

4th Homework Problem

- In this homework problem, we wish to exercise the tearing and relaxation methods by means of a slightly larger problem than that presented in the lecture.
- We also wish to compare the computational efficiency of the simulation codes obtained by the two methods.

October 11, 2012

© Prof. Dr. François E. Cellier

Start Presentation



- Tearing Algorithm
- Relaxation Algorithm
- Tearing vs. Relaxation

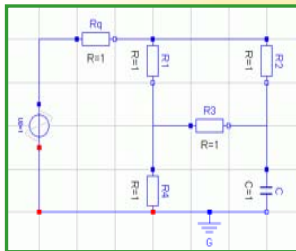
October 11, 2012

© Prof. Dr. François E. Cellier

Start Presentation



Tearing Algorithm I



Given the electrical circuit of the figure on the left, determine a complete set of equations in currents and Voltages (by use of both node and mesh equations).

Make the equation system causal, while trying to get by with two tearing variables (and residual equations).

October 11, 2012

© Prof. Dr. François E. Cellier

Start Presentation



Tearing Algorithm II


- Solve symbolically for the tearing variables, and find replacement equations for the two residual equations, which permit to make the entire set of equations causal.
- Find an explicit DAE system by completely sorting the equations both horizontally and vertically.

October 11, 2012

© Prof. Dr. François E. Cellier

Start Presentation





ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems


Relaxation Algorithm I


- Apply the relaxation algorithm to the same electrical circuit.
- For determining the sequence of equations and variables, make use of the following heuristics:
 - Make the equations causal in the same way as for *Problem 4.1*.
 - Start with the first residual equation. It is being placed as the *last equation*, whereby the corresponding tearing variable is the *last variable*.
 - Number the equations, which can be made causal on the basis of the assumption that the tearing variable is already known, starting with *equation #1*, and set the variables for which these equations are being solved also at the beginning of the list of variables, starting with *variable #1*. In this way, the diagonal elements can be normalized to *1*.

October 11, 2012

© Prof. Dr. François E. Cellier

Start Presentation





ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems

Relaxation Algorithm II


- Set the second residual equation as the second to last equation, whereby the corresponding tearing variable is also the second to last variable.
- Number the equations, which can be causalized based on the assumption that the second tearing variable is already known, in increasing order following the first set of equations (with the exception of the first residual equation, which comes at the very end).
- The resulting equation system in matrix form has diagonal elements that are all normalized to 1, and contains exactly two non-zero elements above the diagonal. These are located in the columns of the tearing variables and in the rows of the first equation of the causalized equation system.


- Consequently, the problem of minimizing the number of non-zero elements above the diagonal in the case of the relaxation algorithm is indeed identical with the search for suitable tearing variables.

October 11, 2012

© Prof. Dr. François E. Cellier

Start Presentation





ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems

Tearing vs. Relaxation

- Apply the relaxation algorithm to the resulting set of equations, and determine an explicit DAE system in this way.
- Count the number of additions, subtractions, multiplications, and divisions of the two sets of equations that result from the two algorithms, and determine, which of the two algorithms is more economical in the case of the given example.

October 11, 2012

© Prof. Dr. François E. Cellier

Start Presentation

