

$$\begin{bmatrix} \bar{V}(s) & -\bar{U}(s) \\ -\bar{N}(s) & \bar{M}(s) \end{bmatrix} \cdot \begin{bmatrix} M(s) & U(s) \\ N(s) & V(s) \end{bmatrix} = I^{(u)}$$

Algorithm:

(I) Find $M(s), N(s)$:

$$P(s) = \left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & D_p \end{array} \right]$$

$$[P(s) = C_p \cdot (sI - A_p)^{-1} B_p + D_p]$$

Find a stabilizing state feedback

f :

$$\left. \begin{array}{l} \dot{x}_p = A_p \cdot x_p + B_p \cdot u_p \\ y_p = C_p \cdot x_p + D_p \cdot u_p \\ u_p = r + f \cdot x_p \end{array} \right|$$

$$\Rightarrow \left. \begin{array}{l} \dot{x}_p = (A_p + B_p \cdot f) \cdot x_p + B_p \cdot r \\ y_p = (C_p + D_p \cdot f) \cdot x_p + D_p \cdot r \\ u_p = f \cdot x_p + r \end{array} \right|$$

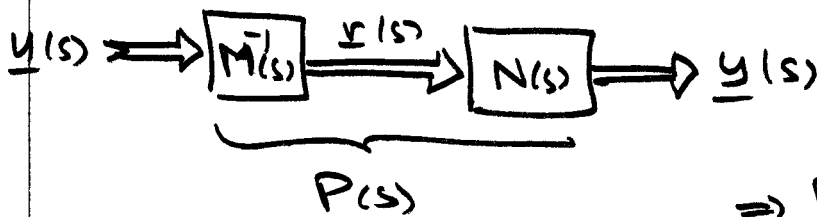
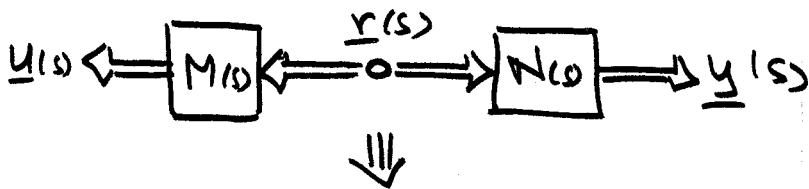


Let: $\underline{y}(s) = M(s) \cdot \underline{r}(s)$

$$\Rightarrow M(s) = \left[\begin{array}{c|c} A_p + B_p \cdot F & B_p \\ \hline F & I(m) \end{array} \right]$$

Let: $\underline{y}(s) = N(s) \cdot \underline{v}(s)$

$$\Rightarrow N(s) = \left[\begin{array}{c|c} A_p + B_p \cdot F & B_p \\ \hline C_p + D_p \cdot F & D_p \end{array} \right]$$



$$\Rightarrow \underline{P}(s) = N(s) \cdot M^{-1}(s)$$

- $N(s), M(s)$ are both stable, since $\text{eig}(A_p + B_p \cdot F) < \phi$ by design.
- $N(s), M(s)$ are coprime (proof later).

Superposition principle:

Set $w_1 = r = \phi$ for now:

$$\begin{aligned} \underline{w}^0 &= (A_p + B_p \cdot F + L \cdot C_p + L \cdot D_p \cdot F) \underline{w} - L \cdot \underline{u} \\ \underline{u} &= F \cdot \underline{w} \end{aligned}$$

$$\Rightarrow K(s) = \left[\begin{array}{c|c} A_p + B_p \cdot F + L \cdot C_p + L \cdot D_p \cdot F & -L \\ \hline F & \phi \end{array} \right]$$

$$K(s) \equiv U(s) \cdot V(s)^{-1}$$

Introduce:

$$\underline{y} = (C_p + D_p \cdot F) \underline{w} - \underline{u}$$

$$\Rightarrow \underline{w} = (A_p + B_p \cdot F) \underline{w} + L \cdot \underline{y}$$

13-782 500 SHEETS, FILLER, 5 SQUARE
 42-381 50 SHEETS, EYE-EASE, 5 SQUARE
 42-382 100 SHEETS, EYE-EASE, 5 SQUARE
 42-383 100 SHEETS, EYE-EASE, 5 SQUARE
 42-384 100 SHEETS, EYE-EASE, 5 SQUARE
 42-385 100 RECYCLED WHITE, 5 SQUARE
 42-386 200 RECYCLED WHITE, 5 SQUARE
 MADE IN U.S.A.



Let: $\underline{y}_2(s) = (-U(s)) \cdot \underline{y}(s)$

$$\Rightarrow -U(s) = \left[\begin{array}{c|c} A_p + B_p \cdot F & L \\ \hline F & \emptyset \end{array} \right]$$

$$\Rightarrow U(s) = \left[\begin{array}{c|c} A_p + B_p \cdot F & -L \\ \hline F & \emptyset \end{array} \right]$$

Let: $\underline{y}_1(s) = (-V(s)) \cdot \underline{y}(s)$

$$\underline{y} = (C_p + D_p \cdot F) \underline{y} - \underline{y}_1$$

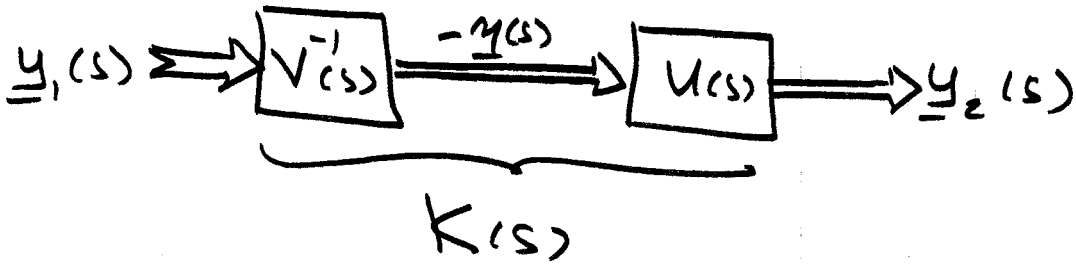
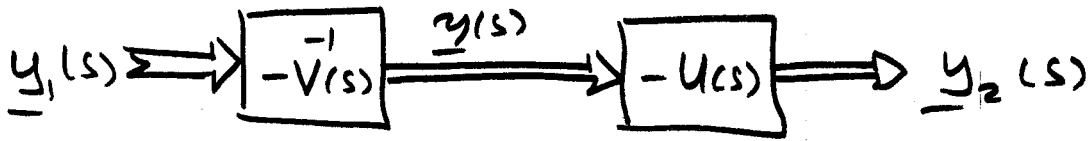
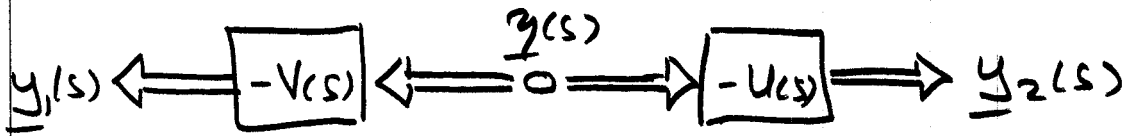
$$\Rightarrow \underline{y}_1 = (C_p + D_p \cdot F) \underline{y} - \underline{y}$$

$$\Rightarrow -V(s) = \left[\begin{array}{c|c} A_p + B_p \cdot F & L \\ \hline C_p + D_p \cdot F & -I^{(p)} \end{array} \right]$$

$$\Rightarrow V(s) = \left[\begin{array}{c|c} A_p + B_p \cdot F & -L \\ \hline C_p + D_p \cdot F & I^{(p)} \end{array} \right]$$

13/82 500 SHEETS FULLER 5 SQUARE
 42-381 100 SHEETS EYE-EASE 5 SQUARE
 42-382 100 SHEETS EYE-EASE 5 SQUARE
 42-383 100 SHEETS EYE-EASE 5 SQUARE
 42-389 100 RECYCLED WHITE 5 SQUARE
 42-390 200 RECYCLED WHITE 5 SQUARE
 Made in U.S.A.





$\Rightarrow \underline{\underline{K(s) = U(s) \cdot V^{-1}(s)}}$

✓



Left-Coprime Factorization:

⇒ Use duality principle:

$$\vec{M}^*(s) = \left[\begin{array}{c|c} A_p^* + C_p^* \cdot L^* & C_p^* \\ \hline L^* & I^{(p)} \end{array} \right]$$

etc.

$$\Rightarrow \vec{M}(s) = \left[\begin{array}{c|c} A_p + L \cdot C_p & L \\ \hline C_p & I^{(p)} \end{array} \right]$$

$$\vec{N}(s) = \left[\begin{array}{c|c} A_p + L \cdot C_p & B_p + L \cdot D_p \\ \hline C_p & D_p \end{array} \right]$$

$$\vec{U}(s) = \left[\begin{array}{c|c} A_p + L \cdot C_p & L \\ \hline -F & \emptyset \end{array} \right]$$

$$\vec{V}(s) = \left[\begin{array}{c|c} A_p + L \cdot C_p & B_p + L \cdot D_p \\ \hline -F & I^{(m)} \end{array} \right]$$

13-782 500 SHEETS, FILLER, 5 SQUARE
 42-381 60 SHEETS, FILLER, 5 SQUARE
 42-382 100 SHEETS, FILLER, 5 SQUARE
 42-383 150 SHEETS, FILLER, 5 SQUARE
 42-384 200 SHEETS, FILLER, 5 SQUARE
 42-385 200 SHEETS, FILLER, 5 SQUARE
 42-386 100 RECYCLED WHITE, 5 SQUARE
 42-387 200 RECYCLED WHITE, 5 SQUARE
 MADE IN U.S.A.



```
function [N,M,U,V,K] = rcf(G,PcQ,PoRin,Rout)
%
% This function operates on POLPAC system descriptions stored in modes 4..6.
% It computes the right coprime factorization (RCF) of G. It also computes
% the optimal feedback matrix K.
%
% Input Parameter:
% -----
%
% G      := Transfer function matrix (modes 4..6)
% Pc/Q   := Pc is a column vector of controller pole locations
%         Q is a state weighting matrix for LQR
% Po/Rin := Po is a column vector of observer pole locations
%         Rin is an input weighting matrix for LQR
% Rout   := Rout is an output weighting matrix for LQR
%
% Output Parameter:
% -----
%
% N, M   := Right coprime factorization of G
% U, V   := Right coprime factorization of K
% K      := The optimal feedback system
%
% Explanation:
% -----
%
% Given a minimal transfer function matrix. G is first converted to the
% time domain. Optimal feedback matrices F and L are computed using LQR
% with Q = I, Rin = I, and Rout = I.. System representations are then found
% for M, N, U, V. These are converted back to the frequency domain (same
% mode as G). Then:  $G(s) = N(s)*inv(M(s))$ , and
%                   $K(s) = U(s)*inv(V(s))$ .
%
% Defaults:
% -----
%
% Only the first input parameter is mandated. By default, the stabilizing
% output feedback controller will be computed by LQR using identity matrices
% for Q, Rin, and Rout.
%
% If three input parameters are provided, the second should be the pole
% locations of the controller poles, and the third should be the pole
% locations of the observer poles. The PLACE routine will be used to
% compute the stabilizing output feedback controller.
%
% If four input parameters are provided, the second should be the state
% weighting matrix, the third the input wieghting matrix, and the fourth
% the output weighting matrix. LQR will be used to compute the stabilizing
% output feedback controller.
%
push('RCF')
global debg
%
% Start by unpacking the structural information
%
[mode,logcol,dtype,stype] = unpack(G(1,1));
[p,m] = logdim(G);
dom = domain(G);
%
% Check for consistency
```



```
%
if debg == 3,
    if mode < 4,
        disp('RCF: Error - Operates on modes 4..6 only'),
        abort,
    end,
    if mode > 6,
        disp('RCF: Error - Operates on modes 4..6 only'),
        abort,
    end,
end
%
% Convert to time domain
%
Sg = tfm2ss(G);
Sg = minreals(Sg);
%
% Extract system matrices
%
[A,B,C,D] = oldsysst(Sg);
[n,n] = size(A);
%
% Compute optimal control
%
if nargin == 3,
    F = -place(A,B,PcQ);
    L = -place(A',C',PoRin)';
else
    if nargin == 1,
        PcQ = eye(n);
        PcRin = eye(m);
        Rout = eye(p);
    end,
    F = -lqr(A,B,PcQ,PcRin);
    L = -lqr(A',C',PcQ,Rout)';
end
%
% Now compute the four systems.
%
Am = A + B*F;
Bm = B;
Cm = F;
Dm = eye(m);
%
An = Am;
Bn = B;
Cn = C + D*F;
Dn = D;
%
Au = Am;
Bu = -L;
Cu = F;
Du = zeros(m,p);
%
Av = Am;
Bv = Bu;
Cv = Cn;
Dv = eye(p);
%
% Make into systems
```

```
%  
Sm = newsyst (Am, Bm, Cm, Dm, stype) ;  
Sn = newsyst (An, Bn, Cn, Dn, stype) ;  
Su = newsyst (Au, Bu, Cu, Du, stype) ;  
Sv = newsyst (Av, Bv, Cv, Dv, stype) ;  
%  
% Convert back to the frequency domain  
%  
M = ss2tfm (Sm, mode, dom) ;  
N = ss2tfm (Sn, mode, dom) ;  
U = ss2tfm (Su, mode, dom) ;  
V = ss2tfm (Sv, mode, dom) ;  
M = reduceg (M) ;  
N = reduceg (N) ;  
U = reduceg (U) ;  
V = reduceg (V) ;  
%  
% Check whether we are done  
%  
if nargin == 5,  
    %  
    % We still need to compute K  
    %  
    Ak = A + B*F + L*C + L*D*F ;  
    Bk = -L ;  
    Ck = F ;  
    Dk = zeros (m, p) ;  
    Sk = newsyst (Ak, Bk, Ck, Dk, stype) ;  
    K = ss2tfm (Sk, mode, dom) ;  
    K = reduceg (K) ;  
end  
%  
pull  
return
```

```
function [N,M,U,V,K] = lcf(G,PcQ,PoRin,Rout)
```

```
%  
% This function operates on POLPAC system descriptions stored in modes 4..6.  
% It computes the left coprime factorization (RCF) of G. It also computes  
% the optimal feedback matrix K.
```

```
% Input Parameter:
```

```
-----
```

```
% G := Transfer function matrix (modes 4..6)  
% Pc/Q := Pc is a column vector of controller pole locations  
% Q is a state weighting matrix for LQR  
% Po/Rin := Po is a column vector of observer pole locations  
% Rin is an input weighting matrix for LQR  
% Rout := Rout is an output weighting matrix for LQR
```

```
% Output Parameter:
```

```
-----
```

```
% N, M := Left coprime factorization of G  
% U, V := Left coprime factorization of K  
% K := The optimal feedback system
```

```
% Explanation:
```

```
-----
```

```
% Given a minimal transfer function matrix. G is first converted to the  
% time domain. Optimal feedback matrices F and L are computed using LQR  
% with Q = I, Rin = I, and Rout = I.. System representations are then found  
% for M, N, U, V. These are converted back to the frequency domain (same  
% mode as G). Then:  $G(s) = \text{inv}(M(s))*N(s)$ , and  
%  $K(s) = \text{inv}(V(s))*U(s)$ .
```

```
% Defaults:
```

```
-----
```

```
% Only the first input parameter is mandated. By default, the stabilizing  
% output feedback controller will be computed by LQR using identity matrices  
% for Q, Rin, and Rout.
```

```
% If three input parameters are provided, the second should be the pole  
% locations of the controller poles, and the third should be the pole  
% locations of the observer poles. The PLACE routine will be used to  
% compute the stabilizing output feedback controller.
```

```
% If four input parameters are provided, the second should be the state  
% weighting matrix, the third the input weighting matrix, and the fourth  
% the output weighting matrix. LQR will be used to compute the stabilizing  
% output feedback controller.
```

```
% push('LCF')
```

```
% global dbg
```

```
% Start by unpacking the structural information
```

```
% [mode,logcol,dtype,stype] = unpack(G(1,1));
```

```
% [p,m] = logdim(G);
```

```
% dom = domain(G);
```

```
% Check for consistency
```

```
%
if debug == 3,
    if mode < 4,
        disp('LCF: Error - Operates on modes 4..6 only'),
        abort,
    end,
    if mode > 6,
        disp('LCF: Error - Operates on modes 4..6 only'),
        abort,
    end,
end
%
% Convert to time domain
%
Sg = tfm2ss(G);
Sg = minreals(Sg);
%
% Extract system matrices
%
[A,B,C,D] = oldsysst(Sg);
[n,n] = size(A);
%
% Compute optimal control
%
if nargin == 3,
    F = -place(A,B,PcQ);
    L = -place(A',C',PoRin)';
else
    if nargin == 1,
        PcQ = eye(n);
        PcRin = eye(m);
        Rout = eye(p);
    end,
    F = -lqr(A,B,PcQ,PcRin);
    L = -lqr(A',C',PcQ,Rout)';
end
%
% Now compute the four systems.
%
Am = A + L*C;
Bm = L;
Cm = C;
Dm = eye(p);
%
An = Am;
Bn = B + L*D;
Cn = C;
Dn = D;
%
Au = Am;
Bu = L;
Cu = -F;
Du = zeros(m,p);
%
Av = Am;
Bv = Bn;
Cv = Cu;
Dv = eye(m);
%
% Make into systems
```

```
%  
Sm = newsyst (Am, Bm, Cm, Dm, stype) ;  
Sn = newsyst (An, Bn, Cn, Dn, stype) ;  
Su = newsyst (Au, Bu, Cu, Du, stype) ;  
Sv = newsyst (Av, Bv, Cv, Dv, stype) ;  
%  
% Convert back to the frequency domain  
%  
M = ss2tfm (Sm, mode, dom) ;  
N = ss2tfm (Sn, mode, dom) ;  
U = ss2tfm (Su, mode, dom) ;  
V = ss2tfm (Sv, mode, dom) ;  
M = reduceg (M) ;  
N = reduceg (N) ;  
U = reduceg (U) ;  
V = reduceg (V) ;  
%  
% Check whether we are done  
%  
if nargin == 5,  
%  
% We still need to compute K  
%  
Ak = A + B*F + L*C + L*D*F ;  
Bk = -L ;  
Ck = F ;  
Dk = zeros (m, p) ;  
Sk = newsyst (Ak, Bk, Ck, Dk, stype) ;  
K = ss2tfm (Sk, mode, dom) ;  
K = reduceg (K) ;  
end  
%  
pull  
return
```

```

%
% Test [5.3]: Coprime factorizations of a POLPAC System
% -----
%
polpac_init
global debg
debg = 3;
%
% Enter the system
%
A = [ 0      1      0
      2      1      0
      0      0      1 ];
B = [ 0      0
      1      0
      2      1 ];
C = [ 1      0      1
      0      1      1 ];
D = zeros(2);
n = 3;
m = 2;
p = 2;
stype = 0;
mode = 4;
dom = 0;
S = newsyst(A,B,C,D,stype);
Gm = ss2tfm(S,mode,dom);
Gm = reduceeg(Gm);
show(Gm,'Gm')
Gm(s) =
      (2*s^2 - s - 5)          (1)
-----
(s^3 - 2*s^2 - s + 2)      (s - 1)
      (3*s^2 - 3*s - 4)          (1)
-----
(s^3 - 2*s^2 - s + 2)      (s - 1)

%
[N,M,U,V,K] = rcf(Gm);
show(N,'N')
N(s) =
      (2*s^2 + 2.936*s + 0.3798)          (s^2 + 5.707*s + 5.625)
-----
(s^3 + 4.278*s^2 + 5.599*s + 2.384)      (s^3 + 4.278*s^2 + 5.599*s + 2.384)
      (3*s^2 + 2.424*s - 0.1088)          (s^2 + 6.666*s + 4.667)
-----
(s^3 + 4.278*s^2 + 5.599*s + 2.384)      (s^3 + 4.278*s^2 + 5.599*s + 2.384)

show(M,'M')
M(s) =
(s^3 - 0.5115*s^2 - 2.489*s - 0.9771)      (0.9588*s^2 - 0.9588*s - 1.918)
-----
(s^3 + 4.278*s^2 + 5.599*s + 2.384)      (s^3 + 4.278*s^2 + 5.599*s + 2.384)
      (0.9588*s^2 + 2.932*s + 2.063)          (s^3 + 2.79*s^2 + 0.877*s - 0.8316)
-----
(s^3 + 4.278*s^2 + 5.599*s + 2.384)      (s^3 + 4.278*s^2 + 5.599*s + 2.384)

```

show(U, 'U')

$$\begin{array}{r}
 U(s) = \\
 \frac{(56.21s^2 + 37.9s - 18.27)}{(s^3 + 4.278s^2 + 5.599s + 2.384)} \quad \frac{(-135.2s^2 - 117.6s + 17.71)}{(s^3 + 4.278s^2 + 5.599s + 2.384)} \\
 \frac{(-40.04s^2 - 126.4s - 86.93)}{(s^3 + 4.278s^2 + 5.599s + 2.384)} \quad \frac{(83.13s^2 + 262.3s + 180.9)}{(s^3 + 4.278s^2 + 5.599s + 2.384)}
 \end{array}$$

show(V, 'V')

$$\begin{array}{r}
 V(s) = \\
 \frac{(s^3 + 9.695s^2 + 103.8s + 12)}{(s^3 + 4.278s^2 + 5.599s + 2.384)} \quad \frac{(2.629s^2 - 167.6s - 222.1)}{(s^3 + 4.278s^2 + 5.599s + 2.384)} \\
 \frac{(2.629s^2 + 137.9s + 110.6)}{(s^3 + 4.278s^2 + 5.599s + 2.384)} \quad \frac{(s^3 + 12.8s^2 - 258.3s - 220.2)}{(s^3 + 4.278s^2 + 5.599s + 2.384)}
 \end{array}$$

show(K, 'K')

$$\begin{array}{r}
 K(s) = \\
 \frac{(56.21s^2 + 872.2s + 866.6)}{(s^3 + 18.21s^2 - 120.9s - 785.9)} \quad \frac{(-135.2s^2 - 998s - 810.9)}{(s^3 + 18.21s^2 - 120.9s - 785.9)} \\
 \frac{(-40.04s^2 - 686s - 360.1)}{(s^3 + 18.21s^2 - 120.9s - 785.9)} \quad \frac{(83.13s^2 + 817.9s + 1009)}{(s^3 + 18.21s^2 - 120.9s - 785.9)}
 \end{array}$$

%
[Nt,Mt,Ut,Vt,Kt] = lcf(Gm);

show(Nt, 'Nt')

$$\begin{array}{r}
 Nt(s) = \\
 \frac{(2s^2 + 8.147s + 13.28)}{(s^3 + 11.93s^2 + 35.22s + 31.1)} \quad \frac{(s^2 + 4.888s + 5.824)}{(s^3 + 11.93s^2 + 35.22s + 31.1)} \\
 \frac{(3s^2 + 7.993s - 5.975)}{(s^3 + 11.93s^2 + 35.22s + 31.1)} \quad \frac{(s^2 + 1.788s - 0.2789)}{(s^3 + 11.93s^2 + 35.22s + 31.1)}
 \end{array}$$

show(Mt, 'Mt')

$$\begin{array}{r}
 Mt(s) = \\
 \frac{(s^3 + 6.517s^2 + 4.195s - 3.259)}{(s^3 + 11.93s^2 + 35.22s + 31.1)} \quad \frac{(-2.629s^2 - 3.259s - 2.566)}{(s^3 + 11.93s^2 + 35.22s + 31.1)} \\
 \frac{(-2.629s^2 + 7.138s + 10.83)}{(s^3 + 11.93s^2 + 35.22s + 31.1)} \quad \frac{(s^3 + 3.417s^2 - 9.205s - 10.56)}{(s^3 + 11.93s^2 + 35.22s + 31.1)}
 \end{array}$$

show(Ut, 'Ut')

$$\begin{array}{r}
 Ut(s) = \\
 \frac{(56.21s^2 + 826.9s + 858.3)}{(s^3 + 11.93s^2 + 35.22s + 31.1)} \quad \frac{(-135.2s^2 - 876.4s - 653.8)}{(s^3 + 11.93s^2 + 35.22s + 31.1)} \\
 \frac{(-40.04s^2 - 548.2s - 568.5)}{(s^3 + 11.93s^2 + 35.22s + 31.1)} \quad \frac{(83.13s^2 + 549.4s + 406.2)}{(s^3 + 11.93s^2 + 35.22s + 31.1)}
 \end{array}$$

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$$(s^3 + 11.93s^2 + 35.22s + 31.1)$$

$$(s^3 + 11.93s^2 + 35.22s + 31.1)$$

show(Vt, 'Vt')

$$Vt(s) = (s^3 + 16.72s^2 - 189.5s - 851)$$

$$\text{-----}$$
$$(s^3 + 11.93s^2 + 35.22s + 31.1)$$

$$(-0.9588s^2 + 153s + 576.7)$$

$$\text{-----}$$
$$(s^3 + 11.93s^2 + 35.22s + 31.1)$$

$$(-0.9588s^2 - 91.44s - 174.6)$$

$$\text{-----}$$
$$(s^3 + 11.93s^2 + 35.22s + 31.1)$$

$$(s^3 + 13.42s^2 + 97.56s + 147.1)$$

$$\text{-----}$$
$$(s^3 + 11.93s^2 + 35.22s + 31.1)$$

show(Kt, 'Kt')

$$Kt(s) = (56.21s^2 + 872.2s + 866.6)$$

$$\text{-----}$$
$$(s^3 + 18.21s^2 - 120.9s - 785.9)$$

$$(-40.04s^2 - 686s - 360.1)$$

$$\text{-----}$$
$$(s^3 + 18.21s^2 - 120.9s - 785.9)$$

$$(-135.2s^2 - 998s - 810.9)$$

$$\text{-----}$$
$$(s^3 + 18.21s^2 - 120.9s - 785.9)$$

$$(83.13s^2 + 817.9s + 1009)$$

$$\text{-----}$$
$$(s^3 + 18.21s^2 - 120.9s - 785.9)$$

```
%
B11 = sub(multg(Vt,M),multg(Ut,N));
B11 = reduceg(B11);
show(B11, 'B11')
B11(s) =
(1) (0)
(0) (1)
```

```
%
B12 = sub(multg(Vt,U),multg(Ut,V));
B12 = reduceg(B12);
show(B12, 'B12')
B12(s) =
(0) (0)
(0) (0)
```

```
%
B21 = sub(multg(Mt,N),multg(Nt,M));
B21 = reduceg(B21);
show(B21, 'B21')
B21(s) =
(0) (0)
(0) (0)
```

```
%
B22 = sub(multg(Mt,V),multg(Nt,U));
B22 = reduceg(B22);
show(B22, 'B22')
B22(s) =
(1) (0)
(0) (1)
```

```
%
show(Gm, 'Gm')
Gm(s) =
(2*s^2 - s - 5) (1)
-----
(s^3 - 2*s^2 - s + 2) (s - 1)
(3*s^2 - 3*s - 4) (1)
-----
(s^3 - 2*s^2 - s + 2) (s - 1)
```



```
Gm2 = multg(N, invg(M));
Gm2 = reduceeg(Gm2);
show(Gm2, 'Gm2')
```

$$\begin{array}{r} \text{Gm2}(s) = \\ \frac{(2*s^2 - s - 5)}{(s^3 - 2*s^2 - s + 2)} \quad (1) \\ \hline \frac{(3*s^2 - 3*s - 4)}{(s^3 - 2*s^2 - s + 2)} \quad (1) \end{array}$$

```
Gm3 = multg(invg(Mt), Nt);
Gm3 = reduceeg(Gm3);
show(Gm3, 'Gm3')
```

$$\begin{array}{r} \text{Gm3}(s) = \\ \frac{(2*s^2 - s - 5)}{(s^3 - 2*s^2 - s + 2)} \quad (1) \\ \hline \frac{(3*s^2 - 3*s - 4)}{(s^3 - 2*s^2 - s + 2)} \quad (1) \end{array}$$

```
%
show(K, 'K')
```

$$\begin{array}{r} \text{K}(s) = \\ \frac{(56.21*s^2 + 872.2*s + 866.6)}{(s^3 + 18.21*s^2 - 120.9*s - 785.9)} \quad \frac{(-135.2*s^2 - 998*s - 810.9)}{(s^3 + 18.21*s^2 - 120.9*s - 785.9)} \\ \hline \frac{(-40.04*s^2 - 686*s - 360.1)}{(s^3 + 18.21*s^2 - 120.9*s - 785.9)} \quad \frac{(83.13*s^2 + 817.9*s + 1009)}{(s^3 + 18.21*s^2 - 120.9*s - 785.9)} \end{array}$$

```
K2 = multg(U, invg(V));
K2 = reduceeg(K2);
show(K2, 'K2')
```

K2(s) =
SHOW1: Warning - Column too broad. Cannot properly display
Col. 1

$$\begin{array}{r} \frac{(56.21*s^2 + 872.2*s + 866.6)}{(s^3 + 18.21*s^2 - 120.9*s - 785.9)} \\ \hline \frac{(-40.04*s^2 - 686*s - 360.1)}{(s^3 + 18.21*s^2 - 120.9*s - 785.9)} \end{array}$$

Col. 2

$$\begin{array}{r} \frac{(-135.2*s^8 - 2155*s^7 - 1.334e+04*s^6 - 4.35e+04*s^5 - 8.349e+04*s^4 - 9.797e+04*s^3 - 2.859e+04*s^2 - 4.716e+04*s - 810.9)}{(s^9 + 26.77*s^8 + 64.42*s^7 - 1231*s^6 - 9281*s^5 - 2.859e+04*s^4 - 4.716e+04*s^3 - 810.9)} \\ \hline \frac{(83.13*s^2 + 817.9*s + 1009)}{(s^3 + 18.21*s^2 - 120.9*s - 785.9)} \end{array}$$

$$(s^3 + 18.21s^2 - 120.9s - 785.9)$$

K3 = multg(invg(Vt),Ut);

K3 = reduceg(K3);

show(K3,'K3')

K3(s) =

SHOW1: Warning - Column too broad. Cannot properly display
Col. 1

$$\frac{(56.21s^2 + 872.2s + 866.6)}{(s^3 + 18.21s^2 - 120.9s - 785.9)}$$

$$\frac{(-40.04s^2 - 686s - 360.1)}{(s^3 + 18.21s^2 - 120.9s - 785.9)}$$

Col. 2

$$\frac{(-135.2s^2 - 998s - 810.9)}{(s^3 + 18.21s^2 - 120.9s - 785.9)}$$

$$\frac{(83.13s^8 + 2802s^7 + 3.823e+04s^6 + 2.732e+05s^5 + 1.118e+06s^4 + 2.714e+07s^3 + 42.08s^2 + 526.7s + 1108)}{(s^9 + 42.08s^8 + 526.7s^7 + 1108s^6 - 2.607e+04s^5 - 2.382e+05s^4 - 9.08e+06s^3 - 7.859e+07s^2 - 1.209e+08s - 7.859e+08)}$$

%
diary off