Advanced Algorithms 2024	05/10, 2023
Exercise 02	
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1 Rounding for Bin Packing

For the construction of a PTAS for the Bin packing problem we needed a tricky way of rounding. Here we investigate why this is necessary.

Construct a set of items with sizes $0 < x_1, \ldots, x_n < 1$ with the following property: while it is possible to pack these into some number m of bins, if we for any $\varepsilon > 0$ consider the set of items with sizes $(1+\varepsilon)x_1, \ldots, (1+\varepsilon)x_n$, we need at least αm bins for some $\alpha > 1$. What is the best α you can get and how does it relate to the issue of designing a PTAS for Bin packing?

2 Target shooting

Suppose we have a ground set S and we wish to estimate the size of a subset $T \subseteq S$. We can do it by repeatedly sampling from S: we sample m times a uniform element from S and let X_i to be an indicator for the event of the i-th sample being an element of T. One can use the Chernoff bound to prove that the variable $(X_1 + X_2 + \ldots X_m)/m$ is within $(1+\varepsilon)$ multiplicative error of $|T|/|S|$ with probability at least $1 - \delta$ if we set

$$m = \Theta\left(\frac{|S|}{|T|}\varepsilon^{-2}\log(1/\delta)\right).$$

1. Argue that the lemma really follows from the Chernoff bound.
2. Argue that if we sample only $O(\frac{|S|}{|T|})$ elements, with constant probability, we never sample an element from T throughout the whole procedure and, hence, we cannot get a reasonable estimate.
3. Suppose that we have a FPRAS that returns value that is in the correct range $[(1-\varepsilon)OPT, (1+\varepsilon)OPT]$ with probability 2/3. Recall the median trick from the lecture and show how to amplify the probability to $1-\delta$ with $O(\log 1/\delta)$ calls of the original algorithm.

3 Counting satisfying assignments

In the lecture we have seen a FPRAS that estimates the number of satisfying assignments of a given DNF formula. Suppose that we wish to solve this problem and we know that one of the clauses contains only 10 variables. Can you propose a faster algorithm than the one you know from the lecture?