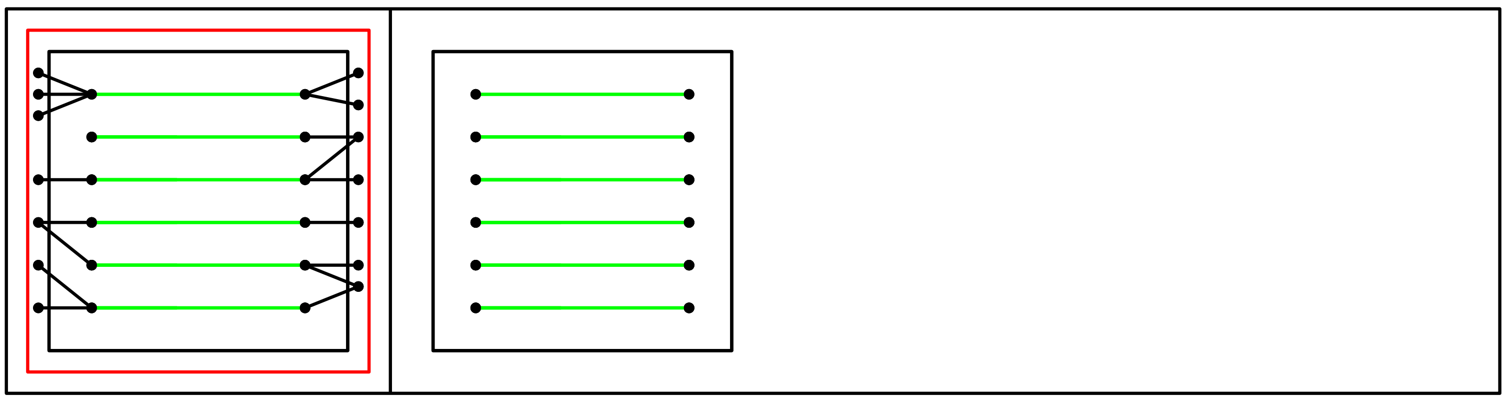
G $\qquad$ G_1

M^* $\qquad$ $|M_1^*| \leq \left(1 - \frac{1}{c}\right) |M^*|$

$|M| \geq \frac{1}{c} |M^*|$

Improving Approximation Ratio

Observation: matching size in remainder graph reduces by $O(1)$-factor



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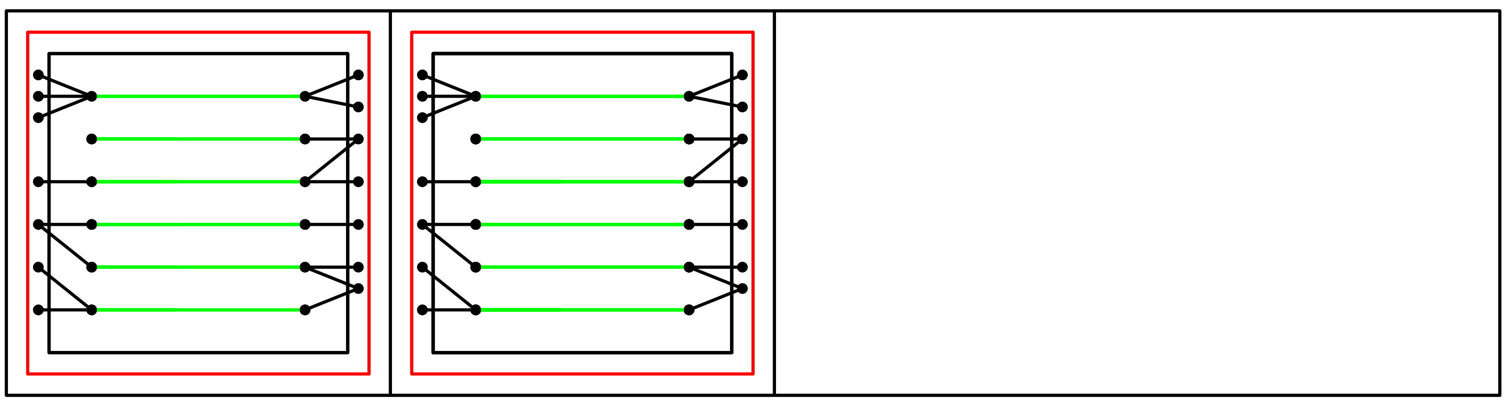

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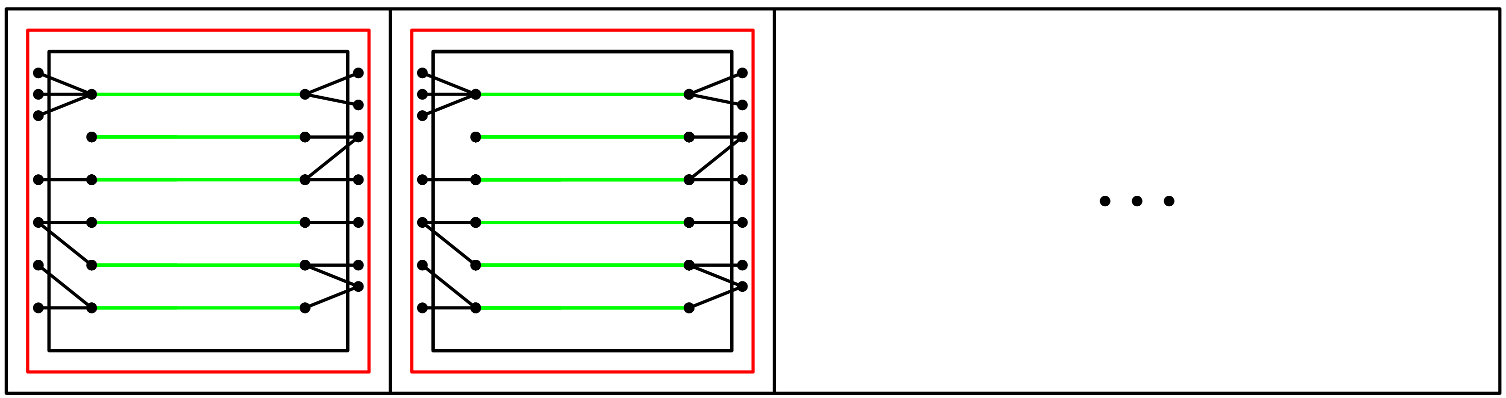

G | G_1

M^*

$|M_1^*| \leq \left(1 - \frac{1}{c}\right) |M^*|$

$|M| \geq \frac{1}{c} |M^*|$

$|M_1| \geq \frac{1}{c} |M_1^*|$

inductively: $|M_i^*| \leq \left(1 - \frac{1}{c}\right)^i |M^*|$

Improving Approximation Ratio

$$|M_i^*| \leq \left(1 - \frac{1}{c}\right)^i |M^*|$$

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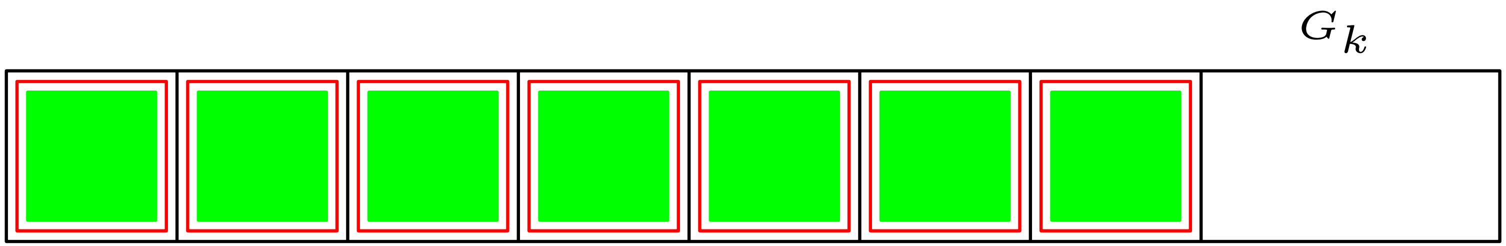
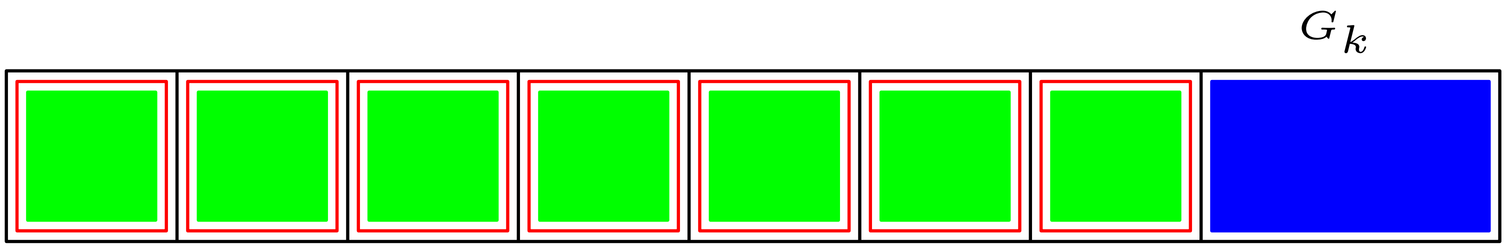


Improving Approximation Ratio

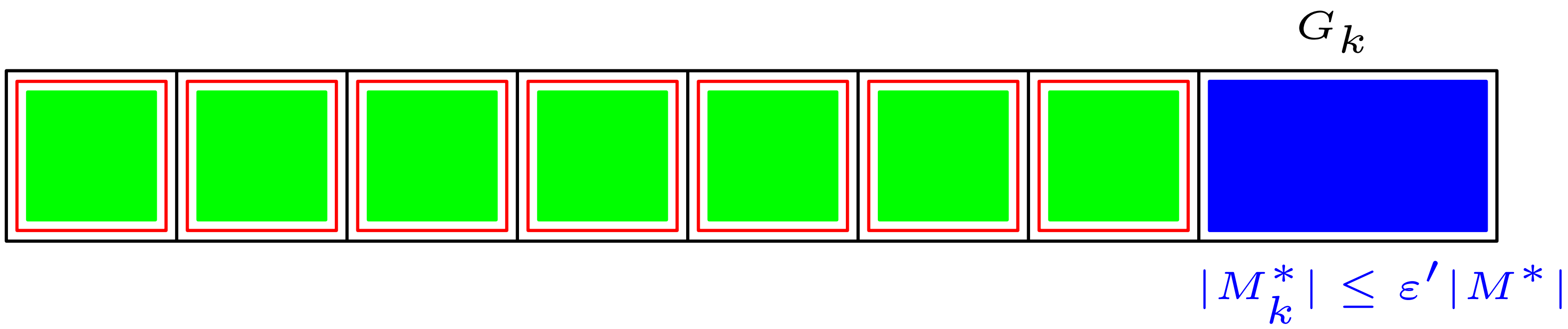
$|M_i^*| \leq \left(1 - \frac{1}{c}\right)^i |M^*|$



$|M_k^*| \leq \varepsilon' |M^*|$

Improving Approximation Ratio

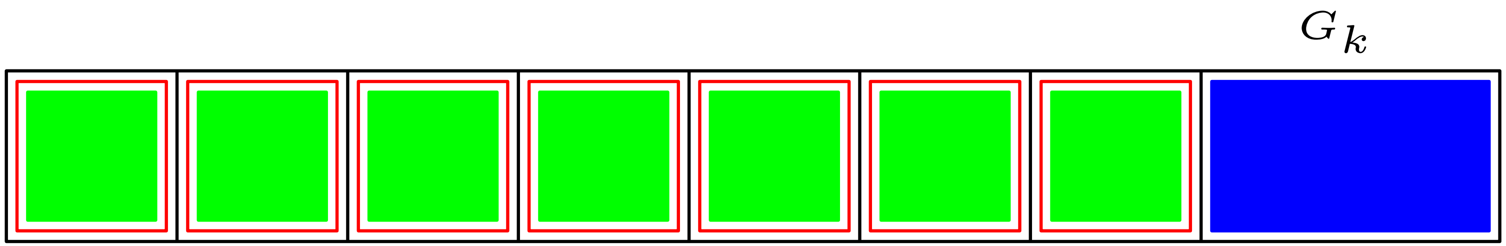
$$|M_i^*| \leq \left(1 - \frac{1}{c}\right)^i |M^*|$$



$M_k^* \cup \bigcup_{i=0}^{k-1} M_i$ is maximal in G

Improving Approximation Ratio

$|M_i^*| \leq \left(1 - \frac{1}{c}\right)^i |M^*|$



$|M_k^*| \leq \varepsilon' |M^*|$

$M_k^* \cup \bigcup_{i=0}^{k-1} M_i$ is maximal in G

$|M_k^*| + \sum_{i=0}^{k-1} |M_i| \geq \frac{1}{2} |M^*|$

Improving Approximation Ratio

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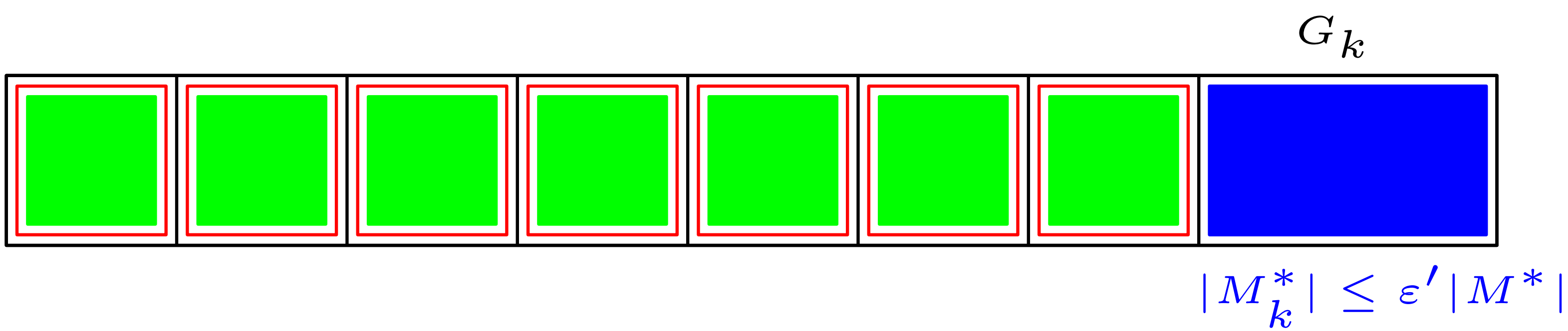

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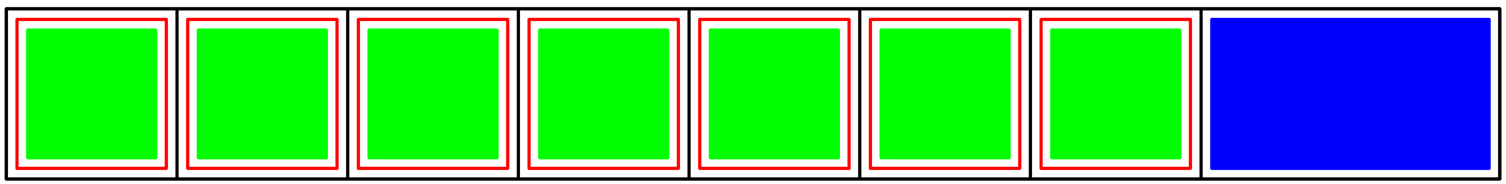
- $O(\log^2 \Delta \cdot \log \frac{1}{\varepsilon} + \log^* n)$ rounds for $(2 + \varepsilon)$-approximation

Improving Approximation Ratio

$$|M_i^*| \leq \left(1 - \frac{1}{c}\right)^i |M^*|$$

G_k



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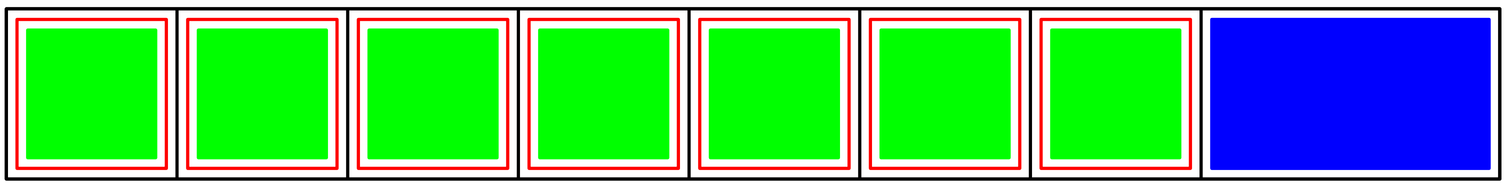
- $O(\log^2 \Delta \cdot \log \frac{1}{\varepsilon} + \log^* n)$ rounds for $(2+\varepsilon)$-approximation

  direct: $O((\log^2 \Delta + \log^* n) \log \frac{1}{\varepsilon})$

Improving Approximation Ratio

$|M_i^*| \leq \left(1 - \frac{1}{c}\right)^i |M^*|$



$|M_k^*| \leq \varepsilon' |M^*|$

$M_k^* \cup \bigcup_{i=0}^{k-1} M_i$ is maximal in G

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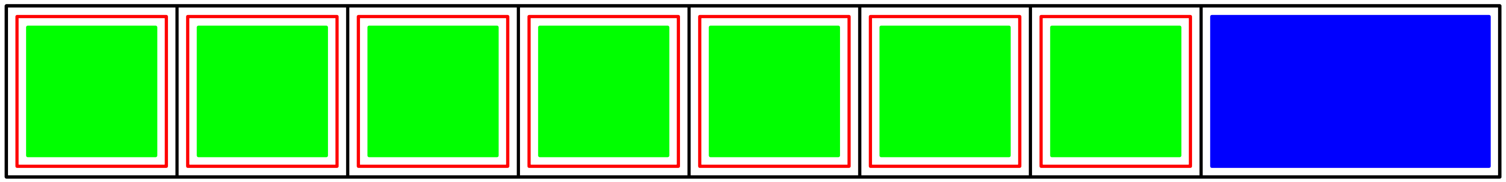
- $O(\log^2 \Delta \cdot \log \frac{1}{\varepsilon} + \log^* n)$ rounds for $(2+\varepsilon)$-approximation

  direct: $O((\log^2 \Delta + \log^* n) \log \frac{1}{\varepsilon})$
- $O(\log^2 \Delta \cdot \log n)$ rounds for maximal matching

Improving Approximation Ratio

$|M_i^*| \leq \left(1 - \frac{1}{c}\right)^i |M^*|$



$|M_k^*| \leq \varepsilon' |M^*|$

$M_k^* \cup \bigcup_{i=0}^{k-1} M_i$ is maximal in G $\qquad |M_k^*| + \sum_{i=0}^{k-1} |M_i| \geq \frac{1}{2} |M^*|$

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  after $k = O(\log n)$ iterations: $|M_k^*| \leq \frac{1}{2n} |M^*| < 1$

Recap: Our Results

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Our Result [F., Ghaffari 2017]

$O(\log^2 \Delta + \log^* n)$	for constant approximation
$O(\log^2 \Delta \cdot \log \frac{1}{\varepsilon} + \log^* n)$	for $(2+\varepsilon)$-approximation
$O(\log^2 \Delta \cdot \log n)$	for maximal matching (2-approximation)

using an $O\left(\log^2 \Delta\right)$-round $O(1)$-approximation algorithm for bipartite graphs

$O(\log^4 n)$	Hańćkowiak, Karoński, Panconesi [PODC'99]
$O(\Delta + \log^* n)$	Panconesi, Rizzi [DIST'01]
$\Omega\left(\frac{\log \Delta}{\log \log \Delta} + \log^* n\right)$	Kuhn, Moscibroda, Wattenhofer [PODC'04], Linial [FOCS'87]

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$$O(\log^2 \Delta \cdot \log n)$$

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  - deterministic poly $\log n$-round algorithm for $(2\Delta - 1)$-edge-coloring (F., Ghaffari, Kuhn (2017)) using deterministic rounding (for hypergraph matching)

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- ## Deterministic Complexity of Matching
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- ## Rounding Method
  - completeness of rounding (Ghaffari, Kuhn, Maus [STOC'17])

    matching admits an efficient deterministic algorithm because
    matching admits efficient deterministic rounding
  - deterministic poly $\log n$-round algorithm for $(2\Delta - 1)$-edge-coloring (F., Ghaffari, Kuhn (2017)) using deterministic rounding (for hypergraph matching)
  - more general deterministic rounding method?