

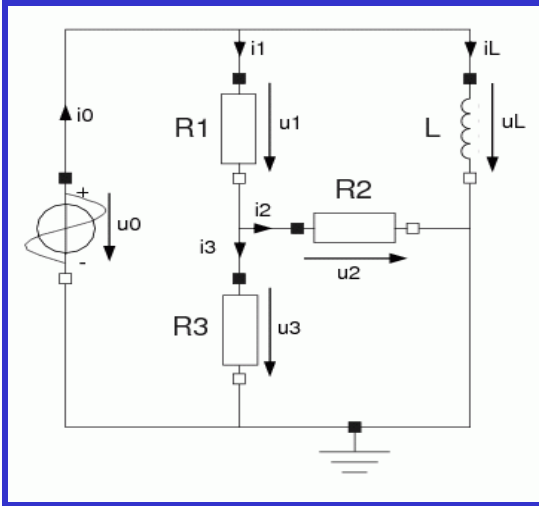
# Bond Graphs II

- In this class, we shall deal with the effects of algebraic loops and structural singularities on the bond graphs of physical systems.
- We shall also analyze the description of mechanical systems by means of bond graphs.

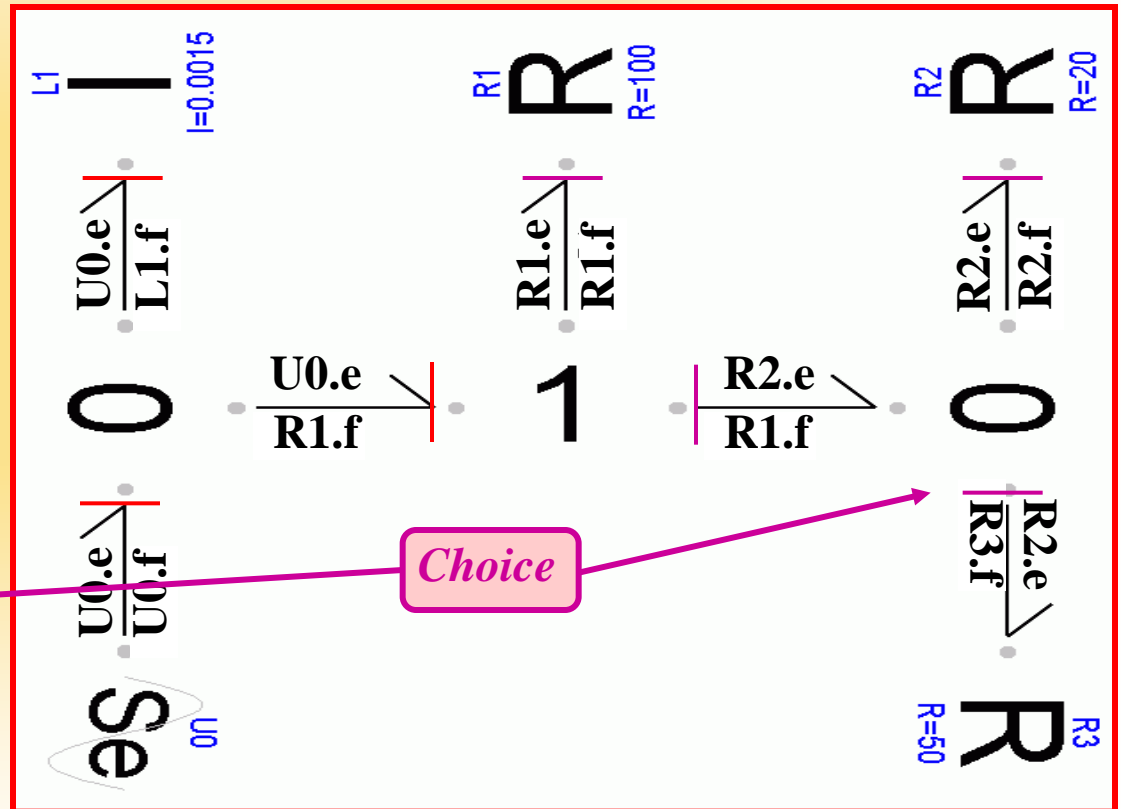
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# Algebraic Loops

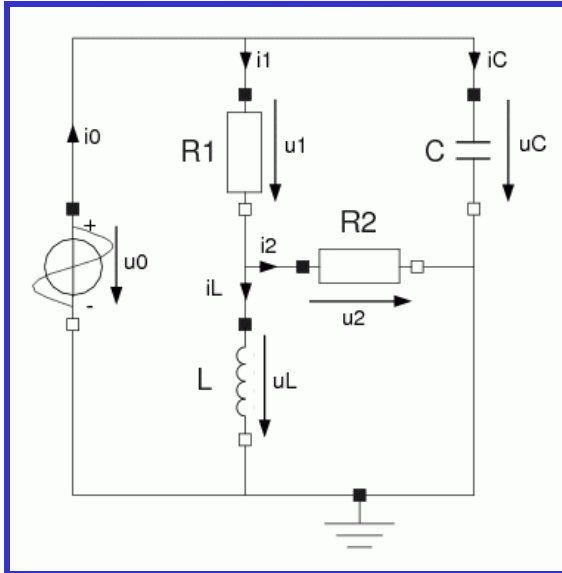


$$\begin{aligned}
 U0.e &= f(t) \\
 U0.f &= L1.f + R1.f \\
 dL1.f/dt &= U0.e / L1 \\
 R3.f &= R1.f - R2.f \\
 R2.e &= R3 \cdot R3.f \\
 R2.f &= R2.e / R2 \\
 R1.f &= R1.e / R1 \\
 R1.e &= U0.e - R2.e
 \end{aligned}$$



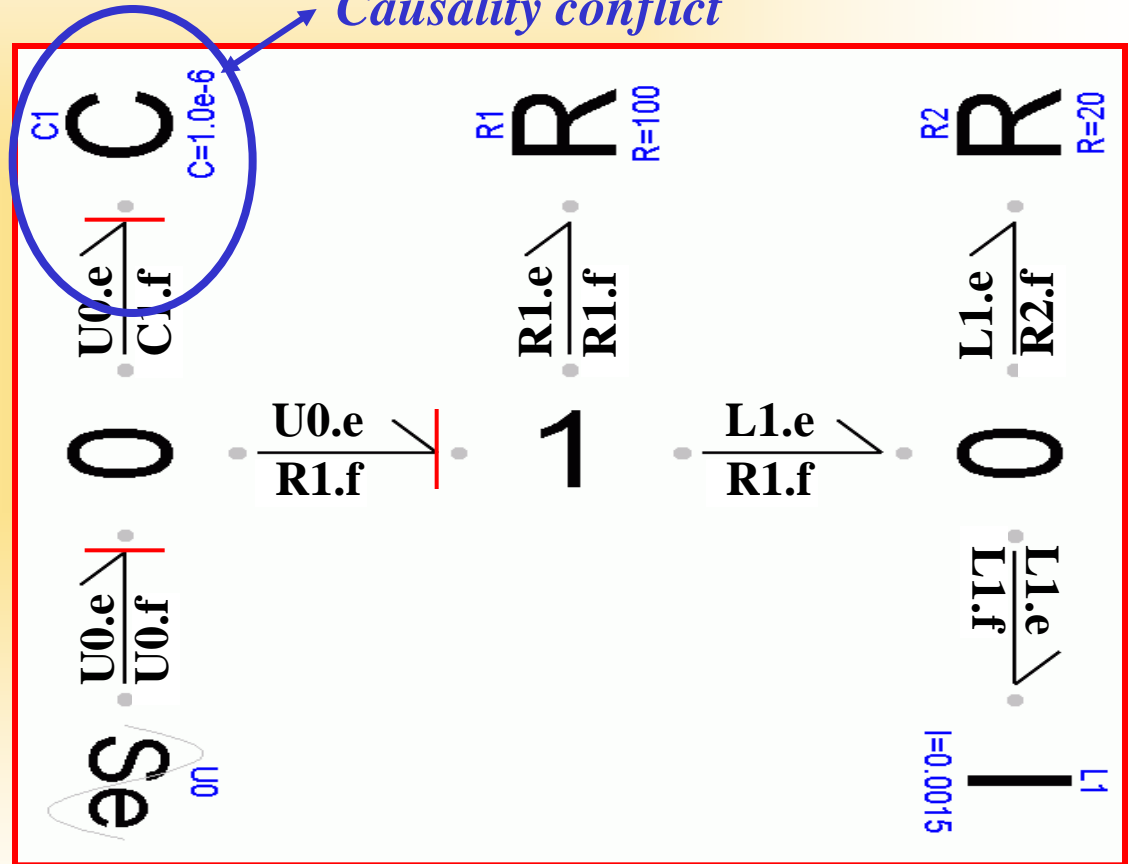
# Structural Singularities

⇒ *Structural Singularity*



$$U0.e = f(t)$$

$$U0.f = C1.f + R1.f$$

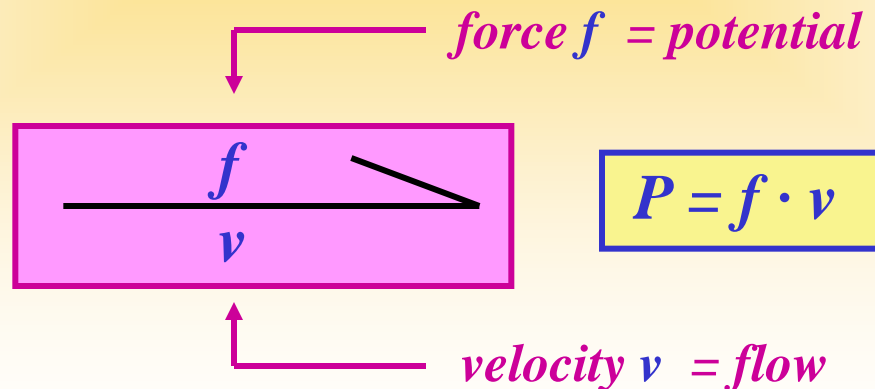


# Bond Graphs of Mechanical Systems I

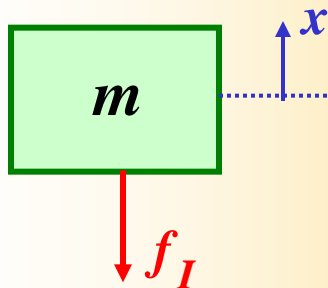
- The two adjugate variables of the mechanical translational system are the *force*  $f$  as well as the *velocity*  $v$ .
- You certainly remember the classical question posed to students in grammar school: *If one eagle flies at an altitude of 100 m above ground, how high do two eagles fly?* Evidently, *position* and *velocity* are *intensive variables* and therefore should be treated as *potentials*.
- However, if one eagle can carry one sheep, two eagles can carry two sheep. Consequently, the *force* is an *extensive variable* and therefore should be treated as a *flow variable*.

# Bond Graphs of Mechanical Systems II

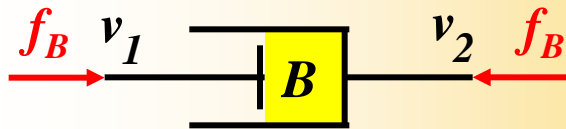
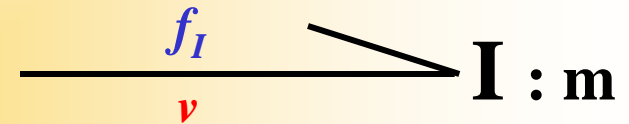
- Sadly, the bond graph community chose the reverse definition. “*Velocity*” gives the impression of a *movement* and therefore of a *flow*.
- We shall show that it is always possible mathematically to make either of the two assumptions (*duality principle*).
- Therefore:



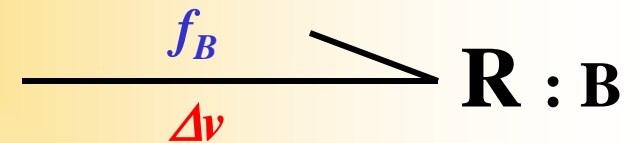
# Passive Mechanical Elements in Bond Graph Notation



$$f_I = m \cdot dv / dt$$

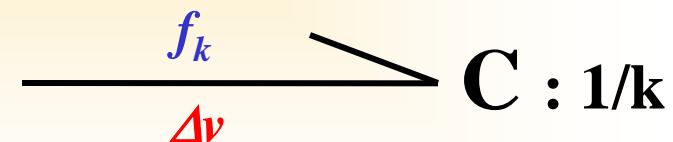


$$f_B = B \cdot \Delta v$$



$$\Delta x = f_k / k$$

$$\Rightarrow \Delta v = (1 / k) \cdot df_k / dt$$

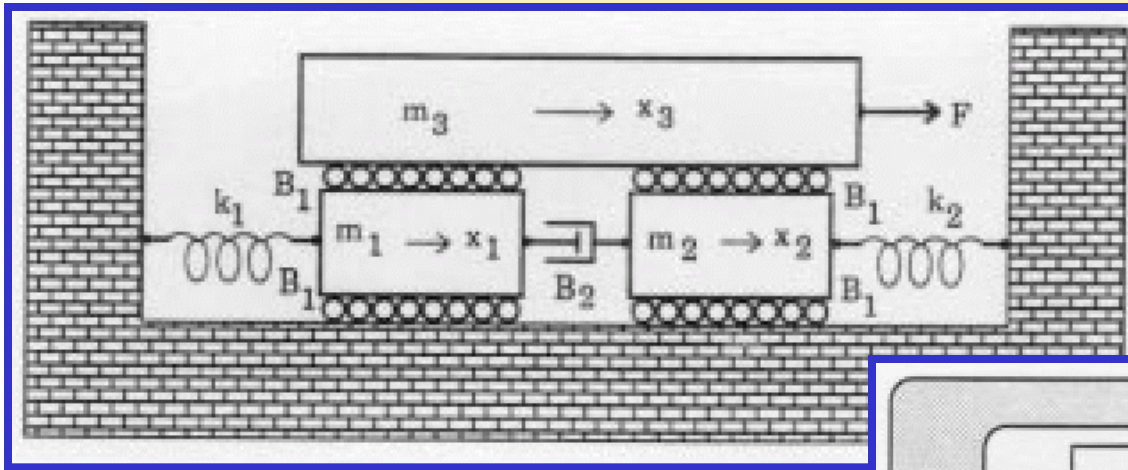


# Selection of State Variables

- The “*classical*” *representation of mechanical systems* makes use of the *absolute motions of the masses* (position and velocity) as its state variables.
- The *multi-body system representation in Dymola* makes use of the *relative motions of the joints* (position and velocity) as its state variables.
- The *bond graph representation* selects the *absolute velocities of masses* as one type of state variable, and the *spring forces* as the other.

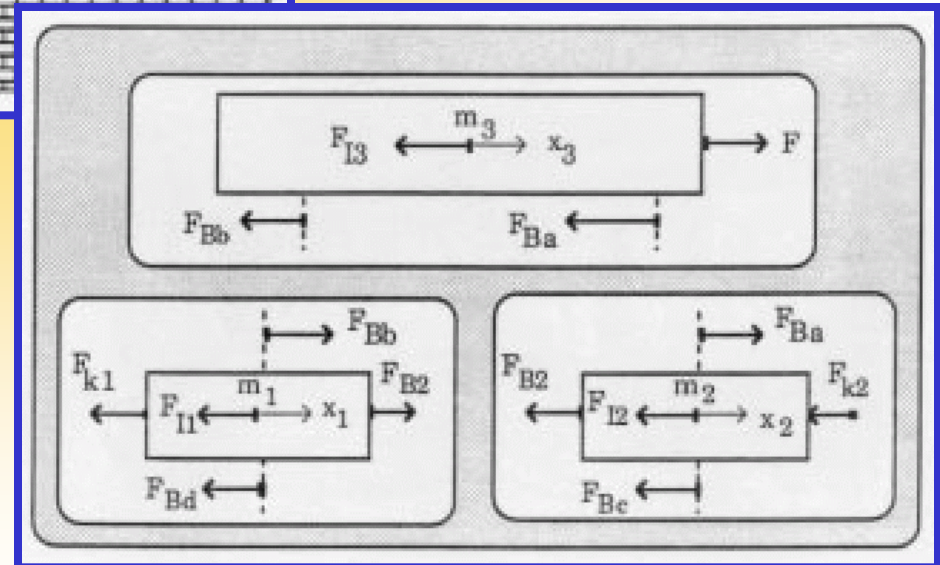


# An Example I

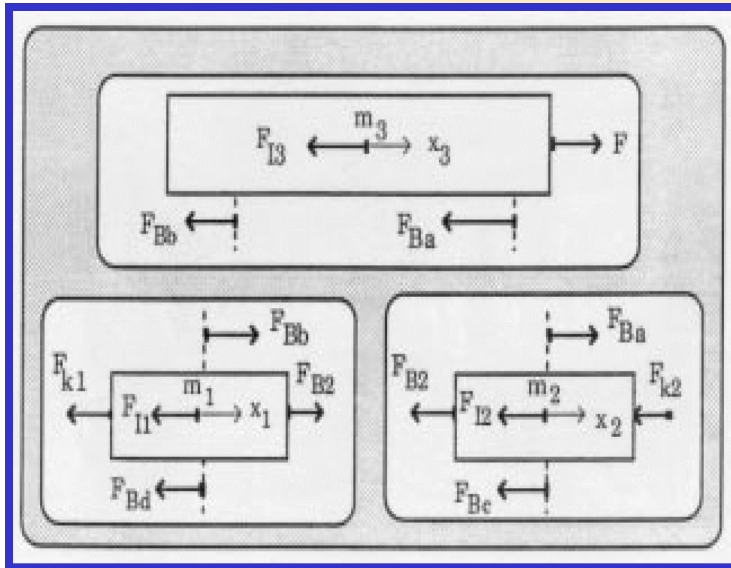


*The cutting forces are represented by springs and friction elements that are placed between bodies at a 0-junction.*

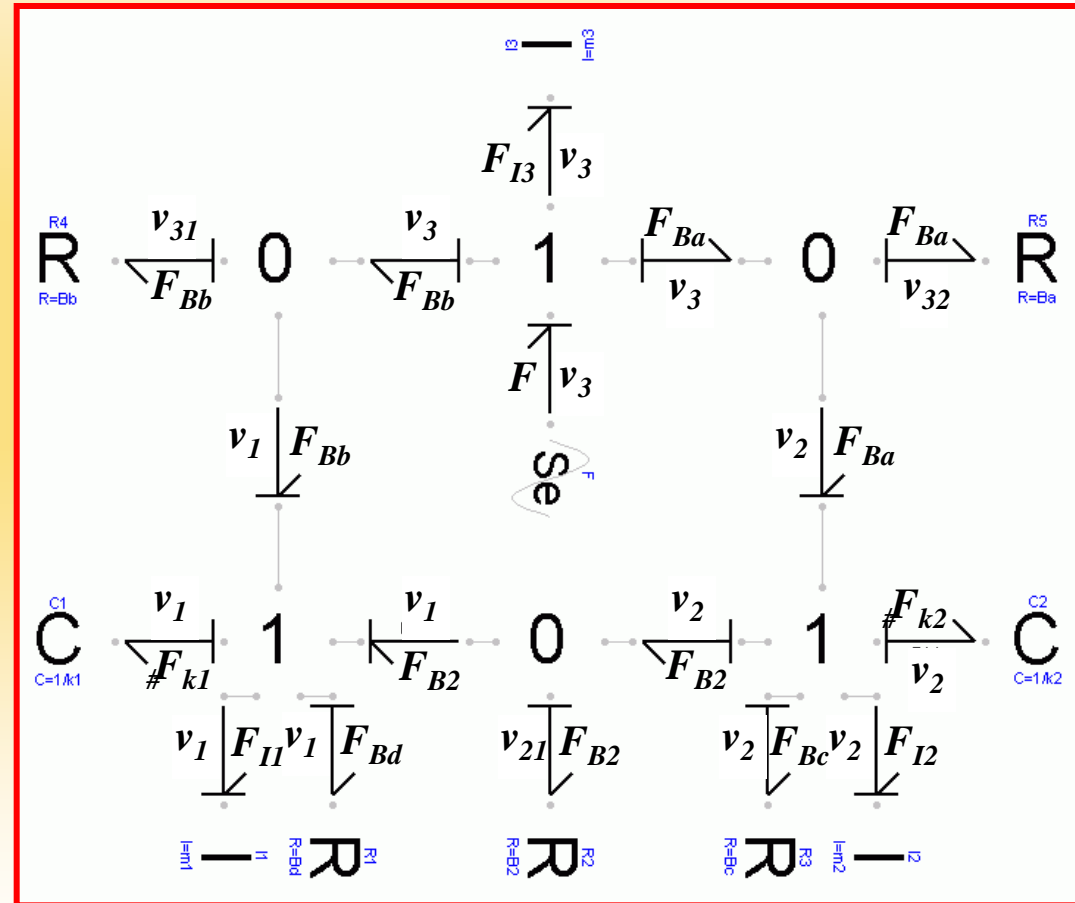
*The D'Alembert principle is formulated in the bond graph representation as a grouping of all forces that attack a body around a junction of type 1.*



# An Example II



*The sign rule follows here automatically, and the modeler rarely makes any mistake relating to it.*



# References

- Borutzky, W. and F.E. Cellier (1996), “Tearing Algebraic Loops in Bond Graphs,” *Trans. of SCS*, **13**(2), pp. 102-115.
- Borutzky, W. and F.E. Cellier (1996), “Tearing in Bond Graphs With Dependent Storage Elements,” *Proc. Symposium on Modelling, Analysis, and Simulation, CESA'96, IMACS MultiConference on Computational Engineering in Systems Applications, Lille, France*, vol. 2, pp. 1113-1119.