Exercise 4 – Solutions

Assigment 4.1

The Galerkin orthogonality property states that

$$a(e, v_h) = a(u - u_h, v_h) = 0 \qquad \text{for all } v_h \in S_0^h. \tag{1}$$

Our problem is formulated such that the search space equals the test space: $S_E^h = S_0^h$. This means that we can set v_h in (1) to be the solution u_h! Therefore,

$$a(e, u_h) = 0, \tag{2}$$

which is not true in general. Using (2) twice we get the desired result:

$$\begin{aligned}
a(e, e) &= a(e, u - u_h) = a(e, u) - a(e, u_h) \\
&\overset{(2)}{=} a(e, u) \\
&= a(u - u_h, u) = a(u, u) - a(u_h, u) \\
&= a(u, u) - a(u_h, u_h + e) = a(u, u) - a(u_h, u_h) - a(u_h, e) \\
&\overset{(2)}{=} a(u, u) - a(u_h, u_h)
\end{aligned}$$