Color Reconfiguration with Token Swapping

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Abstract

A reconfiguration problem gives two instances of a problem I and J together with a simple operation, defining a valid transition to transform an instance I into I'. The question is if there exists a sequence of valid transitions that transform I into J without violating any given restrictions. We study graph coloring as reconfiguration problem where the simple operation is to choose an edge $e = (u, v)$ and swap the colors of the vertices u and v without generating an infeasible coloring, like presented in Figure 1. So, the number of appearance for each color does not change during the reconfiguration.

The question if an instance I can be transformed into J is equal to the question if the two corresponding vertices in the reconfiguration graph are in the same connected component. For our color reconfiguration problem, the vertex set of the reconfiguration graph consists of every feasible coloring that has the same number of appearance for each color as the given instances I and J. Two vertices of the reconfiguration graph are connected by an edge if one reconfiguration step transforms one into the other, e.g., if there is one edge $e = (u, v)$ where the colors of the vertices u and v are swapped. Thus, solving the reconfiguration problem means finding a path between two vertices in the reconfiguration graph.

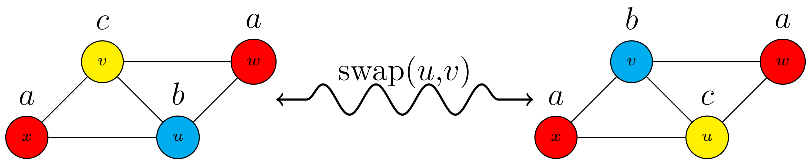

Figure 1: One reconfiguration step is choosing an edge and exchange the color of the vertices without creating an infeasible coloring.

Graph coloring as reconfiguration problem is already in focus of research [1, 9, 6, 9, 7, 3, 4, 5], but not under the swapping configuration rule. The common approach is that two vertices in the reconfiguration graph are connected by an edge, if the corresponding graphs differ in just one color. Our approach is more related to the swapping colored tokens problem [10, 2]. The difference to the colored token problem is that we additionally demand that the coloring is feasible in each step, e.g., no two tokens with the same color are allowed to be next to each other.

We analyze the color reconfiguration problem with token swapping rules on a set of simple graph classes. For two 3-colored paths, we prove that it can be decided in $\mathcal{O}(n^2)$ steps if they can be transformed into each other, where n is the number of vertices. We can transfer this result to proper interval graphs with bounded clique size by 3. More over we show that the problem can be solved on c-colored cographs for arbitrary c. If we parameterize the problem by the number of colors c and the modular width k the problem is in FPT. Additionally, we use the Nondeterministic Constraint Logic (NCL) model, introduced by Hearn and Demaine [8], to prove that the CSP is PSPACE-complete on planar graphs.

We also introduce a variation of the color reconfiguration problem with token swapping, where each swap operation needs to use a specific token. Due to this restriction, the problem becomes on cycle-free graphs (trees) trivial. Although the restriction seems to make the problem easier, it is PSPACE-complete on general graphs.

References

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