How to Write Fast Numerical Code

Spring 2011

Lecture 13

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Miscellaneous

- **No class next Monday, April 11th (Sechseläuten)**
- **Midterm exam: Friday, April 15th**

Today

- **Solving linear systems**
- **Matrix inversion**
- **PLU factorization**
- **Determinant**

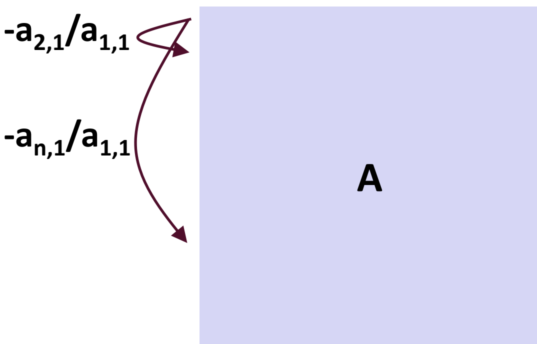
Linear system solving
Matrix inversion
Singular value decomposition
... and more

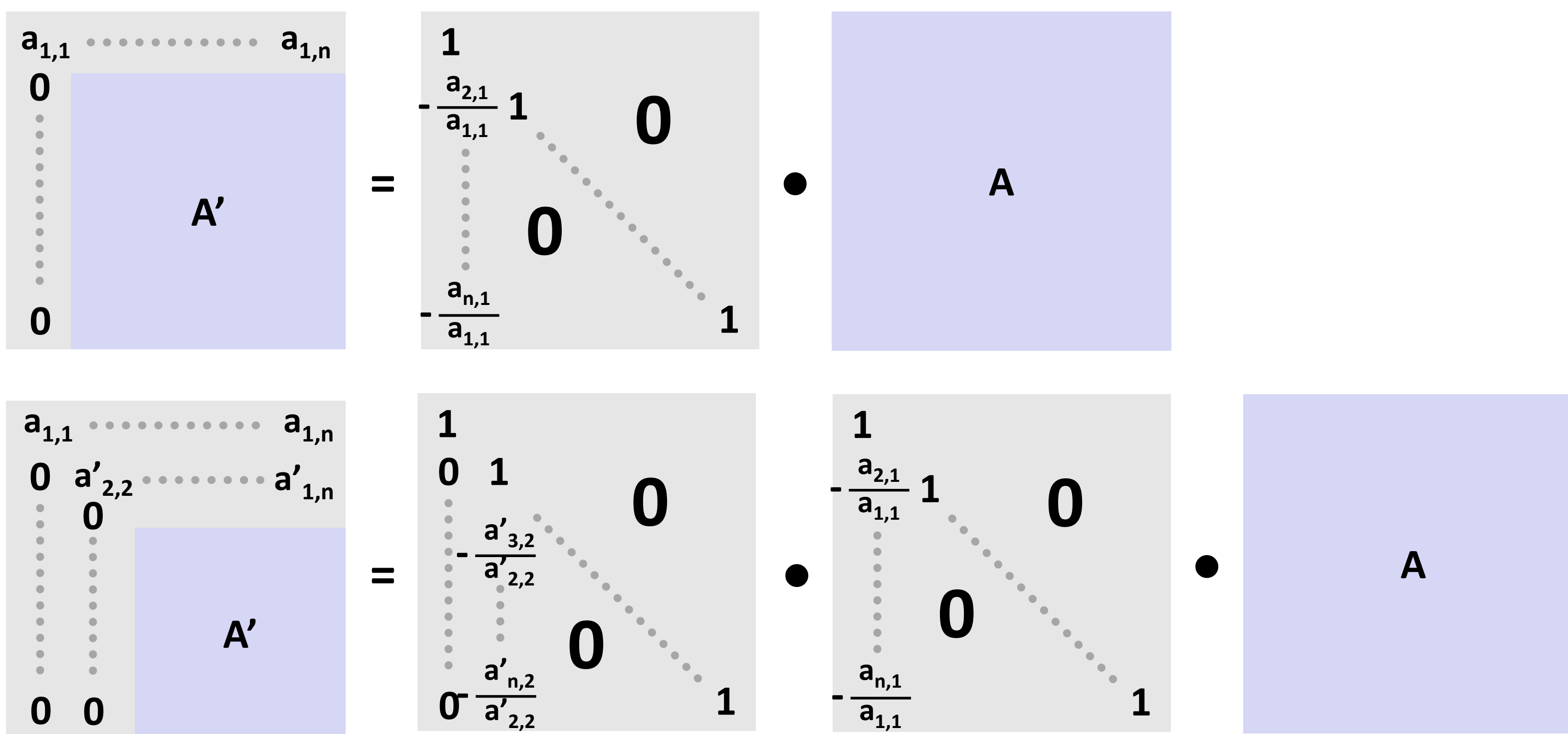
BLAS 1: vector-vector ops
BLAS 2: matrix-vector ops
BLAS 3: matrix-matrix ops

Chapter 2 in James W. Demmel, Applied Numerical Linear Algebra, SIAM, 1997

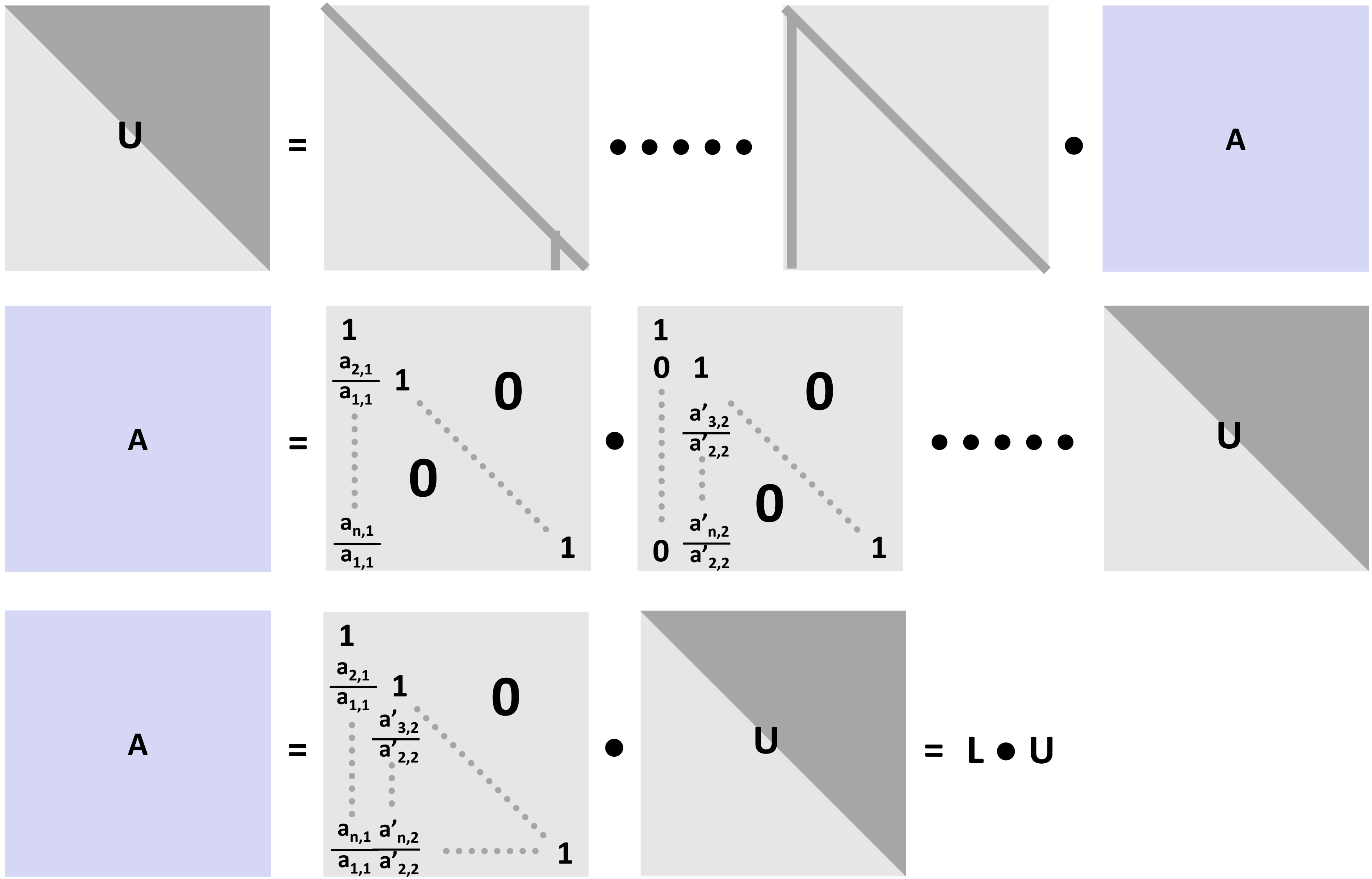
Gauss Elimination and LU Factorization

$A = [a_{k,l}]_{1 \le k,l \le n}$





after n-1 steps ….



Summary

- **Gauss elimination is LU factorization**
  - We assume that the occurring diagonal elements $a_{1,1}$, $a'_{2,2}$, … are all $\neq 0$ (otherwise the LU factorization does not exist)
  - U = upper triangular
  - L = lower triangular (1's on diagonal)
  - L contains the multipliers occurring in Gauss elimination

- **Now Ax = b is LUx = b and can be solved in two steps:**
  - Solve Ly = b
  - Solve Ux = y
  - Cost: $n^2 + O(n)$ for each step = $2n^2 + O(n)$

LU Factorization: Algorithm

- **From straightforward algorithm to optimized BLAS 3 based one (blackboard)**

Chapter 2 in James W. Demmel, Applied Numerical Linear Algebra, SIAM, 1997