From Operational Models to Information Theory
Side Channels in pGCL with Isabelle

David Cock

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NSW GOVERNMENT Trade & Investment
THE UNIVERSITY OF SYDNEY
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Overview

A pen-and-paper proof formalised in Isabelle.

- Linking operational semantics and information theory.
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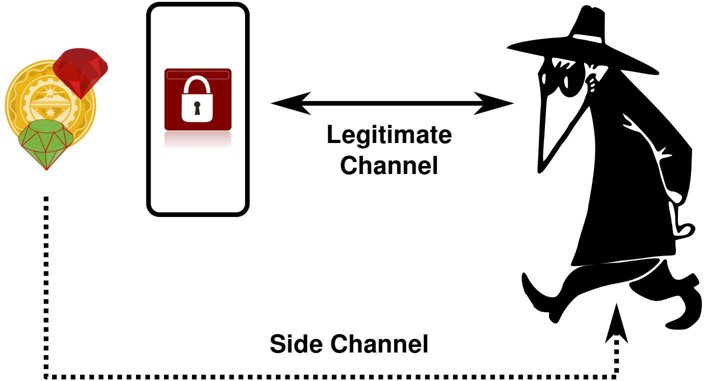
- What was the original result, and why formalise it?
- How was it formalised?
- What did we learn?

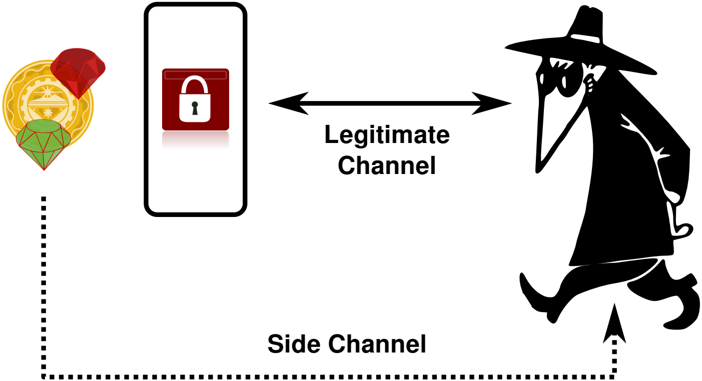
Outline

The Problem



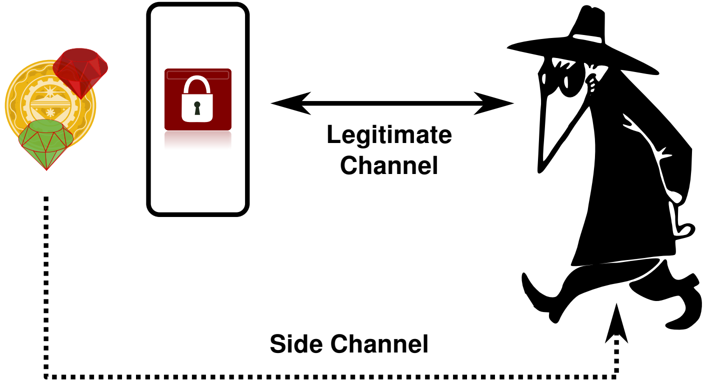
The attacker tries to guess the lock combination.

The Problem



After n tries he's locked out.

The Problem



Every guess leaks something about the combination.

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- For the best guess, maximise $P(ol, s)$.

The Result

$$V \leq \sup_{\sigma} \sum_{ol[..n]} \sum_{s \in \Gamma\ \sigma\ ol} P(ol, s)$$

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…by the supremum over attack strategies
…of the sum over possible lists of observations
…and over the attacker's guesses
…of the joint probability of observations and guess.

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The Important Point

We didn't **assume** optimality, we **proved** it.

Why Formalise?

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That's why.

That's one of 40 or so delicate manipulations in the original proof — I ran out of whiteboard, and I don't trust my penmanship enough.

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There are also a few more technically-justified reasons:

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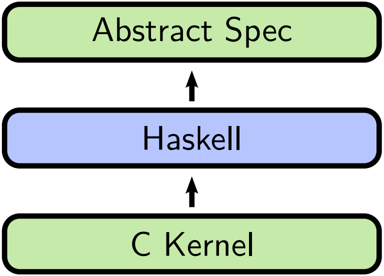
- Replace an ad-hoc operational model with a well-known formalism: pGCL.
- It tested the limits of the pGCL formalisation.
- We've shown before that the refinement order is compatible with L4.verified.
- This is a simple example. Scaling a paper proof is **hard**.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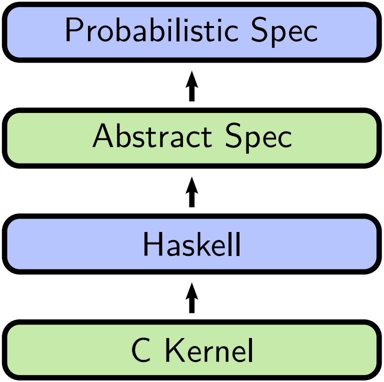
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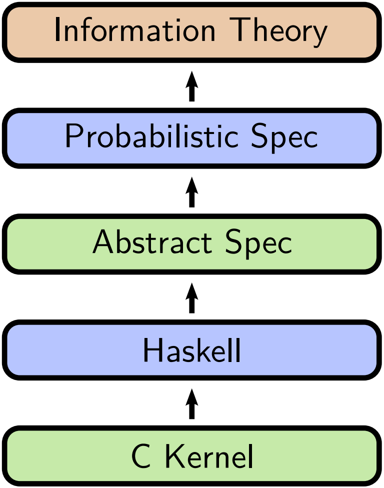
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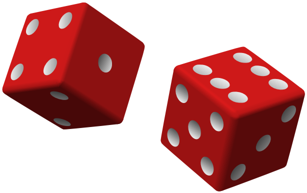
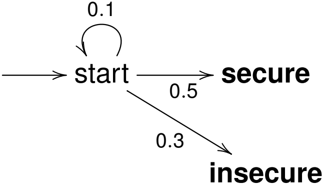
Outline

- The Original Proof
- How It's Formalised and Why
- Outcomes

pGCL





- pGCL is a language of probabilistic automata.
- It models both demonic and probabilistic choice.
- We previously formalised it in Isabelle.

Why Guessing Attacks?

Security properties are often **hyperproperties**:

- Defined over sets of traces.
- Not preserved by refinement.

For a guessing attack, security is a property of the current state.

Security Predicate

Has the attacker guessed the secret yet?

Modelled as a loop:

$$\mathbf{do}\ g \neq s \longrightarrow \text{guess}$$

Termination

The Original Proof
How It's Formalised and Why
Outcomes

$$9, 9, 9, 9, 9, 9, 9, \ldots$$

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What happens if the loop doesn't terminate?

Termination

$$9, 9, 9, 9, 9, 9, 9, \ldots$$

$$\mathbf{do}\ g \neq s \longrightarrow \text{guess}$$

What happens if the loop doesn't terminate?

The probability of establishing the predicate (secure) is 0!

By default, nontermination acts the wrong way.

Partial Correctness

- The solution:

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- The solution: the liberal (wlp) interpretation.
- "Correct if terminating"
- A nonterminating program establishes any predicate with probability 1.
- The probability of remaining secure is:

$$\text{wlp}\ \ (\textbf{do}\ g \neq s \longrightarrow \text{guess})\ \ \text{«secure»}$$

Refinement Soundness

- pGCL is a **probabilistic** logic.
- Refinement increases probabilities:

$$\frac{a \sqsubseteq b}{\text{wlp } a\ Q \vDash \text{wlp } b\ Q}$$

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Refinement preserves probabilistic security predicates.

$$V_n = 1 - \text{wlp } \left(\textbf{do } g \neq s \longrightarrow \text{guess}\right) \text{ «secure»}$$

The Attack in pGCL

$$\textbf{do } g \neq s \longrightarrow \text{guess}$$

Guess until we get the secret, ...

The Attack in pGCL

$\textbf{choose } s \textbf{ at } P(s)$

$\textbf{do } g \neq s \longrightarrow \text{guess}$

...which is chosen randomly.

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Every guess leaks an observation.

The Attack in pGCL

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The Original Proof

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$$\textbf{choose } s \textbf{ at } P(s)$$
$$\textbf{do } g \neq s \longrightarrow$$
$$\quad \textbf{bind } o \textbf{ at } P(o|s) \text{ in}$$
$$\quad ol := o{:}ol$$

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The Attack in pGCL

$ol := [\,]$

choose s **at** $P(s)$

do $g \neq s \longrightarrow$

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  $ol := o{:}ol$

Initially, there are none.

The Attack in pGCL

$$
\begin{array}{l}
ol := [\,] \\
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The attacker uses some strategy, σ . . .

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The Security Predicate

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What does it mean to terminate in a secure state?
The attacker has used all n guesses, without guessing correctly:

$$n < |ol| \wedge \forall i \leq n.\ \sigma\ (\text{tail } i\ ol) \neq s$$

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Probabilistic Security Predicate

What is the **probability** that we end in a secure state?

Classical Invariants

In a classical logic, we annotate loops using **invariants**:

$$\frac{\{I \wedge G\}\ \text{body}\ \{I\}}{\{I\}\ \textbf{do}\ G \rightarrow \text{body}\ \{I \wedge \neg G\}}$$

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A classical invariant becomes 'more true':

$$\frac{G\ s}{I\ s \longrightarrow \text{wlp body}\ I\ s}$$

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Probabilistic loops are almost exactly equivalent:

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A **probabilistic** invariant gets 'bigger':

$$\frac{G\ s}{I\ s \leq \text{wlp body}\ I\ s}$$

The Loop Invariant

$$I = \prod_{i=0}^{n \sqcap |ol|} «\sigma\ (\text{tail}\ i\ ol) \neq s»$$
$$* \sum_{ol'[..n-|ol|]} \prod_{i=|ol|+1}^{n} \left(\begin{array}{l} P((ol' \mathbin{@} ol)\,!(n-i)|s)\ * \\ \quad «\sigma\ (\text{tail}\ i\ (ol' \mathbin{@} ol)) \neq s» \end{array} \right)$$

This consists of two parts:

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- The **probability** that it will continue to hold.

The Postcondition

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The Original Proof

How It's Formalised and Why

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The Precondition

$$\inf_{\sigma} \left(1 - \sum_{ol[..n]} \sum_{s \in \Gamma\ \sigma\ ol} P(ol, s) \right)$$

$$\begin{aligned}
&\textbf{any } \sigma \,;; \\
&\textbf{choose } s \textbf{ at } P(s) \,;; \\
&ol := [\,] \,;; \\
&\textbf{do } g \neq s \to \text{guess}
\end{aligned}$$

The Precondition

$$V_n = 1 - \inf_{\sigma} \left(1 - \sum_{ol[..n]} \sum_{s \in \Gamma\ \sigma\ ol} P(ol, s) \right)$$

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The Original Proof

How It's Formalised and Why

Outcomes

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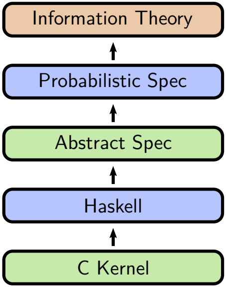
Outline

- The Original Proof
- How It's Formalised and Why
- Outcomes

A Verified Chain From C to Info Theory



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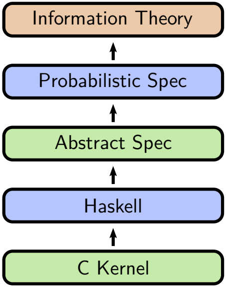

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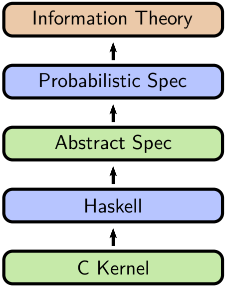

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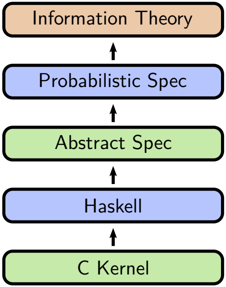
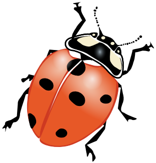


- We've shown that we can embed seL4 into a probabilistic logic.
- Now there's another step: quantitative information flow.
- Vulnerability is preserved by refinement all the way down.

There Were No Proof Bugs



The pen-and-paper proof was correct.
That's great for my self-confidence, but makes this slide rather dull. ;)

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We did have to make some changes to the existing formalisation:

- The existing VCG wasn't powerful enough.
- We had to fully treat recursion — It is now as powerful as the published results.
- The theory is now in a usable state — Under submission to AFP.

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- We've shown how to formally verify a probabilistic property,

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  . . . reusing a real, large-scale proof.
- This particular result can be instantiated with a model for $P(o|s)$ (c.f. my thesis).
- The approach can also be used for any state-based probabilistic property of the correct form.

Questions?