Dynamic Variants of Red-Blue Dominating Set

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Abstract

We introduce a parameterized dynamic version of *Red-Blue Dominating Set* and its partial version. We prove the fixed-parameter tractability of most dynamic versions with respect to the edit-parameter while they remain $W[2]$-hard with respect to the increment-parameter.

1 Introduction

In the *Red-Blue Dominating Set problem* (henceforth *RBDS*) we are given a graph $G = (R \cup B, E)$ such that $R \cap B = \emptyset$, together with an integer $s \geq 0$, and we are asked whether R contains a subset S of cardinality at most s such that every element of B has at least one neighbor in S. In this case we say that G is a red-blue graph and S is a red-blue dominating set of G.

We consider the parameterized dynamic version of *RBDS*. In such dynamic setting, originally defined in [3] in the context of *Dominating Set*, we assume the edges of the input graph G can disappear with time, so an initially feasible *RBDS* solution S may no longer dominate all of B and we want to construct another solution S' so that the Hamming distance between S and S' is minimized. The problem is formally defined as follows.

Dynamic Red-Blue Dominating Set (*DRBDS*)
Given: Red-blue graphs $G = (R \cup B, E)$ and $G' = (R \cup B, E')$, a red-blue dominating set $S \subset R$ (in G), integers k and r such that $|E| - |E'| \leq k$;
Question: Is there a subset S' of R such that $d_H(S, S') \leq r$ and S' is a red-blue dominating set of G'?

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2 Parameterized Complexity of DRBDS

The NP-hardness of *DRBDS* follows immediately from that of *RBDS* itself. To address its parameterized complexity, we define the *Need-Based Red Blue Dominating Set* problem which is mainly used to obtain our positive results.

Need-Based Red-Blue Dominating Set (*NB-RBDS*)
Given: a red-blue graph $G = (R \cup B, E)$, $s \geq 0$, and $\eta : B \longrightarrow \{0, 1, \ldots, q\}$.
Question: Does R contain a subset D such that $|D| \leq s$ and every element v of B has at least $\eta(v)$ neighbors in D?

Theorem 2.1. *NB-RBDS is fixed-parameter tractable (FPT) with respect to parameters $|B|$ and q.*

Corollary 2.2. *Dynamic q-RBDS is FPT with respect to the edit-parameter.*

Corollary 2.3. *Dynamic RBDS is FPT with respect to the edit-parameter.*

It was shown in [1, 3] that Dominating Set is $W[2]$-hard when parameterized by the increment parameter r only. We show the same for *q-RBDS*.

Theorem 2.4. *For any $q \geq 1$, Dynamic q-RBDS is $W[2]$-hard with respect to the increment-parameter.*

Corollary 2.5. *Dynamic Red-Blue Dominating Set is $W[2]$-hard with respect to the increment-parameter.*

Next, we consider the partial version of *RBDS* where we are given an additional parameter t and the objective is to find (whether there is) a subset of R that dominates at least t elements of B. The problem is known to be fixed-parameter tractable with respect to t [2]. We show the following.

Theorem 2.6. *Dynamic Partial RBDS is fixed-parameter tractable with respect to the edit parameter.*

Theorem 2.7. *Partial q-RBDS, parameterized by t and q, is W[1]-hard.*

Observe that usually, the partial variants of domination-like problems tend to be in *FPT*. To the best of our knowledge, this is the first problem variant where this question turns out to be W-hard.

References

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