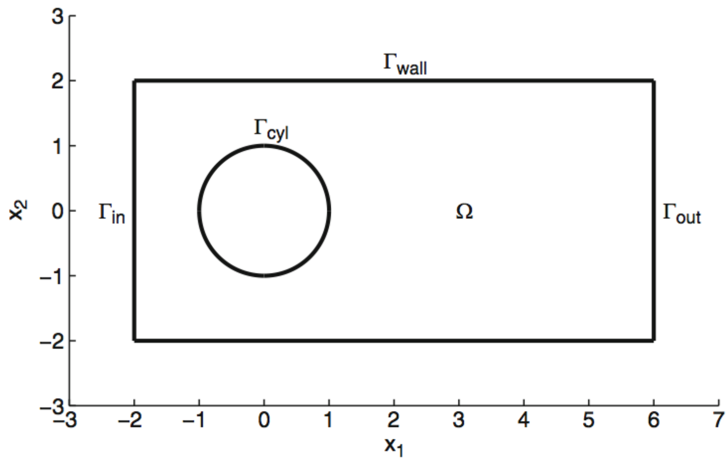
Exercise 5

The purpose of this exercise is the generation of a nonsymmetric matrix to test solvers in later examples. The problem is introduced in Chapter 10 of the book by Larson & Bengzon.

1. Solve the 'real-world application' mentioned in Lecture 5, the ***heat transfer in a fluid flow*** by the Galerkin least squares (GLS) finite element approximation. The strong form of the equation is

$$-\nu\Delta u(x,y) + \boldsymbol{w}(x,y)\cdot\mathbf{grad}\,u(x,y) = f(x,y), \qquad (x,y)\in\Omega,$$

where $\Omega = \{(-2,6)\times(2,2)\setminus\{x^2+y^2\le 1\}\}$, see the graphics below.



The boundary conditions are

$$\begin{aligned} -\nu\Delta u + \boldsymbol{w}\cdot\nabla u &= 0, && \text{in } \Omega,\\ u &= 0, && \text{on } \partial\Omega_{\text{in}},\\ u &= 1, && \text{on } \partial\Omega_{\text{cyl}},\\ -\nu n\cdot\nabla u &= 0, && \text{on } \partial\Omega_{\text{out}},\\ n\cdot(-\nu\nabla u + \boldsymbol{w}u) &= 0, && \text{on } \partial\Omega_{\text{wall}}. \end{aligned}$$

The wind is given by

$$\boldsymbol{w} = U_\infty \begin{pmatrix} 1 - \dfrac{x_1^2 - x_2^2}{(x_1^2+x_2^2)^2} \\ \dfrac{-2x_1x_2}{(x_1^2+x_2^2)^2} \end{pmatrix}$$

Please submit your solution via e-mail to Peter Arbenz (`arbenz@inf.ethz.ch`) by October 26, 2017. (12:00). Please specify the tag **FEM17** in the subject field.

where $U_\infty = 1$ is the free stream velocity. We set the diffusion coefficient $\nu = 0.01$.

Derive the weak form of the problem and solve it. Make use of the subroutines provided on the web page. They are taken from the book by Larson & Bengzon. The most important functions are `HeatFlowSolver2D` and `ConvectionAssembler2D`. (Note that in these codes the wind is denoted $\boldsymbol{b}$.)

Treat the boundary conditions such that the degrees of freedom corresponding to the Dirichlet portion of the boundary are removed from the linear system of equations. (In contrast to the approach of the book where artificial Robin boundary conditions are introduced.)

Modify the function `HeatFlowSolver2D` such that the system matrix A and the right-hand side vector $\boldsymbol{b}$ are returned.

2. Stabilize the procedure above by ***streamline diffusion*** with $\delta = h$, see slide 13 of Lecture 5. This is done in function `SDAssembler2D`.

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