Exercise 6

The goal of this exercise is to get some experience with sparse matrix formats.

1. Consider the following ***skyline*** (aka: *profile*) way of storing a sparse symmetric matrix $\boldsymbol{A}$. Stored are the elements of the lower triangular part of the matrix row by row, starting with the first non-zero element to and including the diagonal element. Zero elements are included! Find a way to encode this storage format using two arrays.

   (a) If no pivoting occurs, show that the triangular factors have the same skyline as $\boldsymbol{A}$, symbolically. This means that the Cholesky factorization can be executed without changing the storage format!

   (b) Write a MATLAB function
   ```
   [val, ptr] = skyline(A)
   ```
   that transforms a dense matrix into your skyline storage format.

   (c) Write a MATLAB function
   ```
   [y] = skylinemult(val, ptr, x)
   ```
   that executes the matrix-vector multiplication $\boldsymbol{y} \leftarrow \boldsymbol{A}\boldsymbol{x}$ where $\boldsymbol{A}$ is stored in the skyline storage format.

   (d) Check the correctness of the results by, e.g., the following procedure
   ```
   A=sprandsym(20,.2,.2,1);
   [val,ptr]=skyline(A);
   b=rand(20,1);
   x = skylinemult(val, ptr, b);
   x1=A*b;
   norm(x-x1)
   ```

2. It often happens that the elements of a sparse matrix come in small blocks, like 2×2 or 3×3. It can be useful to exploit this structure. We consider a matrix that has all its nonzero entries in 2×2 blocks. Write code that constructs the block-CRS (BCRS) format for such a matrix. That is, construct arrays `row_ptr`, `col_ind` and an array `vals` whose entries are 2×2 matrices, that admit to execute the matrix-vector product.

   The m-file `ElasticSolver.m` is available from the website. It lets you generate matrices with 2×2 block-elements. The matrix size is controlled by `hmax` which is a parameter of `initmesh` (see the code).

   Check if the matrix-vector product is executed correctly. Compare the memory requirements and execution times of CRS and BCRS storage formats for matrices generated with `hmax` = .1, .03, .01.

Please submit your solution via e-mail to Peter Arbenz (`arbenz@inf.ethz.ch`) by November 2, 2017. (12:00). Please specify the tag **FEM17** in the subject field.