#### **How to Write Fast Numerical Code**

Spring 2013

Lecture: Benchmarking, Compiler Limitations

Instructor: Markus Püschel

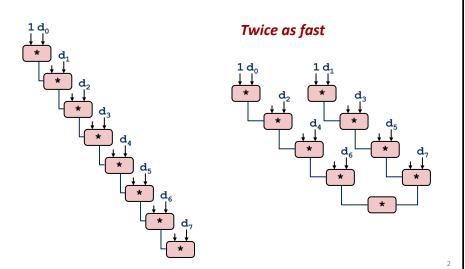
TA: Georg Ofenbeck & Daniele Spampinato

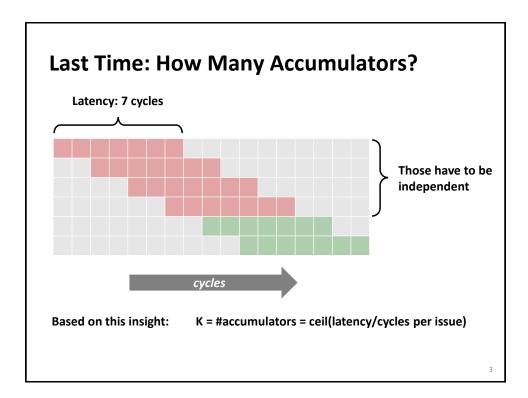
ETH

Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

# **Last Time: ILP**

Latency/throughput (Pentium 4 fp mult: 7/2)





# **Compiler Limitations**

```
void combine4(vec_ptr v, data_t *dest)
{
  int i;
  int length = vec_length(v);
  data_t *d = get_vec_start(v);
  data_t t = IDENT;
  for (i = 0; i < length; i++)
    t = t OP d[i];
  *dest = t;
}</pre>
```

```
void unroll2_sa(vec_ptr v, data_t *dest)
{
    int length = vec_length(v);
    int limit = length-1;
    data_t *d = get_vec_start(v);
    data_t x0 = IDENT;
    data_t x1 = IDENT;
    int i;

/* Combine 2 elements at a time */
    for (i = 0; i < limit; i+=2)
        x0 = x0 OP d[i];
        x1 = x1 OP d[i+1];

/* Finish any remaining elements */
    for (j i < length; i++)
        x0 = x0 OP d[i];
    *dest = x0 OP x1;
}</pre>
```

- Associativity law does not hold for floats: illegal transformation
- No good way of handling choices (e.g., number of accumulators)
- More examples of limitations today

#### **Today**

Measuring performance & benchmarking

Section 3.2 in the tutorial

http://spiral.ece.cmu.edu:8080/pub-spiral/abstract.jsp?id=100

- Optimizing compilers and optimization blockers
  - Overview
  - Removing unnecessary procedure calls
  - Code motion
  - Strength reduction
  - Sharing of common subexpressions
  - Optimization blocker: Procedure calls
  - Optimization blocker: Memory aliasing
  - Summary

Chapter 5 in **Computer Systems: A Programmer's Perspective**, 2<sup>nd</sup> edition, Randal E. Bryant and David R. O'Hallaron, Addison Wesley 2010

Part of these slides are adapted from the course associated with this book

5

#### **Benchmarking**

- First: Validate/test your code!
- Measure runtime (in [s] or [cycles]) for a set of relevant input sizes
  - seconds: actual runtime
  - cycles: abstracts from CPU frequency
- Usually: Compute and show performance (in [flop/s] or [flop/cycle])
- Careful: Better performance ≠ better runtime (why?)
  - Op count could differ
  - Never show in one plot performance of two algorithms with substantially different op count

#### **How to Measure Runtime?**

- C clock()
  - process specific, low resolution, very portable
- gettimeofday
  - measures wall clock time, higher resolution, somewhat portable
- Performance counter (e.g., TSC on Intel)
  - measures cycles (i.e., also wall clock time), highest resolution, not portable
- Careful:
  - measure only what you want to measure
  - ensure proper machine state
     (e.g., cold or warm cache = input data is or is not in cache)
  - measure enough repetitions
  - check how reproducible; if not reproducible: fix it
- Getting proper measurements is not easy at all!

7

#### **Example: Timing MMM**

■ Assume MMM (A,B,C,n) computes

C = C + AB, A,B,C are nxn matrices

```
double time_MMM(int n)
{ // allocate
    double *A=(double*)malloc(n*n*sizeof(double));
    double *B=(double*)malloc(n*n*sizeof(double));
    double *C=(double*)malloc(n*n*sizeof(double));

// initialize
    for (int i = 0; i < n*n; i++){
        A[i] = B[i] = C[i] = 0.0;
    }

init_MMM(A,B,C,n); // if needed

// warm up cache (for warm cache timing)

MMM(A,B,C,n);

// time
ReadTime(t0);
for (int i = 0; i < TIMING_REPETITIONS; i++)
        MMM(A,B,C,n);
ReadTime(t1);

// compute runtime
return (double)((t1-t0)/TIMING_REPETITIONS);
}</pre>
```

ŏ

#### **Problems with Timing**

- Too few iterations: inaccurate non-reproducible timing
- Too many iterations: system events interfere
- Machine is under load: produces side effects
- Multiple timings performed on the same machine
- Bad data alignment of input/output vectors:
  - align to multiples of cache line (on Core: address is divisible by 64)
  - sometimes aligning to page boundaries (address divisible by 4096) makes sense
- Machine was not rebooted for a long time: state of operating system causes problems
- Computation is input data dependent: choose representative input data
- Computation is inplace and data grows until an exception is triggered (computation is done with NaNs)
- You work on a computer that has dynamic frequency scaling (e.g., turbo boost)
- Always check whether timings make sense, are reproducible

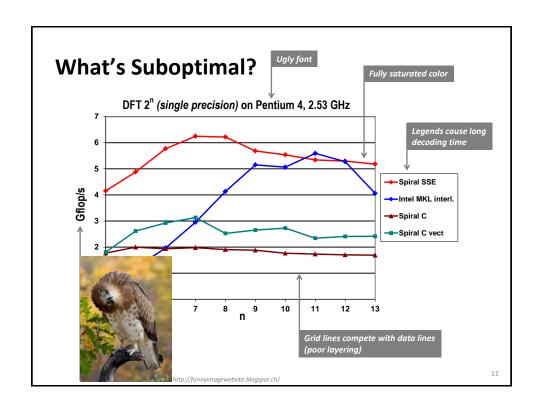
0

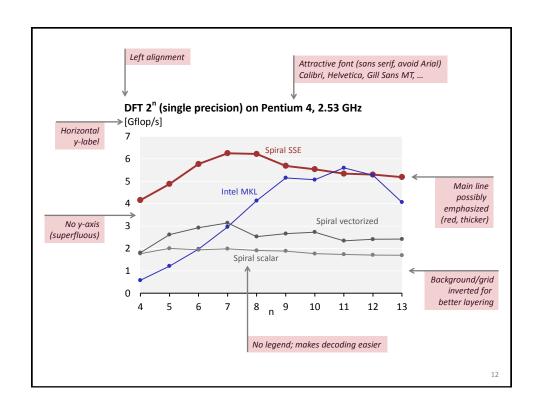
#### **Benchmarks in Writing**

- Specify experimental setup
  - platform
  - compiler and version
  - compiler flags used
- Plot: Very readable
  - Title, x-label, y-label should be there
  - Fonts large enough
  - Enough contrast (no yellow on white please)
  - Proper number format

**No:** 13.254687; **yes:** 13.25 **No:** 2.0345e-05 s; **yes:** 20.3 μs

No: 100000 B; maybe: 100,000 B; yes: 100 KB





# **Today**

- Measuring performance & benchmarking
- Optimizing compilers and optimization blockers
  - Overview
  - Removing unnecessary procedure calls
  - Code motion
  - Strength reduction
  - Sharing of common subexpressions
  - Optimization blocker: Procedure calls
  - Optimization blocker: Memory aliasing
  - Summary

1

# **Optimizing Compilers**



- Always use optimization flags:
  - gcc: default is no optimization (-00)!
  - icc: some optimization is turned on
- Good choices for gcc/icc: -O2, -O3, -march=xxx, -mSSE3, -m64
  - Read in manual what they do
  - Try to understand the differences
- Try different flags and maybe different compilers

# **Example (On Core 2 Duo)**

```
double a[4][4];
double b[4][4];
double c[4][4];

/* Multiply 4 x 4 matrices c = a*b + c */
void mmm(double *a, double *b, double *c) {
  int i, j, k;

for (i = 0; i < 4; i++)
  for (j = 0; j < 4; j++)
  for (k = 0; k < 4; k++)
      c[i*4+j] += a[i*4 + k]*b[k*4 + j];
}</pre>
```

Compiled without flags:

~1300 cycles

■ Compiled with -O3 -m64 -march=... -fno-tree-vectorize ∠ ~150 cycles

15

Prevents use of SSE

MMX:					
Multimedia extension	Intel x	86	Processors		
SSE:	x86-16				
Streaming SIMD extension		X80-10	8086		
ou canning only extension			200		
AVX:			286		
Advanced vector extensions		x86-32	386		
			486		
			Pentium		
		MMX	Pentium MMX		
		SSE	Pentium III		
		SSE2	Pentium 4	time	
		3322	· citiaiii ·		
		SSE3	Pentium 4E		
	x86-	64 / em64t	Pentium 4F		
	AGG !	047 6111046			
			Core 2 Duo		
		SSE4	Penryn		
			Core i7 (Nehalem)		
		AVX	Sandy Bridge	V	
		, and a	cana, in age		
Use architecture flags					
				16	

# **Optimizing Compilers**

- Compilers are *good* at: mapping program to machine
  - register allocation
  - code selection and ordering (instruction scheduling)
  - dead code elimination
  - eliminating minor inefficiencies
- Compilers are not good at: algorithmic restructuring
  - for example to increase ILP, locality, etc.
  - cannot deal with choices
- Compilers are not good at: overcoming "optimization blockers"
  - potential memory aliasing
  - potential procedure side-effects

17

## **Limitations of Optimizing Compilers**

- If in doubt, the compiler is conservative
- Operate under fundamental constraints
  - Must not change program behavior under any possible condition
  - Often prevents it from making optimizations when would only affect behavior under pathological conditions
- Most analysis is performed only within procedures
  - Whole-program analysis is too expensive in most cases
- Most analysis is based only on static information
  - Compiler has difficulty anticipating run-time inputs
  - Not good at evaluating or dealing with choices

# Organization

- Instruction level parallelism (ILP): an example
- Optimizing compilers and optimization blockers
  - Overview
  - Removing unnecessary procedure calls
  - Code motion
  - Strength reduction
  - Sharing of common subexpressions
  - Optimization blocker: Procedure calls
  - Optimization blocker: Memory aliasing
  - Summary

19

# **Example: Data Type for Vectors**

```
/* data structure for vectors */
typedef struct{
  int len;
  double *data;
} vec;
len     0 1     len
  data
```

```
/* retrieve vector element and store at val */
int get_vec_element(vec *v, int idx, double *val)
{
   if (idx < 0 || idx >= v->len)
      return 0;
   *val = v->data[idx];
   return 1;
}
```

# **Example: Summing Vector Elements**

```
/* retrieve vector element and store at val */
int get_vec_element(vec *v, int idx, double *val)
{
  if (idx < 0 || idx >= v->len)
    return 0;
  *val = v->data[idx];
  return 1;
}
```

```
/* sum elements of vector */
double sum_elements(vec *v, double *res)
{
   int i;
   n = vec_length(v);
   *res = 0.0;
   double t;

   for (i = 0; i < n; i++) {
      get_vec_element(v, i, &t);
      *res += t;
   }
   return res;
}</pre>
```

#### Overhead for every fp +:

- One fct call
- One <
- One >=
- One ||
- One memory variable access

#### Slowdown:

probably 10x or more

21

# **Removing Procedure Call**

```
/* sum elements of vector */
double sum_elements(vec *v, double *res)
 n = vec_length(v);
 *res = 0.0;
 double t;
 for (i = 0; i < n; i++) {</pre>
   get_vec_element(v, i, &t);
    *res += t;
 return res;
/* sum elements of vector */
double sum_elements(vec *v, double *res)
 int i;
 n = vec_length(v);
 *res = 0.0;
 double *data = get_vec_start(v);
 for (i = 0; i < n; i++)
   *res += data[i];
 return res;
```

## **Removing Procedure Calls**

- Procedure calls can be very expensive
- Bound checking can be very expensive
- Abstract data types can easily lead to inefficiencies
  - Usually avoided in superfast numerical library functions
- Watch your innermost loop!
- Get a feel for overhead versus actual computation being performed

23

# Organization

- Instruction level parallelism (ILP): an example
- Optimizing compilers and optimization blockers
  - Overview
  - Removing unnecessary procedure calls
  - Code motion
  - Strength reduction
  - Sharing of common subexpressions
  - Optimization blocker: Procedure calls
  - Optimization blocker: Memory aliasing
  - Summary

Compiler is likely to do that

#### **Code Motion**

- Reduce frequency with which computation is performed
  - If it will always produce same result
  - Especially moving code out of loop (loop-invariant code motion)
- Sometimes also called precomputation

```
void set_row(double *a, double *b,
    int i, int n)
{
    int j;
    for (j = 0; j < n; j++)
        a[n*i+j] = b[j];
}</pre>
```



```
int j;
int ni = n*i;
for (j = 0; j < n; j++)
   a[ni+j] = b[j];</pre>
```

-

# Organization

- Instruction level parallelism (ILP): an example
- Optimizing compilers and optimization blockers
  - Overview
  - Removing unnecessary procedure calls
  - Code motion
  - Strength reduction
  - Sharing of common subexpressions
  - Optimization blocker: Procedure calls
  - Optimization blocker: Memory aliasing
  - Summary

Compiler is likely to do that

# **Strength Reduction**

- Replace costly operation with simpler one
- Example: Shift/add instead of multiply or divide 16\*x → x << 4</p>
  - Utility machine dependent
- Example: Recognize sequence of products

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    a[n*i + j] = b[j];

int ni = 0;
  for (i = 0; i < n; i++) {
    for (j = 0; j < n; j++)
        a[ni + j] = b[j];
    ni += n;
}</pre>
```

27

# Organization

- Instruction level parallelism (ILP): an example
- Optimizing compilers and optimization blockers
  - Overview
  - Removing unnecessary procedure calls
  - Code motion
  - Strength reduction
  - Sharing of common subexpressions
  - Optimization blocker: Procedure calls
  - Optimization blocker: Memory aliasing
  - Summary

Compiler is likely to do that

## **Share Common Subexpressions**

- Reuse portions of expressions
- Compilers often not very sophisticated in exploiting arithmetic properties

#### 3 mults: i\*n, (i-1)\*n, (i+1)\*n

```
/* Sum neighbors of i,j */
up = val[(i-1)*n + j ];
down = val[(i+1)*n + j ];
left = val[i*n + j-1];
right = val[i*n + j+1];
sum = up + down + left + right;
```

#### 1 mult: i\*n

20

# Organization

- Instruction level parallelism (ILP): an example
- Optimizing compilers and optimization blockers
  - Overview
  - Removing unnecessary procedure calls
  - Code motion
  - Strength reduction
  - Sharing of common subexpressions
  - Optimization blocker: Procedure calls
  - Optimization blocker: Memory aliasing
  - Summary

# **Optimization Blocker #1: Procedure Calls**

Procedure to convert string to lower case

```
void lower(char *s)
{
   int i;
   for (i = 0; i < strlen(s); i++)
      if (s[i] >= 'A' && s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}

/* My version of strlen */
size_t strlen(const char *s)
{
   size_t length = 0;
   while (*s != '\0') {
      s++;
      length++;
   }
   return length;
}</pre>
O(n²) instead of O(n)
```

# **Improving Performance**

```
void lower(char *s)
{
  int i;
  for (i = 0; i < strlen(s); i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
        s[i] -= ('A' - 'a');
}

void lower(char *s)
{
  int i;
  int len = strlen(s);
  for (i = 0; i < len; i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
        s[i] -= ('A' - 'a');
}</pre>
```

- Move call to strlen outside of loop
- Since result does not change from one iteration to another
- Form of code motion/precomputation

# **Optimization Blocker: Procedure Calls**

- Why couldn't compiler move strlen out of inner loop?
  - Procedure may have side effects
- Compiler usually treats procedure call as a black box that cannot be analyzed
  - Consequence: conservative in optimizations
- In this case the compiler may actually do it if strlen is recognized as built-in function whose properties are known

33

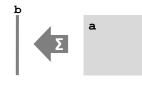
# Organization

- Instruction level parallelism (ILP): an example
- Optimizing compilers and optimization blockers
  - Overview
  - Removing unnecessary procedure calls
  - Code motion
  - Strength reduction
  - Sharing of common subexpressions
  - Optimization blocker: Procedure calls
  - Optimization blocker: Memory aliasing
  - Summary

## **Optimization Blocker: Memory Aliasing**

```
/* Sums rows of n x n matrix a
    and stores in vector b */
void sum_rows1(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}</pre>
```



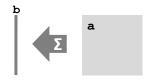
Code updates b[i] (= memory access) on every iteration

35

# **Optimization Blocker: Memory Aliasing**

```
/* Sums rows of n x n matrix a
    and stores in vector b */
void sum_rows1(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
        b[i] += a[i*n + j];
    }
}</pre>
```



```
/* Sums rows of n x n matrix a
    and stores in vector b */
void sum_rows2(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        double val = 0;
        for (j = 0; j < n; j++)
            val += a[i*n + j];
        b[i] = val;
}</pre>
```

Does compiler optimize this? No! Why?

## **Reason: Possible Memory Aliasing**

- If memory is accessed, compiler assumes the possibility of side effects
- Example:

```
/* Sums rows of n x n matrix a
    and stores in vector b */
void sum_rows1(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}</pre>
```

# double A[9] = { 0, 1, 2, 4, 8, 16}, 32, 64, 128}; double B[3] = A+3; sum\_rows1(A, B, 3);

#### Value of B:

```
init: [4, 8, 16]
i = 0: [3, 8, 16]
i = 1: [3, 22, 16]
i = 2: [3, 22, 224]
```

37

# **Removing Aliasing**

```
/* Sums rows of n x n matrix a
    and stores in vector b */
void sum_rows2(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        double val = 0;
        for (j = 0; j < n; j++)
            val += a[i*n + j];
        b[i] = val;
    }
}</pre>
```

- Scalar replacement:
  - Copy array elements that are reused into temporary variables
  - Perform computation on those variables
  - Enables register allocation and instruction scheduling
  - Assumes no memory aliasing (otherwise possibly incorrect)

## **Optimization Blocker: Memory Aliasing**

■ Memory aliasing:

Two different memory references write to the same location

- Easy to have happen in C
  - Since allowed to do address arithmetic
  - Direct access to storage structures
- Hard to analyze = compiler cannot figure it out
  - Hence is conservative
- Solution: Scalar replacement in innermost loop
  - Copy memory variables that are reused into local variables
  - Basic scheme:

```
Load: t1 = a[i], t2 = b[i+1], ....
Compute: t4 = t1 * t2; ....
Store: a[i] = t12, b[i+1] = t7, ...
```

39

## **Example: MMM**

Which array elements are reused? All of them! But how to take advantage?

```
void mmm(double const * A, double const * B, double * C, size_t N) {
for( size_t k6 = 0; k6 < N; k6++ )
for( size_t i5 = 0; i5 < N; i5++ )
for( size_t j7 = 0; j7 < N; j7++ )
C[N*i5 + j7] = C[N*i5 + j7] + A[N*i5 + k6] * B[j7 + N*k6]; }</pre>
```

tile each loop (= blocking MMM)

```
void mmm(double const * A, double const * B, double * C, size_t N) {
for( size_t i21 = 0; i21 < N; i21+=2 )
   for( size_t j23 = 0; j23 < N; j23+=2 )
    for( size_t k22 = 0; k22 < N; k22+=2 )
    for( size_t k25 = 0; kk25 < 2; kk25++ )
    for( size_t i24 = 0; ii24 < 2; ii24++ )
    for( size_t jj26 = 0; jj26 < 2; jj26++ )
        C[N*121 + N*ii24 + j23 + jj26] = C[N*i21 + N*ii24 + j23 + jj26] +
        A[N*i21 + N*ii24 + k22 + kk25] * B[j23 + jj26 + N*k22 + N*kk25]; }</pre>
```

unroll inner three loops

unroll inner three loops

Now the reuse becomes apparent (every elements used twice)

41

Now the reuse becomes apparent (every elements used twice)

unroll inner three loops

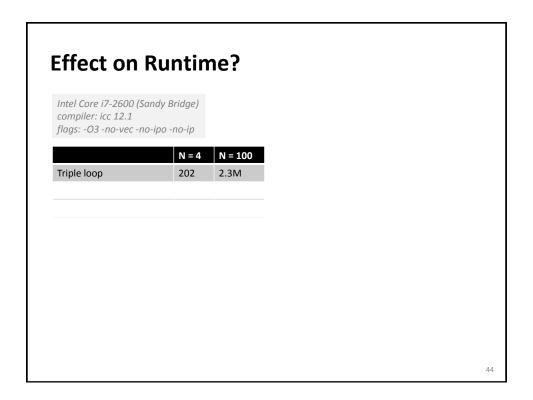
void mmm(double const \* A, double const \* B, double \* C, size\_t N) {

for( size\_t i21 = 0; i21 < N; i21+=2 )
 for( size\_t j23 = 0; j23 < N; j23+=2 )
 for( size\_t j23 = 0; j23 < N; j23+=2 )
 for( size\_t k22 = 0; k22 < N; k22+=2 ) {
 C[N\*i21 + j23] = C[N\*i21 + j23] + A[N\*i21 + k22] \* B[j23 + N\*k22];
 C[N\*i21 + j23 + 1] = C[N\*i21 + j23 + 1] + A[N\*i21 + k22] \* B[j23 + N\*k22 + 1];
 C[N\*i21 + N + j23] = C[N\*i21 + N + j23] + A[N\*i21 + N + k22] \* B[j23 + N\*k22];
 C[N\*i21 + N + j23 + 1] = C[N\*i21 + N + j23] + A[N\*i21 + N + k22] \* B[j23 + N\*k22 + 1];
 C[N\*i21 + j23] = C[N\*i21 + j23] + A[N\*i21 + k22 + 1] \* B[j23 + N\*k22 + N];
 C[N\*i21 + j23 + 1] = C[N\*i21 + j23 + 1] + A[N\*i21 + k22 + 1] \* B[j23 + N\*k22 + N + 1];
 C[N\*i21 + N + j23] = C[N\*i21 + N + j23] + A[N\*i21 + N + k22 + 1] \* B[j23 + N\*k22 + N];
 C[N\*i21 + N + j23 + 1] =
 C[N\*i21 + N + j23 + 1] + A[N\*i21 + N + k22 + 1] \* B[j23 + N\*k22 + N + 1];
 }
}</pre>

scalar replacement

```
void mmm(double const * A, double const * B, double * C, size_t N) {
    for( size_t i 21 = 0; i21 < N; i21+=2 )
        for( size_t i 22 = 0; i22 < N; i23+=2 )
        for( size_t k 22 = 0; k22 < N; b22+=2 ) {
            double t0_0, t0_1, t0_2, t0_3, t0_4, t0_5, t0_6, t0_7, t0_8, t0_9, t0_10, t0_11, t0_12;

            t0_7 = A[N*121 + k22];
            t0_6 = A[N*121 + k22];
            t0_6 = A[N*121 + k22];
            t0_4 = A[N*121 + N + k22];
            t0_4 = A[N*121 + N + k22];
            t0_4 = A[N*121 + N + k22];
            t0_1 = B[j23 + N*k22] + 1];
            t0_9 = B[j23 + N*k22 + N];
            t0_9 = B[j23 + N*k22 + N];
            t0_9 = B[j23 + N*k22 + N];
            t0_9 = C[N*121 + N + j23];
            t0_1 = C[N*1
```



#### **Effect on Runtime?**

Intel Core i7-2600 (Sandy Bridge) compiler: icc 12.1 flags: -03 -no-vec -no-ipo -no-ip

	N = 4	N = 100
Triple loop	202	2.3M
Six-fold loop	144	2.3M
+ Inner three unrolled	166	2.4M
+ scalar replacement	106	1.6M

45

# **Can Compiler Remove Aliasing?**

```
for (i = 0; i < n; i++)
  a[i] = a[i] + b[i];</pre>
```

Potential aliasing: Can compiler do something about it?

Compiler can insert runtime check:

```
if (a + n < b | | b + n < a)
   /* further optimizations may be possible now */
...
else
   /* aliased case */
...</pre>
```

## **Removing Aliasing With Compiler**

- Globally with compiler flag:
  - -fno-alias, /Oa
  - -fargument-noalias, /Qalias-args- (function arguments only)
- For one loop: pragma

```
void add(float *a, float *b, int n) {
    #pragma ivdep
    for (i = 0; i < n; i++)
        a[i] = a[i] + b[i];
}</pre>
```

■ For specific arrays: restrict (needs compiler flag -restrict, /Qrestrict)

```
void add(float *restrict a, float *restrict b, int n) {
  for (i = 0; i < n; i++)
    a[i] = a[i] + b[i];
}</pre>
```

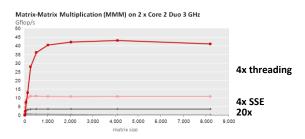
47

# Organization

- Instruction level parallelism (ILP): an example
- Optimizing compilers and optimization blockers
  - Overview
  - Removing unnecessary procedure calls
  - Code motion
  - Strength reduction
  - Sharing of common subexpressions
  - Optimization blocker: Procedure calls
  - Optimization blocker: Memory aliasing
  - Summary

# **Summary**

■ One can easily loose 10x, 100x in runtime or even more



- What matters besides operation count:
  - Coding style (unnecessary procedure calls, unrolling, reordering, ...)
  - Algorithm structure (instruction level parallelism, locality, ...)
  - Data representation (complicated structs or simple arrays)

49

# **Summary: Optimize at Multiple Levels**

- Algorithm:
  - Evaluate different algorithm choices
  - Restructuring may be needed (ILP, locality)
- Data representations:
  - Careful with overhead of complicated data types
  - Best are arrays
- Procedures:
  - Careful with overhead
  - They are black boxes for the compiler
- Loops:
  - Often need to be restructured (ILP, locality)
  - Unrolling often necessary to enable other optimizations
  - Watch the innermost loop bodies

# **Numerical Functions**

- Use arrays if possible
- Unroll to some extent
  - To make ILP explicit
  - To enable scalar replacement and hence register allocation for variables that are reused