

Exercise 12: BiCG, QMR, and BiCGStab

The purpose of this exercise is to compare iterative solvers for nonsymmetric linear systems of equations: BiCG, QMR and BiCGStab, without and with preconditioning, as provided by MATLAB.

1. We generate a linear system corresponding to the *stabilized* convection-diffusion equation, that we used already in Exercise 5. It concerns a heated circular object submerged into

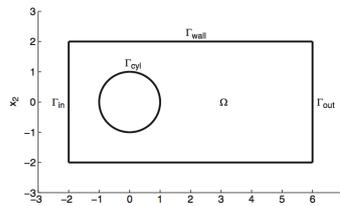


Figure 1: Ball in a channel

a rectangular channel, cf. Fig. 1. Fluid is flowing from left to right round the object. You can use the MATLAB function `HeatFlowSolver2D` of Exercise 5 to generate matrix and right-hand side.

2. MATLAB provides the iterative linear solvers BiCG, QMR and BiCGStab for unsymmetric matrices. Read the documentation of these solvers. Try out these solvers and also `gmres` on the just generated matrix, and check the convergence behavior by plotting the relative residuals ($\text{relres} = \text{norm}(\mathbf{b} - \mathbf{A}\mathbf{x}) / \text{norm}(\mathbf{b})$) at each iteration.

Set `maxit=1000` and `tol = 1e-6`.

Use the solvers without and with the ILU(0) preconditioner.

What do you observe?

The MATLAB function `ilu` has a parameter called `droptol`. Check how the convergence changes as you select the values `droptol = 1, .5, .1, .05, .01`. Do not forget to set the `type` of `ilu` as `ilutp`.

3. Note that MATLAB provides the source code for its iterative solvers. You can look at it using the `type` command. Try to understand the implementation as much as possible.
4. Try to implement your own BiCG solver. Your BiCG solver should compute the projected matrix of A in each iteration step, i.e. $T_k = W_k^T A V_k$, where V_k is the search subspace and W_k is the test subspace. Get the condition number of the projected matrix T_k using the command `cond`, and then plot the condition number at each iteration step. What do you observe? Can you use this plot to explain the convergence behavior of BiCG method which you just observed?

Please submit your solution via e-mail to Peter Arbenz (arbenz@inf.ethz.ch) by December 14, 2017. (12:00). Please specify the tag **FEM17** in the subject field.