

Parallelizing the WHT.

$$\text{WHT}_{nm} = \underbrace{(\text{WHT}_n \otimes \mathbb{I}_m)}_{\downarrow} \underbrace{(\mathbb{I}_n \otimes \text{WHT}_m)}_{\downarrow}$$

Needs to be rewritten
for parallelization.

Naturally parallelized
for n processors.

(See rules in previous
lecture.)

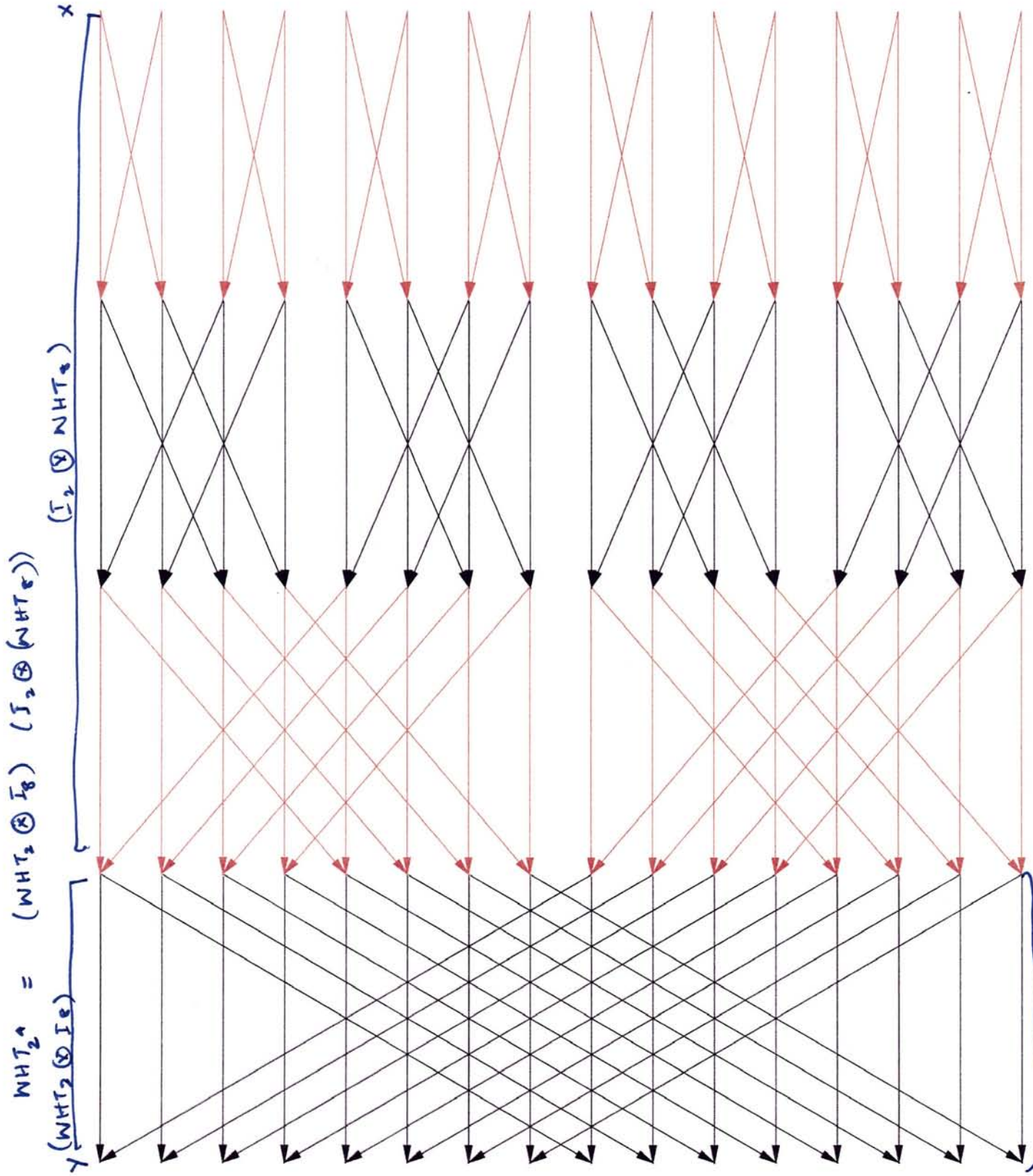
One way to express $(\text{WHT}_n \otimes \mathbb{I}_m)$ is:

$$(\text{WHT}_n \otimes \mathbb{I}_m) = L_n^{nm} (\mathbb{I}_m \otimes \text{WHT}_n) L_m^{nm}$$

The L s (stride permutations) simply
become data exchanges between processors
in the implementation.

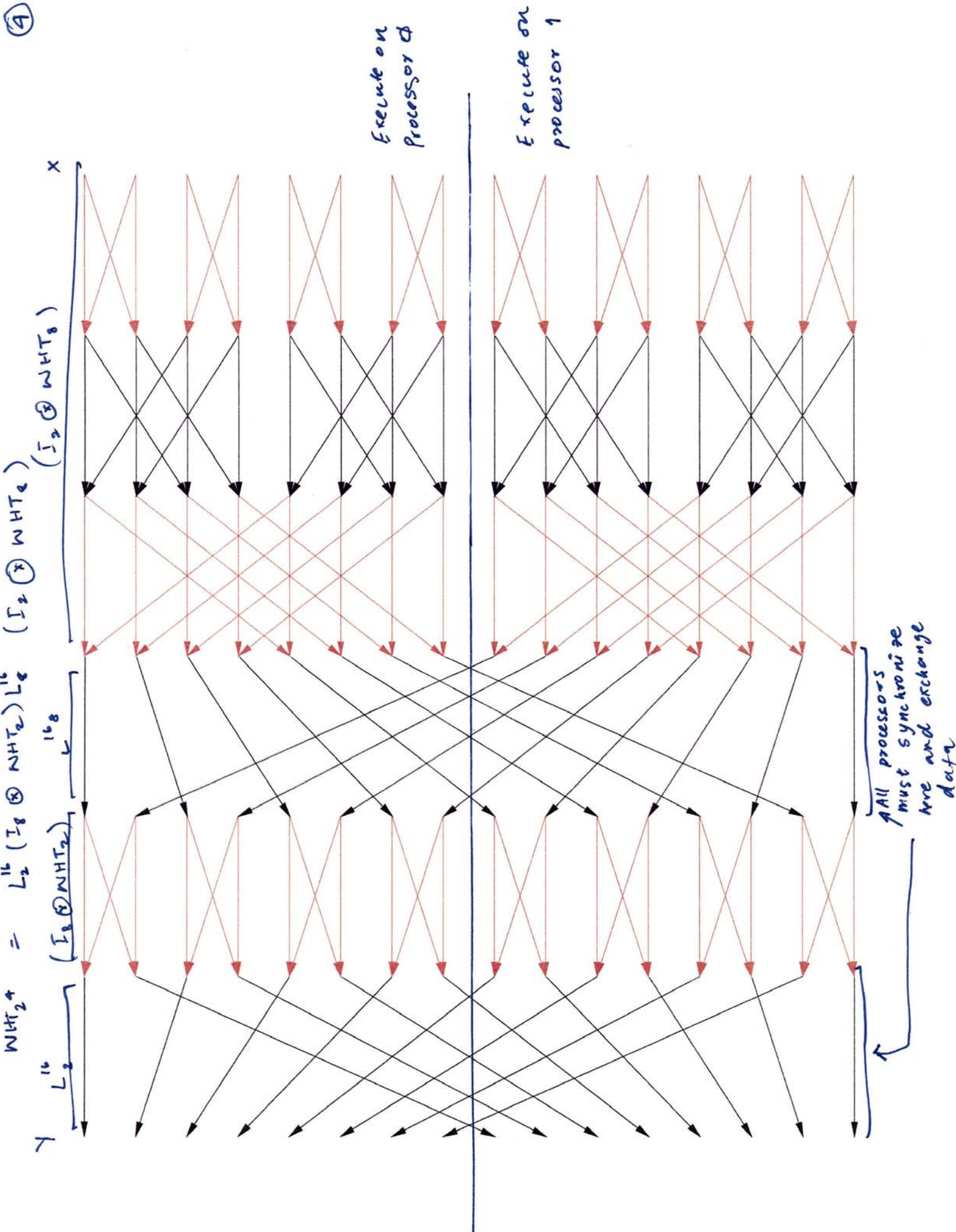
The following figures let us visualize this idea.

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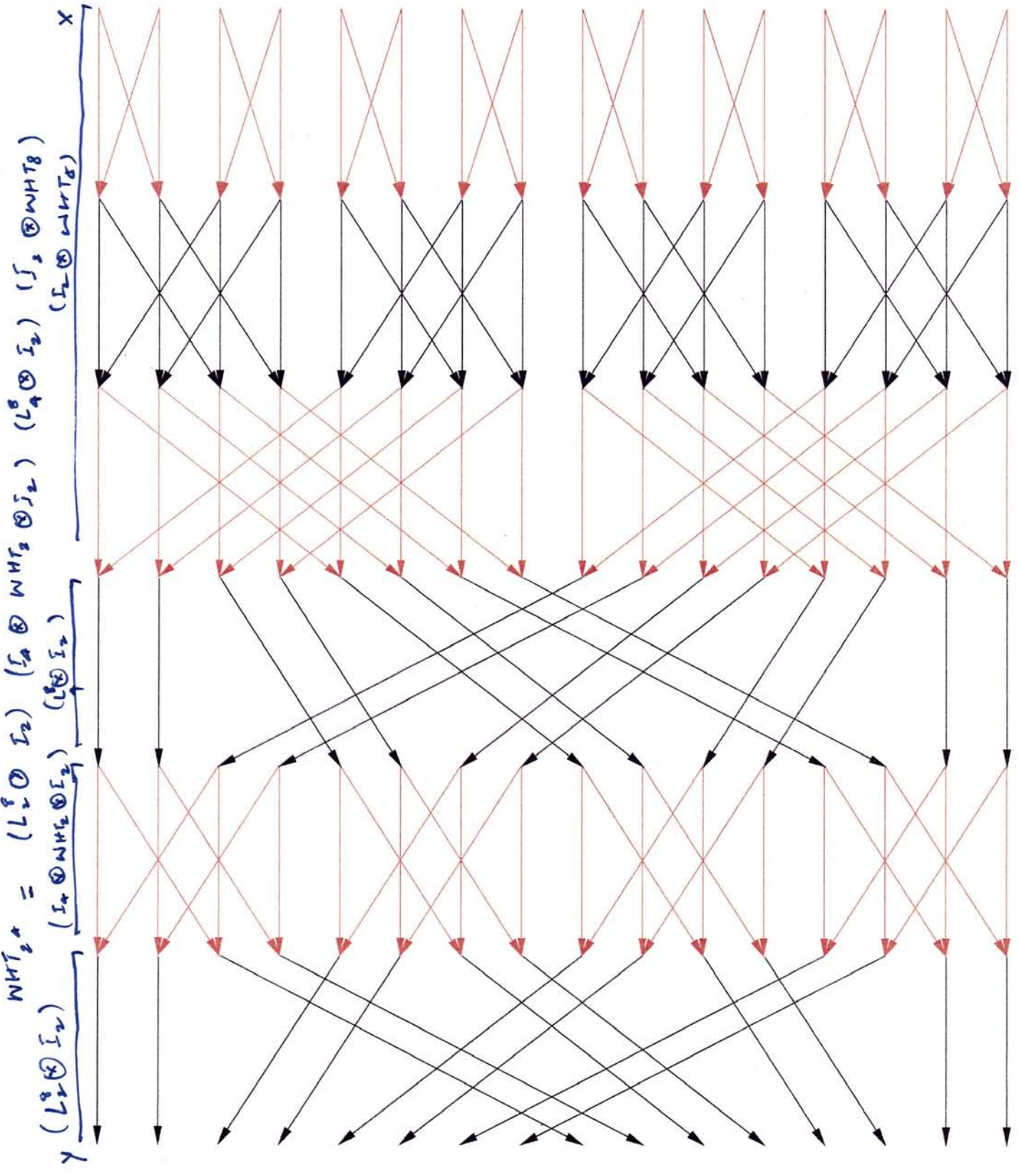


This part cannot be executed in parallel in this form.

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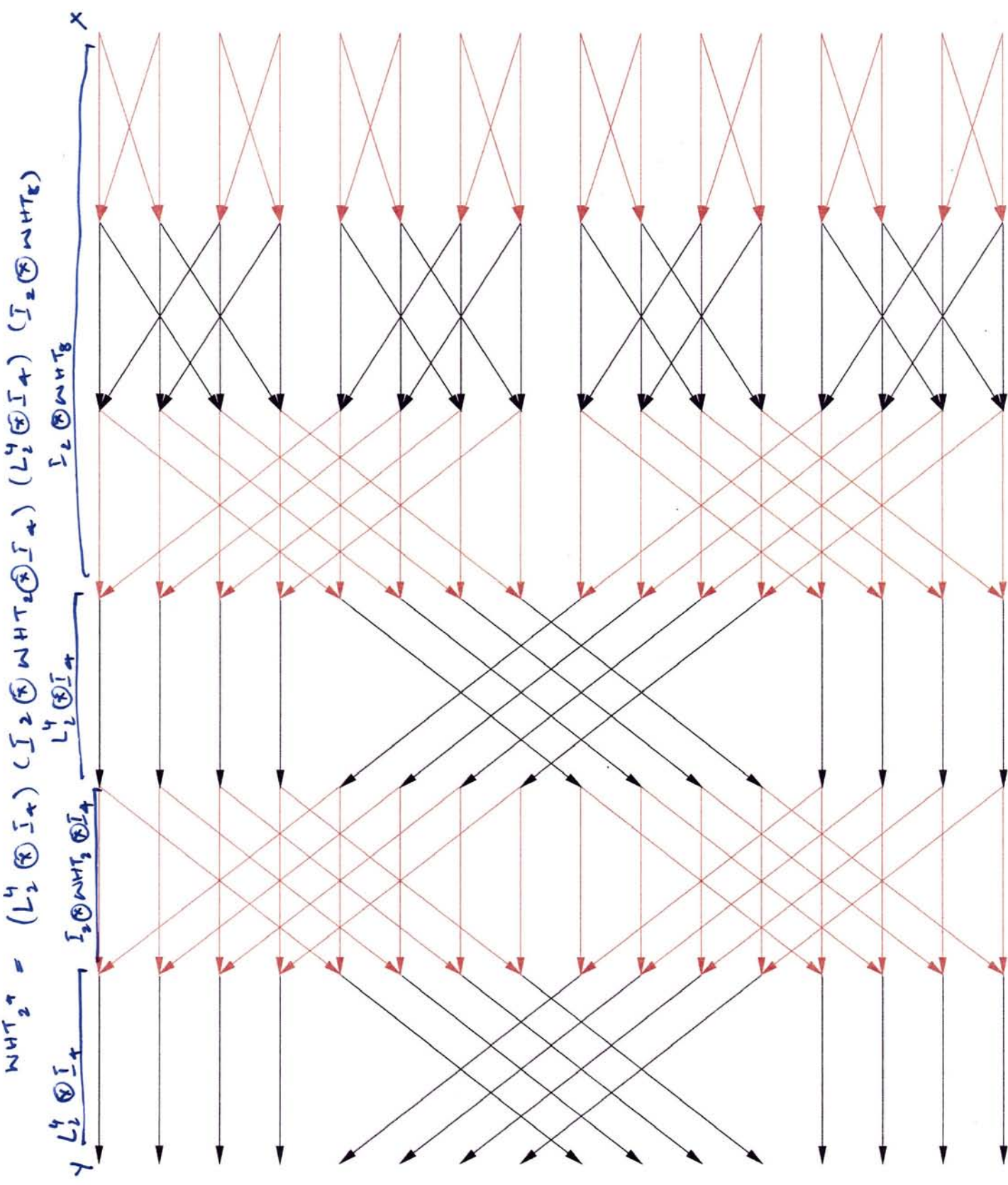


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$(WHI_2 \otimes I_8)$ can also be rewritten as $(L_2^8 \otimes I_2)(I_4 \otimes WHI_2 \otimes I_2)(L_4^8 \otimes I_2)$

Data exchanges in the $(L \otimes I_n)$ stage exchanges n elements at a time, thus avoiding false sharing for cache line lengths of $\geq n$.



$$WHT_{2^4} = (L_2^4 \otimes I_4) (I_2 \otimes WHT_2 \otimes I_4) (L_2^4 \otimes I_4) (I_2 \otimes WHT_2)$$

Similarly, $(WHT_2 \otimes I_8)$ can be expressed as:

$$(L_2^4 \otimes I_4) (I_2 \otimes WHT_2 \otimes I_4) (L_2^4 \otimes I_4)$$