Exercise 10

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1   L1 embedding of cycle

You are given an unweighted cycle C on n vertices.

1. Find a randomized algorithm that embeds the cycle C to $\mathbb{R}$ such that the expected stretch of every edge is constant. That is, your randomized algorithm maps each vertex $u \in C$ to some number $f(u)$. For every pair $u, v \in C$ it has to be the case that $d_C(u,v)/K \leq \mathrm{Exp}[|f(u) - f(v)|] \leq d_C(u,v)$ for some constant K.

2. Find a deterministic algorithm that embeds C to $\mathbb{R}^2$ with $L1$ norm such that the stretch of every edge is constant. That is, you should map each vertex $u \in C$ to some number $f(u)$. For every pair $u, v \in C$ it has to be the case that $d_C(u,v)/K \leq ||f(u) - f(v)||_1 \leq d_C(u,v)$ for some constant K.

2   Minimum bisection cut

A *bisection cut* is a cut (S, S') such that $|S| = |S'| = n/2$. An *r-balanced cut* is a cut where $r \cdot n \leq |S| \leq (1-r) \cdot n$. A size of a cut is the number of edges that go across the cut.

Give a polynomial-time algorithm that, given a graph G as input, outputs a 1/3-balanced cut whose size is $\mathcal{O}(\log n)$ factor from the size of the smallest-size bisection cut of G.

Hint:

Find a black box reduction to the result you saw in the lecture via a greedy algorithm.