### 6<sup>th</sup> Homework - Solution

- In this homework, we shall model and simulate a thermal system.
- We shall model heat conduction along a well-insulated copper rod.
- We shall furthermore study the effects of different spatial discretization schemes on the accuracy of the simulation results.



- Heat conduction in copper rod
- Influence of asymmetric entropy feed
- Influence of discretization

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## Heat Conduction in a Copper Rod I

- A copper rod of length l = 1 m with a radius of r = 1 cm is initially in thermo-dynamical equilibrium at T = 298 K.
- At *Time* = 0, the left end of the rod is brought in contact with a body that had been pre-heated to a temperature of  $T_L = 390 K$ .
- We wish to model the rod using 10 segments, each with a length of  $\Delta x = 10$  cm. The boundary conditions are to be modeled such that the body to the left is replaced by a temperature source.
- It is assumed that no heat flows out at the right end of the rod, and that the rod is thermally so well insulated that no heat is lost anywhere along the rod.

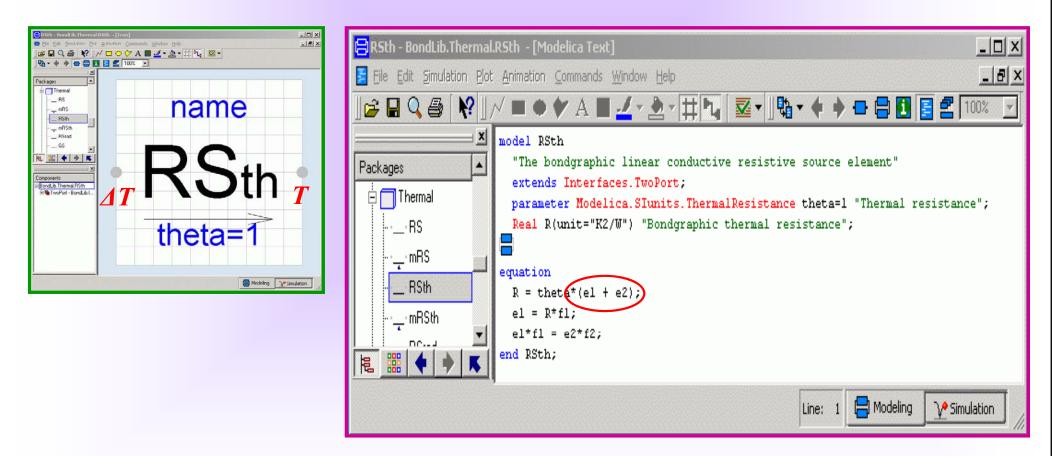


# Heat Conduction in a Copper Rod II

- The density of copper is  $\rho = 8960 \text{ kg} \cdot m^{-3}$ . Its specific thermal conductivity is  $\lambda = 401 \text{ J} \cdot m^{-1} \cdot s^{-1} \cdot K^{-1}$ . Its specific heat capacity is  $c = 386 \text{ J} \cdot kg^{-1} \cdot K^{-1}$ .
- The heat conduction is modeled using the symmetric heat conduction element presented in class. This element is made available as part of the *BondLib* thermal sub-library.
- Simulate the system during *5 hours*.

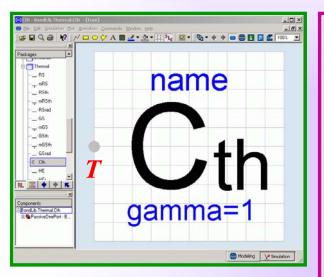


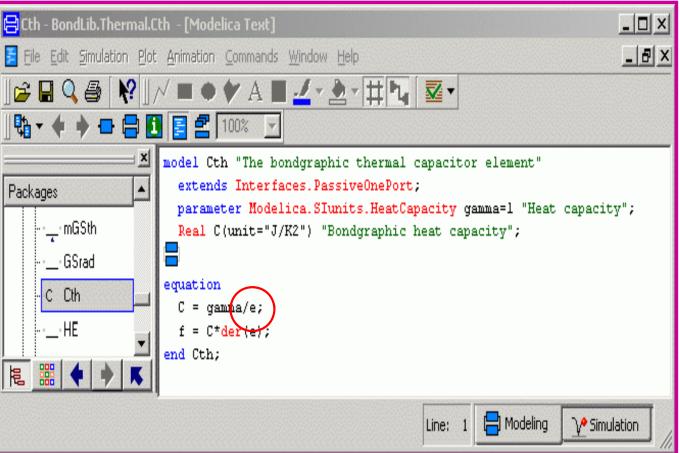
Let us start by looking at some thermal models in the *Dymola BondLib library* that we haven't used before. These are stored in its thermal sub-library.



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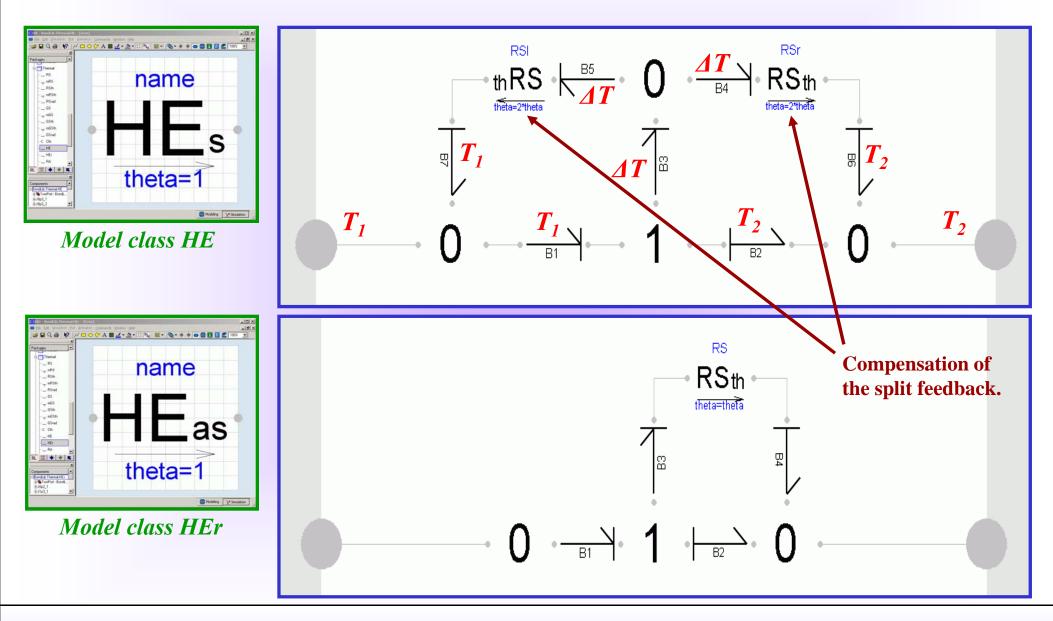


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### We can now start creating the individual chain links.

Ti s Ti+1	General       Add modifiers         General       Add modifiers         Component       Name         Name       C         Comment       Comment         Model       Path         BondLib. Thermal. Cth       Comment The bondgraphic thermal capacitor element         Parameters       e         gamma       gamma         DK       Info	Icon       Cth       Gam   Bondgraphic effort Heat capacity       Cancel
The capacity obtains initial condition.	here its	The 0 on the right is contained in the model, the one one the left isn't. In this way, elements can be cascaded.

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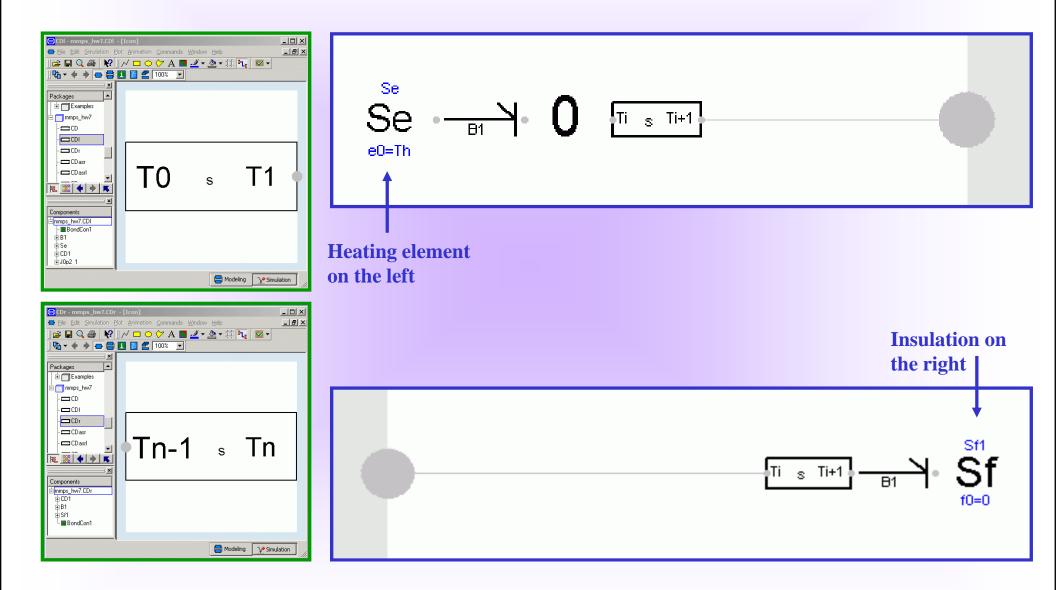
C in mmps\_hw7.CD ? × General Add modifiers Component Icon C Name Cth Comment Cth Model Path BondLib, Thermal, Cth Comment The bondgraphic thermal capacitor element Parameters e(start=T0) e Bondgraphic effort gamma gamma J/K Heat capacity 😑 CD - mmps\_hw7.CD - [Modelica Text] \_ 🗆 X OK Info - 8 X File Edit Simulation Plot Animation Commands Window Help 」 ∕ ■ ● **/** A **■ <u>/</u> · <u>A</u> · <b>拱 ኪ ⊠** • 🖻 🖪 4 🚍 🚺 📑 🛃 100% 🔻 model CD The initial condition is passed on parameter Modelica.SIunits.ThermalResistance theta= 1 "Thermal Resistance"; . Packages as a parameter. parameter Modelica.SIunits.HeatCapacity gamma=1 "Thermal Capacity"; parameter Modelica.SIunits.Temperature TO=298 "Initial temperature"; 🗄 🥅 mmps\_hw7 equation ۹, end CD; 挹 Hodeling mmps\_hw7.CD **₩** Simulation Line: 7

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#### Symmetric heat conduction ₅ T1 TO ₅ Ti+1≬ Tn-1 ₅ Tn Ti **Asymmetric heat conduction** Τ0 Ti ₃₅ Ti+1 as T1 Tn-1 ₃ Tn with preference to the right Asymmetric heat conduction T0 ₃₅ T1 Tn-1 ₃ Tn ₃₅ Ti+1 Ti with preference to the left

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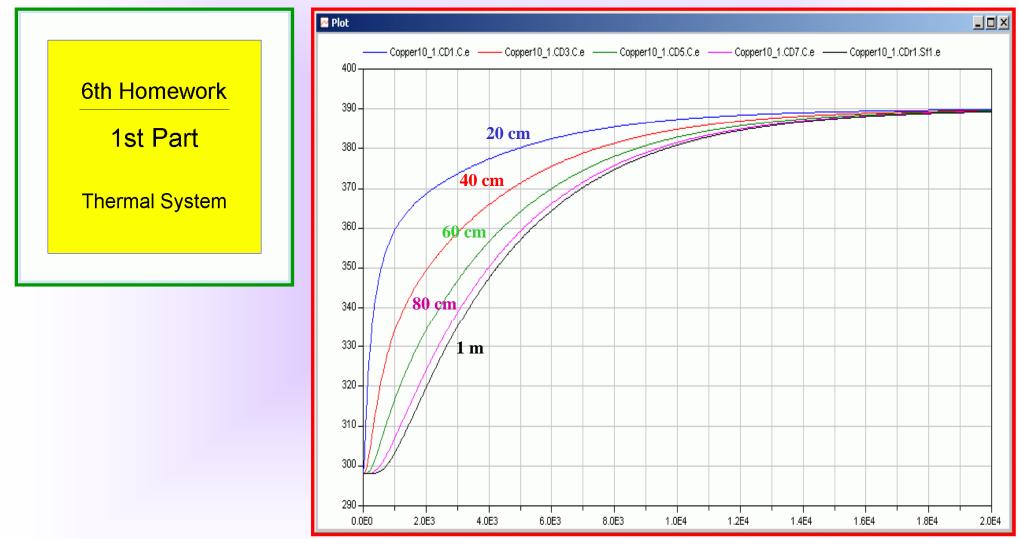
TO s T1 Ti s Ti+1 Tn-1 s Tn

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#### **Temperature values as functions of time and space**



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## **Influence of Asymmetric Entropy Feed**

- Replace the symmetric heat conduction element by two asymmetric elements; one, in which the generated entropy is fed only to the right, the other, in which it is fed exclusively to the left.
- The *BondLib* library offers such an element as well.
- Simulate the so modified model, and present, on a single plot, the results of the three simulation models.
- You may either calculate the three models sequentially while preserving the results from one to the next, or you may simulate the three models in parallel.



🔁 Plot [1\*] Copper10\_1.CDr1.Sf1.e ----- Copper10I1.CDasir1.Sf1.e ----- Copper10r1.CDasrr1.Sf1.e 400 **Three independent models** 10 Segments computed in parallel 390 370 Asymmetric heat conduction with preference to the left 350 Symmetric heat 10 Segments conduction 340 330 Asymmetric heat conduction 320 with preference to the right 310 300 10 Segments 290 -0.0E0 2.0E3 4.0E3 1.4E4 1.6E4 6.0E3 8.0E3 1.0E4 1.2E4 1.8E4

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**Start Presentation** 



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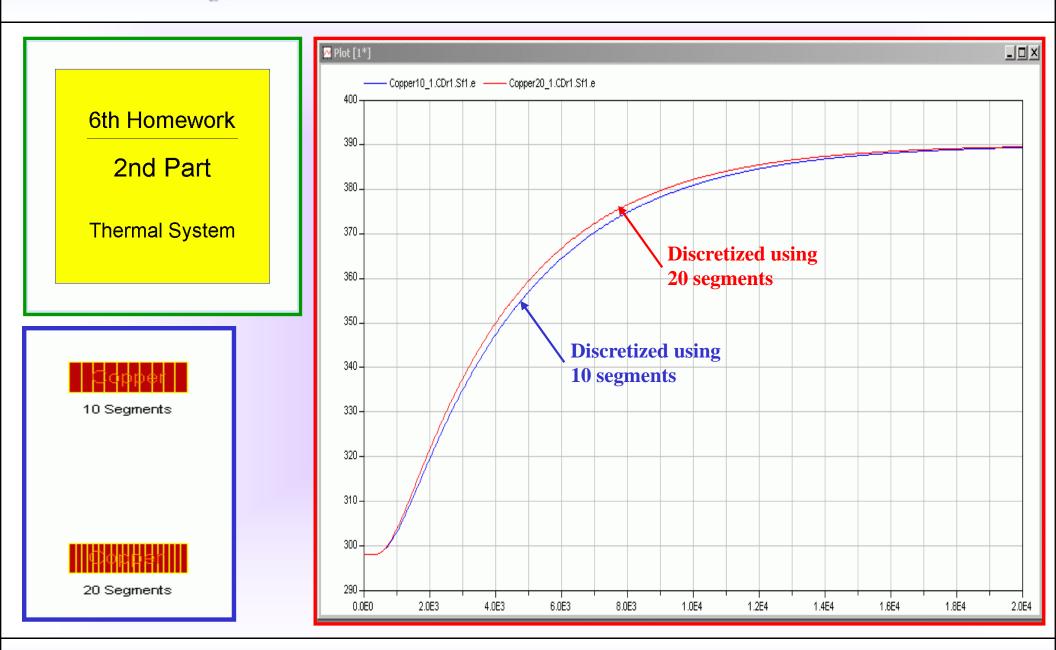
### **Influence of Discretization**

- We return to using the symmetric model. However this time, we wish to model the system using 20 segments, each with a length of  $\Delta x = 5$  cm.
- Simulate the so modified model, and present the results obtained in this way graphically together with the original simulation results.



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### Mathematical Modeling of Physical Systems



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