A Unified Framework for Verification Techniques for Object Invariants (Full Paper)

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Abstract. Verification of object-oriented programs relies on object invariants which express consistency criteria of objects. The semantics of object invariants is subtle, mainly because of call-backs, multi-object invariants, and subclassing.
Several verification techniques for object invariants have been proposed. These techniques are complex and differ in restrictions on programs (*e.g.,* which fields can be updated), restrictions on invariants (what an invariant may refer to), use of advanced type systems (such as Universe types or ownership), meaning of invariants (in which execution states are invariants assumed to hold), and proof obligations (when should an invariant be proven). As a result, it is difficult to understand whether/why these techniques are sound and to compare their expressiveness. This general lack of understanding also hampers the development of new approaches.
In this paper, we develop and formalise a unified framework to describe verification techniques for object invariants. We distil seven parameters, which characterise a verification technique, and identify sufficient conditions on these parameters under which a verification technique is sound. We also give an informal argument demonstrating the necessity of these conditions. To illustrate the generality of our framework, we instantiate it with six verification techniques from the literature. We show how our framework facilitates the assessment and comparison of the soundness and expressiveness of these techniques.

1 Introduction

Object invariants play a crucial role in the verification of object-oriented programs, and have been an integral part of all major contract languages such as Eiffel [?], the Java Modeling Language JML [?], and Spec# [?]. Object invariants express consistency criteria for objects, which guarantee their correct working. These criteria range from simple properties of single objects (for instance, that a field is non-null) to complex properties of whole object structures (for instance, the sorting of a tree).

Most of the existing verification techniques expect object invariants to hold in the pre-state and post-state of method executions, often referred to as *visible states* [?]. Invariants may be violated temporarily between visible states. This semantics is illustrated by class C in Fig. 1. The invariant is established by the constructor. It may be assumed in the pre-state of method m. Therefore, the first statement in m's body can be proven not to cause a division-by-zero error. The invariant might temporarily be violated by the subsequent assignment to a, but it is later reestablished by m's last statement; thus, the invariant holds in m's post-state.

```
class C {
  int a, b;
  invariant 0 <= a < b;

  C() { a := 0; b := 3; }

  void m() {
    int k := 100 / (b − a);
    a := a + 3;
    n();
    b := (k + 4) ∗ b;
  }
  void n() { m(); }
}
```

```
class Client {
  C c;
  invariant c.a <= 10;

  /∗ methods omitted ∗/
}
```

```
class D extends C {
  invariant a <= 10;

  /∗ methods omitted ∗/
}
```

Fig. 1. An example (adapted from [**?**]) illustrating the three main challenges for the verification of object invariants.

While the basic idea of object invariants is simple, verification techniques for practical OO-programs face challenges. These challenges are made more daunting by the common expectation that classes should be verified modularly, that is, without knowledge of their clients and subclasses:

Call-backs: Methods that are called while the invariant of an object o is temporarily broken might call back into o and find the object in an inconsistent state. In our example (Fig. 1), during execution of **new** C().m() the assignment to a violates the invariant, and the call-back via n() leads to a division by zero.

Multi-object invariants: When the invariant of an object p depends on the state of another object o, modifications of o potentially violate the invariant of p. In our example, a call o.m might break the invariant of a Client object p where p.c $= o$. Aliasing makes the proof of preservation of p's invariant difficult. In particular, when verifying o, the invariant of p may not be known and, if not, cannot be expected to be preserved.

Subclassing: When the invariant of a subclass D refers to fields declared in the superclass C then methods of C potentially violate D's invariant by assigning to C's fields. In particular, when verifying a class, its subclass invariants are not known in general, and so cannot be expected to be preserved.

A number of verification techniques have been suggested to address some or all of these problems [?,?,?,?,?,?,?,?,?,?]. These techniques share many commonalities, but differ in the following important aspects:

1. *Invariant semantics:* What invariants are expected to hold in which execution states? Some techniques require all invariants to hold in all visible states, whereas others address the multi-object invariant challenge by excluding certain invariants.
2. *Proof obligations:* What is required to be proven? Some techniques require proofs for invariants relating to the current active object whereas others require invariant proofs for all objects in the heap.
3. *Invariant restrictions:* What objects may invariants depend on? Some techniques use unrestricted invariants, whereas others address the subclassing challenge by preventing invariants from referring to inherited fields.
4. *Program restrictions:* What objects may be used as receivers of field updates and method calls? Some techniques permit arbitrary field updates, whereas others simplify verification by allowing updates to fields of the current receiver only.
5. *Type systems:* What syntactic information is used for reasoning? Some techniques are designed for arbitrary programs, whereas others use ownership types to facilitate verification.
6. *Specification languages:* In what syntax are specifications for programs (and methods) written? Different techniques may use different (sometimes customised) logical syntax to express invariants, for example.
7. *Verification logics:* How should proof obligations be discharged? What logical inferences are permitted? If a technique employs a custom-made logic, the modes of reasoning available will not usually be standard.

These differences, together with the fact that most verification techniques are not formally specified, complicate the comparison of verification techniques, and hinder the understanding of why these techniques satisfy claimed properties such as soundness. For these reasons, it is hard to decide which technique to adopt, or to develop new sound techniques.

In this paper, we present a unified framework for verification techniques for object invariants. This framework formalises verification techniques in terms of seven parameters, which abstract away from differences pertaining to language features (type system, specification language, and logics) and highlight characteristics intrinsic to the techniques, thereby aiding comparisons. Subsets of these parameters describe aspects applicable to all verification techniques; for example, a generic definition of *soundness* is given in terms of two framework parameters, expressiveness is captured by three other parameters.

We concentrate on techniques that require invariants to hold in the pre-state and post-state of a method execution (visible states) while temporary violations between visible states are permitted. These techniques constitute the vast majority of those described in the literature.

Contributions. The contributions of this paper are:

1. We present a unified formalism for object invariant verification techniques.
2. We identify conditions on the framework that guarantee soundness of a verification technique.
3. We separate type system concerns from verification strategy concerns.
4. We show how our framework describes some advanced verification techniques for visible state invariants.
5. We prove soundness for a number of techniques, and, guided by our framework, discover an unsoundness in one technique.

Our framework allows the extraction of comparable data from techniques that were presented using different concepts, terminology and styles. Comparative value judgements concerning the techniques are beyond the scope of our paper.

Outline. Sec. 2 gives an overview of our work, explaining the important concepts. Sec. 3 formalises program and invariant semantics. Sec. 4 describes our framework and defines soundness. Sec. 5 presents sufficient conditions for a verification technique to be sound, and a general soundness theorem, along with an informal argument for the necessity of these conditions under some reasonable additional assumptions. Sec. 6 instantiates our framework with existing verification techniques. Sec. 7 discusses related work.

This paper extends our ECOOP paper [**?**] with full proofs and more explanations and examples.

2 Example and Approach

Example. The following example will be used throughout the paper. Consider a scenario, in which a Person holds an Account, and has a salary. An Account has a balance, an interestRate and an associated DebitCard. We give the code in Fig. 2.

Account's interestRate is required to be zero when the balance is negative (I1). A further invariant (the two can be read as conjuncts of the full invariant for the class) ensures that the DebitCard associated with an account has a consistent reference back to the account (I2). A SavingsAccount is a special kind of Account, whose balance must be non-negative (I3). Person's invariant (I4) requires that the sum of salary and account's balance is positive. Finally, DebitCard's invariant (I5) requires dailyCharges not to exceed the balance of the associated account. Thus, I2, I4, and I5 are multi-object invariants.

To illustrate the challenges faced by verification techniques, suppose that p is an object of class Person, which holds the Account a with DebitCard d:

Call-backs: When p executes its method spend, this results in a call of withdraw on a, which (via a call to sendReport) eventually calls back notify on p; the call notify might reach p in a state where I4 does not hold.

```
class Account {
  Person holder;
  DebitCard card;
  int balance, interestRate;

  // invariant I1: balance < 0 ==>
       interestRate == 0;
  // invariant I2: card.acc == this;

  void withdraw(int amount) {
    balance -= amount;
    if (balance < 0) {
      interestRate = 0;
      this.sendReport();
    }
  }

  void sendReport()
    { holder.notify(); }
}

class SavingsAccount
            extends Account {
  // invariant I3: balance >= 0;
}
```

```
class Person {
  Account account;
  int salary;

  // invariant I4:
  //   account.balance + salary > 0;

  void spend(int amount)
    { account.withdraw(amount); }

  void notify()
    { ... }
}

class DebitCard {
  Account acc;
  int dailyCharges;

  // invariant I5:
  //   dailyCharges <= acc.balance;
}
```

Fig. 2. An account example illustrating the main challenges for the verification of object invariants. We assume that fields hold non-null values.

Multi-object invariants: When a executes its method withdraw, it may temporarily break its invariant I1, since its balance is debited before any corresponding change is made to its interestRate. This violation is not important according to the visible state semantics; the **if** statement immediately afterwards ensures that the invariant is restored before the next visible state. However, by making an unrestricted reduction of the account balance, the method potentially breaks the invariants of other objects as well. In particular, p's invariant I4, and d's invariant I5 may be broken.

Subclassing: Further to the previous point, if a is a SavingsAccount, then calling the method withdraw may break the invariant I3, which was not necessarily known during the verification of class Account.

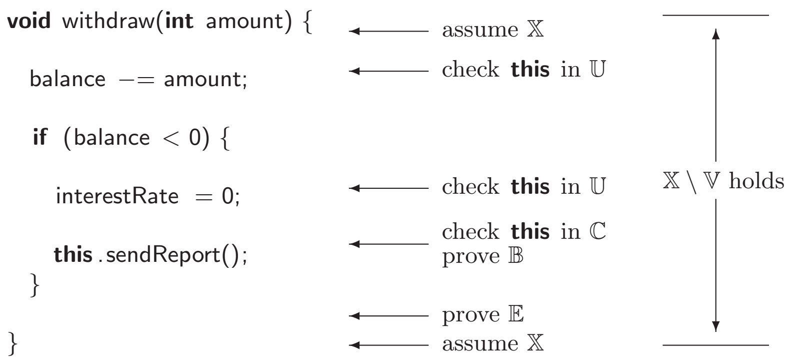
These points are addressed in the literature by striking various trade-offs between the first four differing aspects listed in the introduction (they are somewhat orthogonal to the choice of type system, specification language and verification logic, however).

Approach. Our framework uses seven parameters to capture the first four aspects in which verification techniques differ, *i.e.,* invariant semantics, invariant restrictions, proof obligations and program restrictions. To describe these parameters we use two abstract notions, which we call *regions* and *properties*. A *region* (when interpreted semantically) describes a set of objects (*e.g.,* those on which a method may be called), while a property describes a set of invariants (*e.g.,* the invariants that have to be proven before a method call). We deal with the aspects identified in the previous section as follows:

1. *Invariant semantics:* The property $\mathbb{X}$ describes the invariants e$\underline{\text{x}}$pected to hold in visible states. The property $\mathbb{V}$ describes the invariants $\underline{v}$*ulnerable* to a given method, *i.e.,* those which are potentially broken by the method through field updates or nested calls.
2. *Invariant restrictions:* The property $\mathbb{D}$ describes the invariants that may $\underline{\text{d}}$epend on a given heap location. This also characterises indirectly the locations an invariant may depend on.
3. *Proof obligations:* The properties $\mathbb{B}$ and $\mathbb{E}$ describe the invariants that must be proven to hold $\underline{\text{b}}$efore a method call and at the $\underline{\text{e}}$nd of a method body, respectively.
4. *Program restrictions:* The regions $\mathbb{U}$ and $\mathbb{C}$ describe the permitted receivers for field $\underline{\text{u}}$pdates and method $\underline{\text{c}}$alls, respectively.
5. *Type systems:* We parameterise our framework by the type system. We state requirements on the type system, but leave abstract its concrete definition. We require that types are formed of a region-class pair so that we can handle types that express heap topologies (such as ownership types).
6. *Specification languages:* Rather than describing invariants concretely, we assume a judgement that expresses that an object satisfies the invariant of a class in a heap.
7. *Verification logics:* We express proof obligations via a special construct prv $\mathbb{p}$, which throws an exception if the invariants in property $\mathbb{p}$ cannot be proven, and has an empty effect otherwise. We leave abstract how the actual proofs are constructed and checked.

Fig. 3 illustrates the parameters of our framework by annotating the body of the method withdraw. $\mathbb{X}$ may be assumed to hold in the pre- and post-states of the method. Between these visible states, some object invariants may be broken (so long as they fall within $\mathbb{V}$), but $\mathbb{X}\backslash\mathbb{V}$ is known to hold throughout the method body. Field updates and method calls are allowed if the receiver object (here, **this**) is in $\mathbb{U}$ and $\mathbb{C}$, respectively. Before a method call, $\mathbb{B}$ must be proven. At the end of the method body, $\mathbb{E}$ must be proven. Finally, $\mathbb{D}$ (not shown in Fig. 3) constrains the effects of field updates on invariants. Thus, assignments to balance and interestRate affect at most $\mathbb{D}$.

The seven parameters capture concepts explicitly or implicitly found in all verification techniques, defined either through words [?,?,?,?] or typing rules [?]. For example, $\mathbb{V}$ is implicit in [?], but is crucial for their soundness argument. $\mathbb{X}$ and $\mathbb{V}$ are explicit in [?], while $\mathbb{U}$ and $\mathbb{C}$ are implicitly expressed as constraints in their typing rules. Subsets of these seven parameters characterise verification



```
void withdraw(int amount) {
  balance -= amount;
  if (balance < 0) {
    interestRate = 0;
    this.sendReport();
  }
}
```

assume $\mathbb{X}$
check **this** in $\mathbb{U}$
check **this** in $\mathbb{U}$
check **this** in $\mathbb{C}$
prove $\mathbb{B}$
prove $\mathbb{E}$
assume $\mathbb{X}$
$\mathbb{X} \setminus \mathbb{V}$ holds

Fig. 3. Role of framework parameters for method withdraw from Fig. 2.

technique concepts *e.g.*, soundness (through $\mathbb{X}$ and $\mathbb{V}$), expressiveness ($\mathbb{D}$, $\mathbb{X}$ and $\mathbb{V}$), and proof obligations ($\mathbb{B}$ and $\mathbb{E}$).

Developing our framework was challenging because different verification techniques (1) use different type systems to restrict programs and invariants, and do not make a clear distinction between the type system and the verification technique, (2) use different specification languages to express invariants, and (3) use different verification logics. To deal with this diversity within one unified framework, we take the following approach:

1. We make a clear delineation between the framework and the type system and instead of describing one particular type system, we state requirements on the type systems used with our framework.
2. We assume a judgement that describes that an object satisfies the invariant of a class in a heap. We require that a field update preserves the invariant if it does not fall within $\mathbb{D}$.
3. We express proof obligations via a special construct prv $\mathbb{p}$, which throws an exception if the invariants in property $\mathbb{p}$ cannot be proven, and has an empty effect otherwise.

3 Invariant Semantics

We formalise invariant semantics through an operational semantics, defining at which execution points invariants are required to hold. In order to cater for the different techniques, the semantics is parameterised by properties to express proof obligations and which invariants are expected to hold. In this section, we focus on the main ideas of our semantics and relegate the less interesting definitions to App. A. We assume sets of identifiers for class names $\textsc{Cls}$, field names $\textsc{Fld}$, and method names $\textsc{Mthd}$, and use variables $c \in \textsc{Cls}$, $f \in \textsc{Fld}$ and $m \in \textsc{Mthd}$.

$$\begin{array}{rlll|lll}
e ::= & \mathsf{this} & \textit{(this)} & & x & \textit{(variable)} \\
| & \mathsf{null} & \textit{(null)} & & \mathsf{new}\, t & \textit{(new object)} \\
| & e.f & \textit{(access)} & & e.f := e & \textit{(assignment)} \\
| & e.m(e) & \textit{(method call)} & & e \,\mathsf{prv}\, \mathbb{p} & \textit{(proof annotat.)} \\
\\
e_r ::= & \dots & \textit{(as source exprs.)} & & v & \textit{(value)} \\
| & \mathsf{verfExc} & \textit{(verif exc.)} & & \mathsf{fatalExc} & \textit{(fatal exc.)} \\
| & \sigma \cdot e_r & \textit{(nested call)} & & \mathsf{call}\, e_r & \textit{(launch)} \\
| & \mathsf{ret}\, e_r & \textit{(return)} & & &
\end{array}$$

Fig. 4. Source and runtime expression syntax.

Runtime Structures. A *runtime structure* is a tuple consisting of a set of heaps HP, a set of addresses ADR, and a set of values $\text{VAL} = \text{ADR} \cup \{\mathsf{null}\}$, using variables $h \in \text{HP}$, $\iota \in \text{ADR}$, and $v \in \text{VAL}$. A runtime structure provides the following operations. The operation $dom(h)$ represents the domain of the heap. $cls(h, \iota)$ yields the class of the object at address ι. The operation $fld(h, \iota, f)$ yields the value of a field f of the object at address ι. Finally, $upd(h, \iota, f, v)$ yields the new heap after a field update, and $new(h, \iota, t)$ yields the heap and address resulting from the creation of a new object of type t. We leave abstract how these operations work, but require properties about their behaviour, for instance that upd only modifies the corresponding field of the object at the given address, and leaves the remaining heap unmodified. See Def. 8 in App. A for details.

A stack frame $\sigma \in \text{STK} = \text{ADR} \times \text{ADR} \times \text{MTHD} \times \text{CLS}$ is a tuple of a receiver address, an argument address, a method identifier, and a class. The latter two indicate the method currently being executed and the class where it is defined.

Regions, Properties and Types. A region $\mathbb{r} \in \mathbf{R}$ is a syntactic representation for a set of objects; a property $\mathbb{p} \in \mathbf{P}$ is a syntactic representation for a set of assertions about particular objects. It is crucial that our syntax is parametric with the specific regions and properties; we use different regions and properties to model different verification techniques.[1]

We define a type $t \in \text{TYP}$, as a pair of a region and a class. The region allows us to cater for types that express the topology of the heap, without being specific about the underlying type system.

Expressions. In Fig. 4, we define source expressions $e \in \text{EXPR}$. In order to simplify our presentation (but without loss of generality), we restrict methods to always have exactly one argument. Besides the usual basic object-oriented constructs, we include proof annotations $e \,\mathsf{prv}\, \mathbb{p}$. As we will see later, such a proof annotation executes the expression e and then imposes a proof obligation for the invariants characterised by the property $\mathbb{p}$. To maintain generality, we avoid being precise about the actual syntax and checking of proofs.

[1] For example, in Universe types, **rep** and **peer** are regions, while in ownership types, ownership parameters such as X, and also **this**, are regions (more in Sec. 6).

In Fig. 4, we also define runtime expressions $e_r \in \text{RExpr}$. A runtime expression is a source expression, a value, a nested call with its stack frame σ, an exception, or a decorated runtime expression. A verification exception $\mathsf{verfExc}$ indicates that a proof obligation failed. A fatal exception $\mathsf{fatalExc}$ indicates that an expected invariant does not hold. Runtime expressions can be decorated with $\mathsf{call}\, e_r$ and $\mathsf{ret}\, e_r$ to mark the beginning and end of a method call, respectively.

In Def. 10 (App. A), we define evaluation contexts, $E[\cdot]$, which describe contexts within one activation record and extend these to runtime contexts, $F[\cdot]$, which also describe nested calls.

Programming Languages. We define a programming language as a tuple consisting of a set Prg of programs, a runtime structure, a set of regions, and a set of properties (see Def. 11 in App. A). Each $P \in \text{Prg}$ comes equipped with the following operations. $\mathcal{F}(c, f)$ yields the type of field f in class c as well as the class in which f is declared (c or a superclass of c). $\mathcal{M}(c, m)$ yields the type signature of method m in class c. $\mathcal{B}(c, m)$ yields the expression constituting the body of method m in class c as well as the class in which m is declared. Moreover, there are operators to denote subclasses and subtypes ($<:$), inclusion of regions ($\sqsubseteq$), and interpretation ($[\![\cdot]\!]$) of regions and properties.

Regions and properties are interpreted wrt. a heap, and from the *viewpoint* of a "current object"; therefore, their definitions depend on heap and address parameters: $[\![\ldots]\!]_{h,\iota}$. The interpretation of a region produces a set of objects. We characterise each invariant by an object-class pair, with the intended meaning that the invariant[2] specified in the class holds for the object.[3] Therefore, the interpretation of a property produces a set of object-class pairs, specifying all the invariants of interest.

Each program also comes with typing judgements $\Gamma \vdash e : t$ and $h \vdash e_r : t$ for source and runtime expressions, respectively. An environment $\Gamma \in \text{Env}$ is a tuple of the class containing the current method, the method identifier, and the type of the sole argument.

Finally, the judgement $h \models \iota, c$ expresses that in heap h, the object at address ι satisfies the invariant declared in class c. We define that the judgement trivially holds if the object is not allocated ($\iota \notin dom(h)$) or is not an instance of c ($cls(h, \iota) \not<: c$). We say that the property $\mathbb{p}$ is *valid* in heap h wrt. address ι if all invariants in $[\![\mathbb{p}]\!]_{h,\iota}$ are satisfied. We denote validity of properties by $h \models \mathbb{p},\, \iota$:

$$h \models \mathbb{p},\, \iota \quad \Leftrightarrow \quad \forall (\iota', c) \in [\![\mathbb{p}]\!]_{h,\iota}.\ h \models \iota', c$$

Operational Semantics. The framework parameter $\mathbb{X}$ describes which invariants are expected to hold at visible states. Given a program P and a set of properties $\mathbb{X}_{c,m}$, each characterising the property that needs to hold at the beginning and end of a method m of class c, the *runtime semantics* is the relation $\longrightarrow \subseteq (\text{RExpr} \times \text{Hp}) \times (\text{RExpr} \times \text{Hp})$ defined in Fig. 5.

[2] or conjunction of invariants, if one prefers to write multiple invariants in the same class

[3] An object may have different invariants for each of the classes it belongs to [?].

(rVarThis)
$$\frac{\sigma = (\iota, v, \_, \_)}{\begin{array}{c}\sigma\cdot\mathsf{this},\ h \longrightarrow \sigma\cdot\iota,\ h\\ \sigma\cdot x,\ h \longrightarrow \sigma\cdot v,\ h\end{array}}$$

(rNew)
$$\frac{\begin{array}{c}\sigma = (\iota, \_, \_, \_)\\ h', \iota' = new(h, \iota, t)\end{array}}{\sigma\cdot\mathsf{new}\,t,\ h \longrightarrow \sigma\cdot\iota',\ h'}$$

(rDer)
$$\frac{v = fld(h, \iota, f)}{\sigma\cdot\iota.f,\ h \longrightarrow \sigma\cdot v,\ h}$$

(rAss)
$$\frac{h' = upd(h, \iota, f, v)}{\sigma\cdot\iota.f := v,\ h \longrightarrow \sigma\cdot v,\ h'}$$

(rCall)
$$\frac{\mathcal{B}(m, cls(h, \iota)) = e, c \quad \sigma' = (\iota, v, c, m)}{\sigma\cdot\iota.m(v),\ h \longrightarrow \sigma\cdot\sigma'\cdot\mathsf{call}\,e,\ h}$$

(rCxtEval)
$$\frac{\sigma\cdot \mathrm{e}_r,\ h \longrightarrow \sigma\cdot \mathrm{e}'_r,\ h'}{\sigma\cdot E[\mathrm{e}_r],\ h \longrightarrow \sigma\cdot E[\mathrm{e}'_r],\ h'}$$

(rCxtFrame)
$$\frac{\mathrm{e}_r,\ h \longrightarrow \mathrm{e}'_r,\ h'}{\sigma\cdot \mathrm{e}_r,\ h \longrightarrow \sigma\cdot \mathrm{e}'_r,\ h'}$$

(rLaunch)
$$\frac{\begin{array}{l}\sigma = (\iota, \_, c, m)\\ h \models \mathbb{X}_{c,m},\ \iota\end{array}}{\sigma\cdot\mathsf{call}\,e,\ h \longrightarrow \sigma\cdot\mathsf{ret}\,e,\ h}$$

(rLaunchEx)
$$\frac{\begin{array}{l}\sigma = (\iota, \_, c, m)\\ h \not\models \mathbb{X}_{c,m},\ \iota\end{array}}{\sigma\cdot\mathsf{call}\,e,\ h \longrightarrow \sigma\cdot\mathsf{fatalExc},\ h}$$

(rFrame)
$$\frac{\begin{array}{l}\sigma = (\iota, \_, c, m)\\ h \models \mathbb{X}_{c,m},\ \iota\end{array}}{\sigma\cdot\mathsf{ret}\,v,\ h \longrightarrow v,\ h}$$

(rFrameEx)
$$\frac{\begin{array}{l}\sigma = (\iota, \_, c, m)\\ h \not\models \mathbb{X}_{c,m},\ \iota\end{array}}{\sigma\cdot\mathsf{ret}\,v,\ h \longrightarrow \mathsf{fatalExc},\ h}$$

(rPrf)
$$\frac{\sigma = (\iota, \_, \_, \_) \quad h \models \mathbb{p},\ \iota}{\sigma\cdot v\ \mathsf{prv}\ \mathbb{p},\ h \longrightarrow \sigma\cdot v,\ h}$$

(rPrfEx)
$$\frac{\sigma = (\iota, \_, \_, \_) \quad h \not\models \mathbb{p},\ \iota}{\sigma\cdot v\ \mathsf{prv}\ \mathbb{p},\ h \longrightarrow \sigma\cdot\mathsf{verfExc},\ h}$$

Fig. 5. Reduction rules of operational semantics.

The first seven rules are standard for object-oriented languages. Note that in rNew, a new object is created using the function *new*, which takes a type as parameter rather than a class, thereby making the semantics parametric wrt. the type system: different type systems may use different regions and definitions of *new* to describe heap-topological information. Similarly, *upd* and *fld* do not fix a particular heap representation. Rule rCall describes method calls; it stores the class in which the method body is defined in the new stack frame σ, and introduces the "marker" call e_r at the beginning of the method body.

Our reduction rules abstract away from program verification and describe only its effect. Thus, rLaunch, rLaunchExc, rFrame, and rFrameExc check whether $\mathbb{X}_{c,m}$ is valid at the beginning and end of any execution of a method m defined in class c, and generate a fatal exception, fatalExc, if the check fails. This represents the visible state semantics discussed in the introduction. Proof obligations e prv $\mathbb{p}$ are verified once e reduces to a value (rPrf and rPrfExc); if $\mathbb{p}$ is not found to be valid, a verification exception verfExc is generated.

Verification using visible state semantics amounts to showing all proof obligations in some program logic, based on the assumption that expected invariants hold in visible states. Informally then, a specific verification technique described in our framework is sound if it guarantees that a fatalExc is never encountered. Verification technique soundness does allow verfExc to be generated, but this will never happen in a correctly verified program. We give a formal definition of soundness at the end of the next section.

This semantics allows us to be parametric wrt. the syntax of invariants and the logic of proofs. We also define properties that permit us to be parametric wrt. a sound type system (*cf.* Def. 15 in App. A). Thus, we can concentrate entirely on verification concerns.

4 Verification Techniques

In this section, we formalise verification techniques and their connection to programs. Within this formalisation, we define what it means for a verification technique to be sound.

A verification technique is described by a 7-tuple, where the *components* of the tuple provide instantiations for the seven parameters of our framework. These instantiations are expressed in terms of the regions and properties provided by the programming language. To allow the instantiations to refer to the program, for instance, to look up field declarations, we define a verification technique as a mapping from programs to 7-tuples.

Definition 1 (Verification Technique) *A* verification technique $\mathcal{V}$ for a programming language *is a mapping from programs into a tuple:*

$$
\begin{array}{l}
\mathcal{V} \,:\, \textsc{Prg} \to \textsc{eXp} \times \textsc{Vul} \times \textsc{Dep} \times \textsc{Pre} \times \textsc{End} \times \textsc{Cll} \times \textsc{Ass} \\
\textit{where} \\
\begin{array}{rl}
\textsc{eXp} = & \textsc{Cls} \times \textsc{Mthd} \to \mathbf{P} \\
\textsc{Vul} = & \textsc{Cls} \times \textsc{Mthd} \to \mathbf{P} \\
\textsc{Dep} = & \textsc{Cls} \to \mathbf{P} \\
\textsc{Pre} = & \textsc{Cls} \times \textsc{Mthd} \times \mathbf{R} \to \mathbf{P} \\
\textsc{End} = & \textsc{Cls} \times \textsc{Mthd} \to \mathbf{P} \\
\textsc{Cll} = & \textsc{Cls} \times \textsc{Mthd} \times \textsc{Cls} \to \mathbf{R} \\
\textsc{Ass} = & \textsc{Cls} \times \textsc{Mthd} \times \textsc{Cls} \times \textsc{Mthd} \to \mathbf{R}
\end{array}
\end{array}
$$

To describe a verification technique applied to a program, we write the application of the components to class, method names, *etc.*, as $\mathbb{X}_{c,m}$, $\mathbb{V}_{c,m}$, $\mathbb{D}_c$, $\mathbb{B}_{c,m,\mathbb{r}}$, $\mathbb{E}_{c,m}$, $\mathbb{U}_{c,m,c'}$, $\mathbb{C}_{c,m,c',m'}$. The meaning of these components is:

$\mathbb{X}_{c,m}$: the property expected to be valid at the beginning and end of the body of method m in class c. The parameters c and m allow a verification technique to expect different invariants in the visible states of different methods. For instance, JML's helper methods [**?**,**?**] do not expect any invariants to hold.

$\mathbb{V}_{c,m}$: the property vulnerable to method m of class c, that is, the property whose validity may be broken by m or a nested call. Method m can break an invariant by updating a field or by calling a method that breaks, but does not reestablish the invariant (for instance, a helper method). The parameters c and m allow a verification technique to require that invariants of certain classes (for instance, c's subclasses) are not vulnerable.

$\mathbb{D}_c$: the property that depends on a field declared in class c. The parameter c is used, for instance, to prevent invariants from depending on fields declared in c's superclasses [**?**,**?**].

$\mathbb{B}_{c,m,\mathbb{r}}$: the property whose validity has to be proven before calling a method on a receiver in region $\mathbb{r}$ from the execution of a method m in class c. The parameters allow a verification technique to impose proof obligations depending on the calling method and the ownership relation between caller and callee.

$\mathbb{E}_{c,m}$: the property whose validity has to be proven at the end of method m in class c. The parameters allow a verification technique to require different proofs for different methods to exclude subclass invariants or helper methods.

$\mathbb{U}_{c,m,c'}$: the region of allowed receivers for an update of a field in class c', within the body of method m in class c. The parameters allow a verification technique, for instance, to prevent field updates within pure methods.

$\mathbb{C}_{c,m,c',m'}$: the region of allowed receivers for a call to method m' of class c', within the body of method m of class c. The parameters allow a verification technique to permit calls depending on attributes (such as purity or effect specifications) of the caller and the callee.

Role of the Seven Components. The operational semantics uses a verification technique to specify the invariants expected in visible states, whereas the static analysis imposed by the verification technique describes program restrictions and proof obligations. More precisely, the operational semantics uses $\mathbb{X}$, to be checked at visible states; soundness requires that the invariants $\mathbb{X}\setminus\mathbb{V}$ hold during a method activation. Static analysis describes proof obligations using $\mathbb{B}$ and $\mathbb{E}$, and ensures program restrictions, through $\mathbb{U}$ and $\mathbb{C}$, are respected. Finally, $\mathbb{D}$ restricts invariants for well-verified programs (*cf.* Def. 2 below). Sec. 5 gives five conditions on these components that guarantee soundness.

It might be initially surprising that we need as many as seven components. This number is justified by the variety of concepts used by modern verification techniques, such as accessibility of fields, purity, helper methods, ownership, and effect specifications. Note for instance that $\mathbb{V}$ would be redundant if all methods were to reestablish the invariants they break; in such a setting, a method could break invariants only through field updates, and $\mathbb{V}$ could be derived from $\mathbb{U}$ and $\mathbb{D}$. However, in the presence of helper methods and ownership, methods may break but not reestablish invariants.

The class and method identifiers used as parameters to our components can be extracted from an environment Γ or a stack frame σ in the obvious way. Thus, for $\Gamma=(c,m,\_)$ or for $\sigma=(\iota,\_,c,m)$, we use $\mathbb{X}_\Gamma$ and $\mathbb{X}_\sigma$ as shorthands for $\mathbb{X}_{c,m}$; we also use $\mathbb{B}_{\Gamma,\mathbb{r}}$ and $\mathbb{B}_{\sigma,\mathbb{r}}$ as shorthands for $\mathbb{B}_{c,m,\mathbb{r}}$.

Well-Verified Programs. The judgement $\Gamma \vdash_{\mathcal{V}} e$ expresses that expression e is well-verified according to verification technique $\mathcal{V}$. The rules for this judgement are shown in Fig. 6.

$$\text{(vs-null)}\ \frac{}{\Gamma \vdash_{\mathcal{V}} \mathsf{null}} \qquad \text{(vs-Var)}\ \frac{}{\Gamma \vdash_{\mathcal{V}} x} \qquad \text{(vs-this)}\ \frac{}{\Gamma \vdash_{\mathcal{V}} \mathsf{this}} \qquad \text{(vs-new)}\ \frac{}{\Gamma \vdash_{\mathcal{V}} \mathsf{new}\, t} \qquad \text{(vs-fld)}\ \frac{\Gamma \vdash_{\mathcal{V}} e}{\Gamma \vdash_{\mathcal{V}} e.f}$$

$$\text{(vs-ass)}\ \frac{\Gamma \vdash e : \mathbb{r}\, c' \quad \mathcal{F}(c',f) = \_, c \quad \mathbb{r} \sqsubseteq \mathbb{U}_{\Gamma,c} \quad \Gamma \vdash_{\mathcal{V}} e \quad \Gamma \vdash_{\mathcal{V}} e'}{\Gamma \vdash_{\mathcal{V}} e.f := e'}$$

$$\text{(vs-call)}\ \frac{\Gamma \vdash e : \mathbb{r}\, c' \quad \mathcal{B}(c',m) = \_, c \quad \mathbb{r} \sqsubseteq \mathbb{C}_{\Gamma,c,m} \quad \Gamma \vdash_{\mathcal{V}} e \quad \Gamma \vdash_{\mathcal{V}} e'}{\Gamma \vdash_{\mathcal{V}} e.m(e'\ \mathsf{prv}\, \mathbb{B}_{\Gamma,\mathbb{r}})}$$

$$\text{(vs-class)}\ \frac{\left.\begin{array}{l}\mathcal{B}(c,m) = e, c \\ \mathcal{M}(c,m) = t, t'\end{array}\right\} \Rightarrow \left\{\begin{array}{l} e = e'\ \mathsf{prv}\, \mathbb{E}_{c,m} \\ c, m, t \vdash_{\mathcal{V}} e'\end{array}\right.}{\vdash_{\mathcal{V}} c}$$

Fig. 6. Well-verified source expressions and classes.

The first five rules express that literals, variable lookup, object creation, and field lookup do not require proofs. The receiver of a field update must fall into $\mathbb{U}$ (vs-ass). The receiver of a call must fall into $\mathbb{C}$ (vs-call). Moreover, we require the proof of $\mathbb{B}$ before a call. Finally, a class is well-verified if the body of each of its methods is well-verified and ends with a proof obligation for $\mathbb{E}$ (vs-class). Note that we use the type judgement $\Gamma \vdash e : t$ without defining it; the definition is given by the underlying programming language, not by our framework.

Fig. 7 defines the judgement $h \vdash_{\mathcal{V}} e_r$ for well-annotated runtime expressions. The rules correspond to those from Fig. 6, with the addition of rules for values and nested calls.[4]

(vd-null)
$$\frac{}{h \vdash_{\mathcal{V}} \sigma\cdot\mathsf{null}}$$

(vd-addr)
$$\frac{\iota \in dom(h)}{h \vdash_{\mathcal{V}} \sigma\cdot\iota}$$

(vd-new)
$$\frac{}{h \vdash_{\mathcal{V}} \sigma\cdot\mathsf{new}\,t}$$

(vd-Var)
$$\frac{}{h \vdash_{\mathcal{V}} \sigma\cdot x}$$

(vd-this)
$$\frac{}{h \vdash_{\mathcal{V}} \sigma\cdot\mathsf{this}}$$

(vd-verEx)
$$\frac{}{h \vdash_{\mathcal{V}} F[\mathsf{verfExc}]}$$

(vd-ass)
$$\frac{\begin{array}{l} h \vdash \sigma\cdot e_r : \mathbb{r}\, c' \\ \mathcal{F}(c', f) = \_, c \\ \mathbb{r} \sqsubseteq \mathbb{U}_{\sigma,c} \\ h \vdash_{\mathcal{V}} \sigma\cdot e_r \\ h \vdash_{\mathcal{V}} \sigma\cdot e'_r \end{array}}{h \vdash_{\mathcal{V}} \sigma\cdot e_r.f := e'_r}$$

(vd-fld)
$$\frac{h \vdash_{\mathcal{V}} \sigma\cdot e_r}{h \vdash_{\mathcal{V}} \sigma\cdot e_r.f}$$

(vd-end)
$$\frac{h \vdash_{\mathcal{V}} \sigma'\cdot v \quad h \models \mathbb{E}_{\sigma'},\, \sigma'}{h \vdash_{\mathcal{V}} \sigma\cdot\sigma'\cdot\mathsf{ret}\, v}$$

(vd-call)
$$\frac{\begin{array}{l} h \vdash \sigma\cdot e_r : \mathbb{r}\, c' \\ \mathcal{B}(c', m) = \_, c \\ \mathbb{r} \sqsubseteq \mathbb{C}_{\sigma,c,m} \\ h \vdash_{\mathcal{V}} \sigma\cdot e_r \\ h \vdash_{\mathcal{V}} \sigma\cdot e'_r \end{array}}{h \vdash_{\mathcal{V}} \sigma\cdot e_r.m(e'_r\ \mathsf{prv}\, \mathbb{B}_{\sigma,\mathbb{r}})}$$

(vd-call-2)
$$\frac{\begin{array}{l} h \vdash \sigma\cdot v : \mathbb{r}\, c' \\ \mathcal{B}(c', m) = \_, c \\ h \models \mathbb{B}_{\sigma,\mathbb{r}},\, \sigma \\ \mathbb{r} \sqsubseteq \mathbb{C}_{\sigma,c,m} \\ h \vdash_{\mathcal{V}} \sigma\cdot v \\ h \vdash_{\mathcal{V}} \sigma\cdot v' \end{array}}{h \vdash_{\mathcal{V}} \sigma\cdot v.m(v')}$$

(vd-start)
$$\frac{h \vdash_{\mathcal{V}} \sigma'\cdot e}{h \vdash_{\mathcal{V}} \sigma\cdot\sigma'\cdot\mathsf{call}\, e\ \mathsf{prv}\, \mathbb{E}_{\sigma'}}$$

(vd-frame)
$$\frac{h \vdash_{\mathcal{V}} \sigma'\cdot e_r}{h \vdash_{\mathcal{V}} \sigma\cdot\sigma'\cdot\mathsf{ret}\, e_r\ \mathsf{prv}\, \mathbb{E}_{\sigma'}}$$

Fig. 7. Well-annotated runtime expressions.

[4] Notice that although the syntax of runtime expressions (Def. 4) allows for unrestricted nesting of stack frames (*i.e.*, a runtime expression might have the form $\sigma_1\cdot\sigma_2\cdot\ldots\cdot\sigma_n\cdot e_r$), such expressions cannot be reached from execution of an initial expression of the form $\sigma\cdot e'_r$ in which e'_r does not contain further stack frames. For this reason, the definition of well-annotated runtime expressions only treats a more-restricted syntax, in which stack frames may only occur in suitable positions. Note that nested calls are not represented by simply nesting stack frames, but instead through the intermediate ret keyword, *e.g.*, $\sigma_1\cdot\mathsf{ret}\, E_1[\sigma_2\cdot\mathsf{ret}\, E_2[\ldots\cdot\sigma_n\cdot\mathsf{ret}\, e_r]]$.

A program P is well-verified wrt. $\mathcal{V}$, denoted as $\vdash_{\mathcal{V}} P$, iff *(1)* all classes are well-verified and *(2)* all class invariants respect the dependency restrictions dictated by $\mathbb{D}$. That is, the invariant of an object ι' declared in a class c' will be preserved by an update of a field of an object of class c if it is not within $\mathbb{D}_c$.

Definition 2 (Well-Verified Programs)

$$\vdash_{\mathcal{V}} P \Leftrightarrow \begin{cases} \textit{(W1)}\ \ \forall c \in P.\, \vdash_{\mathcal{V}} c \\ \\ \left.\begin{array}{l} \qquad \mathcal{F}(\mathit{cls}(h,\iota), f) = \_, c \\ \textit{(W2)}\ (\iota', c') \notin [\![\mathbb{D}_c]\!]_{h,\iota}, \\ \qquad h \models \iota', c' \end{array}\right\} \Rightarrow\ \mathit{upd}(h,\iota,f,v) \models \iota', c' \end{cases}$$

Valid States. The properties $\mathbb{X}$ and $\mathbb{X} \setminus \mathbb{V}$ characterise the invariants that are known to hold in the visible states and between visible states of the current method execution, respectively. That is, they reflect the local knowledge of the current method, but do not describe globally all the invariants that need to hold in a given state.

For any state with heap h and execution stack $\overline{\sigma}$, the function $\mathit{vi}(\overline{\sigma}, h)$ yields the set of *valid invariants*, that is, invariants that are expected to hold :

$$\mathit{vi}(\overline{\sigma}, h) = \begin{cases} \emptyset & \text{if } \overline{\sigma} = \epsilon \\ (\mathit{vi}(\overline{\sigma_1}, h) \cup [\![\mathbb{X}_\sigma]\!]_{h,\sigma}) \setminus [\![\mathbb{V}_\sigma]\!]_{h,\sigma} & \text{if } \overline{\sigma} = \overline{\sigma_1} \cdot \sigma \end{cases}$$

The call stack is empty at the beginning of program execution, at which point we expect the heap to be empty. For each additional stack frame σ, the corresponding method m may assume $\mathbb{X}_\sigma$ at the beginning of the call, therefore we add $[\![\mathbb{X}_\sigma]\!]_{h,\sigma}$ to the valid invariants. The method may break $\mathbb{V}_\sigma$ during the call, and so we remove $[\![\mathbb{V}_\sigma]\!]_{h,\sigma}$ from the valid invariants.

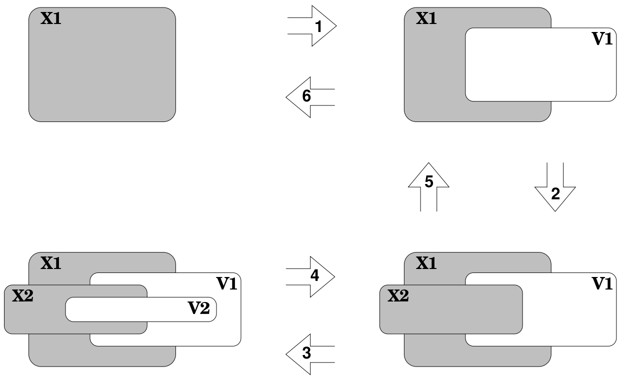


Fig. 8. Open calls and valid invariants in a heap.

Fig. 8 depicts this mechanism of invariant violation and reestablishing for the execution of two consecutive calls. The properties $\mathbb{X}_1$ and $\mathbb{V}_1$ denote the expected and vulnerable properties of the outer call, and $\mathbb{X}_2$ and $\mathbb{V}_2$ are the expected and vulnerable properties of the subcall. The first call violates $\mathbb{V}_1$ but the invariants $\mathbb{X}_1 \setminus \mathbb{V}_1$ hold throughout the call (1). Before making the subcall, it establishes all of $\mathbb{X}_2$ (2). The subcall violates $\mathbb{V}_2$ (3) but reestablishes all of $\mathbb{X}_2$ before returning (4); similarly, after the first call resumes control (5), it reestablishes $\mathbb{X}_1$ at the end of its execution (6).

A state with heap h and stack $\overline{\sigma}$ is *valid* iff:

(1) $\overline{\sigma}$ is a valid stack, denoted by $h \vdash_{\mathcal{V}} \overline{\sigma}$ (Def. 12 in App. A), and meaning that the receivers of consecutive method calls are within the respective $\mathbb{C}$ regions.
(2) The valid invariants $vi(\overline{\sigma}, h)$ hold.
(3) If execution is in a visible state with σ as the topmost frame of $\overline{\sigma}$, then, in addition, the expected invariants $\mathbb{X}_\sigma$ hold.

These properties are formalised in Def. 3. A state is determined by a heap h and a runtime expression e_r; the stack is extracted from e_r using function *stack*, given by Def. 13 in App. A.

Definition 3 *A state with heap* h *and runtime expression* e_r *is valid for a verification technique* $\mathcal{V}$, $e_r \models_{\mathcal{V}} h$, *iff:*

$$\text{(1)}\ \ h \vdash_{\mathcal{V}} \mathit{stack}(e_r) \qquad\qquad \text{(2)}\ \ h \models \mathit{vi}(\mathit{stack}(e_r), h)$$
$$\text{(3)}\ \ e_r = F[\sigma \cdot \mathbf{call}\, e] \ \ \text{or}\ \ e_r = F[\sigma \cdot \mathbf{ret}\, v] \ \ \Rightarrow \ \ h \models \mathbb{X}_\sigma,\, \sigma$$

Soundness. A verification technique is *sound* if verified programs only produce valid states and do not throw fatal exceptions. More precisely, a verification technique $\mathcal{V}$ is sound for a programming language *PL* iff for all well-formed and verified programs $P \in PL$, any well-typed and verified runtime expression e_r executed in a valid state reduces to another verified expression e'_r with a resulting valid state. Note that a verified e'_r contains no fatalExc (see Fig. 7).

Well-formedness of program P is denoted by $\vdash_{\mathbf{wf}} P$ (Def. 14, App. A). Well-typedness of runtime expression e_r is denoted by $h \vdash e_r : t$ and required as part of a sound type system in Def. 11, App. A. These requirements permit separation of concerns, whereby we can formally define verification technique soundness *in isolation*, assuming program well-formedness and a sound type system.

Definition 4 *A verification technique* $\mathcal{V}$ *is* s*ound for a programming language* *PL iff for all programs* $P \in PL$*:*

$$\left.\begin{array}{l} \vdash_{\mathbf{wf}} P, \quad h \vdash e_r : \_, \quad \vdash_{\mathcal{V}} P, \quad e_r \models_{\mathcal{V}} h, \\ h \vdash_{\mathcal{V}} e_r, \quad e_r,\, h \longrightarrow e'_r,\, h' \end{array}\right\} \Rightarrow \ e'_r \models_{\mathcal{V}} h', \quad h' \vdash_{\mathcal{V}} e'_r$$

5 Well-Structured Verification Techniques

In this section, we identify conditions on the components of a verification technique that are sufficient for soundness.

Definition 5 (Well-Structured Verification Methodology) *A verification technique is* well-structured *if, for all programs in the programming language:*

(S1) $\mathbb{r} \sqsubseteq \mathbb{C}_{c,m,c'm'} \Rightarrow (\mathbb{r} \triangleright \mathbb{X}_{c',m'}) \setminus (\mathbb{X}_{c,m} \setminus \mathbb{V}_{c,m}) \subseteq \mathbb{B}_{c,m,\mathbb{r}}$
(S2) $\mathbb{V}_{c,m} \cap \mathbb{X}_{c,m} \subseteq \mathbb{E}_{c,m}$
(S3) $\mathbb{C}_{c,m,c',m'} \triangleright (\mathbb{V}_{c',m'} \setminus \mathbb{E}_{c',m'}) \subseteq \mathbb{V}_{c,m}$
(S4) $\mathbb{U}_{c,m,c'} \triangleright \mathbb{D}_{c'} \subseteq \mathbb{V}_{c,m}$
(S5) $c' <: c \Rightarrow \begin{cases} \mathbb{X}_{c',m} \subseteq \mathbb{X}_{c,m}, \\ \mathbb{V}_{c',m} \setminus \mathbb{E}_{c',m} \subseteq \mathbb{V}_{c,m} \setminus \mathbb{E}_{c,m} \end{cases}$

In the above, the set theoretic symbols have the obvious interpretation in the domain of properties. For example *(S2)* is short for $\forall h, \iota :\ [\![\mathbb{V}_{c,m}]\!]_{h,\iota} \cap ([\![\mathbb{X}_c]\!]_{h,\iota} \subseteq [\![\mathbb{E}_{c,m}]\!]_{h,\iota}$. We use *viewpoint adaptation* $\mathbb{r} \triangleright \mathbb{p}$, defined as:

$$[\![\mathbb{r} \triangleright \mathbb{p}]\!]_{h,\iota} = \bigcup_{\iota' \in [\![\mathbb{r}]\!]_{h,\iota}} [\![\mathbb{p}]\!]_{h,\iota'}$$

meaning that the interpretation of a viewpoint-adapted property $\mathbb{r} \triangleright \mathbb{p}$ wrt. an address ι is equal to the union of the interpretations of $\mathbb{p}$ wrt. each object in the interpretation of $\mathbb{r}$.

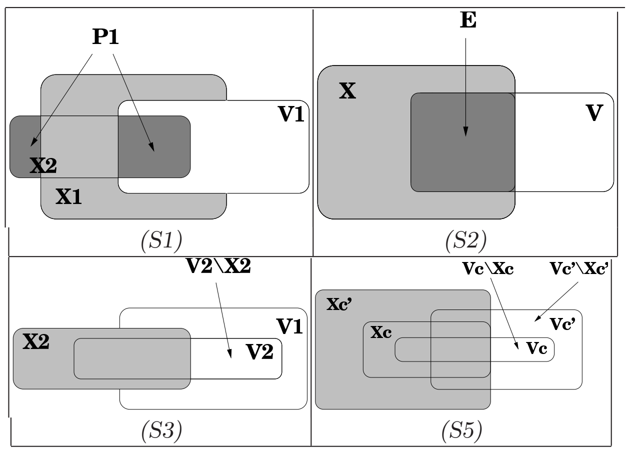


Fig. 9. Well-structured conditions.

The first two conditions relate proof obligations with expected invariants. *(S1)* ensures for a call within the permitted region that the expected invariants of the callee ($\mathbb{r} \triangleright \mathbb{X}_{c',m'}$) minus the invariants that hold throughout the calling method ($\mathbb{X}_{c,m} \setminus \mathbb{V}_{c,m}$) are included in the proof obligation for the call ($\mathbb{B}_{c,m,\mathbb{r}}$). *(S2)* ensures that the invariants that were broken during the execution of a method, but which are required to hold again at the end of the method ($\mathbb{V}_{c,m} \cap \mathbb{X}_{c,m}$) are included in the proof obligation at the end of the method ($\mathbb{E}_{c,m}$).

The third and fourth condition ensure that invariants that are broken by a method m of class c are actually in its vulnerable set. Condition *(S3)* deals with calls and therefore uses viewpoint adaptation for call regions ($\mathbb{C}_{c,m,c',m'} \triangleright \ldots$). It restricts the invariants that may be broken by the callee method m', but are not reestablished by the callee through $\mathbb{E}$. These invariants must be included in the vulnerable invariants of the caller. Condition *(S4)* ensures for field updates within the permitted region that the invariants broken by updating a field of class c' are included in the vulnerable invariants of the enclosing method, m.

Finally, *(S5)* establishes conditions for subclasses. An overriding method m in a subclass c may expect fewer invariants than the overridden m in superclass c'. Moreover, the subclass method must leave less invariants broken than the superclass method.

To further motivate the well-structured requirements of Def. 5, let us refer back to Fig. 8. *(S1)* ensures that by proving $\mathbb{B}$ before making the subcall, we can safely make move (2) and reach a valid visible state at the beginning of the subcall. *(S2)* ensures that by proving $\mathbb{E}$ at the end of both method bodies we can make moves (4) and (6) and reach a valid visible state at the end of both calls. Constraint *(S3)* ensures that when the caller resumes control after the subcall, we can make move (5) and reach a state where the invariants $\mathbb{X}1 \setminus \mathbb{V}1$ hold. *(S4)* guarantees that the invariants $\mathbb{X} \setminus \mathbb{V}$ always hold for any call by ensuring that $\mathbb{V}$ is an adequate upper limit for the effect of a call. Finally, but crucially, *(S5)* permits static analysis of the relationship between $\mathbb{X}$ and $\mathbb{V}$ in the presence of dynamic dispatch of subclass methods because static property information of what is expected to hold at the visible states and what are the residue vulnerable invariants of the superclass imply the corresponding properties for the same method in the subclass.

The five conditions from Def. 5 guarantee soundness, as stated in Def. 4.

Theorem 6 (Soundness For Visible-States Verification Techniques) *A well-structured verification technique built on top of a PL with a sound type system is sound.*

This theorem is one of our main results. It reduces the complex task of proving soundness of a verification technique to checking five fairly simple conditions.

5.1 Proof of Soundness Theorem

The proof of Theorem 6 uses a number of lemmas we briefly discuss here. For a start, we require to show the correspondence between well-verified source expressions (Fig. 6) and well-verified runtime expressions (Fig. 7).

Lemma 1 (Substitution/Instantiation).

$$\Gamma \vdash_{\mathcal{V}} e, \ \ \Gamma \vdash h, \sigma \ \Rightarrow \ h \vdash_{\mathcal{V}} \sigma \cdot e$$

Proof. By induction on the derivation of $\Gamma \vdash_{\mathcal{V}} e$.

We also require the following lemma stating that the adaptation operation is adequate and monotonic.

Lemma 2 (Adaptation Correspondence).

1. $h \vdash \sigma{\cdot}\iota : \mathbb{r}, \_ \;\Rightarrow\; [\![\mathbb{p}]\!]_{h,\iota} \subseteq [\![\mathbb{r} \rhd \mathbb{p}]\!]_{h,\sigma}$
2. $\mathbb{r}_1 \sqsubseteq \mathbb{r}_2 \;\Rightarrow\; \mathbb{r}_1 \rhd \mathbb{p} \subseteq \mathbb{r}_2 \rhd \mathbb{p}$
3. $\mathbb{p}_1 \sqsubseteq \mathbb{p}_2 \;\Rightarrow\; \mathbb{r} \rhd \mathbb{p}_1 \subseteq \mathbb{r} \rhd \mathbb{p}_2$

Proof. The first clause is straightforward from (T6) of Def. 15 and (P5) of Def. 11. The second and third clauses are also immediate as a result of (P5) of Def. 11.

We also require a number of lemmas dealing with the reduction rules and heap validity. For instance, the following lemma states that heaps can only grow as a result of a reduction.

Lemma 3. $e_r,\, h \longrightarrow e_r',\, h' \;\Rightarrow\; h \preceq h'$

Proof. By induction on the derivation of $e_r,\, h \longrightarrow e_r',\, h'$ and Definition 8.

The following lemma states that a well-verified runtime expression remains well-verified in an extended heap and also that well-verified values are independent of any guarding stack frame. The latter property is useful when we consider a return from a subcall in the main proof.

Lemma 4.

1. $h \vdash_{\mathcal{V}} e_r,\; h \preceq h' \;\Rightarrow\; h' \vdash_{\mathcal{V}} e_r$
2. $h \vdash_{\mathcal{V}} \sigma{\cdot}v \;\Rightarrow\; h \vdash_{\mathcal{V}} \sigma'{\cdot}v$

Heap validity depends solely on the stack frames of a runtime expression, thus evaluation contexts are non-influential.

Lemma 5 (Valid States). *If* $\mathit{stack}(e_r) = \sigma_1 \cdot \ldots \cdot \sigma_n$ *then*

1. $\sigma'{\cdot}e_r \models_{\mathcal{V}} h \;\Leftrightarrow\; \sigma'{\cdot}E[e_r] \models_{\mathcal{V}} h$

2. $$\sigma'{\cdot}e_r \models_{\mathcal{V}} h \;\Leftrightarrow\; \begin{cases} h \models \mathbb{X}_{\sigma'} \setminus \mathbb{V}_{\sigma'} \setminus \mathbb{V}_{\sigma_1} \setminus \ldots \setminus \mathbb{V}_{\sigma_n} \\ h \vdash_{\mathcal{V}} \sigma' \cdot \sigma_1 \cdot \ldots \cdot \sigma_n \\ e_r \models_{\mathcal{V}} h \end{cases}$$

Proof. Immediate from Definition 3.

In order to determine the effect of updates on valid invariants in a heap we require the following lemma, which essentially states that, during a field update, no more invariants can be made broken than are described by the property $\mathbb{D}$.

Lemma 6 (Invariant Satisfaction Effect).

$$\left.\begin{array}{l} \vdash_{\mathcal{V}} P \\ h \models [\![\mathbb{p}]\!]_{h,\iota'} \\ \mathit{cls}(h,\iota) <: c' \\ \mathcal{F}(c', f, =)_{\_}, c \\ h' = \mathit{upd}(h, \iota, f, v) \end{array}\right\} \Rightarrow\; h' \models [\![\mathbb{p}]\!]_{h',\iota'} \setminus [\![\mathbb{D}_c]\!]_{h',\iota}$$

Proof. Immediate from (W2) of Def. 2

Thus for a well structured verification technique, we are guaranteed that certain invariants are unaffected by reductions.

Lemma 7 (Computation Effects). *For an arbitrary invariant set*

$$s = \{(\iota_1, c_1), \ldots, (\iota_n, c_n)\}$$

if $\mathcal{V}$ is well-structured then:

$$\left.\begin{array}{l} h \models s \\ e_r,\, h \longrightarrow e'_r,\, h' \\ \mathit{stack}(e'_r) = \sigma_1 \cdot \ldots \cdot \sigma_n \end{array}\right\} \Rightarrow\ h' \models s \setminus \mathbb{V}_{\sigma_1} \setminus \ldots \setminus \mathbb{V}_{\sigma_n}$$

Proof. By induction on the derivation of $e_r,\, h \longrightarrow e'_r,\, h'$, Lemma 6 and ($S4$) of Definition 5.

Finally, we restate the soundness theorem, Theorem 6, in full; a proof of the main cases is presented in App. B.

Theorem 9 Soundness for Visible-States Verification Techniques *If $\mathcal{V}$ is well-structured, then:*

$$\left.\begin{array}{l} \vdash_{\mathit{wf}} P,\ \ h \vdash e_r : t, \\ \vdash_{\mathcal{V}} P,\ \ e_r \models_{\mathcal{V}} h,\ \ h \vdash_{\mathcal{V}} e_r, \\ e_r,\, h \longrightarrow e'_r,\, h' \end{array}\right\} \ \Rightarrow\ \ e'_r \models_{\mathcal{V}} h',\ \ h' \vdash_{\mathcal{V}} e'_r$$

5.2 Necessity of the Soundness Conditions

Having proved that the five conditions presented are sufficient to guarantee soundness of a verification technique, it is natural to consider whether they are *necessary*, or whether they impose restrictions which are sometimes too strong. In particular, we wish to argue that if any of the conditions were *not* satisfied by a verification technique, then there would exist a program execution demonstrating that the technique is unsound.

We make here an informal argument that under certain additional (but, we argue, reasonable) extra assumptions, the soundness conditions are indeed necessary for a verification technique to be sound. Our arguments are by contradiction: we show that if one of the soundness conditions were to be violated, then there would exist a well-verified program and valid state of the program which, when executed further, would produce an invalid state. In particular, we observe that it is necessary that all invariants which may be violated during a method execution are included within the vulnerable invariants $\mathbb{V}$. Otherwise, it would be possible to write a method body in which $\mathbb{X} \setminus \mathbb{V}$ would not be guaranteed during execution, even if $\mathbb{X}$ held initially, violating the definition of valid states.

Since the workings of a verification technique are expressed by the regions $\mathbb{C}$ and $\mathbb{U}$, in order to argue necessity of the five conditions we need to make the assumption that all objects within these regions are in principle accessible by the current receiver (informally, everyone we are allowed to call can be called, and everyone we are allowed to modify can be modified). This does not seem a very strong restriction, since one could always consider a program and heap in which suitable extra fields and references exist, in order to make an object reachable. Under these assumptions, we can then argue that *(S3)* and *(S4)* describe invariants which may in principle be broken by a method execution, and therefore, by the argument of the previous paragraph, these invariants must be included in $\mathbb{V}$ for soundness, *i.e.,* these two conditions are necessary. Similarly, we need to assume that for all invariants allowed to depend on a location (the property $\mathbb{D}$), it is indeed possible for the invariant to be defined to depend on the location (which again requires that suitable reference chains can in principle exist).

More subtly, we observe that the definition of the property $\mathbb{V}$ is not constrained to *only* contain those invariants which may actually be violated by a method execution (and so it need not actually correspond to the 'vulnerable invariants'). To take an extreme example, if we consider a verification technique in which no fields may legally be modified ($\mathbb{U}$ is empty), but the property $\mathbb{V}$ is defined to contain all invariants of all objects in the heap, then there would be no possibility to actually break any invariants in verified programs, but the soundness conditions *(S1)* and *(S2)* would erroneously insist on proof obligations before and after method calls. Therefore, in order to argue completeness, we require the property that $\mathbb{V}$ is not 'too large'; it must contain exactly the invariants which may be violated by a method call. Such a definition of $\mathbb{V}$ could be obtained in terms of the other six parameters by computing the least fixpoint solution of the constraints implied by *(S4)*, *(S3)* and *(S5)*, although it would not be the case in general that a corresponding property would exist with exactly the correct interpretation, unless the language of properties were always extended with arbitrary intersections and unions. However, it does not seem unreasonable to assume that such set operations could be included by default.

If we can assume that all objects in $\mathbb{C}$ and $\mathbb{U}$ are accessible, and that $\mathbb{V}$ contains precisely the invariants which are actually potentially vulnerable to method calls, then we can argue necessity of each of the five soundness conditions. For example, suppose *(S2)*, were not satisfied for a particular program P with a method m in class c. Then, there would exist some heap h and address ι such that $[\![\mathbb{V}_{c,m}]\!]_{h,\iota} \cap [\![\mathbb{X}_{c,m}]\!]_{h,\iota} \not\subseteq [\![\mathbb{E}_{c,m}]\!]_{h,\iota}$. In particular, there must exist some address ι' and class c' such that both $(\iota', c') \in [\![\mathbb{V}_{c,m}]\!]_{h,\iota}$ and $(\iota', c') \in [\![\mathbb{X}_{c,m}]\!]_{h,\iota}$ but $(\iota', c') \notin [\![\mathbb{E}_{c,m}]\!]_{h,\iota}$. Now consider an alternative program P' in which the definitions of invariants have been suitably changed to ensure that, within heap h, the invariants $\mathbb{X}_{c,m}$ hold. Consider an execution of the method m (with no nested call-stack). By our assumptions about $\mathbb{V}$, there exists some definition of the method m which, when executed, invalidates the invariant (ι', c'); let m be suitably redefined so that it does indeed break this invariant and does not reestablish it. Furthermore, let m be guaranteed to terminate (so that we can be sure of reaching the post-state of a method execution). Then, this method definition can be verified: there is no requirement for the invariant (ι', c') to be reestablished by the method implementation, since $(\iota', c') \notin [\![\mathbb{E}_{c,m}]\!]_{h,\iota}$. When such a method execution returns, this invariant, which is within $\mathbb{X}_{c,m}$ will remain false, and so we will reach an invalid state, implying that the technique is unsound.

We can make a similar (but longer) argument for the necessity of *(S1)*, and for the second part of *(S5)* (again, using our assumption that $\mathbb{V}$ is "precise"). The first part of *(S5)* is easily shown to be necessary in the presence of overridden method definitions.

According to the arguments above, so long as we impose a stricter (but intuitive) definition of vulnerable invariants, the five soundness conditions are, in general, necessary. Our stricter definition could be imposed as part of our work, but this would remove the possibility of a verification technique describing its own explicit definition of vulnerable invariants, which is the case, *e.g.*, for *Oval* [**?**].

6 Instantiations

In this section, we instantiate our framework to describe six verification techniques from the literature, and compare their expressiveness. We also prove their soundness using Def. 5 and Theorem 6 from Sec. 5.

An optimal verification technique would allow maximal expressiveness of the invariants (*i.e.*, large $\mathbb{D}$), impose as few program restrictions as possible (*i.e.*, large $\mathbb{U}$ and $\mathbb{C}$), and require as few proof obligations as possible (*i.e.*, small $\mathbb{B}$ and $\mathbb{E}$). Obviously, these are contradictory goals, and some trade-offs need to be struck.

The first three techniques use information about classes to improve the trade-off, whereas the latter three also use information about the topology of the heap. We call them *unstructured heap* and *structured heap* techniques, respectively.

6.1 Verification Techniques for Unstructured Heaps

Unstructured heap techniques make trade-offs by using information about classes, visibility, and access paths used in definitions of invariants. The instantiations are summarised in Fig. 10 whereby the keyword *all* denotes the set of all object invariants.

Poetzsch-Heffter. Poetzsch-Heffter [**?**] devised the first verification technique that is sound for call-backs and multi-object invariants. His technique neither restricts programs nor invariants. To deal with this generality, it requires extremely strong proof obligations.

The absence of restrictions is reflected by the regions and properties needed to model Poetzsch-Heffter's technique. We define a singleton region set $\mathbf{R} = \{\mathsf{any}\}$ and a singleton property set $\mathbf{P} = \{\mathsf{any}\}$ with interpretations respectively as: $[\![\mathsf{any}]\!]_{h,\iota} = dom(h)$ and $[\![\mathsf{any}]\!]_{h,\iota} = all$.

	Poetzsch-Heffter	Huizing & Kuiper	Leavens & Müller
$\mathbb{X}_c$	any	any	any
$\mathbb{V}_{c,m}$	any	vul$\langle c\rangle$	any$\langle c\rangle$
$\mathbb{D}_c$	any	vul$\langle c\rangle$	self$\langle c\rangle$
$\mathbb{B}_{c,m,\mathtt{r}}$	any	vul$\langle c\rangle$	any$\langle c\rangle$
$\mathbb{E}_{c,m,c'}$	any	vul$\langle c\rangle$	any$\langle c\rangle$
$\mathbb{U}_{c,m,c'}$	any	self	any if visF(c', c) empotherwise
$\mathbb{C}_{c,m,c',m'}$	any	any	any

Fig. 10. Verification techniques for unstructured heaps.

As shown in Fig. 10, this technique requires all invariants to hold in visible states. It does not restrict invariants; $\mathbb{D}$ allows a field update to affect any invariant. $\mathbb{U}$ and $\mathbb{C}$ permit arbitrary receivers for field updates and method calls. Consequently, any invariant is vulnerable to each method. This requires proof obligations for all invariants before method calls (to handle call-backs) and at the end of the method.

Huizing & Kuiper. Huizing and Kuiper's technique [**?**] is almost as liberal as Poetzsch-Heffter's, but imposes fewer proof obligations. It achieves this by determining syntactically for each field the set of invariants that are potentially invalidated by updating the field. Proof obligations are imposed only for those vulnerable invariants.

We define the region set $\mathbf{R} = \{\mathsf{self}, \mathsf{any}\}$ with the interpretation $[\![\mathsf{self}]\!]_{h,\iota} = \{\iota\}$ and $[\![\mathsf{any}]\!]_{h,\iota} = \mathit{dom}(h)$. The region self is used to restrict the receivers of field updates to **this** (see Fig. 10). The concept of vulnerability is captured by the property set $\mathbf{P} = \{\mathsf{vul}\langle\mathbf{c}\rangle, \mathsf{any}\}$ with the following interpretation:

$$\begin{aligned}
[\![\mathsf{vul}\langle c\rangle]\!]_{h,\iota} = {} & \{(\iota', c') \mid \text{the invariant of } c' \text{ contains an expression} \\
& \textbf{this}\,.g_1 \ldots g_n.f \ (n \geq 0) \text{ where } \mathcal{F}(c, f) = \_, \_ \wedge \\
& \mathit{fld}(h, \mathit{fld}(h, \mathit{fld}(h, \iota', g_1), \ldots), g_n) = \iota\} \cup \\
& \{(\iota, c') \mid \mathit{cls}(h, \iota) <: c'\} \\
[\![\mathsf{any}]\!]_{h,\iota} = {} & \mathit{all}
\end{aligned}$$

Given an address ι and a class c, the set of vulnerable invariants contains the invariants of all client objects ι' of ι that refer to a field f of c via an access path $g_1 \ldots g_n$, as well as all invariants of ι. The interpretation shows that this technique inspects client invariants syntactically to determine whether they are vulnerable or not.

As shown in Fig. 10, this technique requires all invariants to hold in visible states. It does not restrict invariants; therefore, $\mathbb{D}$ describes exactly the set of vulnerable invariants. These invariants are vulnerable to each method and must be proven before method calls and at the end of each method.

Formalizing Huizing and Kuiper's technique in our framework reveals that it is very similar to Poetzsch-Heffter's. The main difference is that the former technique uses a syntactic analysis and restricts field updates to reduce proof obligations.

Leavens & Müller. Leavens and Müller [**?**] studied information hiding in interface specifications, based on the notion of visibility defined by access control of the programming language. For instance in Java, private field are visible only within their class. Their technique allows classes to declare several invariants and to specify the visibility of these invariants.

Since our formalization does not cover the visibility of fields and assumes exactly one invariant per class, we model a special case of Leavens and Müller's technique. We assume that all fields of a class have the same visibility. The predicate $\mathsf{visF}(c', c)$ yields whether the fields declared in class c' are visible in class c. We assume that each class declares exactly one invariant and specifies its visibility. The predicate $\mathsf{visI}(c', c)$ yields whether the invariant declared in class c' is visible in class c. A generalization is possible, but does not provide any deeper insights.

We define the region set $\mathbf{R} = \{\mathsf{emp}, \mathsf{any}\}$ with the interpretation $[\![\mathsf{emp}]\!]_{h,\iota} = \emptyset$ and $[\![\mathsf{any}]\!]_{h,\iota} = dom(h)$. This technique permits field updates on arbitrary receivers as long as the field is visible in the method performing the update (see Fig. 10). Method calls are not restricted. The visibility of invariants is captured by the property set $\mathbf{P} = \{\mathsf{any}, \mathsf{self}\langle\mathbf{c}\rangle, \mathsf{any}\langle\mathbf{c}\rangle\}$ with the following interpretation:

$$[\![\mathsf{any}]\!]_{h,\iota} = all \qquad [\![\mathsf{any}\langle c\rangle]\!]_{h,\iota} = \{(\iota', c') \mid \mathsf{visI}(c', c)\}$$
$$[\![\mathsf{self}\langle c\rangle]\!]_{h,\iota} = \{(\iota, c) \mid \forall c'.\mathsf{visF}(c, c') \Leftrightarrow \mathsf{visI}(c, c')\}$$

$\mathbb{D}$ allows invariants to depend on fields of the same object declared in the same class, provided that the invariant is visible wherever the field is. This requirement enforces that any method that potentially breaks an invariant can see it and, thus, reestablish it. This requirement is very restrictive, as it disallows multi-object invariants and prevents invariants from depending on inherited fields.

The technique guarantees that only visible invariants are vulnerable; therefore, only visible invariants need to be proven before method calls and at the end of methods. It also supports helper methods, which we omit here for brevity.

Comparison. We compare invariant restrictions, program restrictions, and proof obligations.

Invariant Restrictions ($\mathbb{D}$). Poetzsch-Heffter allows invariants to depend on arbitrary locations, in particular, his technique supports multi-object invariants. Huizing and Kuiper require for multi-object invariants the existence of an access path from the object containing the invariant to the object it depends on. This excludes, for instance, universal quantifications over objects. Leavens and Müller focus on invariants of single objects, and address the subclass challenge by disallowing dependencies on inherited fields.

Program Restrictions ($\mathbb{U}$ and $\mathbb{C}$). All three techniques permit arbitrary method calls. Huizing and Kuiper restrict field updates to the receiver **this**. Leavens and Müller require the updated field to be visible; a requirement enforced by the type system anyway, thus they are not limiting expressiveness.

Proof Obligations ($\mathbb{B}$ and $\mathbb{E}$). Both Poetzsch-Heffter and Huizing and Kuiper impose proof obligations for invariants of essentially all classes of a program (even though Huizing and Kuiper use a syntactic analysis to exclude invariants that are not vulnerable). This makes both techniques highly non-modular. Leavens and Müller's technique requires proof obligations only for visible invariants, which makes this technique modular.

Lemma 8 (Well-Structuredness of Verification Techniques for Unstructured Heaps). *The techniques by Poetzsch-Heffter, by Huizing and Kuiper, and by Leavens and Müller are well-structured.*

Proof. We here outline the proof for Poetzsch-Heffter's technique and leave the proof of the remaining two for the interested reader. According to Def. 5, the components of Poetzsch-Heffter's verification technique, given earlier in Fig. 10, have to satisfy the following 5 criteria:

(S1): From $(\mathbb{r} \triangleright \mathbb{X}_{c',m'}) \setminus (\mathbb{X}_{c,m} \setminus \mathbb{V}_{c,m}) \subseteq \mathbb{B}_{c,m,\mathbb{r}}$ we get:

$$\begin{array}{rll} & \mathsf{any} \triangleright \mathsf{any} \setminus (\mathsf{any} \setminus \mathsf{any}) & \subseteq \mathsf{any} \\ \Leftrightarrow & \mathsf{any} \triangleright \mathsf{any} \setminus \emptyset & \subseteq \mathsf{any} \\ \Leftrightarrow & \mathsf{any} & \subseteq \mathsf{any} \end{array}$$

(S2): From $\mathbb{V}_{c,m} \cap \mathbb{X}_{c,m} \subseteq \mathbb{E}_{c,m}$ we get:

$$\begin{array}{rr} & \mathsf{any} \cap \mathsf{any} \subseteq \mathsf{any} \\ \Leftrightarrow & \mathsf{any} \subseteq \mathsf{any} \end{array}$$

(S3): From $\mathbb{C}_{c,m,c',m'} \triangleright (\mathbb{V}_{c',m'} \setminus \mathbb{E}_{c',m'}) \subseteq \mathbb{V}_{c,m}$ we get:

$$\begin{array}{rr} & \mathsf{any} \triangleright (\mathsf{any} \setminus \mathsf{any}) \subseteq \mathsf{any} \\ \Leftrightarrow & \mathsf{any} \triangleright \emptyset \subseteq \mathsf{any} \\ \Leftrightarrow & \emptyset \subseteq \mathsf{any} \end{array}$$

(S4): From $\mathbb{U}_{c,m,c'} \triangleright \mathbb{D}_{c'} \subseteq \mathbb{V}_{c,m}$ we get:

$$\begin{array}{rr} & \mathsf{any} \triangleright \mathsf{any} \subseteq \mathsf{any} \\ \Leftrightarrow & \mathsf{any} \subseteq \mathsf{any} \end{array}$$

(S5): For $c' <: c$, from $\mathbb{X}_{c',m} \subseteq \mathbb{X}_{c,m}$ we get

$$\mathsf{any} \subseteq \mathsf{any}$$

and from $\mathbb{V}_{c',m} \setminus \mathbb{E}_{c',m} \subseteq \mathbb{V}_{c,m} \setminus \mathbb{E}_{c,m}$ we get

$$\mathsf{any} \setminus \mathsf{any} \subseteq \mathsf{any} \setminus \mathsf{any}$$

6.2 Verification Techniques for Structured Heaps

We consider three techniques which strike a better trade-off by using the heap topology enforced by ownership types, and summarise them in Fig. 11.

To sharpen our discussion wrt. structured heaps, we will be adding annotations to the example from Fig. 2, to obtain a topology where the Person p owns the Account a and the DebitCard d.

	Müller *et al.* (*OT*)	Müller *et al.* (*VT*)	Lu *et al.*(*Oval*)
$\mathbb{X}_{c,m}$	$\mathsf{own}\,;\mathsf{rep}^{+}$	$\mathsf{own}\,;\mathsf{rep}^{+}$	$\mathsf{I}\,;\mathsf{rep}^{*}$
$\mathbb{V}_{c,m}$	$\mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}$	$\mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+}$	$\mathsf{E}\,;\mathsf{own}^{*}$
$\mathbb{D}_{c}$	$\mathsf{self}\langle c\rangle \sqcup \mathsf{own}^{+}$	$\mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+}$	$\mathsf{self}\,;\mathsf{own}^{*}$
$\mathbb{B}_{c,m,\mathbb{r}}$	$\mathsf{super}\langle c\rangle$ if $\mathsf{intrsPeer}(\mathbb{r})$ emp otherwise	$\mathsf{peer}\langle c\rangle$ if $\mathsf{intrsPeer}(\mathbb{r})$ emp otherwise	emp
$\mathbb{E}_{c,m}$	$\mathsf{super}\langle c\rangle$	$\mathsf{peer}\langle c\rangle$	self if $\mathsf{I}=\mathsf{E}$ emp otherwise
$\mathbb{U}_{c,m,c'}$	self	peer	self if $\mathsf{I}=\mathsf{E}$ emp otherwise
$\mathbb{C}_{c,m,c',m'}$	$\mathsf{rep}\langle c\rangle \sqcup \mathsf{peer}$	$\mathsf{rep}\langle c\rangle \sqcup \mathsf{peer}$	$\bigsqcup_{\mathbb{r},\ \text{with } \mathsf{SC}(\mathsf{I},\mathsf{E},\mathsf{I}',\mathsf{E}',\mathcal{O}_{\mathbb{r},c})} \mathbb{r}$

Fig. 11. Components of verification techniques. For *Oval*, $\mathcal{O}_{\mathbb{r},c}$ is the owner of $\mathbb{r}$; we use shorthands $\mathsf{I} = \mathsf{I}(c, m)$, and $\mathsf{E} = \mathsf{E}(c, m)$, and $\mathsf{I}' = \mathbb{r}\,; \mathsf{I}(c', m')$, and $\mathsf{E}' = \mathbb{r}\,; \mathsf{E}(c', m')$.

Müller *et al.* Müller, Poetzsch-Heffter, and Leavens [**?**] present two techniques for multi-object invariants, called ownership technique and visibility technique (*OT* and *VT* for short). Both techniques utilise the hierarchic heap topology enforced by Universe types [**?**,**?**]. Universe types associate reference types with ownership modifiers, which specify ownership relative to the current object. The modifier **rep** expresses that an object is owned by the current object; **peer** expresses that an object has the same owner as the current object; **any** expresses that an object may have any owner.

```
class Account {            class Person {            class DebitCard {
   peer DebitCard card;       rep Account account;      peer Account acc;
   any Person holder;         ...                       ...
   ...                     }                         }
}
```

Fig. 12. Universe modifiers for the Account example from Fig. 2.

Both *OT* and *VT* forbid rep fields f and g declared in different classes c_f and c_g, of the same object o to reference the same object. This *subclass separation* is formalised elegantly by using an ownership model where each object is owned by an object-class pair [?]. In this model, the object referenced from $o.f$ is owned by (o, c_f), whereas the object referenced from $o.g$ is owned by (o, c_g). Since they have different owners, these objects must be different.

We assume a heap operation that yields the owner of an object in a heap: $\mathit{ownr} : \textsc{Hp} \times \textsc{Adr} \to \textsc{Adr} \times \textsc{Cls}$. The set of regions is:

$$\mathbb{r} \in \mathbf{R} ::= \mathsf{emp} \mid \mathsf{self} \mid \mathsf{rep}\langle \mathbf{c} \rangle \mid \mathsf{peer} \mid \mathsf{any} \mid \mathbb{r} \sqcup \mathbb{r}$$

with the following interpretations:

$$\begin{aligned}
[\![\mathsf{self}]\!]_{h,\iota} &= \{\iota\} \qquad [\![\mathsf{any}]\!]_{h,\iota} = \mathit{dom}(h) \qquad [\![\mathsf{emp}]\!]_{h,\iota} = \emptyset \\
[\![\mathsf{rep}\langle c \rangle]\!]_{h,\iota} &= \{\iota' \mid \ \mathit{ownr}(h,\iota') = \iota\, c\} \\
[\![\mathsf{peer}]\!]_{h,\iota} &= \{\iota' \mid \ \mathit{ownr}(h,\iota') = \mathit{ownr}(h,\iota)\} \\
[\![\mathbb{r}_1 \sqcup \mathbb{r}_2]\!]_{h,\iota} &= [\![\mathbb{r}_2]\!]_{h,\iota} \cup [\![\mathbb{r}_2]\!]_{h,\iota}
\end{aligned}$$

In our framework, Universe modifiers intuitively correspond to regions, since they describe areas of the heap. For example, peer describes all objects which share the owner (object-class pair) with the current object. However, because of the subclass separation described above, it is useful to employ richer regions of the form $\mathsf{rep}\langle c \rangle$, describing all objects owned by the current object *and* class c. For regions (and properties) we also include the "union" of two regions (properties). The predicate $\mathsf{intrsPeer}(\mathbb{r})$ checks whether a region intersects the peer region.

The two techniques require a rather rich set of properties to deal with the various aspects of ownership and subclassing:

$$\mathbb{p} \in \mathbf{P} ::= \mathsf{emp} \mid \mathsf{self}\langle c \rangle \mid \mathsf{super}\langle c \rangle \mid \mathsf{peer}\langle c \rangle \mid \mathsf{rep} \mid \mathsf{own} \mid \mathsf{rep}^{+} \mid \mathsf{own}^{+} \mid \mathbb{p}; \mathbb{p}$$

with the following interpretations:

$$\begin{aligned}
[\![\mathsf{emp}]\!]_{h,\iota} &= \emptyset \qquad [\![\mathsf{self}\langle c \rangle]\!]_{h,\iota} = \{(\iota, c) \mid \ \mathit{cls}(h,\iota) <: c\} \\
[\![\mathsf{super}\langle c \rangle]\!]_{h,\iota} &= \left\{(\iota, c') \mid \ c <: c'\right\} \\
[\![\mathsf{peer}\langle c \rangle]\!]_{h,\iota} &= \left\{(\iota', c') \mid \mathit{ownr}(h,\iota') = \mathit{ownr}(h,\iota) \wedge \mathsf{vis}(c', c)\right\} \\
[\![\mathsf{rep}]\!]_{h,\iota} &= \left\{(\iota', c') \mid \mathit{ownr}(h,\iota') {=} \iota\, \_\right\} \qquad [\![\mathsf{own}]\!]_{h,\iota} {=} \{\mathit{ownr}(h,\iota)\} \\
[\![\mathbb{p}_1; \mathbb{p}_2]\!]_{h,\iota} &= \textstyle\bigcup_{(\iota',c) \in [\![\mathbb{p}_1]\!]_{h,\iota}} [\![\mathbb{p}_2]\!]_{h,\iota'} \\
[\![\mathsf{rep}^{+}]\!]_{h,\iota} &= [\![\mathsf{rep}]\!]_{h,\iota} \cup [\![\mathsf{rep}; \mathsf{rep}^{+}]\!]_{h,\iota} \\
[\![\mathsf{own}^{+}]\!]_{h,\iota} &= [\![\mathsf{own}]\!]_{h,\iota} \cup [\![\mathsf{own}; \mathsf{own}^{+}]\!]_{h,\iota}
\end{aligned}$$

Here we exploit that owners and object invariants both are object-class pairs. Therefore, we can use the owner (o, c) of an object to denote the object invariant for object o declared in class c.

For properties, $\mathsf{self}\langle c \rangle$ represents the singleton set containing a pair of the current object with the class c. The property $\mathsf{super}\langle c \rangle$ represents the set of pairs of the current object with all its classes that are superclasses of c. The property $\mathsf{peer}\langle c \rangle$ represents all the objects (paired with their classes) that share the owner with the current object, provided their class is visible in c. There are also properties to describe the invariants of an object's owned objects, its owner, its transitively owned objects, and its transitive owners. A property of the form $\mathbb{p}_1; \mathbb{p}_2$ denotes a composition of properties, which behaves similarly to function composition when interpreted.

Ownership Technique. As shown in Fig. 11, OT requires that in visible states, all objects owned by the owner of **this** must satisfy their invariants ($\mathbb{X}$).

Invariants are allowed to depend on fields of the object itself (at the current class), as in I1 in Fig. 2, and all its **rep** objects, as in I2. Other client invariants (such as I4 and I5) and subclass invariants that depend on inherited fields (such as I3) are not permitted. Therefore, a field update potentially affects the invariants of the modified object and of all its (transitive) owners ($\mathbb{D}$).

A method may update fields of **this** ($\mathbb{U}$). Since an updated field is declared in the enclosing class or a superclass, the invariants potentially affected by the update are those of **this** (for the enclosing class and its superclasses, which addresses the subclass challenge) as well as the invariants of the (transitive) owners of **this** ($\mathbb{V}$).

OT handles multi-object invariants by allowing invariants to depend on fields of owned objects ($\mathbb{D}$). Therefore, methods may break the invariants of the transitive owners of **this** ($\mathbb{V}$). For example, the invariant I2 of Person (Fig. 2) is legal only because account is a **rep** field (Fig. 12). Account's method withdraw need not preserve Person's invariant. This is reflected by the definition of $\mathbb{E}$: only the invariants of **this** are proven at the end of the method, while those of the transitive owners may remain broken; it is the responsibility of the owners to reestablish them, which addresses the multi-object challenge. As an example, the method spend has to reestablish Person's invariant after the call to account.withdraw.

Since the invariants of the owners of **this** might not hold, OT disallows calls except those on **rep** and **peer** references ($\mathbb{C}$). For instance, the call holder . notify () in method sendReport is not permitted because holder is in an ancestor ownership context.

The proof obligations for method calls ($\mathbb{B}$) must cover those invariants expected by the callee that are vulnerable to the caller. This intersection contains the invariant of the caller, if the caller and the callee are peers because the callee might call back; it is otherwise empty (**rep**s cannot callback their owners).

Visibility Technique. VT relaxes the restrictions of OT in two ways. First, it permits invariants of a class c to depend on fields of peer objects, provided that these invariants are visible in c ($\mathbb{D}$). Thus, VT can handle multi-object structures that are not organised hierarchically. For instance, in addition to the invariants permitted by OT, VT permits invariants I4 and I5 in Fig. 2. Visibility is transitive, and so the invariant must also be visible wherever fields of c are updated. Second, VT permits field updates on peers of **this** ($\mathbb{U}$).

These relaxations make more invariants vulnerable. Therefore, $\mathbb{V}$ includes additionally the invariants of the peers of **this**. This addition is also reflected in the proof obligations before peer calls ($\mathbb{B}$) and before the end of a method ($\mathbb{E}$). For instance, method withdraw must be proven to preserve the invariant of the associated DebitCard, which does not in general succeed in our example.

Lemma 9. *OT and VT are well-structured.*

We present the proof in App. C.1.

Lu *et al.* Lu, Potter, and Xue [?] define *Oval*, a verification technique based on ownership types, which support owner parameters for classes [?], thus permitting a more precise description of the heap topology. The distinctive features of *Oval* are: (1) Expected and vulnerable invariants are specific to every method in every class through the notion of *contracts*. (2) Invariant restrictions do not take subclassing into account. (3) Proof obligations are only imposed at the end of calls. (4) To address the call-back challenge, calls are subject to "subcontracting", a requirement that guarantees that the expected and vulnerable invariants of the callee are within those of the caller.

Here we describe *Oval*', an adaptation of *Oval*, where *i*) we omit non-rep fields, a refinement whereby the invariant of the current object cannot depend on such fields (but its owners can), and *ii*) we drop the existential class parameter "*" annotation—both features enhance programming expressiveness of *Oval*, but are deemed as non-central to our analysis. *Oval*' also used different restrictions for method overriding, because the original restrictions defined in *Oval* lead to unsoundness [?], as we discuss later on. In [?] description of the *Oval* verification technique is intertwined with that of the ownership type system it is based on. However, for the presentation of *Oval*', we strive to disentangle the two.

Oval' classes have owner parameters, indicated by X, Y, and the subclass relationship is described through a judgement $c\langle\overline{X}\rangle \lhd c'\langle\overline{X'}\rangle$ defined as:

$$\frac{\texttt{class}\ c\langle\overline{X}\rangle\ \texttt{extends}\ c'\langle\overline{X'}\rangle \ldots}{\begin{array}{c} c\langle\overline{X}\rangle \lhd c\langle\overline{X}\rangle \\ c\langle\overline{X}\rangle \lhd c'\langle\overline{X'}\rangle \end{array}} \qquad \frac{\begin{array}{c} c\langle\overline{X}\rangle \lhd c'\langle\overline{X'}\rangle \\ c'\langle\overline{Y}\rangle \lhd c''\langle\overline{Y'}\rangle \end{array}}{c\langle\overline{X}\rangle \lhd c''\langle\{\overline{X'}/\overline{Y}\}\overline{Y'}\rangle}$$

where $\overline{X}$ are the disjoint formal class parameters of c in the program.

For simplicity we require that the formal class parameters are disjoint for every class. This assumption is very powerful, as, in contrast to usual systems, it allows the $\lhd$ relationship to be context independent. An *Oval* program also defines an "inside" partial order relation, $\preceq$ for parameters of the same class.

Fig. 13 shows our example in *Oval* using ownership parameters [?] to describe heap topologies. The ownership parameter o denotes the owner of the current object; p denotes the owner of o and specifies the position of holder in the hierarchy, more precisely than the **any** modifier in Universe types.

```
class Account[o,p] {                          class Person[o] {
  DebitCard⟨o⟩ card;                            Account⟨this⟩ account;
  Person⟨p⟩ holder;                             ...
  ...                                           void spend(int amount)⟨this,this⟩
  void withdraw(int amount)⟨this,this⟩            { account.withdraw(amount); }
    { ... }                                     void notify ()⟨bot,top⟩
  void sendReport()⟨bot,p⟩                        { ... }
    { ... }                                   }
}
```

Fig. 13. Ownership parameters and method contracts in *Oval*.

Method Contracts. Ownership parameters are also used to describe expected and vulnerable invariants, which are specific to each method. Every *Oval* program extends method signatures with a contract $\langle \mathsf{I}, \mathsf{E}\rangle$: the expected invariants at visible states ($\mathbb{X}$) are the invariants of the object characterised by I and all objects transitively owned by this object; the vulnerable invariants ($\mathbb{V}$) are the object at E and its transitive owners. These properties are syntactically characterised by Ls in the code (and Ks in typing rules), where:

$$L ::= \mathsf{top} \mid \mathsf{bot} \mid \mathsf{this} \mid X \qquad\qquad K ::= L \mid K ; \mathsf{rep}$$

and where X stands for the class' owner parameters. “Contexts” L, obtained from [?], are syntactic descriptions of the standard and vulnerable properties. As in [?], the type system extends L to K to described context abstraction[?], *i.e.,* objects owned by class parameters, and generalises the partial ordering $\preceq$ to K as a lattice bounded by top and bot, using rules from [?]. As in [?], the type system defines the judgement

$$c\langle\overline{K}\rangle <: c'\langle\overline{K'}\rangle \;\Leftrightarrow\; c\langle\overline{X}\rangle \lhd c'\langle\overline{X'}\rangle,\, \forall i.K'_i = \{\overline{K}/\bar{X}\}X'_i$$

As in [?], the type system also requires all classes c and methods m to satisfy

$$\mathsf{I}(c,m) \preceq \mathsf{E}(c,m) \qquad \mathsf{I}(c,m) = \mathsf{E}(c,m) \Rightarrow \mathsf{I}(c,m) = \mathsf{this} \qquad (1)$$

which guarantees that the expected and the vulnerable invariants of every method can intersect at most at the current object. Central to *Oval* is *subcontracting*, which we adopt for *Oval*'(modulo renaming).

In class Account (Fig. 13), withdraw() expects the current object and the objects it transitively owns to be valid (I=**this**) and, during execution, this method may invalidate the current object and its transitive owners (E=**this**). The contract of sendReport() does not expect any objects to be valid at visible states (I=**bot**) but may violate object p and its transitive owners (E=p).

Subcontracting. Call-backs are handled via *subcontracting*, which is defined using the order $L \preceq L'$, which expresses that at runtime the object denoted by L will be transitively owned by the object denoted by L'.

Definition 1 (Subcontracting). *To interpret Oval's subcontracting in our framework, we use* $SC(I, E, I', E', K)$*, which holds iff:*

$$I \prec E \;\Rightarrow\; I' \preceq I \qquad I = E \;\Rightarrow\; I' \prec I \qquad I' \prec E' \;\Rightarrow\; E \preceq E' \qquad I' = E' \;\Rightarrow\; E \preceq K$$

where I*,* E *characterise the caller,* I'*,* E' *characterise the callee, and* K *stands for the callee's owner.*

The first two requirements in the definition above ensure that the caller guarantees the invariant expected by the callee. The other two conditions ensure that the invariants vulnerable to the callee are also vulnerable to the caller. For instance, the call holder. notify () in method sendReport satisfies subcontracting because caller and callee do not expect any invariants, and the callee has no vulnerable invariants. In particular, the receiver of a call may be owned by any of the owners of the current receiver, provided that subcontracting is respected ($\mathbb{C}$).

Given that $\mathsf{I} \preceq \mathsf{E}$ for all well-formed methods, and that $\mathbb{B}_{c,m,\mathbb{r}} = \mathsf{emp}$, the first two requirements of subcontracting exactly give *(S1)*, while the latter two exactly give *(S3)* from Def. 5.

The heap model defines an additional operation *typ* which gives the runtime type of each object, $c\langle\bar{\iota}\rangle$ where:

$$typ(h,\iota) = c\langle\bar{\iota}\rangle \;\Rightarrow\; cls(h,\iota) = c,\; c\langle\overline{X}\rangle \lhd c'\langle\overline{X'}\rangle,\; |\bar{\iota}| = |\overline{X}|$$

The owner of ι above is ι_1. We define address runtime typing and address ownership as:

$$\begin{aligned}
h \vdash \iota : c\langle\bar{\iota}\rangle \;&\Leftrightarrow\; \begin{cases} typ(h,\iota) = c'\langle\overline{\iota'}\rangle,\; c'\langle\overline{X'}\rangle \lhd c\langle\overline{X}\rangle, \\ \forall i. \iota_i = \{\overline{\iota'}/\overline{X'}\} X_i \end{cases} \\
h \vdash \iota' \preceq \iota \;&\Leftrightarrow\; typ(h,\iota') = c\langle\iota,\bar{\iota}\rangle \\
h \vdash \iota' \preceq^* \iota \;&\Leftrightarrow\; \iota' = \iota \vee \exists \iota''. h \vdash \iota' \preceq \iota'',\; h \vdash \iota'' \preceq^* \iota
\end{aligned}$$

Regions and Properties. To express *Oval* in our framework, we define regions and properties as follows:

$$\mathbb{r} \in \mathbf{R} ::= \mathsf{emp} \mid \mathsf{self} \mid c\langle\overline{K}\rangle \mid \mathbb{r} \sqcup \mathbb{r} \qquad \mathbb{p} \in \mathbf{P} ::= \mathsf{emp} \mid \mathsf{self} \mid K \mid K;\mathsf{rep}^* \mid K;\mathsf{own}^*$$

Remark: Note that our definition of regions introduces some redundancy, because a type $t = \mathbb{r}\ c$ would have the shape, *e.g.*, `C<rep,o2> C`. This redundancy is harmless.

The interpretation for regions and properties is based on the interpretation of extended contexts:

$$\begin{aligned}
\{\!|\mathsf{top}|\!\}_{h,\iota} = \{\!|\mathsf{bot}|\!\}_{h,\iota} &= \emptyset \qquad\qquad \{\!|\mathsf{this}|\!\}_{h,\iota} = \{\iota\} \\
\{\!|X|\!\}_{h,\iota} &= \left\{\iota'_i \mid h \vdash \iota : c\langle\bar{\iota}\rangle,\; c\langle\overline{X}\rangle \lhd \_,\; X = X_i\right\} \\
\{\!|K;\mathsf{rep}|\!\}_{h,\iota} &= \{\iota' \mid \iota'' \in \{\!|oEffK|\!\}_{h,\iota},\; h \vdash \iota' \preceq \iota''\}
\end{aligned}$$

The interpretation of regions is:

$$\begin{aligned}
&[\![\mathsf{emp}]\!]_{h,\iota} = \emptyset \qquad [\![\mathsf{this}]\!]_{h,\iota} = \{\iota\} \qquad [\![\mathbb{r} \sqcup \mathbb{r}']\!]_{h,\iota} = [\![\mathbb{r}]\!]_{h,\iota} \cup [\![\mathbb{r}']\!]_{h,\iota} \\
&[\![c\langle\overline{K}\rangle]\!]_{h,\iota} = \{\iota' \mid h \vdash \iota' : c\langle\bar{\iota}\rangle, \forall i.\, \iota_i \in \{\!|K_i|\!\}_{h,\iota}\}
\end{aligned}$$

The interpretation for properties is as follows:

$$\begin{aligned}
[\![\mathsf{emp}]\!]_{h,\iota} &= [\![\mathsf{top}]\!]_{h,\iota} = [\![\mathsf{bot}]\!]_{h,\iota} = \emptyset \\
[\![K]\!]_{h,\iota} &= \{(\iota',c) \mid \iota' \in \{\!|K|\!\}_{h.\iota},\; cls(h,\iota') <: c\} \\
[\![K;\mathbb{p}]\!]_{h,\iota} &= \begin{cases} all \quad K = \mathsf{top}, \mathbb{p} = \mathsf{rep}^* \;\vee K = \mathsf{bot}, \mathbb{p} = \mathsf{own}^* \\ \bigcup_{(\iota',c) \in [\![K]\!]_{h,\iota}} [\![\mathbb{p}]\!]_{h,\iota'} \quad \mathbb{p} \in \{\mathsf{rep}^*, \mathsf{own}^*\} \end{cases} \\
[\![\mathsf{rep}^*]\!]_{h,\iota} &= \{\iota' \mid h \vdash \iota' \preceq^* \iota\} \qquad [\![\mathsf{own}^*]\!]_{h,\iota} = \{\iota' \mid h \vdash \iota \preceq^* \iota'\}
\end{aligned}$$

$$\frac{}{\mathsf{emp} \sqsubseteq \mathbb{r}} \qquad \frac{c\langle\overline{X}\rangle \lhd c\langle\overline{X'}\rangle}{c\langle\overline{K}\rangle \sqsubseteq c\langle\{\overline{K}/\overline{X}\}\overline{X'}\rangle} \qquad \frac{}{\mathbb{r} \sqsubseteq \mathbb{r} \sqcup \mathbb{r}'}$$

$$\frac{}{K \sqsubseteq K\,;\mathsf{rep}^*} \qquad \frac{}{K \sqsubseteq K\,;\mathsf{own}^*}$$

$$\frac{K \preceq K'}{K\,;\mathsf{rep}^* \sqsubseteq K'\,;\mathsf{rep}^*} \qquad \frac{K \preceq K'}{K'\,;\mathsf{own}^* \sqsubseteq K\,;\mathsf{own}^*}$$

Fig. 14. The $\sqsubseteq$ relation for *Oval*'.

Based on the ordering $\preceq$, we define the reflexive and transitive judgement $\sqsubseteq$ for regions and properties in Fig. 14. Based on the viewpoint type adaptation of the *Oval* type system[**?**] we define the "adaptation" operation **;** between regions and contexts L, returning extended contexts K:

$$\mathbb{r}\,;L = \begin{cases} L & \text{if } \mathbb{r} = \mathsf{this} \\ K_i & \text{if } \mathbb{r} = c\langle\overline{K}\rangle, L = X_i \\ K_1\,;\mathsf{rep} & \text{if } \mathbb{r} = c\langle\overline{K}\rangle, L = \mathsf{this} \\ \bot & \text{otherwise} \end{cases}$$

from which we define the viewpoint adaptation operation

$$\mathbb{r} \rhd \mathbb{p} = \begin{cases} \mathsf{emp} & \mathbb{r} = \mathsf{emp} \vee \mathbb{p} = \mathsf{emp} \\ \mathbb{p} & \mathbb{r} = \mathsf{this} \\ \mathbb{p}_1 \sqcup \mathbb{p}_2 & \mathbb{r} = \mathbb{r}_1 \sqcup \mathbb{r}_2,\ \mathbb{p}_i = \mathbb{r}_i \rhd \mathbb{p} \\ K_1;\mathsf{rep} & \mathbb{r} = c\langle\overline{K}\rangle,\ \mathbb{p} = \mathsf{this} \\ K_i & \mathbb{r} = c\langle\overline{K}\rangle,\ \mathbb{p} = X_i \\ (\mathbb{r};K);\mathbb{p}' & \mathbb{r} = c\langle\overline{K}\rangle,\ \mathbb{p} = K;\mathbb{p}',\ \mathbb{p}' \in \{\mathsf{rep}, \mathsf{rep}^*, \mathsf{own}^*\} \end{cases}$$

As already stated, expected and vulnerable properties depend on the contract of the method and express $\mathbb{X}$ as $\mathsf{I}\,;\mathsf{rep}^*$ and $\mathbb{V}$ as $\mathsf{E}\,;\mathsf{own}^*$ (see Fig. 11). Similarly to *OT*, invariant dependencies are restricted to an object and the objects it transitively owns ($\mathbb{D}$). Therefore, I1 and I4 are legal, as well as I3, which depends on an inherited field. *Oval* imposes a restriction on contracts that the expected and vulnerable invariants of every method intersect at most at this. Consequently, at the end of a method, one has to prove the invariant of the current receiver, if $\mathsf{I} = \mathsf{E} = \mathsf{this}$, and nothing otherwise ($\mathbb{E}$). In the former case, the method is allowed to update fields of its receiver; no updates are allowed otherwise ($\mathbb{U}$). Therefore, spend and withdraw are the only methods in our example that are allowed to make field updates. *Oval* does not impose proof obligations on method calls ($\mathbb{B}$ is empty), but addresses the call-back challenge through subcontracting. Therefore, call-backs are safe because the callee cannot expect invariants that are temporarily broken. With the existing contracts in Fig. 13, subcontracting permits spend to call account.withdraw(), and withdraw to call **this**.sendReport(), and also sendReport to call holder . notify (). The last two subcalls may potentially lead to callbacks, but are safe because the contracts of sendReport and notify do not expect the receiver to be in a valid state (I=**bot**).

Subclassing and Subcontracting. *Oval* also requires subcontracting between a superclass method and an overriding subclass method. As we discuss later, this does not guarantee soundness [?], and we found a counterexample (*cf.* Sec. 5). Therefore, we have improved these conditions, and require that a subclass expects no more than the superclass, and vice versa for vulnerable invariants, and that if an expected invariant in the superclass is vulnerable in the subclass, then it must also be expected in the subclass:

$$\mathsf{I}' \preceq \mathsf{I} \preceq \mathsf{E} \preceq \mathsf{E}' \qquad \mathsf{I} = \mathsf{E}' \Rightarrow \mathsf{I}' = \mathsf{E}'$$

where I, E, I', E' characterise the superclass, resp. subclass, method. This requirement gives exactly *(S5)* from Def. 5. It allows $\mathsf{I}' = \mathsf{I} = \mathsf{E} = \mathsf{E}'$ which is forbidden in *Oval*. We use the above requirement for *Oval*$'$.

Results.

Lemma 10. *Oval' is a programming language in the sense of Def. 11. Also, Oval' has a sound type system in the sense of Def. 15.*

Remark: Note also, that usually in ownership type systems, and indeed in most systems with parameterised classes, the field and method lookup functions, $\mathcal{F}$, $\mathcal{M}$ and $\mathcal{B}$ are defined on types, rather than classes. For instance, one would expect to have $\mathcal{F}(c\langle o1, o2\rangle, f)$ rather than $\mathcal{F}(c, f)$ as in our framework. In contrast, in our framework, these functions are defined on classes. Namely, as we have requested the owner parameters to be disjoint across different classes, the meaning of. *e.g.,* $\mathcal{F}(c, f)$ is, implicitly that of $\mathcal{F}(c\langle c1, c2\rangle, f)$ where $c1, c2$ are the formal ownership parameters of class c.

Furthermore, in contrast to usual practice in ownership types, and parameterised classes, the type of an inherited field (or method) remains the same (as required in Def. 11, part *F2* and *F3* of Def. 14. Again, because the owner parameters are disjoint across classes, we can make this simplification. For example, for

```
class C<c1>{  A<c1> f; }
class D<d1> extends C<c1> { }
```

we would have that $\mathcal{F}(\mathsf{C}, \mathsf{f})$=$\mathcal{F}(\mathsf{D}, \mathsf{f})$=`A<c1> A`.

Our framework does not require the underlying type system of the programming language to be expressed in terms of the functions $\mathcal{F}$ and $\mathcal{M}$. Nevertheless, the underlying type system *could* be expressed in terms of these functions. For example, for field access, we would have the underlying type system rule:

$$\frac{\Gamma \vdash e : \mathbb{r}\ c \qquad \mathcal{F}(c, f) = \mathbb{r}'\ c'}{\Gamma \vdash e.f : (\mathbb{r} \rhd \mathbb{r}')\ c'}$$

where we define $\mathcal{R}$, the owner parameter extraction function so that it extracts all owner parameters out of a context sequence, *i.e.,* $\mathcal{R}(\mathsf{top}) = \mathcal{R}(\mathsf{bot}) = \mathcal{R}(\mathsf{this}) = \epsilon$,

$\mathcal{R}(X) = X$, $\mathcal{R}(K; \mathsf{rep}) = \mathcal{R}(K)$, and where $\mathcal{R}(K, \overline{K}) = \mathcal{R}(K), \mathcal{R}(\overline{K})$, and where the formal parameters of a class are defined through $OP(c){=}\overline{X}$ iff class c has formal owner parameters $\overline{X}$, and where we define the region adaptation operator $\triangleright$ as follows:

$$c\langle\overline{K}\rangle \triangleright c'\langle\overline{K'}\rangle = \begin{cases} c'\langle\overline{K'}\rangle & \text{if } \mathcal{R}(\overline{K'}) = \epsilon \\ c'\langle\overline{[\overline{K}/\overline{X''}]K'}\rangle & \text{if } c\langle\overline{X}\rangle \lhd c''\langle\overline{X''}\rangle \\ & \quad \text{and } \ \mathcal{R}(\overline{K'}) \subseteq OP(c'') \\ \perp & \text{otherwise} \end{cases}$$

For example `D<o3>` $\triangleright$ `A<c1>` $=$ `A<o3>`.

We define owner extraction function $\mathcal{O}$ as follows

$$\mathcal{O}_{\mathbb{r},c} = \begin{cases} K_1, & \text{if } \ \mathbb{r} = c\langle\overline{K}\rangle \\ X_1, & \text{if } \ \mathbb{r} = \mathsf{this}, \ c\langle\overline{X}\rangle \lhd \_ \\ \perp & \text{otherwise} \end{cases}$$

These functions are used to describe the *Oval'* verification technique, as shown in Fig. 11.

Lemma 11. *Oval' is well-structured.*

We present the proof in App. C.2.

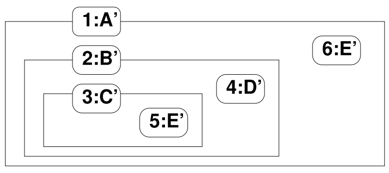


Fig. 15. Heap h_0, with objects at addresses 1–6 belonging to indicated classes. Objects atop a box own those inside it. Assume that A' is a subclass of A and analogously for the other classes.

Comparisons. We first illustrate differences between the techniques for structured heaps using the heap h_0 in Fig. 15. Fig. 16 shows the values of the components of the three techniques for class C and object 3.

OT and *VT* require knowledge of the class at which on object is owned; this information is shown in the last row of Fig. 16. For Oval, the methods have I and E as given in the last row.

Invariant Restrictions ($\mathbb{D}$). All three techniques support multi-object invariants. Both *OT* and Oval achieve this by permitting the invariant of an object o to depend on fields of o and of objects (transitively) owned by o. However, *OT* requires that fields of o are declared in the same class as the invariant to address

	Müller *et al.* (*OT*)	Müller *et al.* (*VT*)	Lu *et al.*
1. $[\![\mathbb{X}_{\mathsf{C,m}}]\!]_{h_0,3}$	{ (4, D), (4, D'), (3, C), (3, C'), (5, E), (5, E') }	{ (4, D), (4, D'), (3, C), (3, C'), (5, E), (5, E') }	{ (3, C), (3, C'), (5, E), (5, E') }
2. $[\![\mathbb{V}_{\mathsf{C,m}}]\!]_{h_0,3}$	{ (3, C), (2, B), (1, A') }	{ (3, C), (2, B), (1, A'), (4, D) }	{ (2, B), (2, B'), (1, A), (1, A') }
3. $[\![\mathbb{D}_{\mathsf{C}}]\!]_{h_0,3}$	{ (3, C), (2, B), (1, A') }	{ (3, C), (2, B), (1, A'), (4, D) }	{ (3, C), (3, C'), (2, B), (2, B'), (1, A), (1, A') }
4. $[\![\mathbb{B}_{\mathsf{C,m},\mathbb{r}}]\!]_{h_0,3}$	$\emptyset$ if $\mathbb{r}$ = rep⟨C⟩ { (3, C) } if $\mathbb{r}$ = peer	$\emptyset$ if $\mathbb{r}$ = rep⟨C⟩ { (3, C), (4, D) } if $\mathbb{r}$ = peer	$\emptyset$
5*a*. $[\![\mathbb{E}_{\mathsf{C,m}}]\!]_{h_0,3}$	{ (3, C) }	{ (3, C), (4, D) }	$\emptyset$
5*b*. $[\![\mathbb{E}_{\mathsf{C,m1}}]\!]_{h_0,3}$	{ (3, C) }	{ (3, C), (4, D) }	{ (3, C), (3,C') }
6*a*. $[\![\mathbb{U}_{\mathsf{C,m,Objct}}]\!]_{h_0,3}$	{ 3 }	{ 3, 4 }	$\emptyset$
6*b*. $[\![\mathbb{U}_{\mathsf{C,m1,Objct}}]\!]_{h_0,3}$	{ 3 }	{ 3, 4 }	{ 3 }
7. $[\![\mathbb{C}_{\mathsf{C,m,Objct,m2}}]\!]_{h_0,3}$	{ 3, 4, 5 }	{ 3, 4, 5 }	{ 1, 2, 3, 4, 5, 6 }
assuming that	C::m not pure $ownr(h_0, 5)$ = 3, C', $ownr(h_0, 3)$ = 2, B, $ownr(h_0, 4)$ = 2, B', $ownr(h_0, 2)$ = 1, A	C::m not pure $ownr(h_0, 5)$ = 3, C', $ownr(h_0, 3)$ = 2, B, $ownr(h_0, 4)$ = 2, B', $ownr(h_0, 2)$ = 1, A vis(C, D), ¬vis(C, D')	I(C, m) = this E(C, m) = X, and X maps to 2 I(C, m1) = E(C, m1) = this I(Obj, m2) = bot E(Obj, m2) = top

Fig. 16. Comparison of techniques for structured heaps; differences are highlighted in grey.

the subclass challenge. For instance, $\mathbb{D}$ for *OT* does not include (3,C'), whereas $\mathbb{D}$ for Oval does.

In addition, *VT* allows dependencies on peers (therefore, $\mathbb{D}$ includes (4,D)) and thus can handle multi-object structures that are not organised hierarchically.

Program Restrictions ($\mathbb{U}$ and $\mathbb{C}$). In *OT* and Oval, an object may only modify its own fields, whereas *VT* also allows modifications of peers; thus, object 4 is part of $\mathbb{U}$ for *VT*. In Oval, an object may only modify its own fields if the I, E annotations are this; this is why $\mathbb{U}$ is empty for m but contains 3 for m1.

Method calls in *OT* and *VT* are restricted to the peers and reps of an object; thus, a call on a rep object o cannot call back into one of o's (transitive) owners, whose invariants might not hold.

In Oval, the receiver of a method call may be *anywhere* within the owners of the current receiver, provided that the I and E annotations of the called method satisfy the subcontract requirement. Therefore, $\mathbb{C}$ for Oval includes for instance object 2, which is not permitted in *OT* and *VT*.

Proof Obligations ($\mathbb{B}$ and $\mathbb{E}$). Since *OT* uses rather restricted invariants, it has a small vulnerable set $\mathbb{V}$ and, thus, few proof obligations. The dependencies on peers permitted by *VT* lead to a larger vulnerable set and more proof obligations. For instance, (4,D) is part of the vulnerable set $\mathbb{V}$ (because executions on 3 might break 4's D-invariant). Hence, of the proof obligations $\mathbb{B}$ and $\mathbb{E}$.

Oval imposes end-of-body proof obligation only when I and E are the same (*i.e.*, m1). Since Oval permits invariants to depend on inherited fields, it requires proof obligations for subclass invariants. For instance, (3,C') is part of $\mathbb{E}$ for m1. *OT* and *VT* disallow such dependencies and their proof obligations do not include (3,C'). This restriction is important for modularity.[5] Oval never impose proof obligations before method calls ($\mathbb{B}$ is empty), and prevents potentially dangerous call-backs through the subcontract requirement.

Implications for our Example. As an alternative comparison, we consider the application of the three techniques to our running example (Fig. 2).

1. *Invariant semantics:* In *OT* and *VT*, the invariants expected at the beginning of withdraw are I1, I2, and I3 for the receiver, as well as I5 for the associated DebitCard (which is a **peer**). For withdraw in *Oval*, I=this, therefore the expected invariants are I1, I2, and I3 for the receiver.
2. *Invariant restrictions:* Invariants I2 and I5 are illegal in *OT* and *Oval*, while they are legal in *VT* (which allows invariants to depend on the fields of **peer**s). Conversely, I3 is illegal in *OT* and *VT* (it mentions a field from a superclass), while it is legal in *Oval*.
3. *Proof obligations:* In *OT*, before the call to **this**.sendReport() and at the end of the body of withdraw, we have to establish I1 and I2 for the receiver. In addition to these, in *VT* we have to establish I5 for the debit card. In *Oval*, the same invariants as for *OT* have to be proven, but only at the end of the method because call-backs are handled through subcontracting. In addition, I3 is required.[6] In all three techniques, withdraw is permitted to leave the invariant I4 of the owning Person object broken. It has to be re-established by the calling Person method.
4. *Program restrictions:* *OT* and *VT* forbid the call holder.notify() (**rep**s cannot call their owners), while *Oval* allows it. On the other hand, if method sendReport required an invariant of its receiver (for instance, to ensure that holder is non-null), then *Oval* would prevent method withdraw from calling it, even though the invariants of the receiver might hold at the time of the call. The proof obligations before calls in *OT* and *VT* would make such a call legal.

6.3 Soundness of Verification Techniques

Instead of proving soundness for every single verification technique discussed in this section, Theorem 6 reduces this complex task to merely checking that the seven components of every instantiations satisfy the five (fairly simple) well-structured conditions of Def. 5. Assuming that the underlying type system is sound, once we show well-structuredness for a technique, verification technique soundness (Def. 4) follows.

[5] The Oval developers plan to solve this modularity problem by requiring that any inherited method has to be re-verified in the subclass [?].

[6] This means that verification of a class requires knowledge of a subclass. The *Oval* developers plan to solve this modularity problem by requiring that any inherited method has to be re-verified in the subclass [?].

Lemma 12 (Type System Soundness for Universes). *The Universe type system satisfies Def. 15.*

Proof. The typing rules together with the soundness proof for the Universes type system has already been given in [?] bar the rules for the (novel) construct $e\ \mathsf{prv}\ \mathbb{p}$ and the exceptions verfExc and fatalExc. The typing of the proof annotation construct however depends exclusively on the typing of the subexpression e; typically this construct would be type-checked using a rule such as the one shown below. Also, the type system should type-check exceptions related to the verification technique as shown below.

$$\frac{\Gamma \vdash e : t}{\Gamma \vdash e\ \mathsf{prv}\ \mathbb{p} : t} \qquad \frac{}{\begin{array}{c}\Gamma \vdash \mathsf{verfExc} : t \\ \Gamma \vdash \mathsf{fatalExc} : t\end{array}}$$

With these additions, it is not hard to check that the type system satisfies the requirements set out by Def. 15.

Lemma 13 (Type System Soundness for *Oval'*). *The Oval' type system satisfies Def. 15.*

Proof. From [?].

Corollary 7 *The verification techniques by Poetzsch-Heffter, by Huizing & Kuiper, by Leavens & Müller, by Müller et al. (OT), by Müller et al. (VT), and Oval' are sound.*

Proof. Immediate from Theorem 6, Lemmas 12 and 13, and Lemmas 8, 9, 11.

These proofs confirm soundness claims from the literature. We found that the semi-formal arguments supporting the original soundness claims at times missed crucial steps. For instance, the soundness proofs for *OT* and *VT* [?] do not mention any condition relating to *(S3)* of Def. 5; in our formal proof, *(S3)* was vital to determine what invariants still hold after a method returns.

Unsoundness of Oval. The original *Oval* proposal [?] is unsound because it requires subcontracting for method overriding. As we said in the previous section, subcontracting corresponds to our *(S1)* and *(S3)*. This gives, for $c' <: c$, the requirements that $\mathbb{X}_{c',m'} \subseteq \mathbb{X}_{c,m} \backslash \mathbb{V}_{c,m}$, and $\mathbb{V}_{c',m'} \backslash \mathbb{E}_{c',m'} \subseteq \mathbb{V}_{c,m}$, which do not imply *(S5)*. We were alerted by this discrepancy, and using only the $\mathbb{X}$, $\mathbb{E}$ and $\mathbb{V}$ components (no type system properties, nor any other component), we constructed the following counterexample.

```
class D[o] {
   C1<this> c = new C2<this>();
   void m() <this,o> { c.mm() }
}
```

```
class C1[o]{
   void mm() <this,this> {...}
}

class C2[o] extends C1<o> {
   void mm() <bot,this> {...}
}
```

The call `c.mm()` is checked using the contract of C1::mm; it expects the callee to reestablish the invariant of the receiver (c), and is type correct. However, the body of C2::mm may break the receiver's invariants, but has no proof obligations ($\mathbb{E}_{\mathsf{C2},\mathsf{mm}}$ = emp). Thus, the call `c.mm()` might break the invariants of c, thus breaking the contract of m. The reason for this problem is, that the—initially appealing—parallel between subcontracting and method overriding does not hold. The authors confirmed our findings [?].

7 Related Work

Object invariants trace back to Hoare's implementation invariants [?] and monitor invariants [?]. They were popularised in object-oriented programming by Meyer [?]. Their work, as well as other early work on object invariants [?,?] did not address the three challenges described in the introduction. Since they were not formalised, it is difficult to understand the exact requirements and soundness arguments (see [?] for a discussion). However, once the requirements are clear, a formalisation within our framework seems straightforward.

The idea of regions and properties is inspired from type and effects systems [?], which have been extremely widely applied, *e.g.*, to support race-free programs and atomicity [?].

The verification techniques based on the Boogie methodology [?,?,?,?] do not use a visible state semantics. Instead, each method specifies in its precondition which invariants it requires. Extending our framework to Spec# requires two changes. First, even though Spec# permits methods to specify explicitly which invariants they require, the default is to require the invariants of its arguments and all their peer objects. These defaults can be modelled in our framework by allowing method-specific properties $\mathbb{X}$. Second, Spec# checks invariants at the end of expose blocks instead of the end of method bodies. Expose blocks can easily be added to our formalism.

In separation logic [?,?], object invariants are generally not as important as in other verification techniques. Instead, predicates specifying consistency criteria can be assumed/proven at *any* point in a program [?]. Abstract predicate families [?] allow one to do so without violating abstraction and information hiding. Parkinson and Bierman [?] show how to address the subclass challenge with abstract predicates. Their work as well as Chin *et al.*'s [?] allow programmers to specify which invariants a method expects and preserves, and do not require subclasses to maintain inherited invariants. The general predicates of separation logic provide more flexibility than can be expressed by our framework.

We know of two techniques based on visible states that cannot be directly expressed in our framework as defined here. Middelkoop *et al.* [?] use proof obligations that certain invariants be *preserved* by a method execution, but are not necessarily required to hold (that is, *if* they hold in the pre-state of a method, then they must hold in the post-state of a method). Similarly, Summers *et al.* recently proposed an extension of the Universe Types methodology to cater for static fields and methods [?], whose design was based on the soundness criteria of this paper. However, in order to formalise their technique they also required proof obligations concerning preserved invariants [?]. In order to formalise these techniques in our framework, we have to generalise our proof obligations to take two properties; one for the pre-state heap and one for the post-state heap. Since this generality is not needed for any of the other techniques, and would obscure the details of, *e.g.,* the soundness conditions, we omitted a formal treatment in this paper.

Some verification techniques exclude the pre- and post-states of so-called helper methods from the visible states [?,?]. Helper methods can easily be expressed in our framework by choosing different parameters for helper and non-helper methods. For instance in JML, $\mathbb{X}$, $\mathbb{B}$, and $\mathbb{E}$ are empty for helper methods, because they neither assume nor have to preserve any invariants.

Once established, strong invariants [?] hold throughout program execution. They are especially useful to reason about concurrency and security properties. Our framework can model strong invariants, essentially by preventing them from occurring in $\mathbb{V}$.

Existing techniques for visible state invariants have only limited support for object initialisation. Constructors are prevented from calling methods because the callee method in general requires all invariants to hold, but the invariant of the new object is not yet established. Fähndrich and Xia developed delayed types [?] to control call-backs into objects that are being initialised. Delayed types support strong invariants. Modelling these in our framework is future work.

8 Conclusions

We presented a framework that describes verification techniques for object invariants in terms of seven parameters and separates verification concerns from those of the underlying type system. Our formalism is parametric wrt. the type system of the programming language and the language used to describe and to prove assumptions. We illustrated the generality of our framework by instantiating it to describe three existing verification techniques. We identified sufficient conditions on the framework parameters that guarantee soundness, and we proved a universal soundness theorem. Our unified framework offers the following important advantages:

1. It allows a simpler understanding and separation of verification concerns. In particular, most of the aspects in which verification techniques differ are distilled in terms of subsets of the parameters of our framework.

2. It facilitates comparisons since relationships between parameters can be expressed at an abstract level (*e.g.*, criteria for well-structuredness in Def. 5), and the interpretations of regions and properties as sets allow formal comparisons of techniques in terms of set operations.
3. It expedites the soundness analysis of verification techniques, since checking the soundness conditions of Def. 5 is significantly simpler than developing soundness proofs from scratch.
4. It captures the design space of *sound* visible states based verification techniques.

We have already used our framework for developing verification techniques for static methods [**?**,**?**], and plan to use it to develop further, more flexible, techniques.

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A Language Definitions

Definition 8 *A runtime structure is a tuple*

$$\text{RS}\textsc{truct} \;=\; (\textsc{Hp}, \textsc{Adr}, \simeq, \preceq, \mathit{dom}, \mathit{cls}, \mathit{fld}, \mathit{upd}, \mathit{new})$$

where $\textsc{Hp}$*, and* $\textsc{Adr}$ *are sets, and where*

$$\begin{array}{lll} \simeq & \subseteq \textsc{Hp} \times \textsc{Hp} & \qquad \preceq\ \subseteq \textsc{Hp} \times \textsc{Hp} \\ \mathit{dom} & : \textsc{Hp} \to \mathcal{P}(\textsc{Adr}) & \\ \mathit{cls} & : \textsc{Hp} \times \textsc{Adr} \rightharpoonup \textsc{Cls} & \\ \mathit{fld} & : \textsc{Hp} \times \textsc{Adr} \times \textsc{Fld} \rightharpoonup \textsc{Val} & \\ \mathit{upd} & : \textsc{Hp} \times \textsc{Adr} \times \textsc{Fld} \times \textsc{Val} \to \textsc{Hp} & \\ \mathit{new} & : \textsc{Hp} \times \textsc{Adr} \times \textsc{Typ} \to \textsc{Hp} \times \textsc{Adr} & \end{array}$$

where $\textsc{Val} = \textsc{Adr} \cup \{\mathsf{null}\}$ *for some element* $\mathsf{null} \notin \textsc{Adr}$*. For all* $h \in \textsc{Hp}$*,* $\iota, \iota' \in \textsc{Adr}$*,* $v \in \textsc{Val}$*, we require:*

$$\textit{(H1)}\ \ \iota \in dom(h) \ \Rightarrow \exists c.cls(h,\iota) = c$$

$$\textit{(H2)}\ \ h \simeq h' \ \Leftrightarrow \begin{cases} dom(h) = dom(h'), \\ cls(h,\iota) = cls(h',\iota) \end{cases}$$

$$\textit{(H3)}\ \ h \preceq h' \ \Leftrightarrow \begin{cases} dom(h) \subseteq dom(h'), \\ \forall \iota \in dom(h). \\ \quad cls(h,\iota) = cls(h',\iota) \end{cases}$$

$$\textit{(H4)}\ \ upd(h,\iota,f,v) = h' \ \Rightarrow \begin{cases} h \simeq h' \\ fld(h',\iota,f) = v \\ \iota \neq \iota' \textit{ or } f \neq f' \Rightarrow \\ \quad fld(h',\iota',f') = fld(h,\iota',f') \end{cases}$$

$$\textit{(H5)}\ \ new(h,\iota,t) = h',\iota' \ \Rightarrow \begin{cases} h \preceq h' \\ \iota' \in dom(h')\backslash dom(h) \end{cases}$$

Definition 9 *A* region/property structure *is a tuple*
$\textsc{AStruct} \ = \ (\mathbf{R}, \mathbf{P}, \triangleright)$
where $\mathbf{R}$ *and* $\mathbf{P}$ *are sets, and* $\triangleright$ *is an operation with signature:*
$\triangleright \ : \ \mathbf{R} \times \mathbf{P} \to \mathbf{P}$

Definition 10 $E[\cdot]$ *and* $F[\cdot]$ *are defined as follows:*

$$\begin{aligned} E[\cdot] ::= & \ [\cdot] \mid E[\cdot].f \mid E[\cdot].f := e \mid \iota.f := E[\cdot] \mid E[\cdot].m(e) \\ & \mid \iota.m(E[\cdot]) \mid E[\cdot]\ \textsf{\textit{prv}}\ \mathbb{p} \mid \textsf{\textit{ret}}\ E[\cdot] \\ F[\cdot] ::= & \ [\cdot] \mid F[\cdot].f \mid F[\cdot].f := e \mid \iota.f := F[\cdot] \mid F[\cdot].m(e) \\ & \mid \iota.m(F[\cdot]) \mid F[\cdot]\ \textsf{\textit{prv}}\ \mathbb{p} \mid \sigma \cdot F[\cdot] \mid \textsf{\textit{call}}\ F[\cdot] \mid \textsf{\textit{ret}}\ F[\cdot] \end{aligned}$$

Definition 11 *A programming language is a tuple*
$PL \ = \ (\textsc{Prg}, \textsc{RStruct}, \textsc{AStruct})$
where $\textsc{Prg}$ *is a set where every* $P \in \textsc{Prg}$ *is a tuple*

$$P \ = \ \begin{pmatrix} \mathcal{F}, \mathcal{M}, \mathcal{B}, <: & \textit{(class definitions)} \\ \sqsubseteq, [\![\cdot]\!] & \textit{(inclusion and projections)} \\ \models, \vdash & \textit{(invariant and type satisfaction)} \end{pmatrix}$$

with signatures:

$$\begin{array}{lll} \mathcal{F} & : & \textsc{Cls} \times \textsc{Fld} \rightharpoonup \textsc{Typ} \times \textsc{Cls} \\ \mathcal{M} & : & \textsc{Cls} \times \textsc{Mthd} \rightharpoonup \textsc{Typ} \times \textsc{Typ} \\ \mathcal{B} & : & \textsc{Cls} \times \textsc{Mthd} \rightharpoonup \textsc{Expr} \times \textsc{Cls} \\ <: & \subseteq & \textsc{Cls} \times \textsc{Cls} \ \cup \ \textsc{Typ} \times \textsc{Typ} \\ \sqsubseteq & \subseteq & \mathbf{R} \times \mathbf{R} \\ [\![\cdot]\!] & : & \mathbf{R} \times \textsc{Hp} \times \textsc{Adr} \to \mathcal{P}(\textsc{Adr}) \\ [\![\cdot]\!] & : & \mathbf{P} \times \textsc{Hp} \times \textsc{Adr} \to \mathcal{P}(\textsc{Adr} \times \textsc{Cls}) \\ \models & \subseteq & \textsc{Hp} \times \textsc{Adr} \times \textsc{Cls} \\ \vdash & \subseteq & (\textsc{Env} \times \textsc{Expr} \cup \textsc{Hp} \times \textsc{Stk} \times \textsc{RExpr}) \times \textsc{Typ} \end{array}$$

where every $P \in \text{PRG}$ *must satisfy the constraints:*
(P1) $\mathcal{F}(c, f) = t, c' \;\Rightarrow c <: c'$
(P2) $\mathcal{B}(c, m) = e, c' \;\Rightarrow c <: c'$
(P3) $\mathcal{F}(cls(h, \iota), f) = t, \_ \;\Rightarrow \exists v. fld(h, \iota, f) = v$
(P4) $\mathbb{r}_1 \sqsubseteq \mathbb{r}_2 \;\Rightarrow [\![\mathbb{r}_1]\!]_{h,\iota} \subseteq [\![\mathbb{r}_2]\!]_{h,\iota}$
(P5) $[\![\mathbb{r} \triangleright \mathbb{p}]\!]_{h,\iota} \;= \bigcup_{\iota' \in [\![\mathbb{r}]\!]_{h,\iota}} [\![\mathbb{p}]\!]_{h,\iota'}$
(P6) $[\![\mathbb{r}]\!]_{h,\iota} \;\subseteq\; dom(h)$
(P7) $h \preceq h' \;\Rightarrow [\![\mathbb{p}]\!]_{h,\iota} \subseteq [\![\mathbb{p}]\!]_{h',\iota}$
(P8) $\mathbb{r}\, c <: \mathbb{r}'\, c' \;\Rightarrow\; \mathbb{r} \sqsubseteq \mathbb{r}',\; c <: c'$

Definition 12 *Stack* $\overline{\sigma}$ *is* valid *wrt. heap* h*, denoted by* $h \models \overline{\sigma}$*, iff:*

$$\overline{\sigma} = \overline{\sigma_1} \cdot \sigma \cdot \sigma' \cdot \overline{\sigma_2} \Rightarrow \begin{cases} \sigma' = (\iota, \_, c', m) \\ h, \sigma \vdash \iota : \mathbb{r}\ \_ \\ c' <: c,\; \mathbb{r} \sqsubseteq \mathbb{C}_{\sigma,c,m} \end{cases}$$

Definition 13 *The function stack* : $\text{REXPR} \rightarrow \text{STK}^*$ *yields the stack of a run-time expression:*

$$stack(E[e_r]) = \begin{cases} \sigma \cdot stack(e'_r) & \textit{if } e_r = \sigma \cdot e'_r \\ \epsilon & \textit{otherwise} \end{cases}$$

Definition 14 *For every program, the judgement:*
$\vdash_{\textit{wf}}\ :\ (\text{HP} \times \text{STK} \times \text{STK} \times \mathbf{R}) \;\cup\; (\text{ENV} \times \text{HP} \times \text{STK}) \;\cup\; \text{PRG}$
is defined as:

– $h, \sigma \vdash_{\textit{wf}} \sigma' : \mathbb{r} \quad\Leftrightarrow\quad \sigma' = (\iota, \_, \_, \_),\quad h, \sigma \vdash \iota : \mathbb{r}\ \_$

– $\Gamma \vdash_{\textit{wf}} h, \sigma \quad\Leftrightarrow\quad \begin{cases} \exists c, m, t, \iota, v. \\ \Gamma = c, m, t,\quad \sigma = (\iota, v, c, m), \\ cls(h, \iota) <: c,\quad h, \sigma \vdash_{\boldsymbol{r}} v : t \end{cases}$

– $\vdash_{\textit{wf}} P \quad\Leftrightarrow\quad \begin{cases} \textit{(F1)}\ \mathcal{M}(c, m) = t,\ t' \;\Rightarrow \\ \qquad \exists e.\ \mathcal{B}(c, m) = e, \_,\quad c, m, t \vdash e : t' \\ \textit{(F2)}\ c <: c',\ \mathcal{F}(c', f) = t, c'' \;\Rightarrow \\ \qquad \mathcal{F}(c, f) = t', c'',\ t' = t \\ \textit{(F3)}\ c <: c',\ \mathcal{M}(c, m) = t, t', \\ \qquad \mathcal{M}(c', m) = t'', t''' \;\Rightarrow \\ \qquad\quad t = t'',\ t' = t'''' \\ \textit{(F4)}\ c <: c',\ \mathcal{B}(c', m) = e', c'' \;\Rightarrow \\ \qquad \exists c'''.\ \mathcal{B}(c, m) = e, c''',\ c''' <: c'' \end{cases}$

The judgement $h, \sigma \vdash_{\mathbf{wf}} \sigma' : \mathbb{r}$ expresses that the receiver of σ' is within $\mathbb{r}$ as seen from the point of view of σ. $\Gamma \vdash_{\mathbf{wf}} h, \sigma$ expresses that h, σ respect the typing environment Γ. $\vdash_{\mathbf{wf}} P$ defines well-formed programs as those where method bodies respect their signatures *(F1)*, fields are not overridden *(F2)*, overridden methods preserve typing *(F3)*, and do not “skip superclasses” *(F4)*.

Definition 15 *A programming language* PL *has a* sound type system *if all programs* $P \in \mathrm{PL}$ *satisfy the constraints:*

$$
\begin{array}{ll}
\textit{(T1)} & \Gamma \vdash e : t,\ \ t <: t' \ \Rightarrow \Gamma \vdash e : t' \\
\textit{(T2)} & h \vdash e_r : t,\ \ t <: t' \ \Rightarrow h \vdash e_r : t' \\
\textit{(T3)} & h \vdash e_r : t,\ \ h \preceq h' \ \Rightarrow h' \vdash e_r : t \\
\textit{(T4)} & h \vdash \sigma \cdot \iota : \_\, c \ \Rightarrow \mathit{cls}(h, \iota) <: c \\
\textit{(T5)} & h \vdash \sigma \cdot \iota.m(v) : t \ \Rightarrow \begin{cases} h \vdash \sigma \cdot \iota : \mathbb{r}\, c \\ \mathcal{M}(c, m) = t', t \\ h \vdash \sigma \cdot v : t' \end{cases} \\
\textit{(T6)} & h \vdash \sigma \cdot \sigma' \cdot \textsf{\textit{ret}}\, e_r\, \textsf{\textit{prv}}\, \mathbb{p} : t \ \ \Rightarrow \ \ h \vdash \sigma' \cdot e_r : t \\
\textit{(T7)} & \sigma = (\iota, \_, \_, \_),\ h \vdash \sigma \cdot \iota' : \mathbb{r}\ \_ \ \Rightarrow \iota' \in [\![\mathbb{r}]\!]_{h,\iota} \\
\textit{(T8)} & \Gamma \vdash e : \mathbb{r}\, c,\ \ \Gamma \vdash h, \sigma \ \Rightarrow h, \sigma \vdash e : \mathbb{r}\, c \\
\textit{(T9)} & \forall \mathbb{X}. \left. \begin{array}{c} \vdash P,\ \ h \vdash e_r : t \\ e_r,\, h \longrightarrow e_r',\, h' \end{array} \right\} \Rightarrow h' \vdash e_r' : t
\end{array}
$$

(T1) and *(T2)* express subsumption. *(T3)* states that runtime expression typing does not depend on the field values assigned in the heap. *(T4)* states that addresses are typed according to their class in the heap. *(T5)* and *(T6)* are a technical constraint stating that method call typing implies that the parameter type and return type set by $\mathcal{M}$ for that method are respected and that proof obligations do not interfere with typing. *(T7)* states that the region component of a type assigned to an address respects the projection given for that region with respect to the same viewpoint of the typing. The most important constraints are *(T8)* and *(T9)*: *(T8)* states the correspondence between typing source expressions and runtime expressions for heaps and stack frames that respect the typing environment; *(T9)* states that for all well-formed programs, reduction preserves typing.

B Soundness Proof for Visible-States Verification Techniques

The proof for Theorem 6 is by induction on the derivation of $e_r,\, h \longrightarrow e_r',\, h'$. As a shorthand, we find it convenient to write $h \models \mathbb{X}_\sigma$ and $h \models \mathbb{r} \triangleright \mathbb{X}_\sigma$ instead of $h \models [\![\mathbb{X}_\sigma]\!]_{h,\sigma}$ and $h \models [\![\mathbb{r} \triangleright \mathbb{X}_\sigma]\!]_{h,\sigma}$ respectively, and similarly for the framework component $\mathbb{V}_\sigma$. For convenience we also enumerate the premises of the theorem as

$$\vdash_{\mathbf{wf}} P, \tag{2}$$
$$h \vdash e_r : t, \tag{3}$$
$$\vdash_{\mathcal{V}} P, \tag{4}$$
$$e_r \models_{\mathcal{V}} h, \tag{5}$$
$$h \vdash_{\mathcal{V}} e_r, \tag{6}$$
$$e_r,\, h \longrightarrow e_r',\, h' \tag{7}$$

We here focus on the main cases for the derivation of (7) and leave the remaining simpler cases for the interested reader.

rAss: From the conclusion and the premises of the rule we know

$$e_r = \sigma\cdot\iota.f := v \tag{8}$$
$$e_r' = \sigma\cdot v \tag{9}$$
$$h' = upd(h, \iota, f, v) \tag{10}$$

From (8) we know (6) could only have been derived using vd-ass and from the premises of this rule we know

$$h \vdash \sigma\cdot\iota : \mathbb{r}\, c' \tag{11}$$
$$\mathcal{F}(c', f) = \_,\, c \tag{12}$$
$$\mathbb{r} \sqsubseteq \mathbb{U}_{\sigma,c} \tag{13}$$
$$h \vdash_{\mathcal{V}} \sigma\cdot\iota \tag{14}$$
$$h \vdash_{\mathcal{V}} \sigma\cdot v \tag{15}$$

From (10) and Def. 8 *(H4)* we know

$$h \simeq h' \text{ which implies } h \preceq h' \tag{16}$$

Thus by (15), (16) and Lemma 5.1 and then by (9) we derive that the resultant configuration is still well-verified, *i.e.*,

$$h' \vdash_{\mathcal{V}} e_r'$$

We still need to show that (7) reduces to a valid state. From (5) and Def. 3 we know

$$h \models \mathbb{X}_\sigma \setminus \mathbb{V}_\sigma \tag{17}$$

Also, from (11) and Def. 15*(T4)* we know

$$cls(h, \iota) <: c' \tag{18}$$

By (4), (17), (18), (12) (10) and Lemma 6 we obtain

$$h' \models \mathbb{X}_\sigma \setminus \mathbb{V}_\sigma \setminus [\![\mathbb{D}_c]\!]_{h,\iota} \tag{19}$$

By (11) and Lemma 2.1 we get

$$[\![\mathbb{D}_c]\!]_{h,\iota} \subseteq [\![\mathbb{r} \triangleright \mathbb{D}_c]\!]_{h,\sigma} \tag{20}$$

By (13) and Lemma 2.2 we get

$$[\![\mathbb{r} \triangleright \mathbb{D}_c]\!]_{h,\sigma} \subseteq [\![\mathbb{U}_{\sigma,c} \triangleright \mathbb{D}_c]\!]_{h,\sigma} \tag{21}$$

Since we assume that our verification technique $\mathcal{V}$ is well-structured, by 5*(S4)* we also get

$$[\![\mathbb{U}_{\sigma,c} \triangleright \mathbb{D}_c]\!]_{h,\sigma} \subseteq [\![\mathbb{V}_\sigma]\!]_{h,\sigma} \tag{22}$$

Thus, from (19), (20), (21), (22) and set inclusion transitivity we obtain

$$h' \models \mathbb{X}_\sigma \setminus \mathbb{V}_\sigma$$

which by (9) and Def. 3 means we get the valid state

$$e'_r \models_{\mathcal{V}} h'$$

rCall: From the conclusion and the premises of the rule we know

$$e_r = \sigma \cdot \iota.m(v) \tag{23}$$
$$e'_r = \sigma \cdot \sigma' \cdot \mathsf{call}\, e_b \tag{24}$$
$$\mathcal{B}(cls(h,\iota), m) = e_b,\, c \tag{25}$$
$$\sigma' = (\iota, v, c, m) \tag{26}$$
$$h' = h \tag{27}$$

From (23) we know that (6) could only have been derived using vd-call-2, and thus from the premises of this rule we get

$$h \vdash \sigma \cdot \iota : \mathbb{r}\, c' \tag{28}$$
$$\mathcal{B}(c', m) = \_,\, c'' \tag{29}$$
$$h \models \mathbb{B}_{\sigma,\mathbb{r}},\, \sigma \tag{30}$$
$$\mathbb{r} \sqsubseteq \mathbb{C}_{\sigma,c'',m} \tag{31}$$
$$h \vdash_{\mathcal{V}} \sigma \cdot \iota \tag{32}$$
$$h \vdash_{\mathcal{V}} \sigma \cdot v \tag{33}$$

From (4), Def. 2*(W1)* and vs-class we know

$$e_b = e\, \mathsf{prv}\, \mathbb{E}_{c,m} \tag{34}$$
$$(c, m, \_) \vdash_{\mathcal{V}} e \tag{35}$$

From (25) and Def. 11*(P2)* we know

$$cls(h,\iota) <: c \tag{36}$$

This allows us to deduce that h, σ' is well-formed wrt. the environment $(c, m, \_)$ since by (26), (36) and Def. 14 we get

$$(c, m, \_) \vdash_{\mathbf{wf}} h, \sigma' \tag{37}$$

By (35), (37) and Lemma 1 we derive

$$h \vdash_{\mathcal{V}} \sigma' \cdot e \tag{38}$$

Hence, by (38) and vd-start we get

$$h \vdash_{\mathcal{V}} \sigma \cdot \sigma' \cdot \mathsf{call}\, e\, \mathsf{prv}\, \mathbb{E}_{\sigma'}$$

and by (27), (24), (34) and (26) we obtain

$$h' \vdash_{\mathcal{V}} e'_r$$

We still need to show that (7) reduces to a valid state, that is $e'_r \models_{\mathcal{V}} h'$. From (28), Def. 15*(T4)* we know

$$\mathit{cls}(h, \iota) <: c' \tag{39}$$

and by (39), (29), (25), (2) and Def. 14*(F4)* we deduce

$$c <: c'' \tag{40}$$

Since $\mathcal{V}$ is well-structured, then by Def. 5*(S5)* and (40) we obtain

$$\mathbb{X}_{c,m} \subseteq \mathbb{X}_{c'',m} \tag{41}$$

which, by Lemma 2.2 yields

$$\mathbb{r} \triangleright \mathbb{X}_{c,m} \subseteq \mathbb{r} \triangleright \mathbb{X}_{c'',m} \tag{42}$$

Moreover from Def. 5*(S1)* and (31) we get

$$(\mathbb{r} \triangleright \mathbb{X}_{c'',m}) \setminus (\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \subseteq \mathbb{B}_{\sigma,\mathbb{r}} \tag{43}$$

From (42) and (43) we obtain

$$(\mathbb{r} \triangleright \mathbb{X}_{c,m}) \setminus (\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \subseteq \mathbb{B}_{\sigma,\mathbb{r}} \tag{44}$$

From (5) and Def. 3, and then from (30) we know

$$h \models \mathbb{X}_\sigma \setminus \mathbb{V}_\sigma \ \cup\ \mathbb{B}_{\sigma,\mathbb{r}} \tag{45}$$

and from (44) and (45) we obtain

$$h \models \mathbb{X}_\sigma \setminus \mathbb{V}_\sigma \ \cup\ (\mathbb{r} \triangleright \mathbb{X}_{c,m}) \tag{46}$$

From (26), (28) and Lemma 2.1 we know

$$[\![\mathbb{X}_{\sigma'}]\!]_{h,\sigma'} \subseteq [\![\mathbb{r} \triangleright \mathbb{X}_{c,m}]\!]_{h,\sigma} \tag{47}$$

and by (46) and (47) we obtain Def. 3*(V2)* and *(V3)*, *i.e.*,

$$h \models \mathbb{X}_\sigma \setminus \mathbb{V}_\sigma \;\cup\; \mathbb{X}_{\sigma'} \tag{48}$$

Also by (26), (28), (31), (40) and Def. 12 we deduce Def. 3*(V1)*, *i.e.*,

$$h \vdash_{\mathcal{V}} \sigma\cdot\sigma' \tag{49}$$

and by (49), (48), Def. 3, and by (24) and (27) we get, as required,

$$\mathrm{e}'_r \models_{\mathcal{V}} h' \tag{50}$$

rCxtFrame: From the conclusion and the premises of the rule we know

$$\mathrm{e}_r = \sigma\cdot\mathrm{e}^1_r \tag{51}$$

$$\mathrm{e}'_r = \sigma\cdot\mathrm{e}^2_r \tag{52}$$

$$\mathrm{e}^1_r,\, h \longrightarrow \mathrm{e}^2_r,\, h' \tag{53}$$

From (51) and (53)[7] we know that (6) could have been derived using either of the following three subcases:

1. vd-start: From the conclusion and premises of this rule we know

$$\mathrm{e}^1_r = \sigma'\cdot\mathsf{call}\, e\, \mathsf{prv}\, \mathbb{E}_{\sigma'} \tag{54}$$

$$h \vdash_{\mathcal{V}} \sigma'\cdot e \tag{55}$$

As a result of (54), we know that (53) could have only been derived using either rLaunch or rLaunchEx. Moreover, because of (5), (51), (54), *i.e.*, $\sigma\cdot\sigma'\cdot\mathsf{call}\, e\, \mathsf{prv}\, \mathbb{E}_{\sigma'} \models_{\mathcal{V}} h$, and 3, we also know

$$h \models \mathbb{X}_\sigma \setminus \mathbb{V}_\sigma \cup \mathbb{X}_{\sigma'} \tag{56}$$

$$h \vdash_{\mathcal{V}} \sigma\cdot\sigma' \tag{57}$$

Now (56), in particular $h \models \mathbb{X}_{\sigma'}$, rules out the use of rLaunchEx to derive (53). Thus, if (53) was derived using rLaunch, we know

$$\mathrm{e}^2_r = \sigma'\cdot\mathsf{ret}\, e\, \mathsf{prv}\, \mathbb{E}_{\sigma'} \quad (\text{where } \mathrm{e}'_r = \sigma\cdot\mathrm{e}^2_r) \tag{58}$$

$$h' = h \tag{59}$$

By (55), (58), (59) and vd-frame we deduce

$$h' \vdash_{\mathcal{V}} \mathrm{e}'_r$$

and by (56), (57), (58), (59) and the fact that $\mathcal{V}$ is well-structured we deduce

$$\mathrm{e}'_r \models_{\mathcal{V}} h'$$

[7] The fact that e^1_r itself reduces means that e^1_r must contain a further stack frame, since all reduction rules in Fig. 5 are defined over runtime expressions of the form $\sigma\cdot\mathrm{e}_r$.

2. $\mathsf{vd\text{-}frame}$: From the conclusion and premises of this rule we know

$$e_r^1 = \sigma' \cdot \mathsf{ret}\, e_r^3\, \mathsf{prv}\, \mathbb{E}_{\sigma'} \tag{60}$$

$$h \vdash_{\mathcal{V}} \sigma' \cdot e_r^3 \tag{61}$$

From (60), we know (53) could have been derived using either of the following 3 subcases:

(a) $\mathsf{rCxtEval}$ with

$$E[\cdot] = \mathsf{ret}\, [\cdot]\, \mathsf{prv}\, \mathbb{E}_{\sigma'} \tag{62}$$

$$\sigma' \cdot e_r^3,\, h \longrightarrow \sigma' \cdot e_r^4,\, h' \tag{63}$$

$$e_r^2 = \sigma' \cdot E[e_r^4] \tag{64}$$

From (5), (51), (60), (62) and Lemma 5.1 and then Lemma 5.2 we obtain

$$\sigma' \cdot e_r^3 \models_{\mathcal{V}} h \tag{65}$$

$$h \vdash_{\mathcal{V}} \sigma \cdot \sigma' \tag{66}$$

$$h \models \mathbb{X}_{\sigma} \setminus \mathbb{V}_{\sigma} \setminus \mathbb{V}_{\sigma'} \setminus \mathbb{V}_{stack(e_r^3)} \tag{67}$$

Also, from (3), (51), (60), (62) and 15*(T6)* we know

$$h \vdash \sigma' \cdot e_r^3 : t \tag{68}$$

Thus by (2), (68), (4), (65), (61), (63) and inductive hypothesis we infer

$$h' \vdash_{\mathcal{V}} \sigma' \cdot e_r^4 \tag{69}$$

$$\sigma' \cdot e_r^4 \models_{\mathcal{V}} h' \tag{70}$$

By (69), (62), (64), (52) and $\mathsf{vd\text{-}frame}$ we derive

$$h' \vdash_{\mathcal{V}} e_r'$$

By (70), (62), (64) and Lemma 5.1 we deduce

$$e_r^2 \models_{\mathcal{V}} h' \tag{71}$$

From (67), (63) and Lemma 7 we get

$$h' \models \mathbb{X}_{\sigma} \setminus \mathbb{V}_{\sigma} \setminus \mathbb{V}_{\sigma'} \setminus \mathbb{V}_{stack(e_r^4)} \tag{72}$$

and by (70), (72), (64), (52) and Lemma 5.2 we obtain, as required,

$$e_r' \models_{\mathcal{V}} h'$$

(b) rCxtEval, rPrf with

$$E[\cdot] = \mathsf{ret}\,[\cdot] \text{ and } e_r^3 = v \tag{73}$$
$$\sigma'\cdot v\,\mathsf{prv}\,\mathbb{E}_{\sigma'},\, h \longrightarrow \sigma'\cdot v,\, h \tag{74}$$
$$e_r^2 = \sigma'\cdot v \text{ and } h' = h \tag{75}$$
$$h \models \mathbb{E}_{\sigma'},\, \sigma' \tag{76}$$

From (73), (75), (52), (61), (76) and vd-end we obtain

$$h' \vdash_{\mathcal{V}} e_r'$$

Since e_r' is a visible state, Def. 3 requires us to prove that more invariants hold for the resultant state to be valid. From $\mathcal{V}$ being well-structured, 5*(S2)* and (76) we deduce

$$h \models \mathbb{X}_{\sigma'} \cap \mathbb{V}_{\sigma'} \tag{77}$$

and by (75), (73), (51), (52), (60), (5) we know

$$h \models ((\mathbb{X}_{\sigma} \setminus \mathbb{V}_{\sigma}) \cup \mathbb{X}_{\sigma'}) \setminus \mathbb{V}_{\sigma'} \tag{78}$$

And thus by (77), (78) and then Def. 3 we deduce $e_r' \models_{\mathcal{V}} h'$

(c) rCxtEval, rPrfEx with

$$E[\cdot] = \mathsf{ret}\,[\cdot] \text{ and } e_r^3 = v$$
$$\sigma'\cdot v\,\mathsf{prv}\,\mathbb{E}_{\sigma'},\, h \longrightarrow \sigma'\cdot\mathsf{verfExc},\, h$$
$$e_r^2 = \sigma'\cdot\mathsf{verfExc} \text{ and } h' = h$$
$$h \not\models \mathbb{E}_{\sigma'},\, \sigma'$$

This case is similar to the previous case and is left for the interested reader.

3. vd-end: from the conclusion and the premises of the rule, we obtain

$$e_r^1 = \sigma'\cdot\mathsf{ret}\,v \tag{79}$$
$$h \vdash_{\mathcal{V}} \sigma'\cdot v \tag{80}$$
$$h \models \mathbb{E}_{\sigma'},\, \sigma' \tag{81}$$

From (79) we know (53) could only have been derived using either rFrame or rFrameEx. However, from (5), (51) and (79) we know that e_r is in a visible state at σ' and thus

$$h \models \mathbb{X}_{\sigma'} \tag{82}$$

which rules out the possibility of using rFrameEx. Thus by rFrame we know

$$e_r^2 = v \tag{83}$$
$$h' = h \tag{84}$$

By (52), (83), (84), (80) and Lemma 4.2 we obtain

$$h' \vdash_{\mathcal{V}} e'_r$$

From (51), (79), (5) and Def. 3 and Def. 12 we know

$$h \models ((\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \cup \mathbb{X}_{\sigma'}) \setminus \mathbb{V}_{\sigma'} \cup \mathbb{X}_{\sigma'} \tag{85}$$
$$\sigma' = (\iota, \_, c', m),\ h \vdash \sigma{\cdot}\iota : \mathbb{r}\ \_,\ c' <: c,\ \mathbb{r} \sqsubseteq \mathbb{C}_{\sigma,c,m} \tag{86}$$

We can rewrite (85) as

$$h \models (\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \setminus (\mathbb{V}_{\sigma'} \setminus \mathbb{X}_{\sigma'}) \cup \mathbb{X}_{\sigma'} \tag{87}$$

We can combine (87) with (81) to obtain

$$h \models (\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \setminus (\mathbb{V}_{\sigma'} \setminus \mathbb{X}_{\sigma'}) \cup \mathbb{E}_{\sigma'} \cup \mathbb{X}_{\sigma'} \tag{88}$$

By standard properties of set theory, we observe that

$$\begin{aligned}
&(\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \setminus (\mathbb{V}_{\sigma'} \setminus \mathbb{E}_{\sigma'})\\
&= ((\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \setminus (\mathbb{V}_{\sigma'})) \cup ((\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \cap (\mathbb{E}_{\sigma'}))\\
&\subseteq ((\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \setminus (\mathbb{V}_{\sigma'})) \cup \mathbb{E}_{\sigma'}\\
&\subseteq ((\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \setminus (\mathbb{V}_{\sigma'} \setminus \mathbb{X}_{\sigma'})) \cup \mathbb{E}_{\sigma'}
\end{aligned}$$

i.e., we have

$$(\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \setminus (\mathbb{V}_{\sigma'} \setminus \mathbb{E}_{\sigma'}) \subseteq (\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \setminus (\mathbb{V}_{\sigma'} \setminus \mathbb{X}_{\sigma'}) \cup \mathbb{E}_{\sigma'} \tag{89}$$

From (88) and (89), we can obtain

$$h \models (\mathbb{X}_\sigma \setminus \mathbb{V}_\sigma) \setminus (\mathbb{V}_{\sigma'} \setminus \mathbb{E}_{\sigma'}) \cup \mathbb{X}_{\sigma'} \tag{90}$$

By (86) and Lemma 2.1 we have

$$[\![\mathbb{V}_{\sigma'} \setminus \mathbb{E}_{\sigma'}]\!]_{h,\sigma'} \subseteq [\![\mathbb{r} \triangleright (\mathbb{V}_{\sigma'} \setminus \mathbb{E}_{\sigma'})]\!]_{h,\sigma} \tag{91}$$

Also by (86) and Lemma 2.2 we have

$$\mathbb{r} \triangleright (\mathbb{V}_{\sigma'} \setminus \mathbb{E}_{\sigma'})\ \subseteq\ \mathbb{C}_{\sigma,c,m} \triangleright (\mathbb{V}_{\sigma'} \setminus \mathbb{E}_{\sigma'}) \tag{92}$$

Since we assume our verification technique $\mathcal{V}$to be well-structured, then by 5*(S3)* we know

$$\mathbb{C}_{\sigma,c,m} \triangleright (\mathbb{V}_{c,m} \setminus \mathbb{E}_{c,m})\ \subseteq\ \mathbb{V}_\sigma \tag{93}$$

and by *(S5)* and (86) we also know

$$(\mathbb{V}_{\sigma'} \setminus \mathbb{E}_{\sigma'})\ \subseteq\ (\mathbb{V}_{c,m} \setminus \mathbb{E}_{c,m}) \tag{94}$$

and by (94) and Lemma 2.3 we get

$$\mathbb{C}_{\sigma,c,m} \triangleright (\mathbb{V}_{\sigma'} \setminus \mathbb{E}_{\sigma'}) \subseteq \mathbb{C}_{\sigma,c,m} \triangleright (\mathbb{V}_{c,m} \setminus \mathbb{E}_{c,m}) \tag{95}$$

By (91), (92), (95) and (93) we can rewrite (90) as

$$h \models (\mathbb{X}_{\sigma} \setminus \mathbb{V}_{\sigma}) \cup \mathbb{X}_{\sigma'} \tag{96}$$

and by (96), (84), (83) and (52) we obtain

$$e'_r \models_{\mathcal{V}} h'$$

as required.

C Proof for Well-Structured Verification Techniques

C.1 Proof for *OT* and *VT*

The proof for Lemma 9 assumes the following definition of region inclusion for the Universe type system, defined as the least relation characterised by the rules below. It is not hard to see that this definition satisfies constraint *(P4)* of Def. 11. Other definitions for universe set inclusion are possible.

$$\text{(u-emp)}\ \frac{}{\mathsf{emp} \sqsubseteq \mathbb{r}} \qquad \text{(u-any)}\ \frac{}{\mathbb{r} \sqsubseteq \mathsf{any}} \qquad \text{(u-self)}\ \frac{}{\mathsf{self} \sqsubseteq \mathsf{peer}}$$

$$\text{(u-union)}\ \frac{}{\begin{array}{c}\mathbb{r}_1 \sqsubseteq \mathbb{r}_1 \sqcup \mathbb{r}_2 \\ \mathbb{r}_2 \sqsubseteq \mathbb{r}_1 \sqcup \mathbb{r}_2\end{array}} \qquad \text{(u-relf)}\ \frac{}{\mathbb{r} \sqsubseteq \mathbb{r}} \qquad \text{(u-trans)}\ \frac{\begin{array}{c}\mathbb{r}_1 \sqsubseteq \mathbb{r}_2 \\ \mathbb{r}_2 \sqsubseteq \mathbb{r}_3\end{array}}{\mathbb{r}_1 \sqsubseteq \mathbb{r}_3}$$

We start by showing that *OT* is well-structured from the components given in Fig. 11.

(S1): There are a number of regions that satisfy $\mathbb{r} \sqsubseteq \mathbb{C}_{c,m,c'm'}$. We here give the proof for the main two cases, *i.e.,* peer and $\mathsf{rep}\langle c\rangle$. For $\mathbb{r} = \mathsf{peer}$ we have to show:

$$(\mathsf{peer} \triangleright \mathsf{own}\,;\mathsf{rep}^{+}) \setminus (\mathsf{own}\,;\mathsf{rep}^{+} \setminus \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}) \subseteq \mathsf{super}\langle c\rangle$$

When we adapt $\mathsf{own}\,;\mathsf{rep}^{+}$, *i.e.,* everything beneath the owner of the current receiver, by peer, *i.e.,* $\mathsf{peer}\triangleright\mathsf{own}\,;\mathsf{rep}^{+}$, we still get $\mathsf{own}\,;\mathsf{rep}^{+}$ since peers share the same owner. Thus we get

$$\mathsf{own}\,;\mathsf{rep}^{+} \setminus (\mathsf{own}\,;\mathsf{rep}^{+} \setminus \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}) \subseteq \mathsf{super}\langle c\rangle$$

Using the set identity

$$A \setminus (B \setminus C) = (A \cap C) \cup (A \setminus B) \tag{97}$$

on the left hand of the inclusion we get:

$$(\mathsf{own}\,;\mathsf{rep}^{+} \cap \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}) \cup (\mathsf{own}\,;\mathsf{rep}^{+} \setminus \mathsf{own}\,;\mathsf{rep}^{+}) \\ \subseteq \mathsf{super}\langle c\rangle$$

and thus

$$(\mathsf{own}\,;\mathsf{rep}^{+} \cap \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}) \cup \emptyset \subseteq \mathsf{super}\langle c\rangle \tag{98}$$

From the interpretations given, we can show that

$$\mathsf{own}\,;\mathsf{rep}^{+} \cap \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+} = \mathsf{super}\langle c\rangle$$

and as a result, from (98) we obtain

$$\mathsf{super}\langle c\rangle \subseteq \mathsf{super}\langle c\rangle$$

For the second case, *i.e.*, $\mathtt{r} = \mathsf{rep}\langle c\rangle$ we have

$$(\mathsf{rep}\langle c\rangle \triangleright \mathsf{own}\,;\mathsf{rep}^{+}) \setminus (\mathsf{own}\,;\mathsf{rep}^{+} \setminus \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}) \\ \subseteq \mathsf{emp}$$

When we adapt $\mathsf{own}\,;\mathsf{rep}^{+}$ by $\mathsf{rep}\langle c\rangle$ we get all objects transitively owned by the current receiver, namely rep^{+} and thus we get

$$\mathsf{rep}^{+} \setminus (\mathsf{own}\,;\mathsf{rep}^{+} \setminus \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}) \subseteq \mathsf{emp}$$

At this point we apply the set identity (97) from the previous case and get

$$(\mathsf{rep}^{+} \cap \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}) \cup (\mathsf{rep}^{+} \setminus \mathsf{own}\,;\mathsf{rep}^{+}) \subseteq \mathsf{emp}$$

Since rep^{+} does not include the current receiver, we know $\mathsf{rep}^{+} \cap \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+} = \emptyset$. Also since $\mathsf{rep}^{+} \subseteq \mathsf{own}\,;\mathsf{rep}^{+}$, we also know $\mathsf{rep}^{+} \setminus \mathsf{own}\,;\mathsf{rep}^{+} = \emptyset$ and hence we get

$$\emptyset \cup \emptyset \subseteq \mathsf{emp}$$

(S2): We require

$$\mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+} \cap \mathsf{own}\,;\mathsf{rep}^{+} \subseteq \mathsf{super}\langle c\rangle \tag{99}$$

From the interpretations, it is not hard to show directly that

$$\mathsf{super}\langle c\rangle \cap \mathsf{own}\,;\mathsf{rep}^{+} = \mathsf{super}\langle c\rangle \tag{100}$$

$$\mathsf{own}^{+} \cap \mathsf{own}\,;\mathsf{rep}^{+} = \emptyset \tag{101}$$

from which (99) follows.

(S3): Instantiating the components of Fig. 11 we need to show

$$\mathsf{rep}\langle c'\rangle \sqcup \mathsf{peer} \rhd ((\mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+})\backslash\mathsf{super}\langle c\rangle) \\ \subseteq \mathsf{super}\langle c'\rangle \sqcup \mathsf{own}^{+}$$

We highlight the different roles played by the classes c and c' in the above statement. It is easy to show that

$$\mathsf{own}^{+} \cap \mathsf{super}\langle c\rangle = \emptyset \tag{102}$$

from which we obtain

$$(\mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}) \setminus \mathsf{super}\langle c\rangle = \mathsf{own}^{+} \tag{103}$$

Therefore, it suffices to show

$$\mathsf{rep}\langle c'\rangle \sqcup \mathsf{peer} \rhd (\mathsf{own}^{+}) \subseteq \mathsf{super}\langle c'\rangle \sqcup \mathsf{own}^{+}$$

From the interpretations we derive the identities:

$$\mathsf{rep}\langle c'\rangle \rhd \mathsf{own}^{+} = \mathsf{self}\langle c'\rangle \sqcup \mathsf{own}^{+} \tag{104}$$

$$\mathsf{peer} \rhd \mathsf{own}^{+} = \mathsf{own}^{+} \tag{105}$$

and thus, from the direct interpretation of $\sqcup$ we obtain

$$\mathsf{self}\langle c'\rangle \cup \mathsf{own}^{+} \subseteq \mathsf{super}\langle c'\rangle \cup \mathsf{own}^{+}$$

which is immediately true since, from the interpretations we know $\mathsf{self}\langle c'\rangle \subseteq \mathsf{super}\langle c'\rangle$

(S4): Once again we have two cases.

– For pure methods we have

$$\mathsf{emp} \rhd \mathsf{self}\langle c\rangle \sqcup \mathsf{own}^{+} \subseteq \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}$$

Since the interpretation of emp is $\emptyset$, anything adapted by the viewpoint emp given emp and thus

$$\mathsf{emp} \subseteq \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}$$

which is trivially true.

– For non-pure methods we have

$$\mathsf{self} \rhd \mathsf{self}\langle c\rangle \sqcup \mathsf{own}^{+} \subseteq \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}$$

Any adaptation by self acts as the identity and thus we obtain

$$\mathsf{self}\langle c\rangle \sqcup \mathsf{own}^{+} \subseteq \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^{+}$$

which is true by the same reasons we gave for case **(S3)** above.

(S5): For $c <: c'$ we have to show

$$1. \qquad \mathsf{own}\,;\mathsf{rep}^+ \subseteq \mathsf{own}\,;\mathsf{rep}^+$$

$$2. \qquad \begin{pmatrix} \mathsf{super}\langle c\rangle \sqcup \mathsf{own}^+ \setminus \\ \mathsf{super}\langle c\rangle \end{pmatrix} \subseteq \begin{pmatrix} \mathsf{super}\langle c'\rangle \sqcup \mathsf{own}^+ \setminus \\ \mathsf{super}\langle c\rangle \end{pmatrix}$$

The first statement is trivially true whereas the second statement is true because (103) is true for any c, so substituting the right hand side of (103) gives us own^+ on both sides.

The proof of well-structuredness of the VT is similar to that of the OT:

(S1): As before, the two cases for $\mathtt{r}$ we consider are peer and $\mathsf{rep}\langle c\rangle$. For peer we have the proof

$$\begin{array}{rl}
\mathsf{peer} \triangleright \mathsf{own}\,;\mathsf{rep}^+ \setminus \begin{pmatrix} \mathsf{own}\,;\mathsf{rep}^+ \setminus \\ \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^+ \end{pmatrix} & \subseteq \mathsf{peer}\langle c\rangle \\
\mathsf{own}\,;\mathsf{rep}^+ \setminus \begin{pmatrix} \mathsf{own}\,;\mathsf{rep}^+ \setminus \\ \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^+ \end{pmatrix} & \subseteq \mathsf{peer}\langle c\rangle \\
\begin{pmatrix} \mathsf{own}\,;\mathsf{rep}^+ \cap \\ \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^+ \end{pmatrix} \cup \begin{pmatrix} \mathsf{own}\,;\mathsf{rep}^+ \setminus \\ \mathsf{own}\,;\mathsf{rep}^+ \end{pmatrix} & \subseteq \mathsf{peer}\langle c\rangle \\
\begin{pmatrix} \mathsf{own}\,;\mathsf{rep}^+ \cap \\ \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^+ \end{pmatrix} \cup \emptyset & \subseteq \mathsf{peer}\langle c\rangle
\end{array}$$

Here we use the property

$$\mathsf{own}\,;\mathsf{rep}^+ \cap \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^+ = \mathsf{peer}\langle c\rangle \tag{106}$$

derived directly from the interpretations and get

$$\mathsf{peer}\langle c\rangle \cup \emptyset \subseteq \mathsf{peer}\langle c\rangle$$

For the case of $\mathsf{rep}\langle c\rangle$ we have the proof:

$$\begin{array}{rl}
\mathsf{rep}\langle c\rangle \triangleright \mathsf{own}\,;\mathsf{rep}^+ \setminus \begin{pmatrix} \mathsf{own}\,;\mathsf{rep}^+ \setminus \\ \mathsf{peer} c \sqcup \mathsf{own}^+ \end{pmatrix} & \subseteq \mathsf{emp} \\
\mathsf{rep}^+ \setminus \begin{pmatrix} \mathsf{own}\,;\mathsf{rep}^+ \setminus \\ \mathsf{peer} c \sqcup \mathsf{own}^+ \end{pmatrix} & \subseteq \mathsf{emp} \\
(\mathsf{rep}^+ \cap \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^+) \cup (\mathsf{rep}^+ \setminus \mathsf{own}\,;\mathsf{rep}^+) & \subseteq \mathsf{emp} \\
\emptyset \cup \emptyset & \subseteq \mathsf{emp}
\end{array}$$

(S2): There are two cases.

– For pure methods we have

$$\begin{array}{r}
\mathsf{emp} \cap \mathsf{own}\,;\mathsf{rep}^+ \sqsubseteq \mathsf{emp} \\
\mathsf{emp} \sqsubseteq \mathsf{emp}
\end{array}$$

– For non-pure methods we have

$$\mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+} \cap \mathsf{own}\,;\mathsf{rep}^{+} \sqsubseteq \mathsf{peer}\langle c\rangle$$

which is true from (106) earlier.

(S3): There are four cases to consider here, depending on the purity of the two methods. We here give the proof for two cases.

– If m is pure and m' is non-pure we have

$$\begin{aligned} &\mathsf{emp} \triangleright (\mathsf{peer}\langle c'\rangle \sqcup \mathsf{own}^{+} \setminus \mathsf{peer}\langle c'\rangle) && \sqsubseteq \mathsf{emp} \\ &\mathsf{emp} \sqsubseteq \mathsf{emp} \end{aligned}$$

– When both m and m' are non-pure, then since we can directly show

$$\mathsf{own}^{+} \cap \mathsf{peer} = \emptyset \tag{107}$$

we have

$$\begin{aligned} &\mathsf{rep}\langle c\rangle \sqcup \mathsf{peer} \triangleright \begin{pmatrix} \mathsf{peer}\langle c'\rangle \sqcup \mathsf{own}^{+} \\ \setminus \mathsf{peer}\langle c'\rangle \end{pmatrix} && \sqsubseteq \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+} \\ &\mathsf{rep}\langle c\rangle \sqcup \mathsf{peer} \triangleright \mathsf{own}^{+} && \sqsubseteq \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+} \end{aligned}$$

At this point we use the identities (104), (105) derived earlier to obtain

$$\mathsf{self}\langle c\rangle \sqcup \mathsf{own}^{+} \sqsubseteq \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+}$$

which is true because we can show $\mathsf{self}\langle c\rangle \sqsubseteq \mathsf{peer}\langle c\rangle$.

(S4): We have two cases.

– If $\mathbb{U}_{c,m,c'} = \mathsf{emp}$ we have two of cases and here we consider the case where m is not pure (the other case is similar).

$$\begin{aligned} \mathsf{emp} \triangleright \mathsf{peer}\langle c'\rangle \sqcup \mathsf{own}^{+} &\sqsubseteq \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+} \\ \mathsf{emp} &\sqsubseteq \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+} \end{aligned}$$

– If $\mathbb{U}_{c,m,c'} = \mathsf{peer}$ then we know that m is not pure and that c is visible from c', *i.e.*, $\mathsf{vis}(c', c)$. We therefore obtain the proof

$$\begin{aligned} \mathsf{peer} \triangleright \mathsf{peer}\langle c'\rangle \sqcup \mathsf{own}^{+} &\sqsubseteq \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+} \\ \mathsf{peer}\langle c'\rangle \sqcup \mathsf{own}^{+} &\sqsubseteq \mathsf{peer}\langle c\rangle \sqcup \mathsf{own}^{+} \end{aligned}$$

and by the symmetric property of $\mathsf{vis}(c', c)$ and the interpretation of $\mathsf{peer}\langle c\rangle$ we derive the identity $\mathsf{peer}\langle c\rangle = \mathsf{peer}\langle c'\rangle$ which make the above true.

(S5): This is similar to *(S5)* for OT.

C.2 Proof for *Oval'*

In the proof for Lemma 11, we use the shorthands $\mathsf{I} = \mathsf{I}(c, m)$, $\mathsf{E} = \mathsf{E}(c, m)$, $\mathsf{I}' = \mathsf{I}(c', m')$ and $\mathsf{E}' = \mathsf{E}(c', m')$ where we recall that they all come from the domain of L. We also use the following lemmas:

Lemma 14. $K \prec K' \Rightarrow \begin{cases} K\,;\mathit{rep}^* \subseteq K'\,;\mathit{rep}^* \\ K'\,;\mathit{own}^* \subseteq K\,;\mathit{own}^* \\ K\,;\mathit{rep}^* \cap K'\,;\mathit{own}^* = \emptyset \end{cases}$

Lemma 15. *If* $\mathbb{r}; L \neq \bot$ *then* $\mathbb{r}; L = \mathbb{r} \triangleright L$

Lemma 16. $\mathit{this}; \mathit{rep}^* \cap \mathit{this}; \mathit{own}^* = \mathit{this}$

Lemma 17. $K \prec \mathit{this} \;\Rightarrow\; K; \mathit{rep}^* \subseteq (\mathit{this}; \mathit{rep}^* \setminus \mathit{this})$

(S1): We need to show

$$\mathbb{r} \triangleright \mathsf{I}'\,; \mathsf{rep}^* \setminus (\mathsf{I}\,; \mathsf{rep}^* \setminus \mathsf{E}\,; \mathsf{own}^*) \subseteq \mathsf{emp} \tag{108}$$

If $\mathbb{r} \sqsubseteq \mathbb{C}_{c,m,c',m'}$ then by Fig. 11 we know

$$\mathsf{SC}(\mathsf{I}, \mathsf{E}, \mathbb{r}\,; \mathsf{I}', \mathbb{r}\,; \mathsf{E}', \mathcal{O}_{\mathbb{r},c}) \tag{109}$$

and from (109) and Def. 1 we obtain two subcases

$\mathsf{I} \prec \mathsf{E}$: From this subcase's clause, *i.e.*, $\mathsf{I} \prec \mathsf{E}$, and Def. 1 we also know

$$\mathbb{r}; \mathsf{I}' \preceq \mathsf{I} \tag{110}$$

and thus, since the ordering $\preceq$ is not defined for $\bot$ values, we conclude

$$\mathbb{r}; \mathsf{I}' \neq \bot \tag{111}$$

From the subcase clause, $\mathsf{I} \prec \mathsf{E}$, and Lemma 14 we obtain

$$\mathsf{I}\,; \mathsf{rep}^* \setminus \mathsf{E}\,; \mathsf{own}^* = \mathsf{I}\,; \mathsf{rep}^*$$

and thus from (108) we get

$$\mathbb{r} \triangleright \mathsf{I}'\,; \mathsf{rep}^* \setminus \mathsf{I}\,; \mathsf{rep}^* \subseteq \mathsf{emp} \tag{112}$$

From (111) and Lemma 15 we can rewrite (110) as $\mathbb{r} \triangleright \mathsf{I}' \preceq \mathsf{I}$ and by Lemma 14 we obtain

$$\mathbb{r} \triangleright \mathsf{I}'\,; \mathsf{rep}^* \subseteq \mathsf{I}\,; \mathsf{rep}^*$$

and thus $\mathbb{r} \triangleright \mathsf{I}'\,; \mathsf{rep}^* \setminus \mathsf{I}\,; \mathsf{rep}^* = \mathsf{emp}$ satisfying (112).

$\mathsf{I} = \mathsf{E} = \mathsf{this}$: Similar to the case before, from $\mathsf{I} = \mathsf{E}$, Def. 1 and Lemma 15 we get

$$\mathbb{r} \rhd \mathsf{I}' \prec \mathsf{this} \tag{113}$$

From the subcase clause, $\mathsf{I} = \mathsf{E} = \mathsf{this}$, and Lemma 16 we can derive

$$\mathsf{this}\,;\mathsf{rep}^* \setminus \mathsf{this}\,;\mathsf{own}^* = \mathsf{this}\,;\mathsf{rep}^* \setminus \mathsf{this}$$

and thus by (108) we obtain

$$\mathbb{r} \rhd \mathsf{I}'\,;\mathsf{rep}^* \setminus (\mathsf{this}\,;\mathsf{rep}^* \setminus \mathsf{this}) \subseteq \mathsf{emp}$$

Finally, from (113) and Lemma 17 we derive that

$$\mathbb{r} \rhd \mathsf{I}'\,;\mathsf{rep}^* \setminus (\mathsf{this}\,;\mathsf{rep}^* \setminus \mathsf{this}) = \mathsf{emp}$$

which satisfies the above.

(S2): Immediate from (1), Lemma 14 and Lemma 16.

(S3): We recall that

$$\mathbb{C}_{c,m,c',m'} = \sqcup \mathbb{r}_i \text{ such that } \mathsf{SC}(\mathsf{I}, \mathsf{E}, \mathbb{r}_i\,;\mathsf{I}', \mathbb{r}_i\,;\mathsf{E}', \mathcal{O}_{\mathbb{r}_i,c})$$

We here prove that for every such $\mathbb{r}_i$

$$\mathbb{r}_i \rhd (\mathbb{V}_{c',m'} \setminus \mathbb{E}_{c',m'}) \subseteq \mathbb{V}_{c,m}$$

From which *(S3)* follows from the monotonicity of $\rhd$. For this proof we find it convenient to distribute the adaptation in *(S3)* and show

$$\mathbb{r}_i \rhd \mathbb{V}_{c',m'} \setminus \mathbb{r}_i \rhd \mathbb{E}_{c',m'} \subseteq \mathbb{V}_{c,m} \tag{114}$$

From the subcontract definition, we have two subcases:

$\mathbb{r}_i\,;\mathsf{I}' \prec \mathbb{r}_i\,;\mathsf{E}'$: From the subcase clause $\mathbb{r}_i\,;\mathsf{I}' \prec \mathbb{r}_i\,;\mathsf{E}'$, along with Lemma 15 and Lemma 14, we deduce

$$\begin{aligned} &\mathbb{r}_i \rhd \mathbb{V}_{c',m'} \setminus \mathbb{r}_i \rhd \mathbb{E}_{c',m'} \\ &\qquad = \mathbb{r}_i \rhd \mathbb{V}_{c',m'} \setminus \qquad \mathbb{r}_i \rhd \mathsf{emp} \\ &\qquad = \mathbb{r}_i \rhd \mathsf{E}'\,;\mathsf{own}^* \setminus \mathsf{emp} \\ &\qquad = \mathbb{r}_i \rhd \mathsf{E}'\,;\mathsf{own}^* \end{aligned}$$

and thus by (114) it suffices to prove

$$\mathbb{r}_i \rhd \mathsf{E}'\,;\mathsf{own}^* \subset \mathsf{E}\,;\mathsf{own}^* \tag{115}$$

From the subcase and Def. 1 we also know $\mathsf{E} \preceq \mathbb{r}_i\,;\mathsf{E}'$, thus by Lemma 15 we have $\mathsf{E} \preceq \mathbb{r}_i \rhd \mathsf{E}'$ and hence by Lemma 14 we obtain (115) as required.

$\mathbb{r}_i\,;\mathsf{I}' = \mathbb{r}_i\,;\mathsf{E}' = \mathsf{this}$: From Lemma 16, Lemma 15 and (114) we obtain

$$\mathbb{r}_i \rhd \mathsf{E}'\,;\mathsf{own}^* \setminus \mathsf{this} \subseteq \mathsf{E}\,;\mathsf{own}^* \tag{116}$$

From the subcase and Def. 1 we also know $\mathsf{E} \preceq \mathcal{O}_{\mathbb{r}_i,c}$ which proves (116) as required.

(S4): By (1) we have two subcases to consider:

$\mathsf{I} \prec \mathsf{E}$**:** From 11 we know $\mathbb{U}_{c,m,c'} = \mathsf{emp}$ thus we have the proof

$$\begin{aligned} \mathsf{emp} \rhd (\mathsf{this}\,;\mathsf{own}^*) &\subseteq \mathsf{E}\,;\mathsf{own}^* \\ \mathsf{emp} &\subseteq \mathsf{E}\,;\mathsf{own}^* \end{aligned}$$

$\mathsf{I} = \mathsf{E} = \mathsf{this}$**:** From 11 we know $\mathbb{U}_{c,m,c'} = \mathsf{this}$ thus we have the proof

$$\begin{aligned} \mathsf{this} \rhd (\mathsf{this}\,;\mathsf{own}^*) &\subseteq \mathsf{this}\,;\mathsf{own}^* \\ \mathsf{this}\,;\mathsf{own}^* &\subseteq \mathsf{this}\,;\mathsf{own}^* \end{aligned}$$

(S5): Suppose $c' \leq c$. Recall that our amended requirements for method overriding are as follows:

$$\mathsf{I}' \preceq \mathsf{I} \preceq \mathsf{E} \preceq \mathsf{E}' \qquad \mathsf{I} = \mathsf{E}' \;\Rightarrow\; \mathsf{I}' = \mathsf{E}'$$

Therefore, it is immediate from Lemma 14 that:

$$\mathbb{X}_{c',m} \subseteq \mathbb{X}_{c,m} \tag{117}$$

holds (i.e., the first part of (S5)), and also that

$$\mathbb{V}_{c',m} \subseteq \mathbb{V}_{c,m} \tag{118}$$

It remains to show that

$$\mathbb{V}_{c',m} \backslash \mathbb{E}_{c',m} \subseteq \mathbb{V}_{c,m} \setminus \mathbb{E}_{c,m} \tag{119}$$

We first eliminate various cases. Firstly, if $\mathsf{I} \neq \mathsf{E}$, then $\mathbb{E}_{c,m} = \mathsf{emp}$, and so (119) follows immediately from (118). Therefore, we consider the remaining case $\mathsf{I} = \mathsf{E} = \mathsf{this}$, for which we know $\mathbb{E}_{c,m} = \mathsf{this}$, and so

$$\mathbb{V}_{c,m} \setminus \mathbb{E}_{c,m} = \mathbb{V}_{c,m} \setminus \mathsf{this} \tag{120}$$

Next, if $\mathsf{E} \prec \mathsf{E}'$, then (119) follows easily from (120). Therefore we consider the remaining case $\mathsf{E} = \mathsf{E}'$. Then, by our second requirement on overriding, we conclude $\mathsf{I}' = \mathsf{I} = \mathsf{E} = \mathsf{E}' = \mathsf{this}$. Therefore, in this case it follows that $\mathbb{V}_{c,m} = \mathbb{V}_{c',m}$ and $\mathbb{E}_{c,m} = \mathbb{E}_{c',m}$, and (119) is trivially satisfied.