How to Write Fast Numerical Code

Spring 2012, Lecture 1

Instructor: *Markus Püschel*

TA: *Georg Ofenbeck*

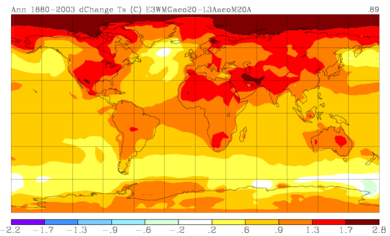
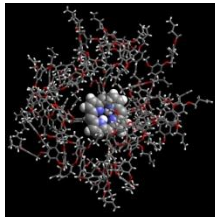
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Picture: www.tapety-na-pulpit.org

Today

- **Motivation for this course**
- **Organization of this course**

Scientific Computing

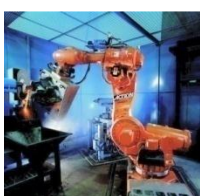
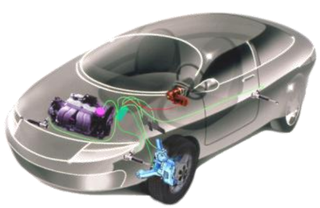



Physics/biology simulations

Consumer Computing


Audio/image/video processing

Embedded Computing

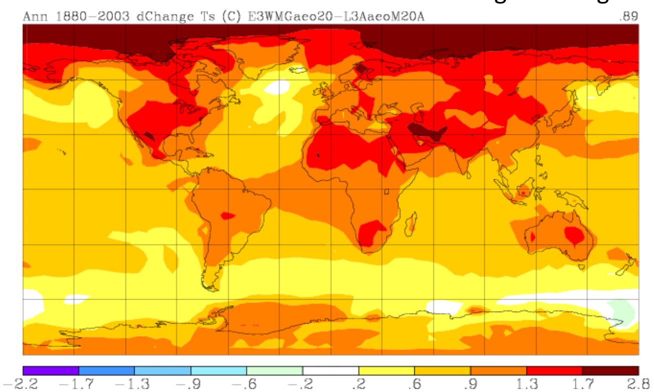
Signal processing, communication, control

Computing

- **Unlimited need for performance**
- **Large set of applications, but …**
- **Relatively small set of critical components (100s to 1000s)**
  - Matrix multiplication
  - Discrete Fourier transform (DFT)
  - Viterbi decoder
  - Shortest path computation
  - Stencils
  - Solving linear system
  - ….

Scientific Computing (Clusters/Supercomputers)

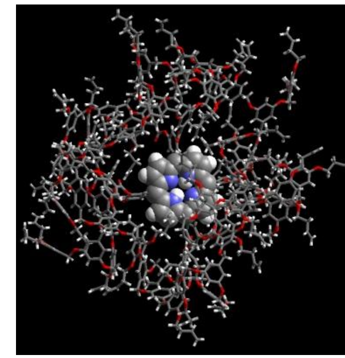
data.giss.nasa.gov



Climate modelling

Finance simulations

www.foresight.org



Molecular dynamics

Other application areas:
- Fluid dynamics
- Chemistry
- Biology
- Medicine
- Geophysics

Methods:
- Mostly linear algebra
- PDE solving
- Linear system solving
- Finite element methods
- Others

Consumer Computing (Desktop, Phone, …)

Photo/video processing

Audio coding

Security

Original

JPEG

JPEG2000

Image compression

Methods:

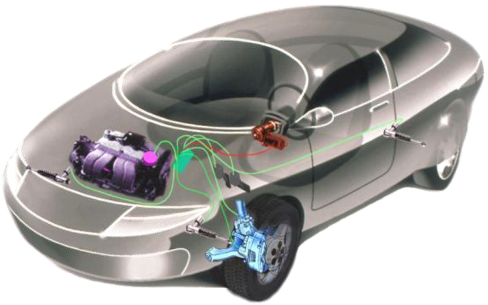
- Linear algebra
- Transforms
- Filters
- Others

Embedded Computing (Low-Power Processors)

www.dei.unipd.it

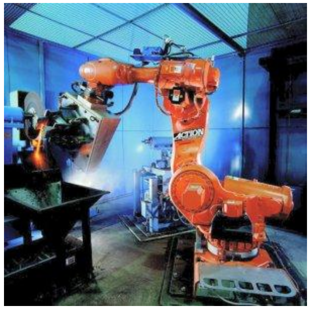
Sensor networks

www.ece.drexel.edu



Cars

www.microway.com.au



Robotics

Computation needed:
- Signal processing
- Control
- Communication

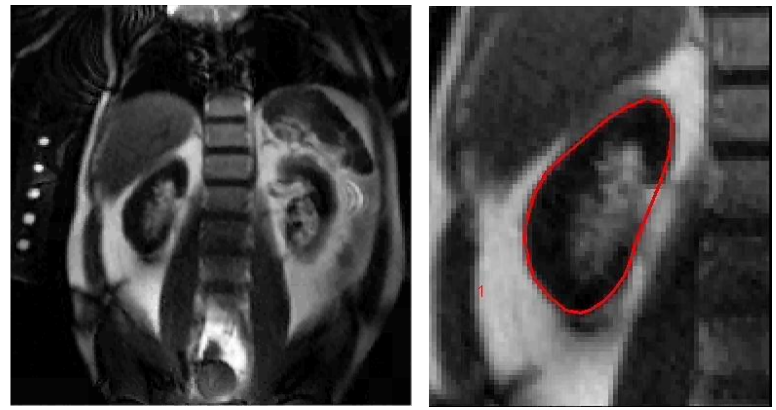
Methods:
- Linear algebra
- Transforms, Filters
- Coding

Research (Examples from Carnegie Mellon)

Bhagavatula/Savvides

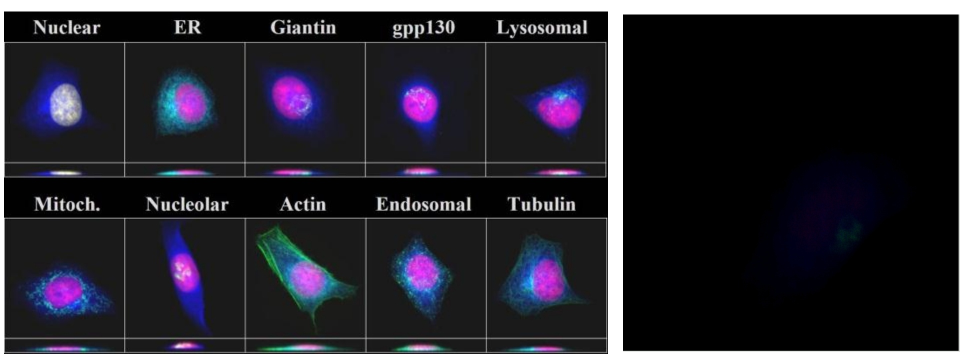
Biometrics

Moura



Medical Imaging

Kovacevic



Bioimaging

Kanade

Computer vision

Classes of Performance-Critical Functions

- **Transforms**
- **Filters/correlation/convolution/stencils/interpolators**
- **Dense linear algebra functions**
- **Sparse linear algebra functions**
- **Coder/decoders**
- **Graph algorithms**
- ***… several others***

See also the 13 dwarfs/motifs in
http://www.eecs.berkeley.edu/Pubs/TechRpts/2006/EECS-2006-183.pdf

How Hard Is It to Get Fast Code?

Problem → *"compute Fourier transform"*

↓

Algorithm theory

↓

Optimal algorithm → *"fast Fourier transform"*
O(nlog(n)) or 4nlog(n) + 3n

↓

Software developer

↓

Source code → *e.g., a C function*

↓

Compiler

↓

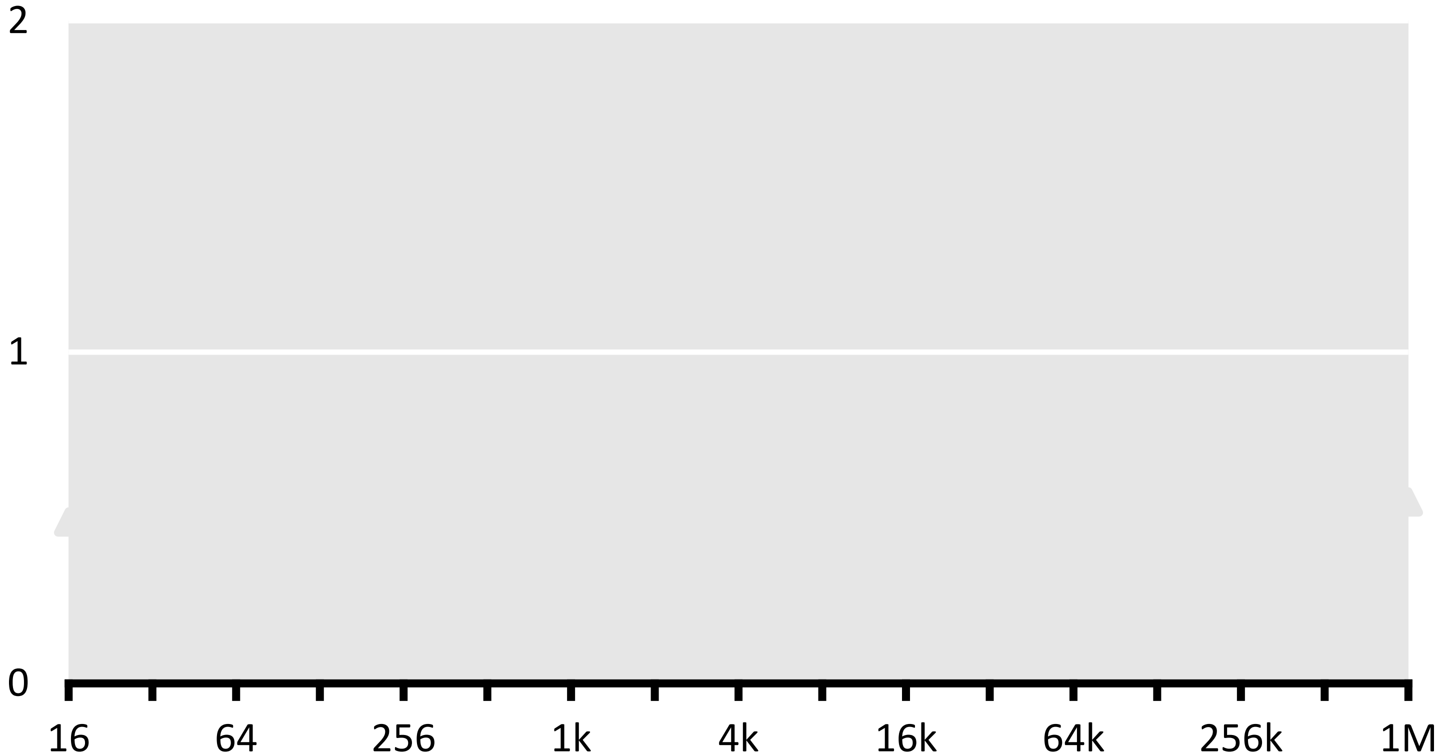
Fast executable

How well does this work?

The Problem: Example 1

DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)

Performance [Gflop/s]



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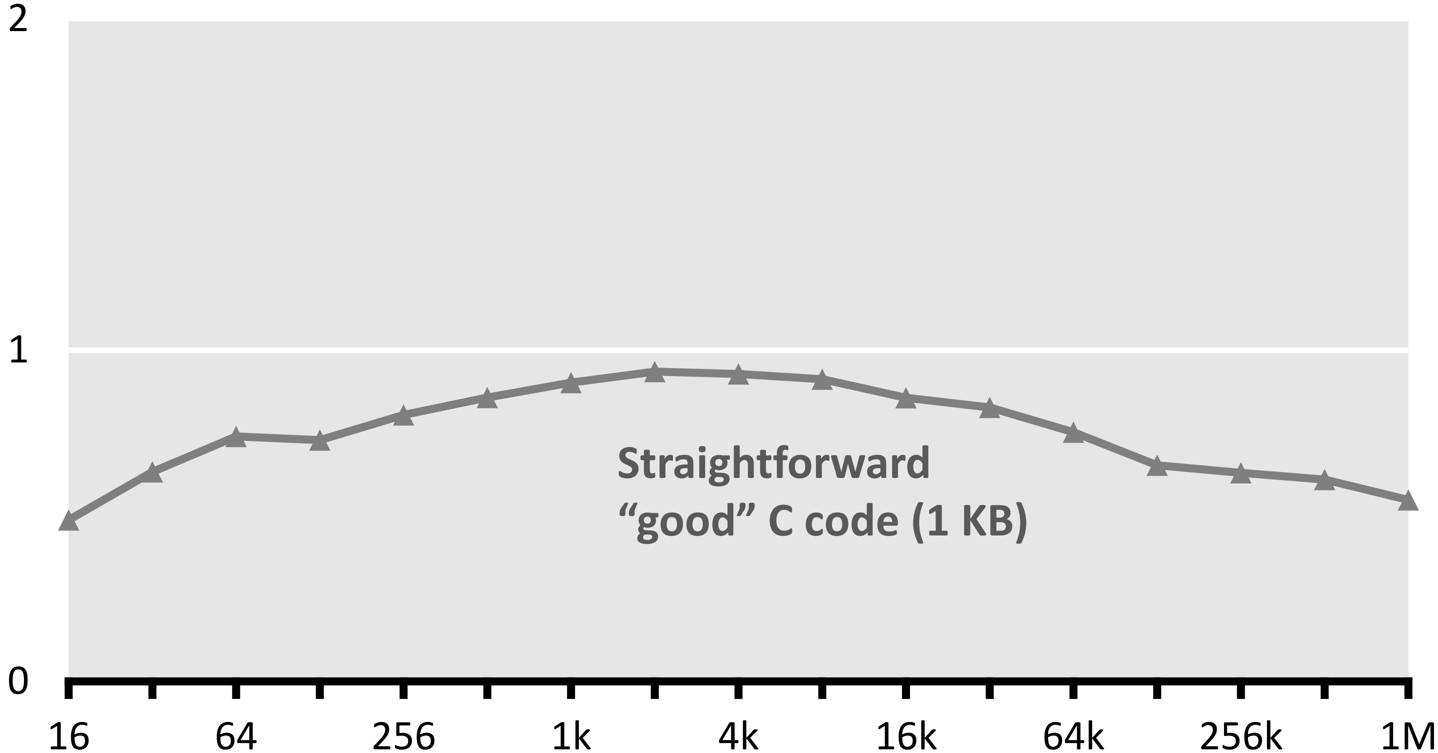

 **or**  **?**

The Problem: Example 1

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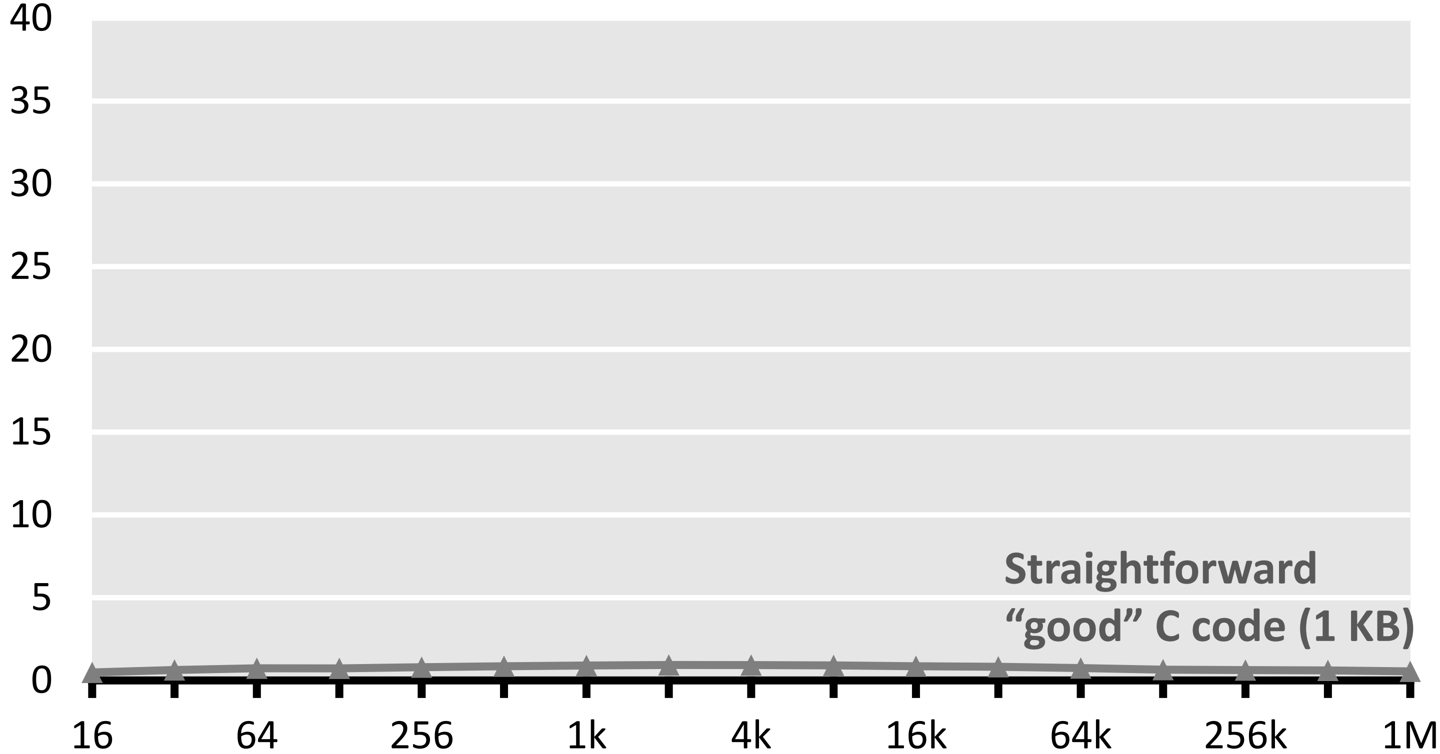

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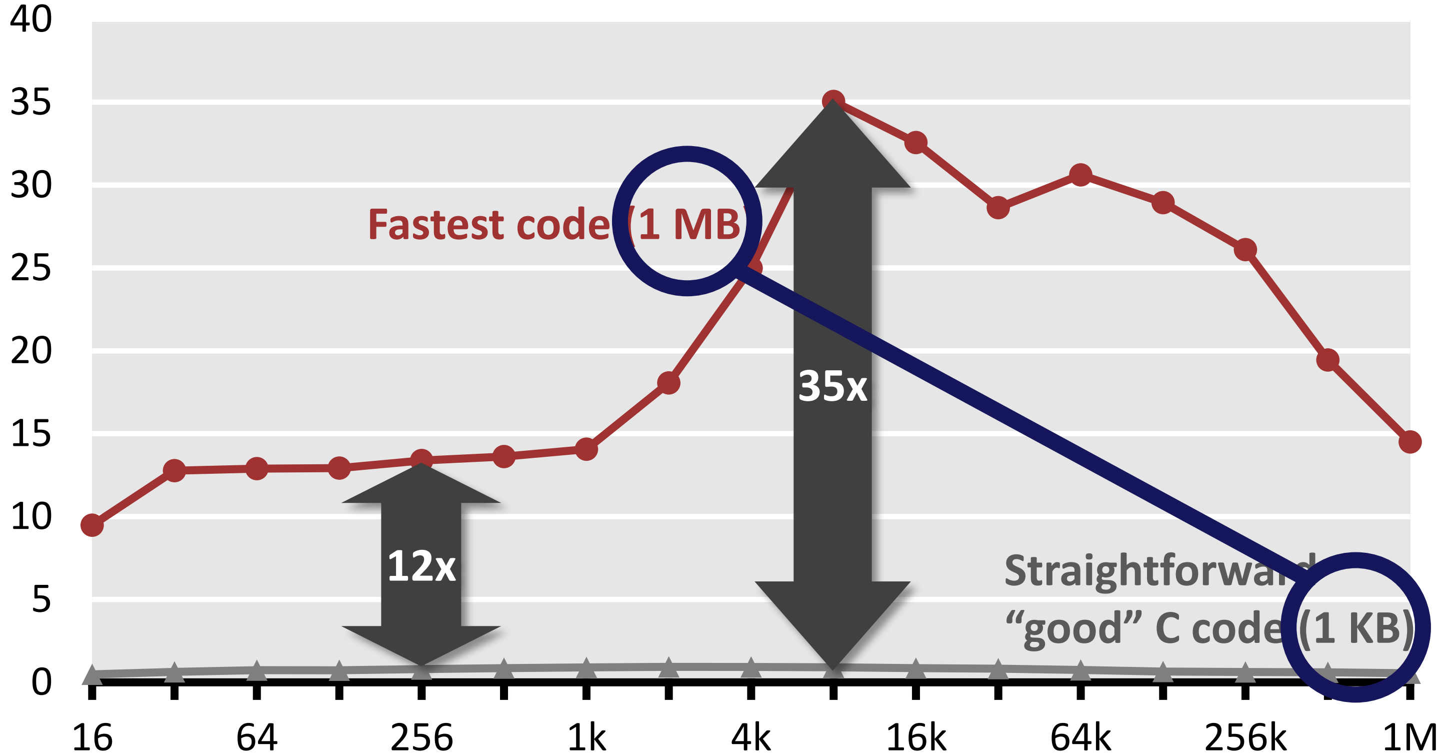
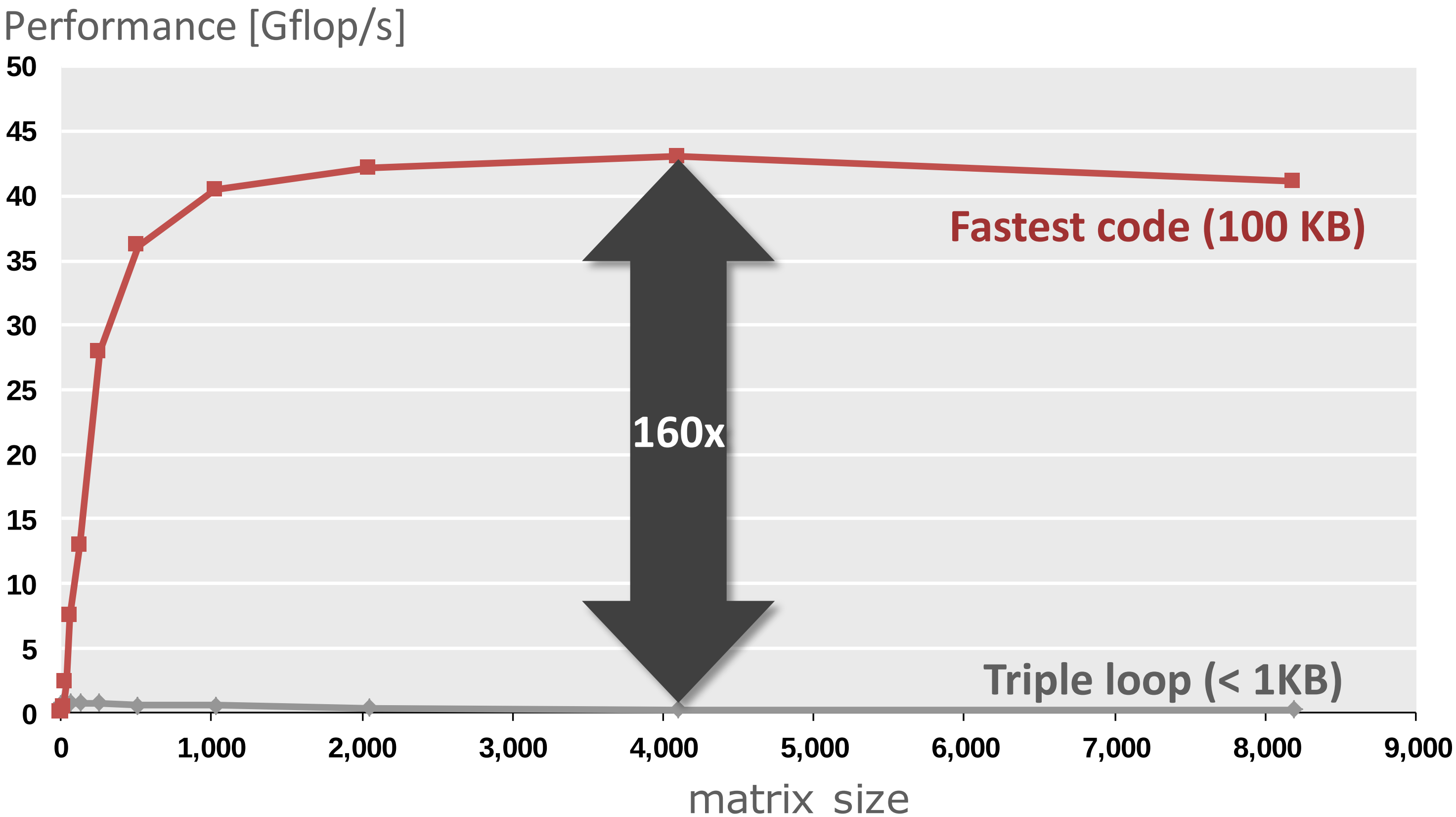


- Vendor compiler, best flags
- Roughly same operations count

The Problem: Example 2

Matrix Multiplication (MMM) on 2 x Core 2 Duo 3 GHz



- Vendor compiler, best flags
- Exact same operations count ($2n^3$)

Model predictive control

Eigenvalues

LU factorization

Optimal binary search organization

Image color conversions

Image geometry transformations

Enclosing ball of points

Metropolis algorithm, Monte Carlo

Seam carving

SURF feature detection

Submodular function optimization

Graph cuts, Edmond-Karps Algorithm

Gaussian filter

Black Scholes option pricing

Disparity map refinement

Singular-value decomposition

Mean shift algorithm for segmentation

Stencil computations

Displacement based algorithms

Motion estimation

Multiresolution classifier

Kalman filter

Object detection

IIR filters

Arithmetic for large numbers

Optimal binary search organization

Software defined radio

Shortest path problem

Feature set for biomedical imaging

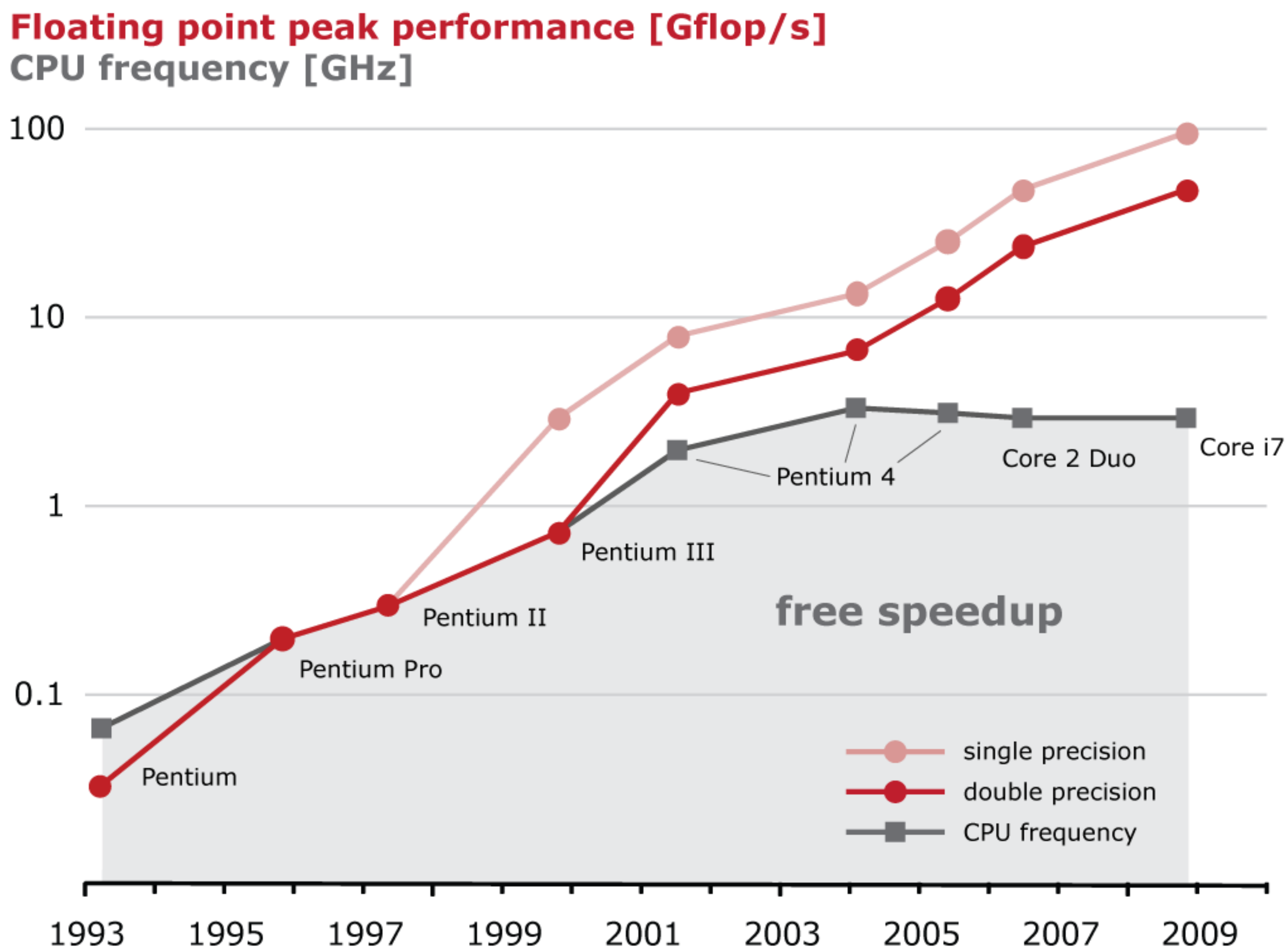
Biometrics identification

“Theorem:”

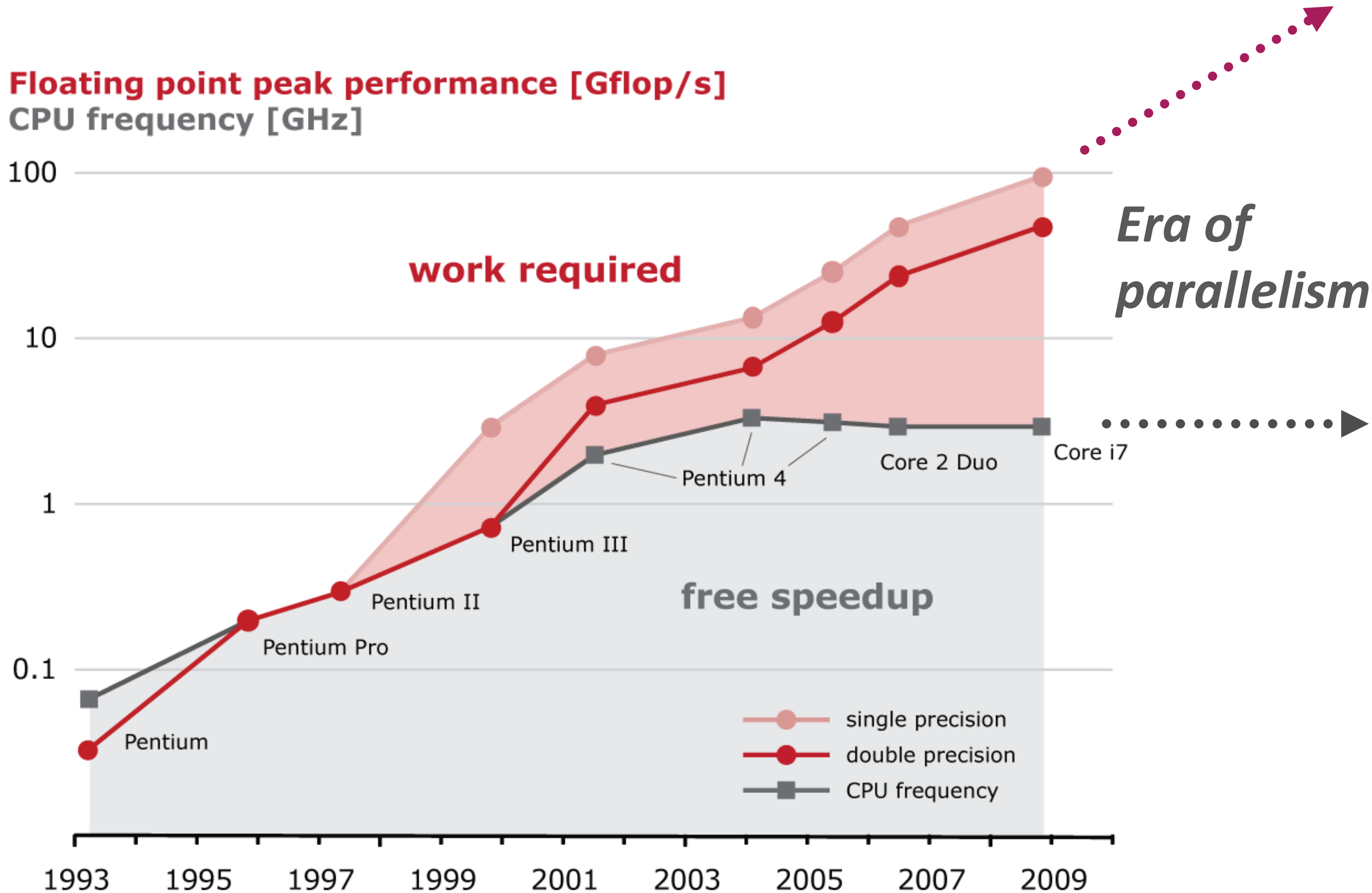
Let f be a mathematical function to be implemented on a state-of-the-art processor. Then

$$\frac{\textit{Performance of optimal implementation of } f}{\textit{Performance of straightforward implementation of } f} \approx \textit{10–100}$$

Evolution of Processors (Intel)

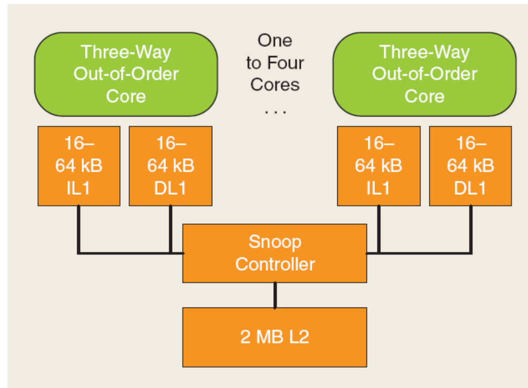


Evolution of Processors (Intel)

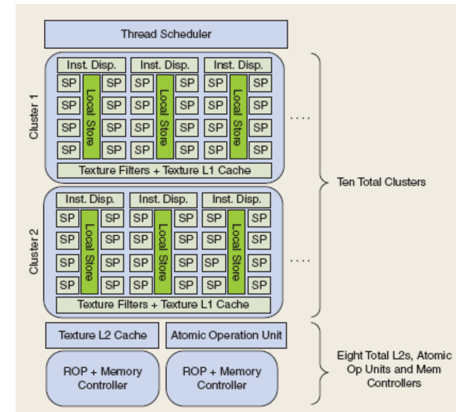


And There Will Be Variety …

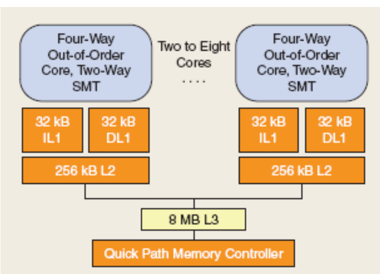
Arm Cortex A9



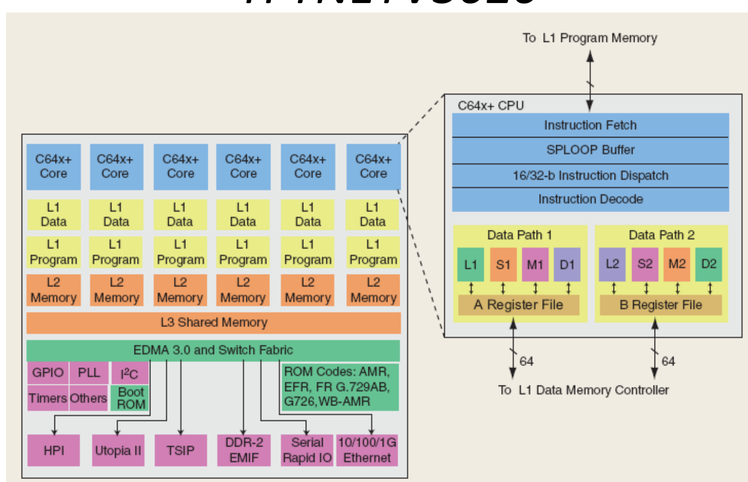
Nvidia G200



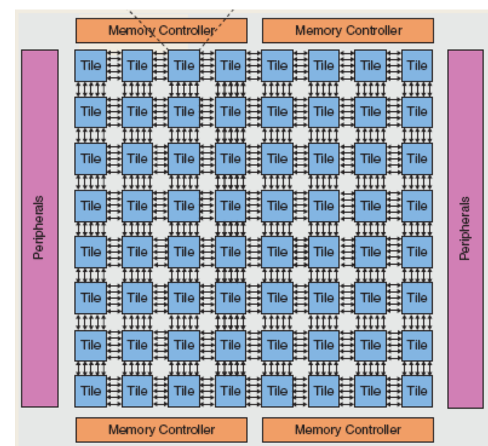
Core i7



TI TNETV3020



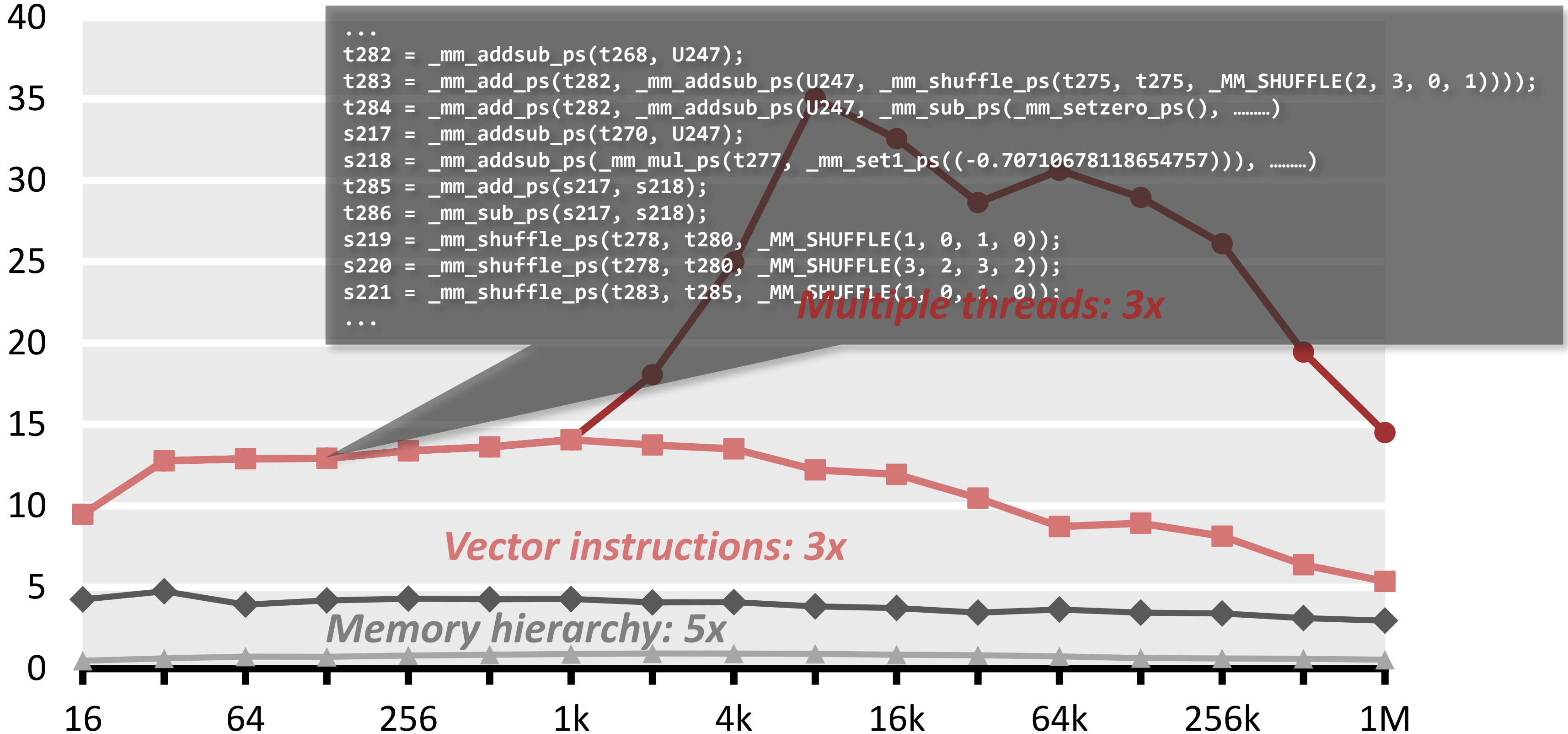
Tilera Tile64



Source: IEEE SP Magazine, Vol. 26, November 2009

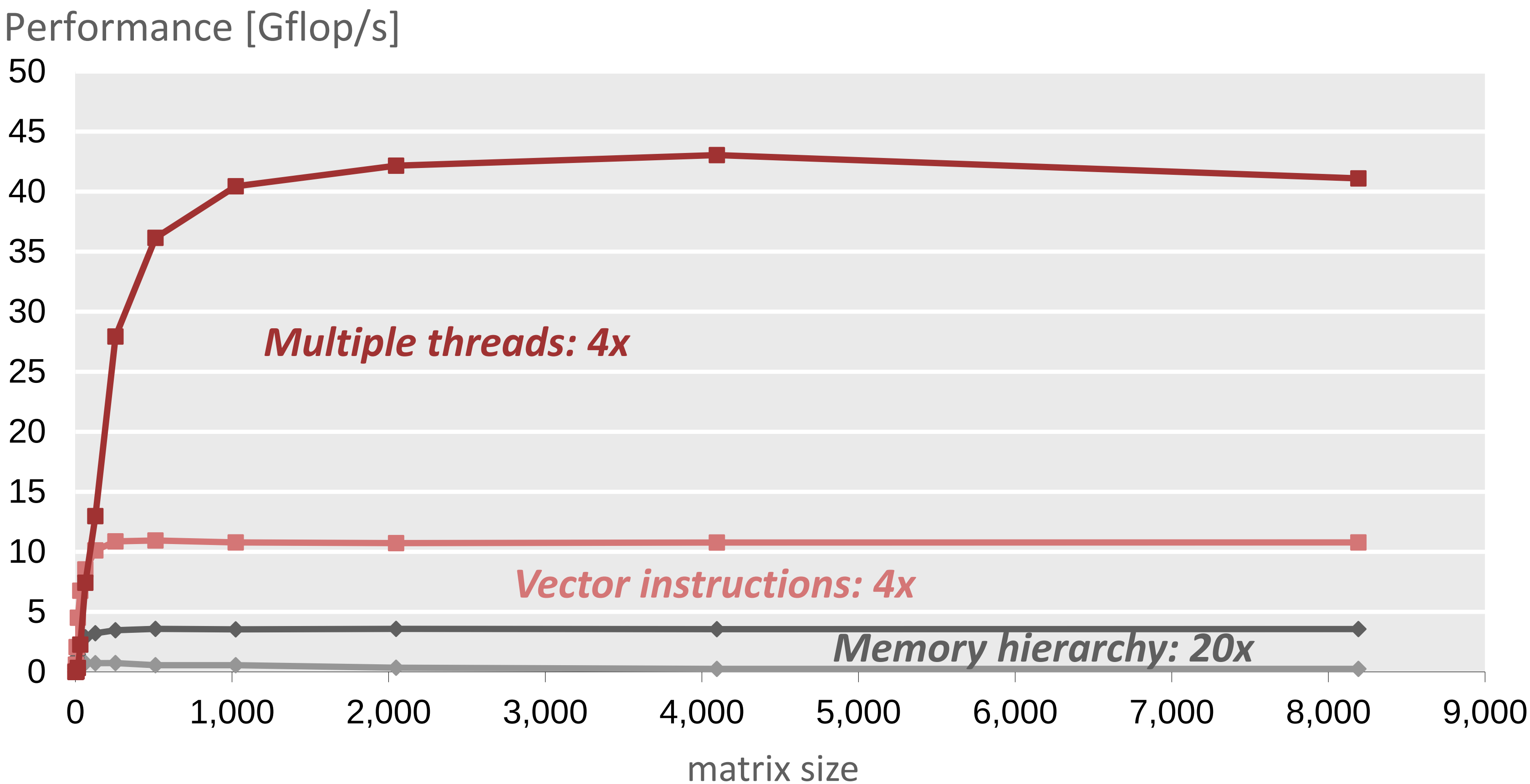
DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)

Performance [Gflop/s]



- Compiler doesn't do the job
- Doing by hand: ***nightmare***

Matrix-Matrix Multiplication (MMM) on 2 x Core 2 Duo 3 GHz



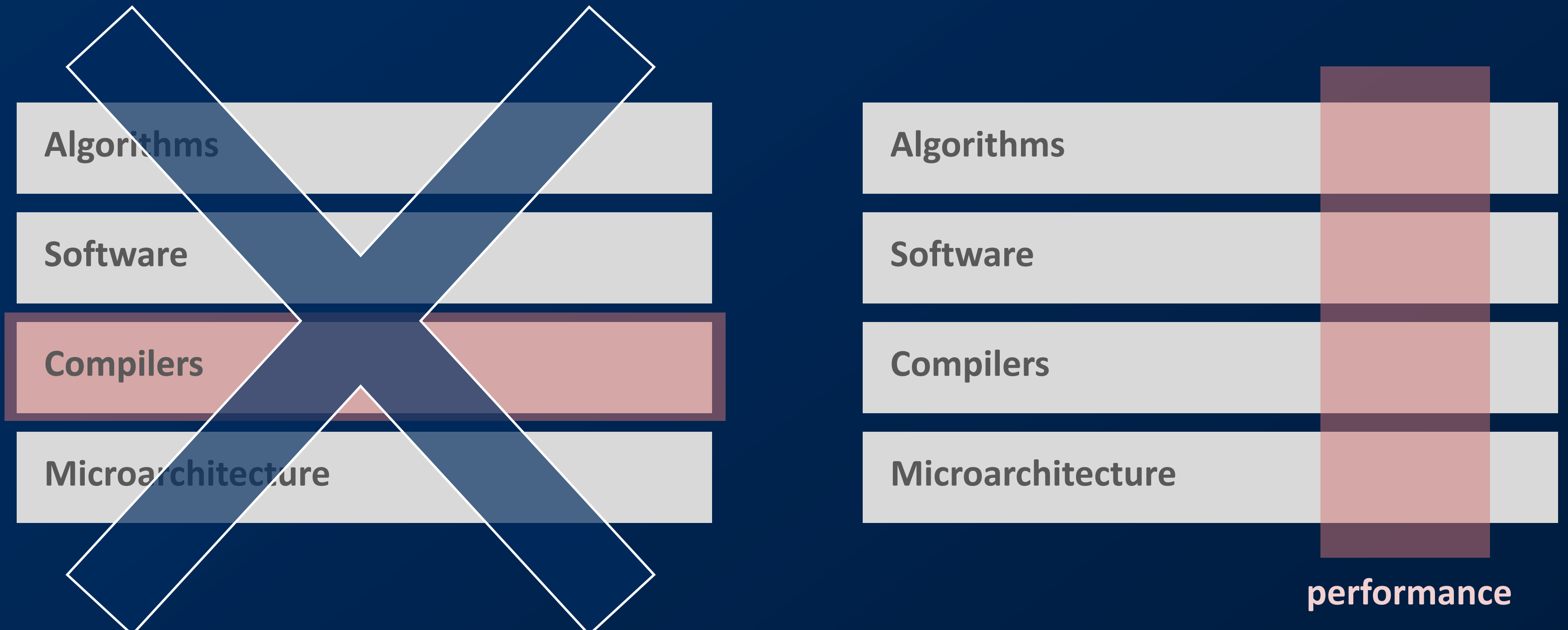
- Compiler doesn't do the job
- Doing by hand: ***nightmare***

Summary and Facts I

- **Implementations with same operations count can have vastly different performance (up to 100x and more)**
  - A cache miss can be 100x more expensive than an operation
  - Vector instructions
  - Multiple cores = processors on one die
- **Minimizing operations count ≠ maximizing performance**
- **End of free speed-up for legacy code**
  - Future performance gains through increasing parallelism

Summary and Facts II

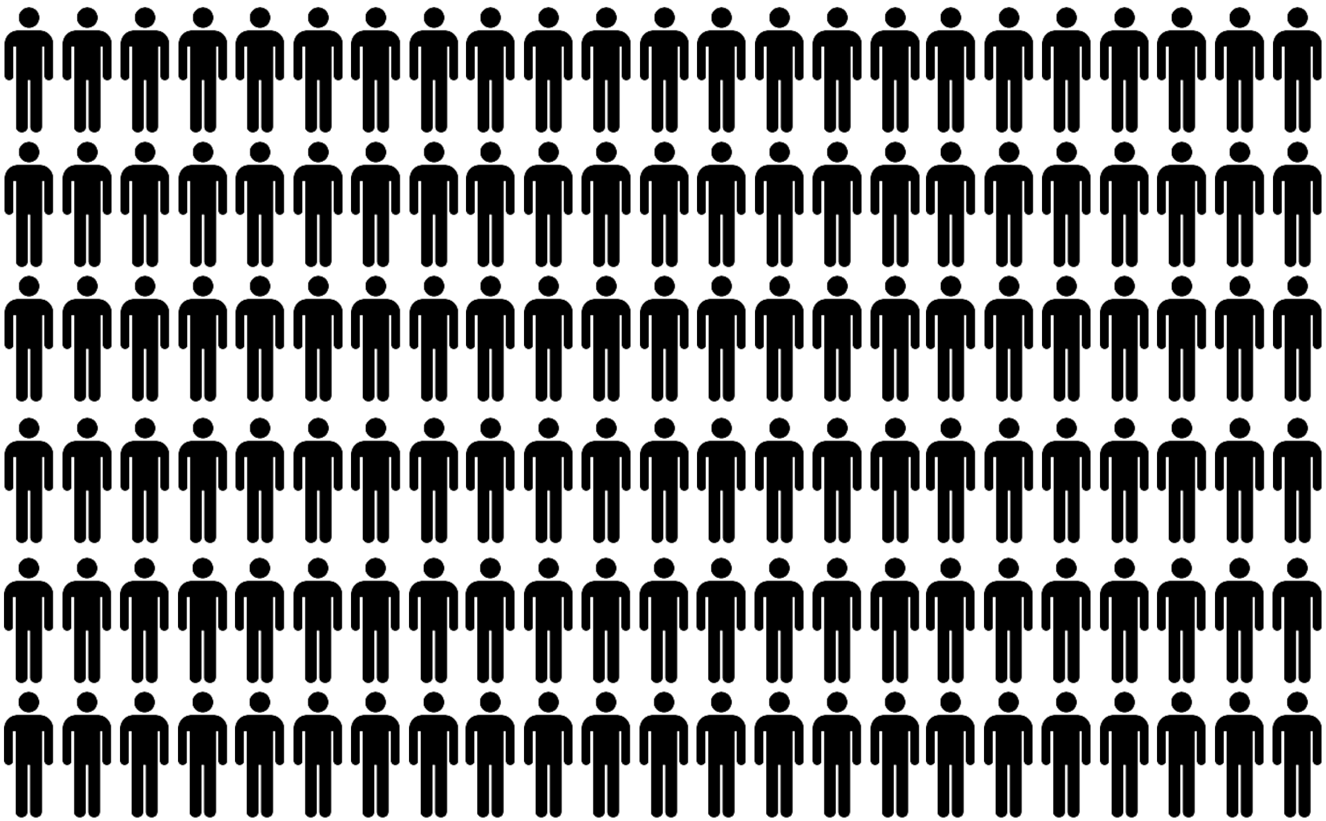
- **It is very difficult to write the fastest code**
  - Tuning for memory hierarchy
  - Vector instructions
  - Efficient parallelization (multiple threads)
  - Requires expert knowledge in algorithms, coding, and architecture
- **Fast code can be large**
  - Can violate "good" software engineering practices
- **Compilers often can't do the job**
  - Often intricate changes in the algorithm required
  - Parallelization/vectorization still unsolved
- **Highest performance is in general non-portable**



Performance is different than other software quality features

Performance/Productivity **Challenge**

Current Solution



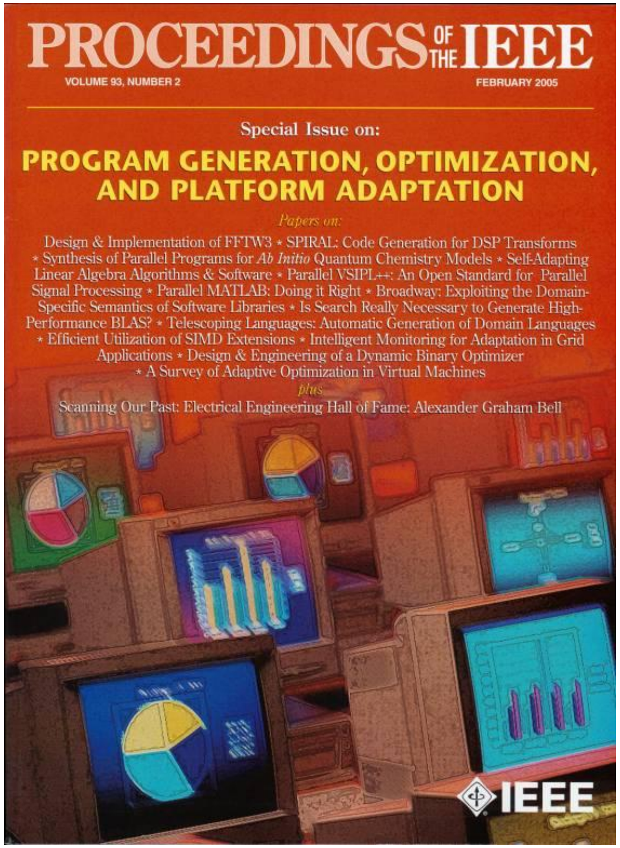
- ***Legions*** of programmers implement and optimize the ***same*** functionality for ***every*** platform and ***whenever*** a new platform comes out.

Better Solution: Autotuning

- **Automate (parts of) the implementation or optimization**

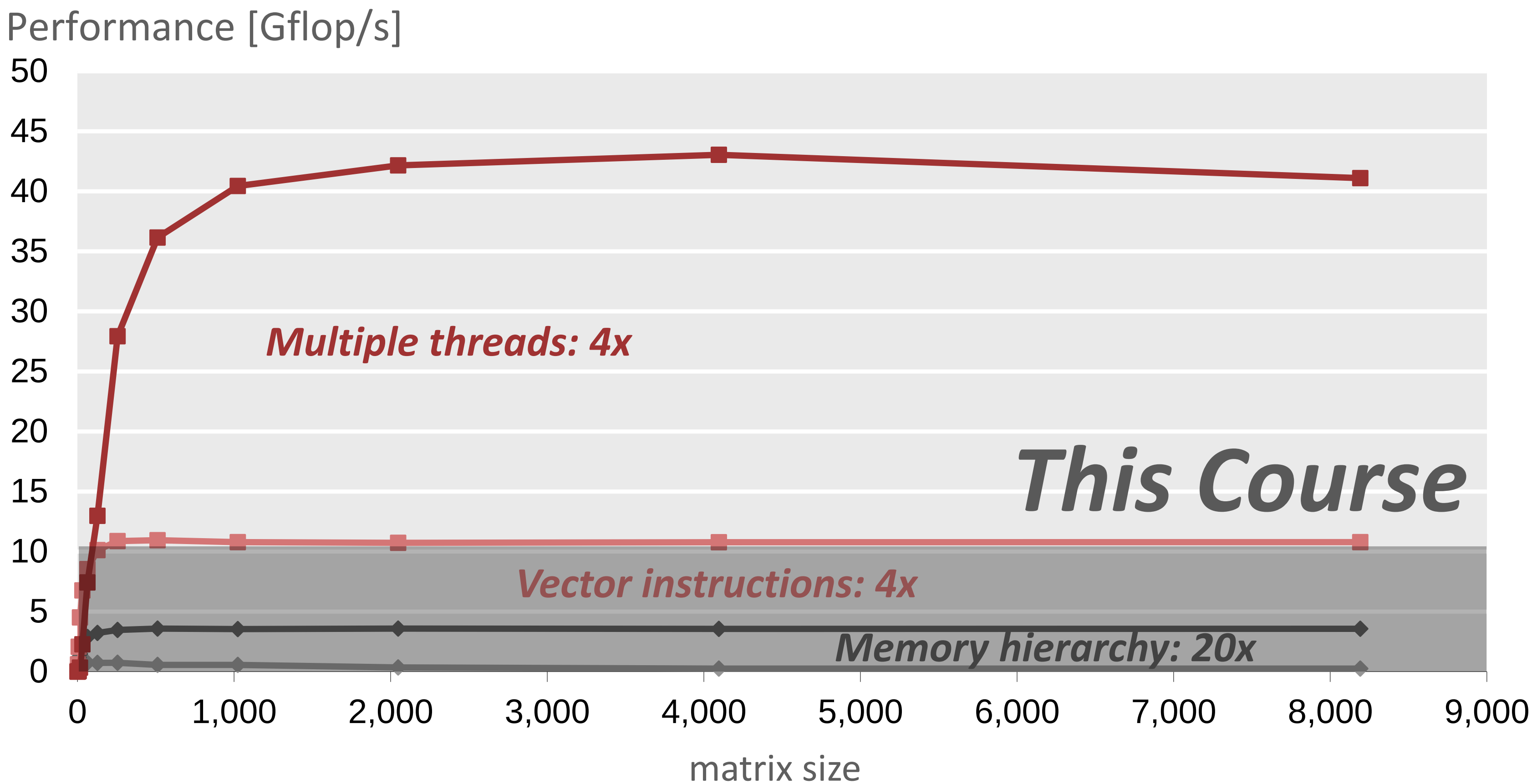
- **Research efforts**
  - Linear algebra: ***Phipac/ATLAS***, LAPACK, ***Sparsity/Bebop/OSKI***, Flame
  - Tensor computations
  - PDE/finite elements: Fenics
  - Adaptive sorting
  - ***Fourier transform: FFTW***
  - ***Linear transforms: Spiral***
  - …others
  - New compiler techniques

Promising new area but much more work needed …

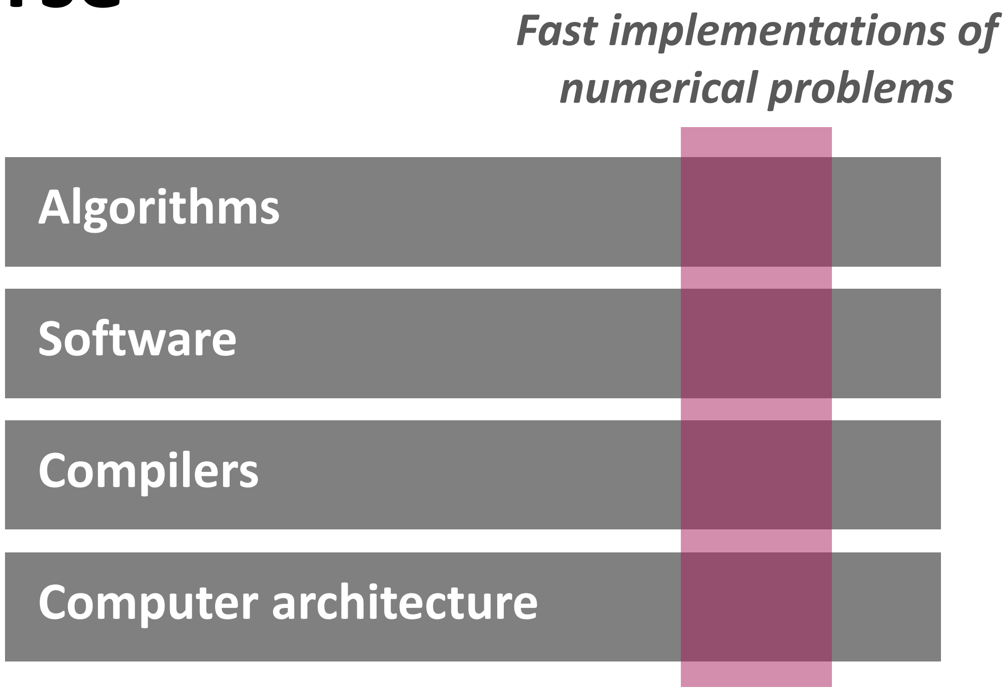


Proceedings of the IEEE special issue, Feb. 2005

Matrix-Matrix Multiplication (MMM) on 2 x Core 2 Duo 3 GHz



This Course



- **Obtain an understanding of performance (runtime)**
- **Learn how to write *fast code* for numerical problems**
  - Focus: Memory hierarchy and vector instructions
  - Principles studied using important examples
  - *Applied in homeworks and a semester-long research project*
- **Learn about autotuning**

Today

- Motivation for this course
- **Organization of this course**

About this Course

- **Team**
  - Me
  - TA: Georg Ofenbeck

- **Office hours: to be determined**
- **Course website has *ALL* information**
- **Questions, finding project partners etc.: Forum (to be set up)**

About this Course (cont'd)

- **Requirements**
  - solid C programming skills
  - matrix algebra
  - Master student or above
- **Grading**
  - 40% research project
  - 20% midterm exam
  - 40% homework
- **Friday slot**
  - Gives you scheduled time to work together
  - Occasionally I will move lecture there

Research Project

- **Team up in pairs**
- ***Topic:*** **Very fast implementation of a numerical problem**
- ***Until March 7th:***
  - *find a project partner*
  - suggest to me a problem or I give you a problem
    Tip: pick something from your research or that you are interested in
- **Show "milestones" during semester**
- **One-on-one meetings**
- **Write 6 page standard conference paper (template will be provided)**
- **Give short presentation end of semester**
- **Submit final code (early semester break)**

Midterm Exam

- **Covers first part of course**

- ***There is no final exam***

Homework

- **Done individually**
- **Exercises on algorithm/performance analysis**
- **Implementation exercises**
  - Concrete numerical problems
  - Study the effect of program optimizations, use of compilers, use of special instructions, etc. (Writing C code + creating runtime/performance plots)
  - Some templates will be provided
  - ***Does everybody have access to an Intel processor?***
- **Homework is scheduled to leave time for research project**
- **Small part of homework grade for neatness**
- **Late homework policy:**
  - ***No deadline extensions***, but
  - 3 late days for the entire semester (at most 2 for one homework)

Academic Integrity

- **Zero tolerance cheating policy (cheat = fail + being reported)**
- **Homeworks**
  - All single-student
  - Don’t look at other students code
  - Don’t copy code from anywhere
  - Ok to discuss things – but then you have to do it alone
- **Code may be checked with tools**
- ***Don’t do copy-paste***
  - code
  - ANY text
  - pictures

Background Material

- **See course website**
- **Chapter 5 in:**
  ***Computer Systems: A Programmer's Perspective*, 2nd edition**
  Randal E. Bryant and David R. O'Hallaron
  (several ones are in the library)
  web: http://csapp.cs.cmu.edu/
- **Prior versions of this course: see website**
- **I post all slides, notes, etc. on the course website**

Class Participation

- **I'll start on time**
- **It is important to attend**
  - Most things I'll teach are not in books
  - I'll use part slides part blackboard
- **Do ask questions**
- **I will provide some anonymous feedback mechanism (maybe after 4 weeks or so)**
- ***I do like to see you in office hours!***