

# Numerical Simulation of Dynamic Systems: Hw2 - Solution

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# [H2.1] Marginal Stability

Given the following linear time-invariant continuous-time system:

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{pmatrix} 1250 & -25113 & -60050 & -42647 & -23999 \\ 500 & -10068 & -24057 & -17092 & -9613 \\ 250 & -5060 & -12079 & -8586 & -4826 \\ -750 & 15101 & 36086 & 25637 & 14420 \\ 250 & -4963 & -11896 & -8438 & -4756 \end{pmatrix} \cdot \mathbf{x} + \begin{pmatrix} 5 \\ 2 \\ 1 \\ -3 \\ 1 \end{pmatrix} \cdot u \\ \mathbf{y} &= (-1 \quad 26 \quad 59 \quad 43 \quad 23) \cdot \mathbf{x}\end{aligned}$$

with initial conditions:

$$\mathbf{x}_0 = \begin{pmatrix} 1 \\ -2 \\ 3 \\ -4 \\ 5 \end{pmatrix}$$

## [H2.1] Marginal Stability II

Determine the step size,  $h_{\text{marg}}$ , for which FE will give marginally stable results.

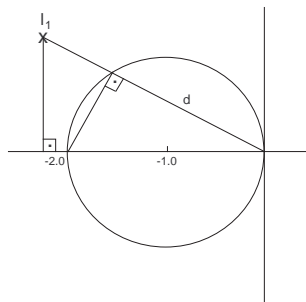
Simulate the system across **10 seconds** of simulated time with step input using the FE algorithm with the following step sizes:

1.  $h = 0.1 \cdot h_{\text{marg}}$ ,
2.  $h = 0.95 \cdot h_{\text{marg}}$ ,
3.  $h = h_{\text{marg}}$ ,
4.  $h = 1.05 \cdot h_{\text{marg}}$ , and
5.  $h = 2 \cdot h_{\text{marg}}$ .

Discuss the results.

## [H2.1] Marginal Stability III

Determine the step size,  $h_{\text{marg}}$ , for which FE will give marginally stable results.



$$d : (-2) = \operatorname{Re}(\lambda_1) : |\lambda_1|$$

Thus:

$$h_{\text{marg}} = \min_{\forall i} \left( \frac{d_i}{|\lambda_i|} \right) = \min_{\forall i} \left( \frac{-2 \cdot \operatorname{Re}(\lambda_i)}{|\lambda_i|^2} \right)$$

$$\Rightarrow I = \operatorname{eig}(A); \quad h_{\text{marg}} = \min(-2 * \operatorname{real}(I) ./ (\operatorname{abs}(I) .* \operatorname{abs}(I)));$$

# [H2.1] Marginal Stability IV

We find:

$$l = \text{eig}(A)$$

$$l =$$

$$-4.0000 + 3.0000i$$

$$-4.0000 - 3.0000i$$

$$-5.0000$$

$$-2.0000$$

$$-1.0000$$

All eigenvalues are in the left-half complex plane, i.e., the system is analytically stable.

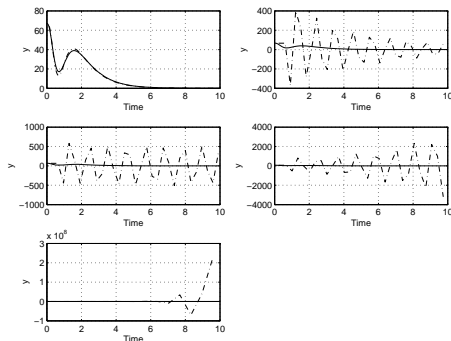
$$h_{\text{marg}} =$$

$$0.3200$$

The numerical solution is marginally stable for  $h_{\text{marg}} = 0.32$ .

## [H2.1] Marginal Stability V

Simulation results:



Even for  $h = 0.032 = 0.1 \cdot h_{\text{marg}}$ , the solution is not very accurate.

The FE algorithm requires very small step sizes in order to generate accurate simulation results.

## [H2.5] Stability Domain

For the *predictor-corrector FE-BE method*, find the stability domains if:

1. no corrector is used,
2. one corrector is used,
3. two correctors are used,
4. three correctors are used, and
5. four correctors are used.

Plot the five stability domains on top of each other, and discuss the results.

# [H2.5] Stability Domain II

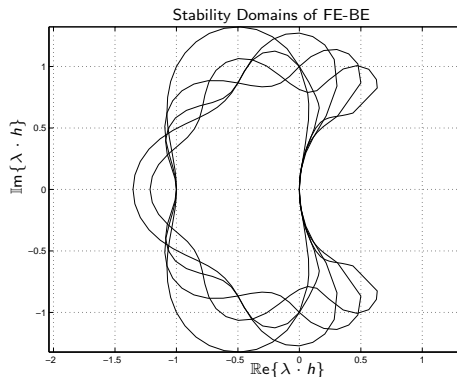


Figure: Stability domains of FE-BE algorithms