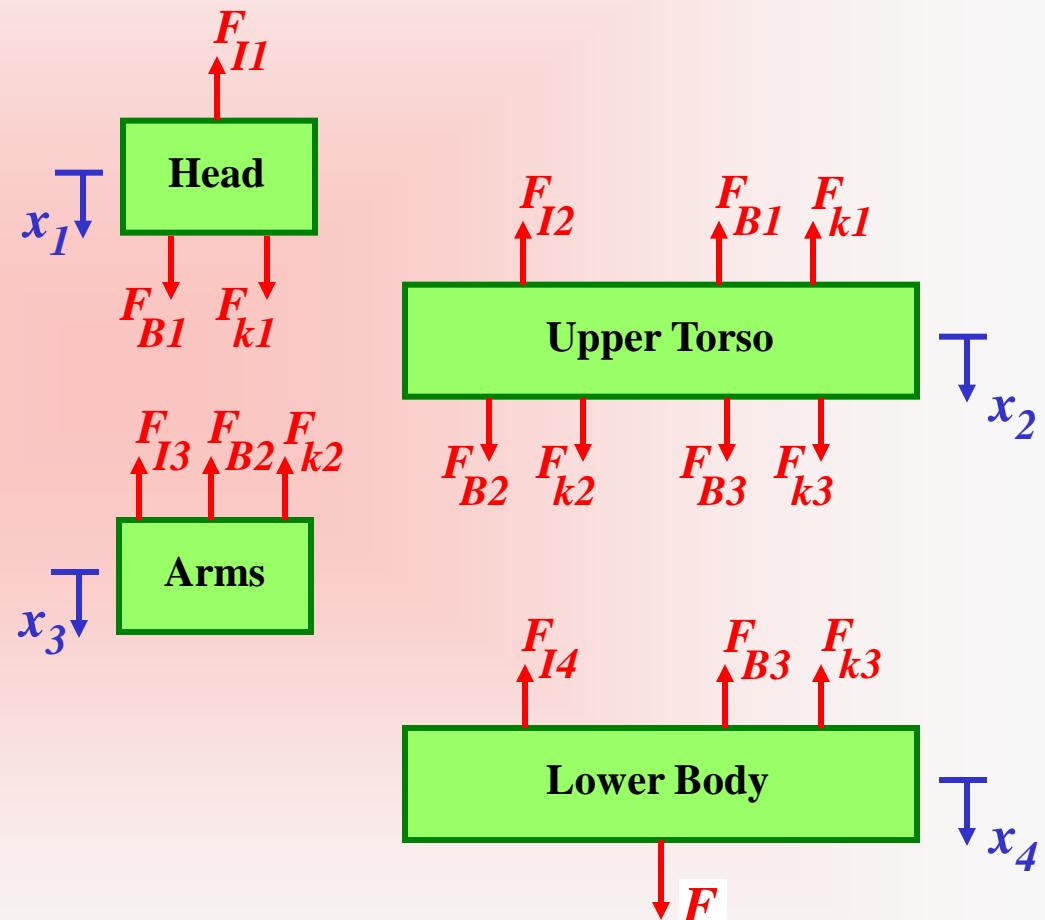
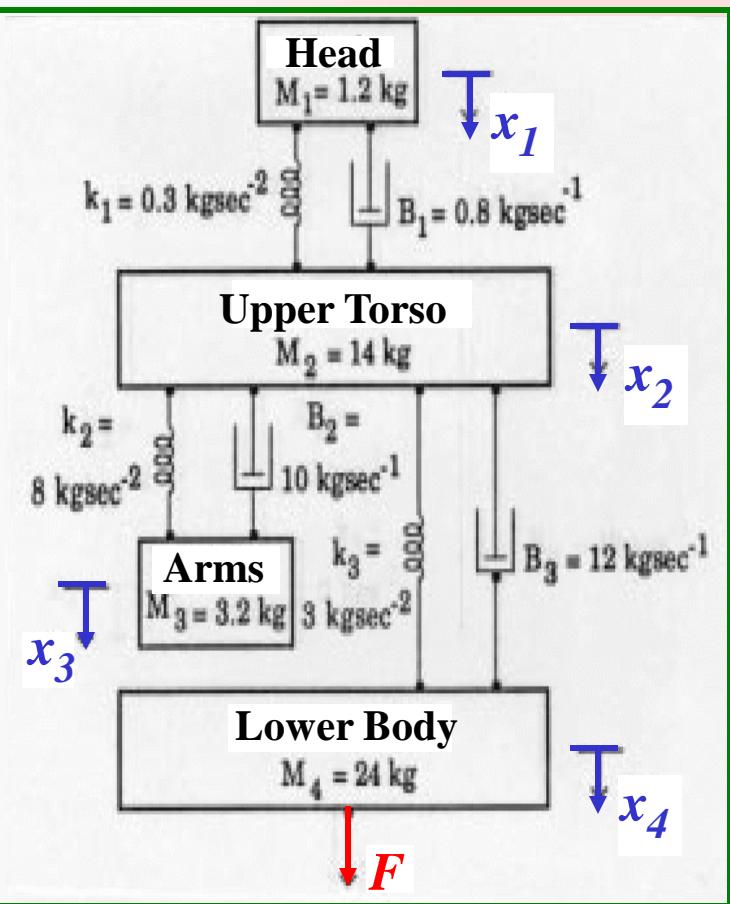


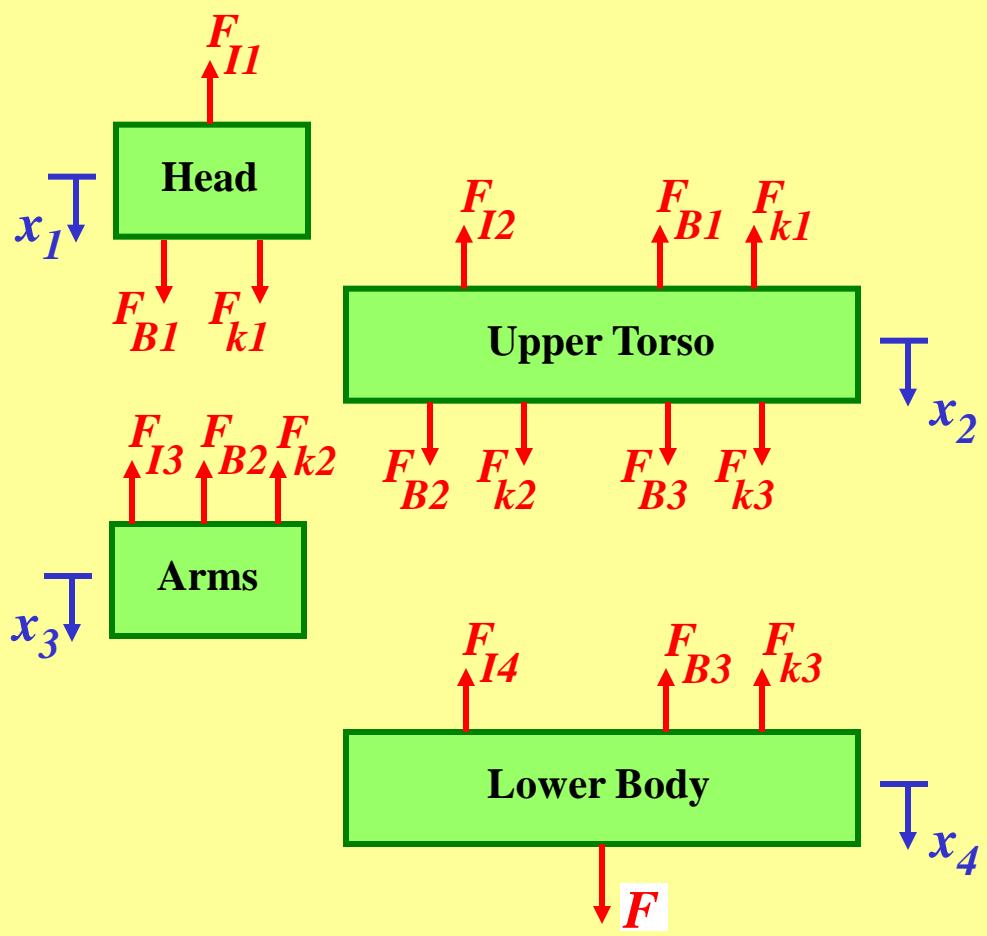
# 2<sup>nd</sup> Homework: Cervical Syndrome



## 1<sup>st</sup> Sub-problem

1. Derive a state-space model for this system. Since this is a linear time-invariant system, put it in linear state-space form and simulate the system in **Matlab**.
2. Simulate the system during *15 seconds*. Use a sinusoidal force ( $F$ ) with a frequency of *1.5 Hz*. As the system is linear, the amplitude of the input signal is irrelevant. 1.0 represents an excellent value. The output is the distance between the head and the shoulder. The initial conditions of all state variables may be assumed as 0.0. This is acceptable, since only the deviation of the output from the stationary position is of relevance.





$$\mathbf{F}(t) = \mathbf{F}_{I4} + \mathbf{F}_{B3} + \mathbf{F}_{k3}$$

$$\mathbf{0} = \mathbf{F}_{I3} + \mathbf{F}_{B2} + \mathbf{F}_{k2}$$

$$\mathbf{F}_{B2} + \mathbf{F}_{k2} + \mathbf{F}_{B3} + \mathbf{F}_{k3} = \mathbf{F}_{I2} + \mathbf{F}_{B1} + \mathbf{F}_{k1}$$

$$\mathbf{F}_{B1} + \mathbf{F}_{k1} = \mathbf{F}_{II}$$

$$\mathbf{F}_{II} = m_1 \cdot \frac{d\mathbf{v}_1}{dt}$$

$$\frac{d\mathbf{x}_1}{dt} = \mathbf{v}_1$$

$$\mathbf{F}_{I2} = m_2 \cdot \frac{d\mathbf{v}_2}{dt}$$

$$\frac{d\mathbf{x}_2}{dt} = \mathbf{v}_2$$

$$\mathbf{F}_{I3} = m_3 \cdot \frac{d\mathbf{v}_3}{dt}$$

$$\frac{d\mathbf{x}_3}{dt} = \mathbf{v}_3$$

$$\mathbf{F}_{I4} = m_4 \cdot \frac{d\mathbf{v}_4}{dt}$$

$$\frac{d\mathbf{x}_4}{dt} = \mathbf{v}_4$$

$$\mathbf{F}_{B1} = \mathbf{B}_1 \cdot (\mathbf{v}_2 - \mathbf{v}_1)$$

$$\mathbf{F}_{B2} = \mathbf{B}_2 \cdot (\mathbf{v}_3 - \mathbf{v}_2)$$

$$\mathbf{F}_{B3} = \mathbf{B}_3 \cdot (\mathbf{v}_4 - \mathbf{v}_3)$$

$$\mathbf{F}_{k1} = \mathbf{k}_1 \cdot (\mathbf{x}_2 - \mathbf{x}_1)$$

$$\mathbf{F}_{k2} = \mathbf{k}_2 \cdot (\mathbf{x}_3 - \mathbf{x}_2)$$

$$\mathbf{F}_{k3} = \mathbf{k}_3 \cdot (\mathbf{x}_4 - \mathbf{x}_3)$$



$$\mathbf{F}(t) = \mathbf{F}_{I4} + \mathbf{F}_{B3} + \mathbf{F}_{k3}$$

$$0 = \mathbf{F}_{I3} + \mathbf{F}_{B2} + \mathbf{F}_{k2}$$

$$\mathbf{F}_{B2} + \mathbf{F}_{k2} + \mathbf{F}_{B3} + \mathbf{F}_{k3} = \mathbf{F}_{I2} + \mathbf{F}_{B1} + \mathbf{F}_{k1}$$

$$\mathbf{F}_{B1} + \mathbf{F}_{k1} = \mathbf{F}_{II}$$

$$\mathbf{F}_{II} = m_1 \cdot \frac{d\mathbf{v}_1}{dt}$$

$$\frac{dx_1}{dt} = \mathbf{v}_1$$

$$\mathbf{F}_{I2} = m_2 \cdot \frac{d\mathbf{v}_2}{dt}$$

$$\frac{dx_2}{dt} = \mathbf{v}_2$$

$$\mathbf{F}_{I3} = m_3 \cdot \frac{d\mathbf{v}_3}{dt}$$

$$\frac{dx_3}{dt} = \mathbf{v}_3$$

$$\mathbf{F}_{I4} = m_4 \cdot \frac{d\mathbf{v}_4}{dt}$$

$$\frac{dx_4}{dt} = \mathbf{v}_4$$

$$\mathbf{F}_{B1} = \mathbf{B}_1 \cdot (\mathbf{v}_2 - \mathbf{v}_1)$$

$$\mathbf{F}_{B2} = \mathbf{B}_2 \cdot (\mathbf{v}_3 - \mathbf{v}_2)$$

$$\mathbf{F}_{B3} = \mathbf{B}_3 \cdot (\mathbf{v}_4 - \mathbf{v}_3)$$

$$\mathbf{F}_{k1} = k_1 \cdot (\mathbf{x}_2 - \mathbf{x}_1)$$

$$\mathbf{F}_{k2} = k_2 \cdot (\mathbf{x}_3 - \mathbf{x}_2)$$

$$\mathbf{F}_{k3} = k_3 \cdot (\mathbf{x}_4 - \mathbf{x}_3)$$



$$\mathbf{F}(t) = \mathbf{F}_{I4} + \mathbf{F}_{B3} + \mathbf{F}_{k3}$$

$$0 = \mathbf{F}_{I3} + \mathbf{F}_{B2} + \mathbf{F}_{k2}$$

$$\mathbf{F}_{B2} + \mathbf{F}_{k2} + \mathbf{F}_{B3} + \mathbf{F}_{k3} = \mathbf{F}_{I2} + \mathbf{F}_{B1} + \mathbf{F}_{k1}$$

$$\mathbf{F}_{B1} + \mathbf{F}_{k1} = \mathbf{F}_{II}$$

$$\mathbf{F}_{II} = m_1 \cdot \frac{d\mathbf{v}_1}{dt}$$

$$\frac{dx_1}{dt} = \mathbf{v}_1$$

$$\mathbf{F}_{I2} = m_2 \cdot \frac{d\mathbf{v}_2}{dt}$$

$$\frac{dx_2}{dt} = \mathbf{v}_2$$

$$\mathbf{F}_{I3} = m_3 \cdot \frac{d\mathbf{v}_3}{dt}$$

$$\frac{dx_3}{dt} = \mathbf{v}_3$$

$$\mathbf{F}_{I4} = m_4 \cdot \frac{d\mathbf{v}_4}{dt}$$

$$\frac{dx_4}{dt} = \mathbf{v}_4$$



$$\mathbf{F}(t) = \mathbf{F}_{I4} + \mathbf{F}_{B3} + \mathbf{F}_{k3}$$

$$0 = \mathbf{F}_{I3} + \mathbf{F}_{B2} + \mathbf{F}_{k2}$$

$$\mathbf{F}_{B2} + \mathbf{F}_{k2} + \mathbf{F}_{B3} + \mathbf{F}_{k3} = \mathbf{F}_{I2} + \mathbf{F}_{B1} + \mathbf{F}_{k1}$$

$$\mathbf{F}_{B1} + \mathbf{F}_{k1} = \mathbf{F}_{II}$$

$$\mathbf{F}_{II} = m_1 \cdot \frac{d\mathbf{v}_1}{dt}$$

$$\frac{dx_1}{dt} = v_1$$

$$\mathbf{F}_{I2} = m_2 \cdot \frac{d\mathbf{v}_2}{dt}$$

$$\frac{dx_2}{dt} = v_2$$

$$\mathbf{F}_{I3} = m_3 \cdot \frac{d\mathbf{v}_3}{dt}$$

$$\frac{dx_3}{dt} = v_3$$

$$\mathbf{F}_{I4} = m_4 \cdot \frac{d\mathbf{v}_4}{dt}$$

$$\frac{dx_4}{dt} = v_4$$

$$\mathbf{F}_{B1} = B_1 \cdot (\mathbf{v}_2 - \mathbf{v}_1)$$

$$\mathbf{F}_{B2} = B_2 \cdot (\mathbf{v}_3 - \mathbf{v}_2)$$

$$\mathbf{F}_{B3} = B_3 \cdot (\mathbf{v}_4 - \mathbf{v}_3)$$

$$\mathbf{F}_{k1} = k_1 \cdot (\mathbf{x}_2 - \mathbf{x}_1)$$

$$\mathbf{F}_{k2} = k_2 \cdot (\mathbf{x}_3 - \mathbf{x}_2)$$

$$\mathbf{F}_{k3} = k_3 \cdot (\mathbf{x}_4 - \mathbf{x}_3)$$



$$\mathbf{F}(t) = \mathbf{F}_{I4} + \mathbf{F}_{B3} + \mathbf{F}_{k3}$$

$$0 = \mathbf{F}_{I3} + \mathbf{F}_{B2} + \mathbf{F}_{k2}$$

$$\mathbf{F}_{B2} + \mathbf{F}_{k2} + \mathbf{F}_{B3} + \mathbf{F}_{k3} = \mathbf{F}_{I2} + \mathbf{F}_{B1} + \mathbf{F}_{k1}$$

$$\mathbf{F}_{B1} + \mathbf{F}_{k1} = \mathbf{F}_{II}$$

$$\mathbf{F}_{II} = m_1 \cdot \frac{d\mathbf{v}_1}{dt}$$

$$\frac{dx_1}{dt} = v_1$$

$$\mathbf{F}_{I2} = m_2 \cdot \frac{d\mathbf{v}_2}{dt}$$

$$\frac{dx_2}{dt} = v_2$$

$$\mathbf{F}_{I3} = m_3 \cdot \frac{d\mathbf{v}_3}{dt}$$

$$\frac{dx_3}{dt} = v_3$$

$$\mathbf{F}_{I4} = m_4 \cdot \frac{d\mathbf{v}_4}{dt}$$

$$\frac{dx_4}{dt} = v_4$$



$$\mathbf{F}(t) = \mathbf{F}_{I4} + \mathbf{F}_{B3} + \mathbf{F}_{k3}$$

$$\mathbf{0} = \mathbf{F}_{I3} + \mathbf{F}_{B2} + \mathbf{F}_{k2}$$

$$\mathbf{F}_{B2} + \mathbf{F}_{k2} + \mathbf{F}_{B3} + \mathbf{F}_{k3} = \mathbf{F}_{I2} + \mathbf{F}_{B1} + \mathbf{F}_{k1}$$

$$\mathbf{F}_{B1} + \mathbf{F}_{k1} = \mathbf{F}_{II}$$

$$\mathbf{F}_{II} = m_1 \cdot \frac{d\mathbf{v}_1}{dt}$$

$$\frac{dx_1}{dt} = v_1$$

$$\mathbf{F}_{I2} = m_2 \cdot \frac{d\mathbf{v}_2}{dt}$$

$$\frac{dx_2}{dt} = v_2$$

$$\mathbf{F}_{I3} = m_3 \cdot \frac{d\mathbf{v}_3}{dt}$$

$$\frac{dx_3}{dt} = v_3$$

$$\mathbf{F}_{I4} = m_4 \cdot \frac{d\mathbf{v}_4}{dt}$$

$$\frac{dx_4}{dt} = v_4$$

$$\mathbf{F}_{B1} = B_1 \cdot (v_2 - v_1)$$

$$\mathbf{F}_{B2} = B_2 \cdot (v_3 - v_2)$$

$$\mathbf{F}_{B3} = B_3 \cdot (v_4 - v_3)$$

$$\mathbf{F}_{k1} = k_1 \cdot (x_2 - x_1)$$

$$\mathbf{F}_{k2} = k_2 \cdot (x_3 - x_2)$$

$$\mathbf{F}_{k3} = k_3 \cdot (x_4 - x_3)$$

⇒

$$\mathbf{F}_{I4} = \mathbf{F}(t) - \mathbf{F}_{B3} - \mathbf{F}_{k3}$$

$$\mathbf{F}_{I3} = -\mathbf{F}_{B2} - \mathbf{F}_{k2}$$

$$\mathbf{F}_{I2} = \mathbf{F}_{B2} + \mathbf{F}_{k2} + \mathbf{F}_{B3} + \mathbf{F}_{k3} - \mathbf{F}_{B1} - \mathbf{F}_{k1}$$

$$\mathbf{F}_{II} = \mathbf{F}_{B1} + \mathbf{F}_{k1}$$

$$\frac{d\mathbf{v}_1}{dt} = \mathbf{F}_{II} / m_1$$

$$\frac{dx_1}{dt} = v_1$$

$$\frac{d\mathbf{v}_2}{dt} = \mathbf{F}_{I2} / m_2$$

$$\frac{dx_2}{dt} = v_2$$

$$\frac{d\mathbf{v}_3}{dt} = \mathbf{F}_{I3} / m_3$$

$$\frac{dx_3}{dt} = v_3$$

$$\frac{d\mathbf{v}_4}{dt} = \mathbf{F}_{I4} / m_4$$

$$\frac{dx_4}{dt} = v_4$$



$$\begin{aligned}\frac{d\mathbf{v}_1}{dt} &= \mathbf{F}_{II}/m_1 \\ &= (\mathbf{F}_{B1} + \mathbf{F}_{k1})/m_1 \\ &= (\mathbf{B}_1 \cdot (\mathbf{v}_2 - \mathbf{v}_1) + k_1 \cdot (\mathbf{x}_2 - \mathbf{x}_1))/m_1 \\ &= -(k_1/m_1) \cdot \mathbf{x}_1 + (k_1/m_1) \cdot \mathbf{x}_2 - (\mathbf{B}_1/m_1) \cdot \mathbf{v}_1 + (\mathbf{B}_1/m_1) \cdot \mathbf{v}_2\end{aligned}$$

$$\begin{aligned}\frac{d\mathbf{v}_2}{dt} &= \mathbf{F}_{I2}/m_2 \\ &= (\mathbf{F}_{B2} + \mathbf{F}_{k2} + \mathbf{F}_{B3} + \mathbf{F}_{k3} - \mathbf{F}_{B1} - \mathbf{F}_{k1})/m_2 \\ &= (\mathbf{B}_2 \cdot (\mathbf{v}_3 - \mathbf{v}_2) + k_2 \cdot (\mathbf{x}_3 - \mathbf{x}_2) + \mathbf{B}_3 \cdot (\mathbf{v}_4 - \mathbf{v}_2) + k_3 \cdot (\mathbf{x}_4 - \mathbf{x}_2) \\ &\quad - \mathbf{B}_1 \cdot (\mathbf{v}_2 - \mathbf{v}_1) - k_1 \cdot (\mathbf{x}_2 - \mathbf{x}_1))/m_2 \\ &= (k_1/m_2) \cdot \mathbf{x}_1 - ((k_1+k_2+k_3)/m_2) \cdot \mathbf{x}_2 + (k_2/m_2) \cdot \mathbf{x}_3 + (k_3/m_2) \cdot \mathbf{x}_4 \\ &\quad + (\mathbf{B}_1/m_2) \cdot \mathbf{v}_1 - ((\mathbf{B}_1+\mathbf{B}_2+\mathbf{B}_3)/m_2) \cdot \mathbf{v}_2 + (\mathbf{B}_2/m_2) \cdot \mathbf{v}_3 + (\mathbf{B}_3/m_2) \cdot \mathbf{v}_4\end{aligned}$$



$$\begin{aligned}\frac{dv_3}{dt} &= F_{I3} / m_3 \\ &= (-F_{B2} - F_{k2}) / m_3 \\ &= (-B_2 \cdot (v_3 - v_2) - k_2 \cdot (x_3 - x_2)) / m_3 \\ &= (k_2/m_3) \cdot x_2 - (k_2/m_3) \cdot x_3 + (B_2/m_3) \cdot v_2 - (B_2/m_3) \cdot v_3\end{aligned}$$

$$\begin{aligned}\frac{dv_4}{dt} &= F_{I4} / m_4 \\ &= (F(t) - F_{B3} - F_{k3}) / m_4 \\ &= (F(t) - B_3 \cdot (v_4 - v_2) - k_3 \cdot (x_4 - x_2)) / m_4 \\ &= F(t) / m_4 + (k_3/m_4) \cdot x_2 - (k_3/m_4) \cdot x_4 + (B_3/m_4) \cdot v_2 - (B_3/m_4) \cdot v_4\end{aligned}$$

$$y = x_1 - x_2$$



## 1<sup>st</sup> Sub-problem

1. Derive a state-space model for this system. Since this is a linear time-invariant system, put it in linear state-space form and simulate the system in *Matlab*.
2. Simulate the system during **15 seconds**. Use a sinusoidal force (**F**) with a frequency of **1.5 Hz**. As the system is linear, the amplitude of the input signal is irrelevant. 1.0 represents an excellent value. The output is the distance between the head and the shoulder. The initial conditions of all state variables may be assumed as 0.0. This is acceptable, since only the deviation of the output from the stationary position is of relevance.



# Matlab Code (1<sup>st</sup> Part)

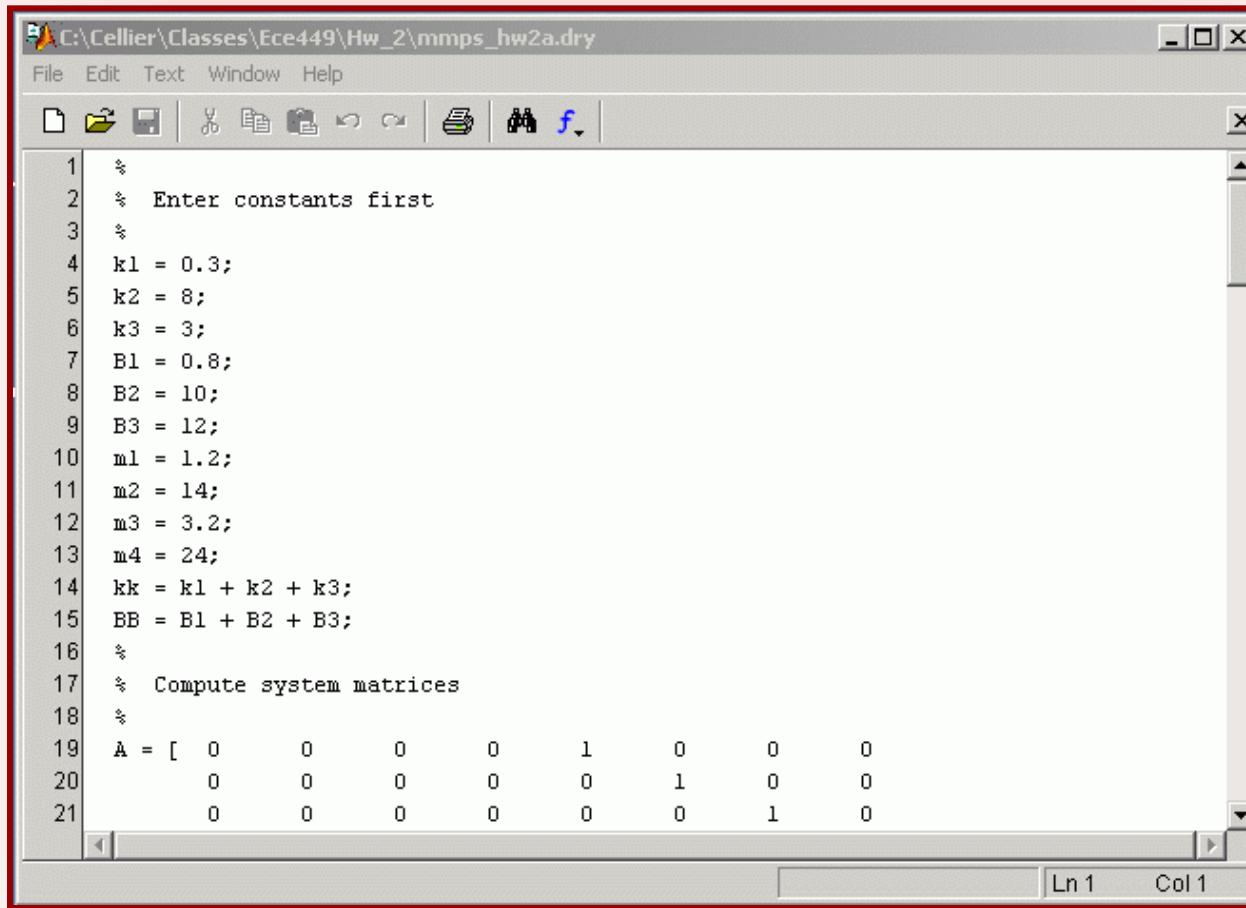
The screenshot shows a Matlab editor window with a red border. The title bar reads "C:\Cellier\Classes\Ece449\Hw\_2\mmmps\_hw2a.m". The menu bar includes File, Edit, Text, Window, and Help. Below the menu is a toolbar with icons for file operations. The main text area contains the following Matlab code:

```
1 % Cervical Syndrome (first part)
2 %
3 %
4 echo on
5 diary mmmps_hw2a.dry
6 %
7 % Enter constants first
8 %
9 k1 = 0.3;
10 k2 = 8;
11 k3 = 3;
12 B1 = 0.8;
13 B2 = 10;
14 B3 = 12;
15 m1 = 1.2;
16 m2 = 14;
17 m3 = 3.2;
18 m4 = 24;
19 kk = k1 + k2 + k3;
20 BB = B1 + B2 + B3;
21 %
```

The status bar at the bottom indicates "script" and "Ln 1 Col 1".



# Matlab Diary File (1<sup>st</sup> Part)

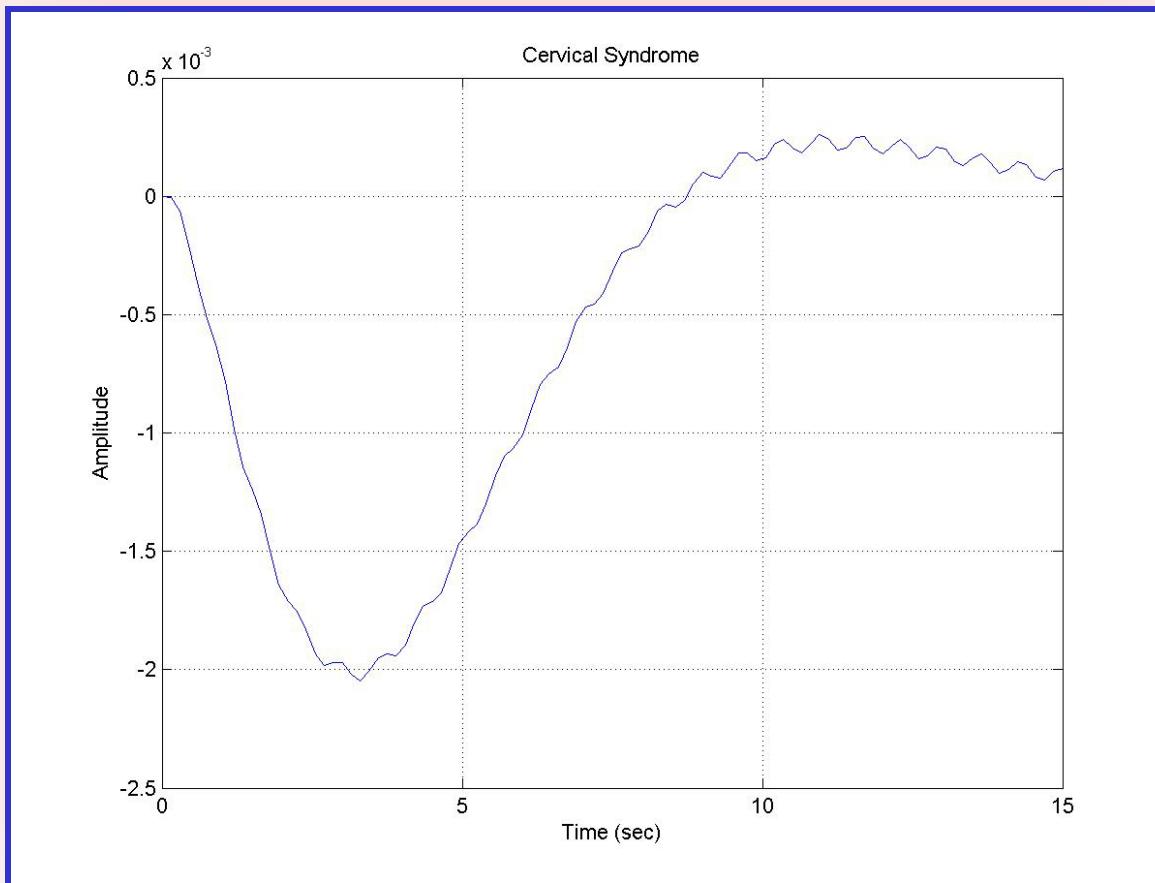


The screenshot shows a Matlab diary file window titled 'C:\Cellier\Classes\Ece449\Hw\_2\mmps\_hw2a.dry'. The window contains the following code:

```
1 %  
2 % Enter constants first  
3 %  
4 k1 = 0.3;  
5 k2 = 8;  
6 k3 = 3;  
7 B1 = 0.8;  
8 B2 = 10;  
9 B3 = 12;  
10 m1 = 1.2;  
11 m2 = 14;  
12 m3 = 3.2;  
13 m4 = 24;  
14 kk = k1 + k2 + k3;  
15 BB = B1 + B2 + B3;  
16 %  
17 % Compute system matrices  
18 %  
19 A = [ 0 0 0 0 1 0 0 0  
20 0 0 0 0 0 1 0 0  
21 0 0 0 0 0 0 1 0 ]
```



# Matlab Simulation (1<sup>st</sup> Part)

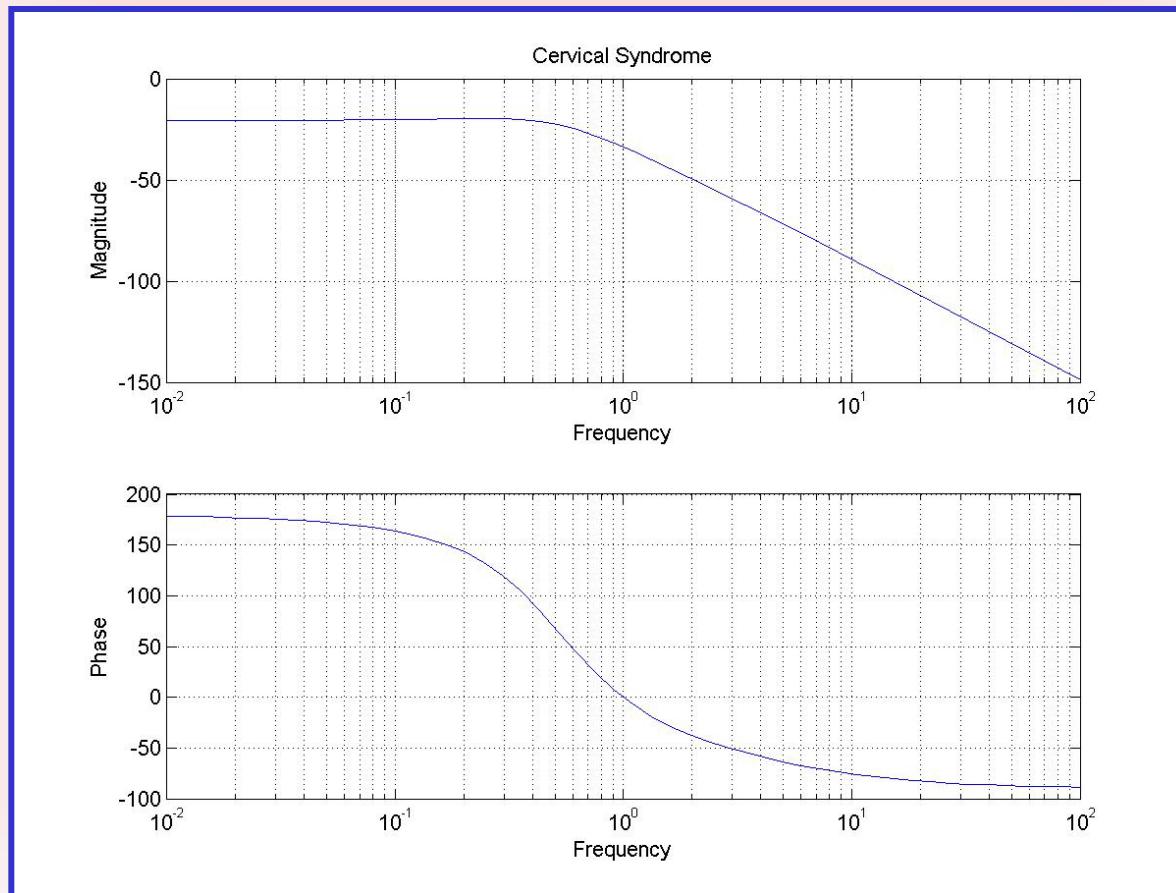


## 2<sup>nd</sup> Sub-problem

3. In order to be able to better analyze the *resonance phenomena*, we wish to obtain a *Bode diagram* of the system. To this end, we generate a logarithmic base of frequency values in the range from 0.01 Hz to 100 Hz by means of Matlab's *logspace* function. The *Bode* function may now be used to compute the Bode diagram. The amplitude needs to be converted to decibels. Using the functions *subplot*, *semilogx*, *grid*, *title*, *xlabel*, and *ylabel*, the Bode diagram shall now be displayed on two separate graphs on the same page.



# Matlab Bode Diagram (1<sup>st</sup> Part)



## 3<sup>rd</sup> Sub-problem

4. Finally, we wish to perform a *sensitivity analysis*. We want to study the variability of the spring constant and the damper between head and upper torso. For this purpose, we assume a variability of these two parameters of  $\pm 50\%$ .
5. Repeat the frequency analysis for the four worst-case combinations of the two parameters.
6. Plot the maxima and minima of the amplitude and phase curves as a sensitivity Bode diagram.



# Matlab Code (2<sup>nd</sup> Part)

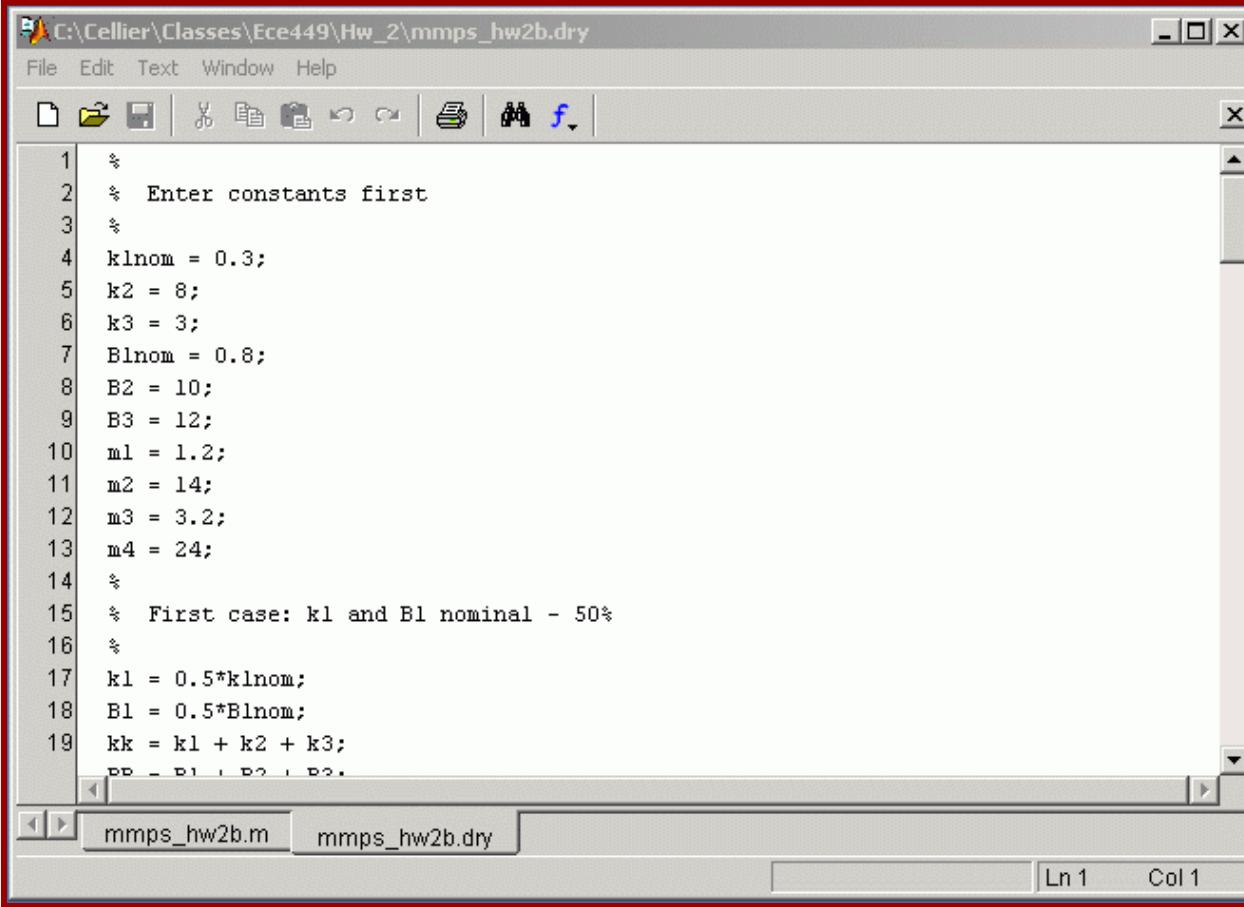
The screenshot shows a Matlab script editor window with a red border. The window title is "C:\Cellier\Classes\Ece449\Hw\_2\mmps\_hw2b.m". The menu bar includes File, Edit, Text, Window, and Help. The toolbar has icons for file operations like Open, Save, and Print. The code area contains the following script:

```
1 % Cervical Syndrome (Second Part)
2 %
3 %
4 echo on
5 diary mmps_hw2b.dry
6 %
7 % Enter constants first
8 %
9 k1nom = 0.3;
10 k2 = 8;
11 k3 = 3;
12 Blnom = 0.8;
13 B2 = 10;
14 B3 = 12;
15 m1 = 1.2;
16 m2 = 14;
17 m3 = 3.2;
18 m4 = 24;
19 %
20 % First case: k1 and Bl nominal - 50%
21 %
```

The status bar at the bottom indicates "script" and "Ln 1 Col 1".



# Matlab Diary File (2<sup>nd</sup> Part)



The screenshot shows a Matlab diary window titled "mmmps\_hw2b.dry". The window contains a script with the following code:

```
1 %  
2 % Enter constants first  
3 %  
4 klnom = 0.3;  
5 k2 = 8;  
6 k3 = 3;  
7 Blnom = 0.8;  
8 B2 = 10;  
9 B3 = 12;  
10 m1 = 1.2;  
11 m2 = 14;  
12 m3 = 3.2;  
13 m4 = 24;  
14 %  
15 % First case: k1 and B1 nominal - 50%  
16 %  
17 k1 = 0.5*klnom;  
18 B1 = 0.5*Blnom;  
19 kk = k1 + k2 + k3;  
pp = p1 + p2 + p3;
```

The diary window has a toolbar at the top with icons for file operations like Open, Save, and Print. Below the toolbar is a menu bar with File, Edit, Text, Window, and Help. The status bar at the bottom right shows "Ln 1 Col 1".



# Matlab Bode Diagram (2<sup>nd</sup> Part)

