

## Modeling of Bipolar Transistors

- In this class, we shall deal with an application of mixed electrical and thermal modeling: the *Bipolar Junction Transistor (BJT)*.
- We shall start out with a *SPICE*-style model of the BJT, then convert the model to a bond graph.
- We shall recognize that the *SPICE*-model of the BJT is problematic.
- We shall convert the bond graph to obtain a modified BJT model that makes sense from a thermodynamic point of view.

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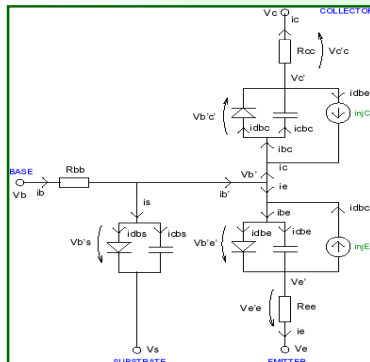
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## SPICE-style BJT Model



*SPICE* models the BJT by three junction diodes, one from the base to the collector, the second from the base to the emitter, and the third to the substrate.

The figure to the left shows a *laterally diffused npn-transistor*.

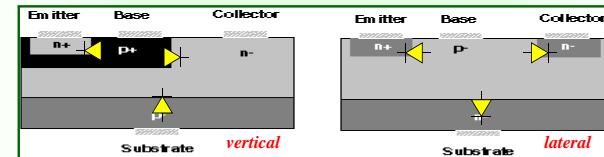
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## Vertical and Lateral npn-Transistors



- The *pn junction diodes* connect positively doped regions with negatively doped regions.
- In the *laterally diffused BJT*, all three junction diodes have their anodes in the base.

**Dopants:** for *p*-region (acceptors): boron or aluminum  
for *n*-region (donors): phosphorus or arsenic

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## Non-linear Current Sources

- The model contains two non-linear current sources that inject currents into the circuit:

$$injC = J_s \left[ \exp\left(\frac{qV_{B'E'}}{kT}\right) - 1 \right] = i_{DBE}$$

$$injE = J_s \left[ \exp\left(\frac{qV_{B'C'}}{kT}\right) - 1 \right] = i_{DBC}$$

- The current injected into the collector is a function of the base-emitter Voltage, and the current injected into the emitter is a function of the base-collector Voltage.

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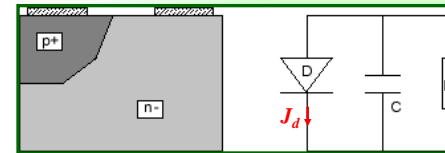
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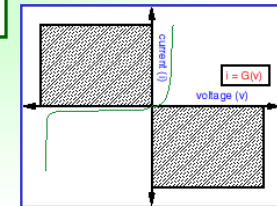


## The Junction Diode Model

- The **pn junction diode** is modeled as follows:



$$J_d = J_s \left[ \exp\left(\frac{qV}{kT}\right) - 1 \right]$$



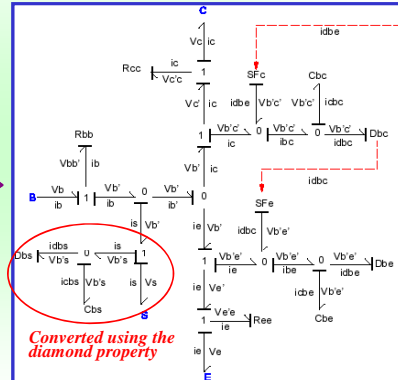
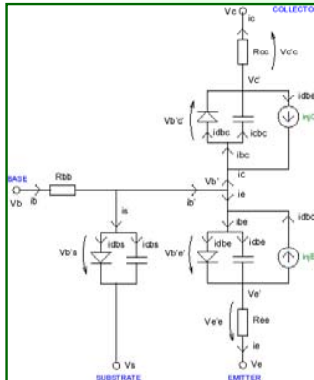
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## The BJT Bond Graph



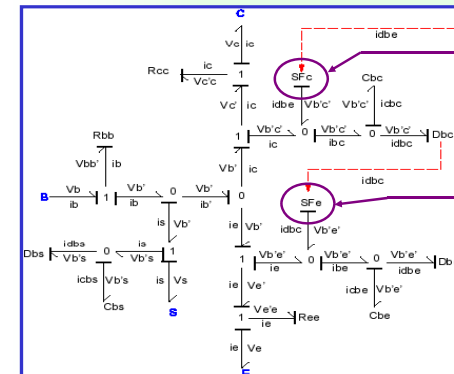
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## Problems With BJT Bond Graph



Where does the power for these current sources come from?

The sources are internal to the model. Hence there is no place where these sources could possibly draw power from.

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## Conversion of the BJT Bond Graph

$i_C + i_{DBE} = i_{BC} = i_{CBC} + i_{DBC}$   
 $\Rightarrow i_C + (i_{DBE} - i_{DBC}) = i_{CBC}$

$i_E + i_{DBC} = i_{BE} = i_{CBE} + i_{DBE}$   
 $\Rightarrow i_E = (i_{DBE} - i_{DBC}) + i_{CBE}$

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## The Non-linear Resistor

*The two current sources are really a power sink, rather than a power source. They can be interpreted as a single non-linear resistor.*

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## Dissipated Power I

- The power dissipated by the RS-element of the junction diodes (i.e., the two former current “sources”) is:

$$P_{BJT} = V_{C'E'} \cdot i_{CE} = (V_{B'E'} - V_{B'C'}) \cdot (i_{DBE} - i_{DBC})$$

- and therefore:

$$P_{BJT} = V_{C'E'} \cdot i_{CE} = T \cdot \dot{S}_D$$

- We still need to show that  $P_{BJT} > 0$ .

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## Dissipated Power II

- We need to show that  $V_{C'E'}$  and  $i_{CE}$  always point in the same direction.

$$V_{C'E'} > 0 \Leftrightarrow V_{B'E'} > V_{B'C'}$$

$$\Rightarrow i_{DBE} > i_{DBC}$$

$$\Rightarrow i_{CE} > 0$$

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- Schweisguth, M.C. (1997), *Semiconductor Modeling with Bondgraphs*, MS Thesis, Dept. of Electr. & Comp. Engr., University of Arizona, Tucson, AZ.
- Schweisguth, M.C. and F.E. Cellier (1999), [A Bond Graph Model of the Bipolar Junction Transistor](#), *Proc. SCS Intl. Conf. on Bond Graph Modeling*, San Francisco, CA, pp. 344-349.