

Eidgenössische Technische Hochschule Zürich

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Mathematical Modeling of Physical Systems


## Thermal Modeling of Buildings II


- This is the second lecture concerning itself with the *thermal modeling of buildings*.
- This second example deals with the thermodynamic budget of *Biosphere 2*, a research project located 50 km north of Tucson.
- Since *Biosphere 2* contains plant life, it is important, not only to consider the *temperature* inside the Biosphere 2 building, but also its *humidity*.
- The entire enclosure is treated like a single room with a single air temperature. The effects of air conditioning are neglected.
- The model considers the weather patterns at the location.

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## Biosphere 2: Original Goals


- Biosphere 2* had been designed as a *closed ecological system*.
- The original aim was to investigate, whether it is possible to build a system that is materially closed, i.e., that exchanges only energy with the environment, but no mass.
- Such systems would be useful e.g. for extended space flights.
- Biosphere 2* contains a number of different *biomes* that communicate with each other.
- The model only considers a single biome. This biome, however, has the size of the entire structure.
- Air flow and air conditioning are being ignored.

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## Biosphere 2: Revised Goals


- Later, *Biosphere 2* was operated in a flow-through mode, i.e., the structure was no longer materially closed.
- Experiments included the analysis of the effects of varying levels of CO<sub>2</sub> on plant grows for the purpose of simulating the effects of the changing composition of the Earth atmosphere on sustainability.
- Unfortunately, research at *Biosphere 2* came to an almost still-stand around 2003 due to lack of funding.
- In 2007, management of *Biosphere 2* was transferred to the University of Arizona. Hopefully, the change in management shall result in a revival of *Biosphere 2* as an exciting experimental research facility for life sciences.

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## Biosphere 2: Construction I





- *Biosphere 2* was built as a frame construction from a mesh of metal bars.
- The metal bars are filled with glass panels that are well insulated.
- During its closed operation, *Biosphere 2* was slightly over-pressurized to prevent outside air from entering the structure. The air loss per unit volume was about 10% of that of the space shuttle!

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## Biosphere 2: Construction II





- The pyramidal structure hosts the jungle biome.
- The less tall structure to the left contains the pond, the marshes, the savannah, and at the lowest level, the desert.
- Though not visible on the photograph, there exists yet one more biome: the agricultural biome.

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## Biosphere 2: Construction III




- The two “lungs” are responsible for pressure equilibration within *Biosphere 2*.
- Each lung contains a heavy concrete ceiling that is flexibly suspended and insulated with a rubber membrane.
- If the temperature within *Biosphere 2* rises, the inside pressure rises as well.


• Consequently, the ceiling rises until the inside and outside pressure values are again identical. The weight of the ceiling is responsible for providing a slight over-pressurization of *Biosphere 2*.

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## Biosphere 2: Biomes I





- The (salt water) pond of *Biosphere 2* hosts a fairly complex maritime ecosystem.
- Visible behind the pond are the marsh lands planted with mangroves. Artificial waves are being generated to keep the mangroves healthy.
- Above the cliffs to the right, there is the high savannah.

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Biosphere 2: Biomes II





- This is the savannah.
- Each biome uses its own soil composition sometimes imported, such as in the case of the rain forest.
- Biosphere 2* has 1800 sensors to monitor the behavior of the system. Measurement values are recorded on average once every 15 Minutes.

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Biosphere 2: Biomes III





- The agricultural biome can be subdivided into three separate units.
- The second lung is on the left in the background.

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Living in Biosphere 2



- The Biosphere 2 library is located at the top level of a high tower with a spiral staircase.





- The view from the library windows over the Sonora desert is spectacular.

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The Rain Maker




- From the commando unit, it is possible to control the climate of each biome individually.
- For example, it is possible to program rain over the savannah to take place at 3 p.m. during 10 minutes.


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## Climate Control I






- The climate control unit (located below ground) is highly impressive. *Biosphere 2* is one of the most complex engineering systems ever built by mankind.

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## Climate Control II






- Beside from the *temperature*, also the *humidity* needs to be controlled.
- To this end, the air must be constantly dehumidified.
- The condensed water flows to the lowest point of the structure, located in one of the two lungs, where the water is being collected in a small lake; from there, it is pumped back up to where it is needed.

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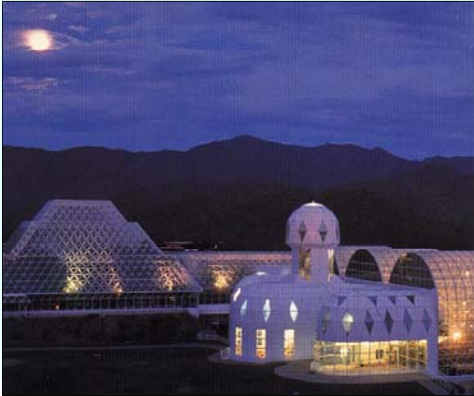


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## Night-time View




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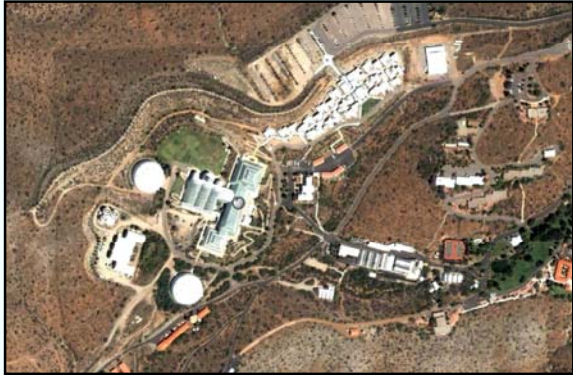


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## Google World




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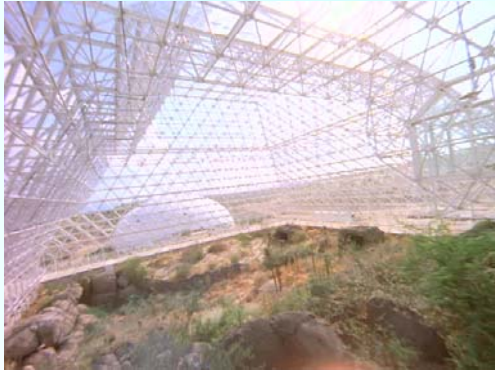


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A Fascinating World




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## A Fascinating World V




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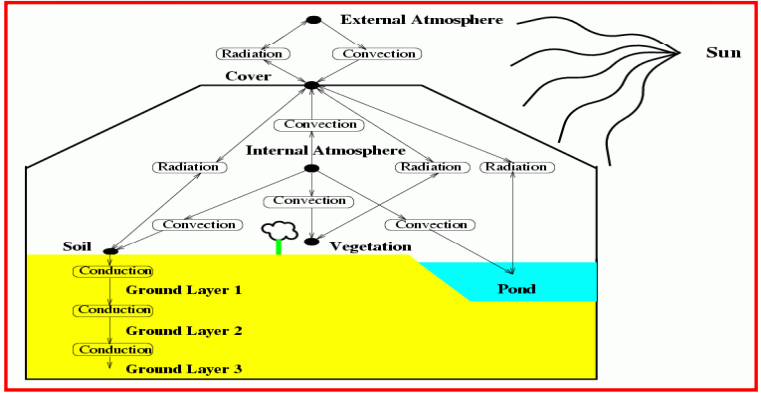


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## The Conceptual Model




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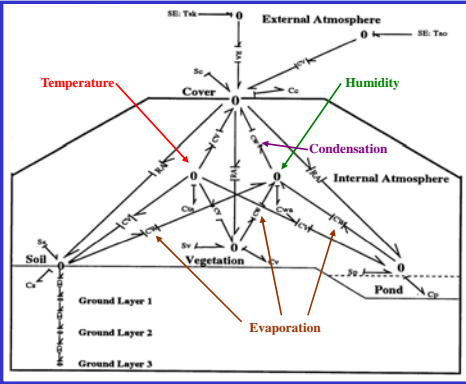


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## The Bond-Graph Model





- For **evaporation**, energy is needed. This energy is taken from the thermal domain. In the process, so-called **latent heat** is being generated.
- In the process of **condensation**, the latent heat is converted back to **sensible heat**.
- The effects of **evaporation** and **condensation** must not be neglected in the modeling of *Biosphere 2*.

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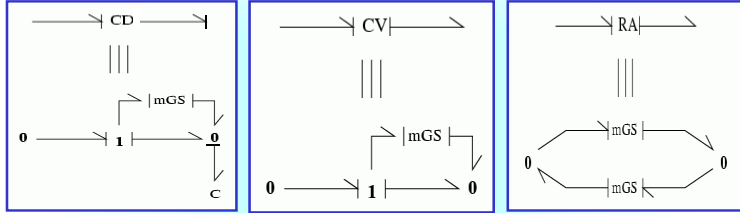


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## Conduction, Convection, Radiation




- These elements have been modeled in the manner presented earlier. Since climate control was not simulated, the convection occurring is not a forced convection, and therefore, it can essentially be treated like a conduction.

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## Evaporation and Condensation

- Both evaporation and condensation can be modeled either as non-linear (modulated) resistors or as non-linear (modulated) transformers.
- Modeling them as transformers would seem a bit better, because they describe reversible phenomena. Yet in the model presented here, they were modeled as resistors.
- These phenomena were expressed in terms of equations rather than in graphical terms, since this turned out to be easier.

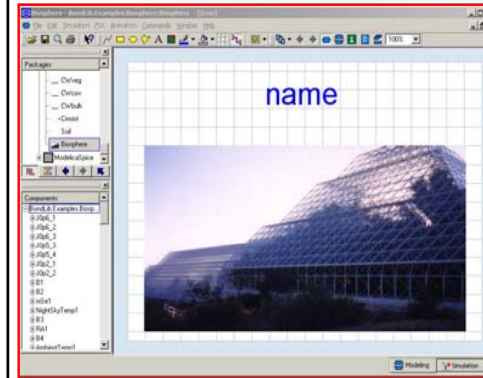
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## The Dymola Model I



- The overall *Dymola* model is shown to the left.
- At least, the picture shown is the top-level icon window of the model.

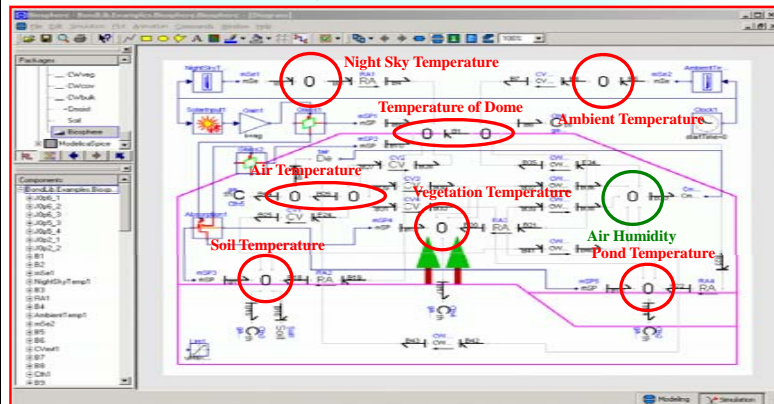
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## The Dymola Model II



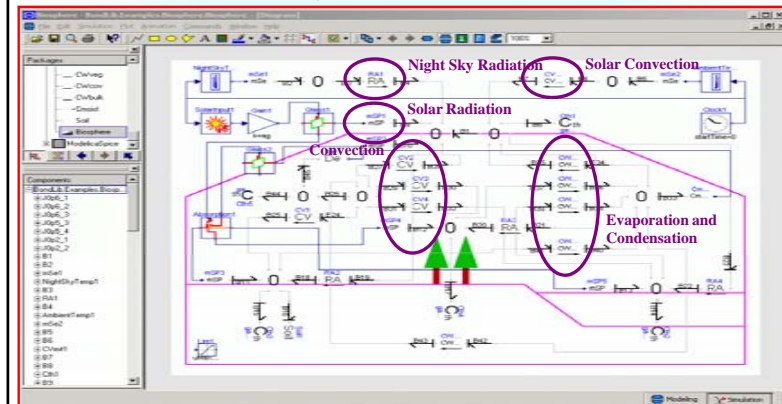
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## The Dymola Model III




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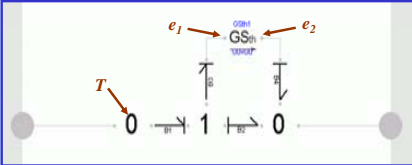
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Convection

name

CV

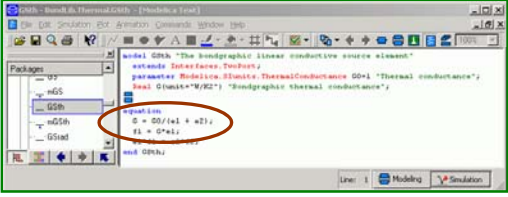


$R_{th} = R \cdot T$   
 $\Rightarrow G_{th} = G / T$   
 $T = e_1 + e_2$

name

GSth


G0=1



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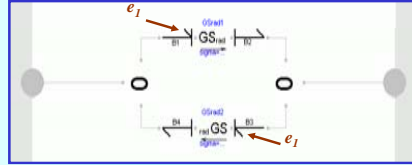
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Radiation

name

RA

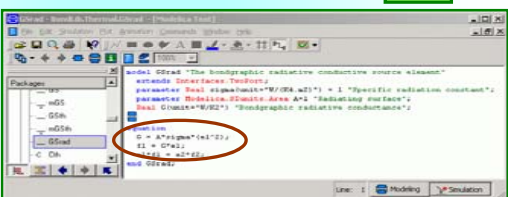


$R_{th} = R / T^2$   
 $\Rightarrow G_{th} = G \cdot T^2$   
 $T = e_1$

name

GSrad


sigma=1



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Evaporation and Condensation II

- We first need to decide, which variables we wish to choose as effort and flow variables for describing humidity.
- A natural choice would have been to select the *mass flow of evaporation* as the flow variable, and the *specific enthalpy of evaporation* as the corresponding effort variable.
- Yet, this won't work in our model, because we aren't tracking any mass flows to start with.
- We don't know, how much water is in the pond or how much water is stored in the leaves of the plants.
- We simply assume that there is always enough water, so that evaporation can take place, when conditions call for it.

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Evaporation and Condensation III

- We chose the *humidity ratio* as the effort variable. It is measured in *kg<sub>water</sub> / kg<sub>air</sub>*.
- This is the only choice we can make. The units of flow must be determined from the fact that *e·f = P*.
- In this model, we did not use standard SI units. Time is here measured in *h*, and power is measured in *kJ/h*.
- Hence the flow variable must be measured in *kJ·kg<sub>air</sub>/(h·kg<sub>water</sub>)*.

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## Evaporation and Condensation IV

- The units of linear resistance follow from the resistance law:  $e = R \cdot f$ . Thus, linear resistance is measured in  $h \cdot \text{kg\_water}^2 / (\text{kJ} \cdot \text{kg\_air}^2)$ .
- Similarly, the units of linear capacitance follow from the capacitive law:  $f = C \cdot \text{der}(e)$ . Hence linear capacitance is measured in  $\text{kJ} \cdot \text{kg\_air}^2 / \text{kg\_water}^2$ .
- $R \cdot C$  is a time constant measured in  $h$ .

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## Evaporation and Condensation V

- Comparing with the literature, we find that our units for  $R$  and  $C$  are a bit off. In the literature, we find that  $R_{\text{hum}}$  is measured in  $h \cdot \text{kg\_water} / (\text{kJ} \cdot \text{kg\_air})$ , and  $C_{\text{hum}}$  is measured in  $\text{kJ} \cdot \text{kg\_air} / \text{kg\_water}$ .
- Hence the same non-linearity applies to the humidity domain that we had already encountered in the thermal domain:  $R = R_{\text{hum}} \cdot e$ , and  $C = C_{\text{hum}} / e$ .

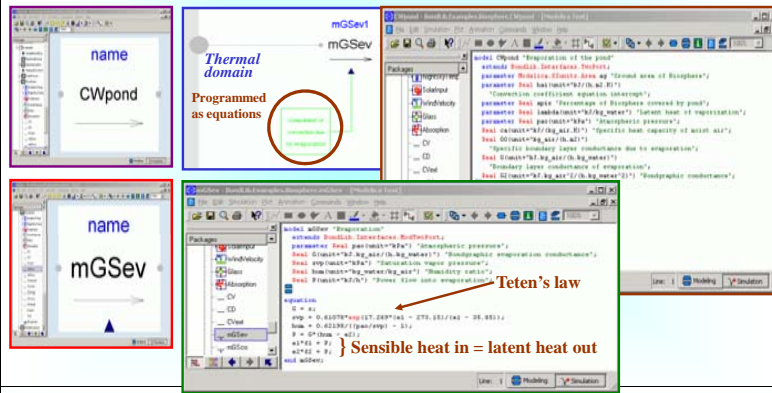
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## Evaporation of the Pond



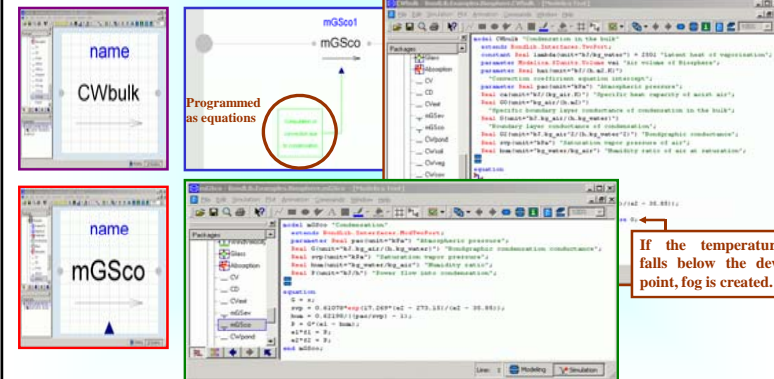
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## Condensation in the Atmosphere




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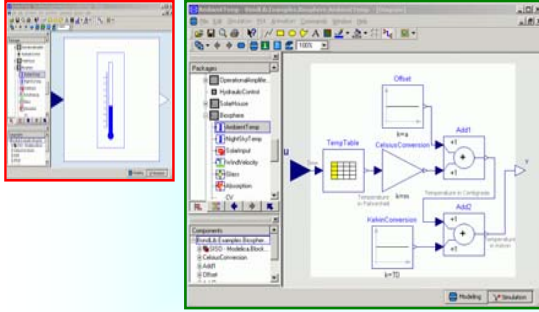


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Ambient Temperature





- In this example, the temperature values are stored as one long binary table.
- The data are given in Fahrenheit.
- Before they can be used, they must be converted to Kelvin.

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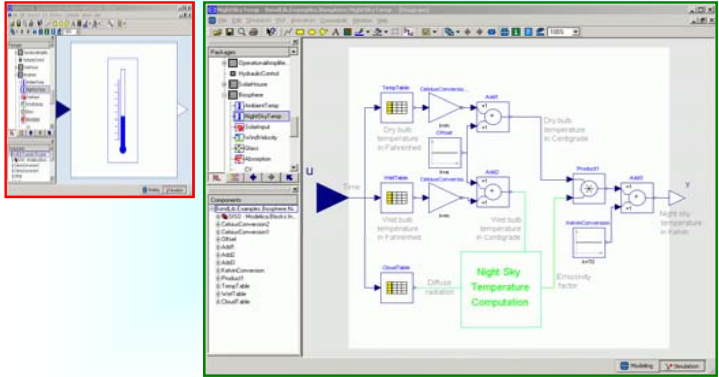


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Night Sky Temperature




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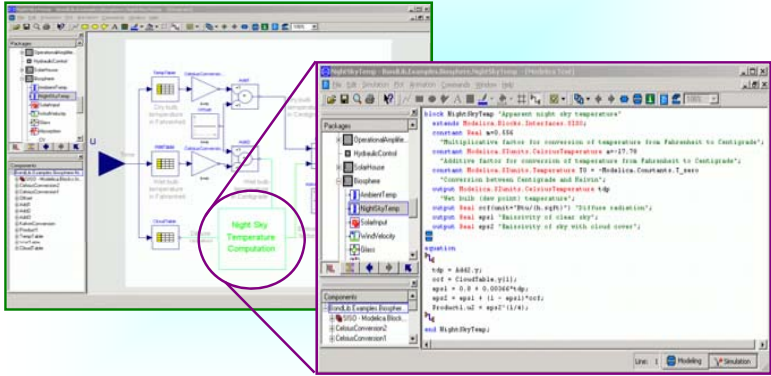


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Night Sky Temperature II




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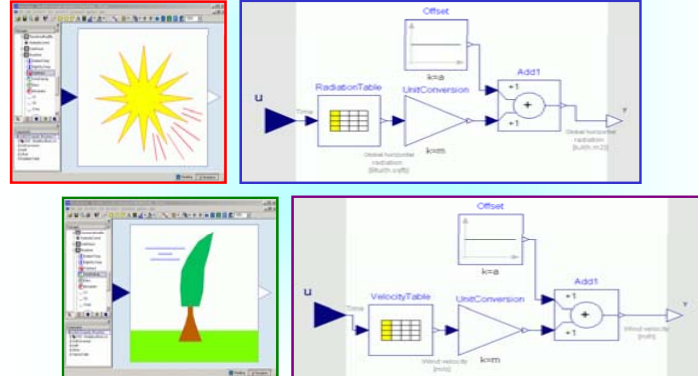


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Solar Input and Wind Velocity



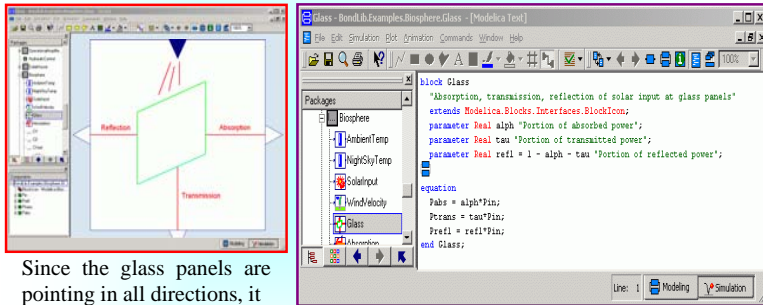
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## Absorption, Reflection, Transmission



Since the glass panels are pointing in all directions, it would be too hard to compute the physics of absorption, reflection, and transmission accurately, as we did in the last example. Instead, we simply divide the incoming radiation proportionally.

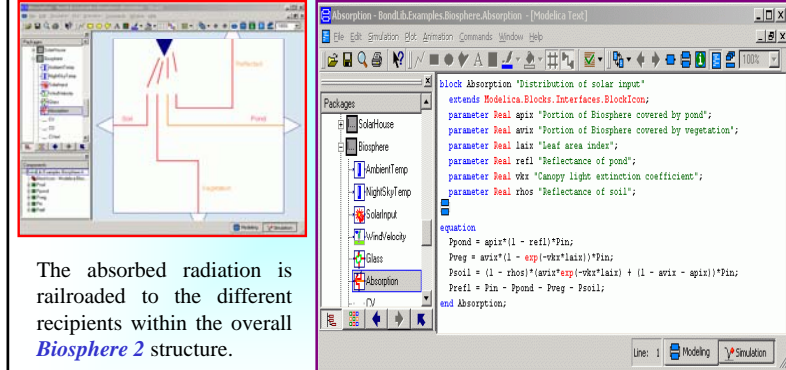
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## Distribution of Absorbed Radiation



The absorbed radiation is railroaded to the different recipients within the overall *Biosphere 2* structure.

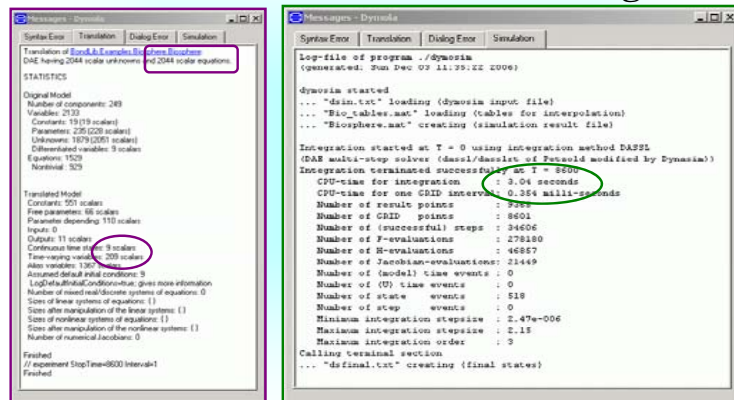
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## Translation and Simulation Logs



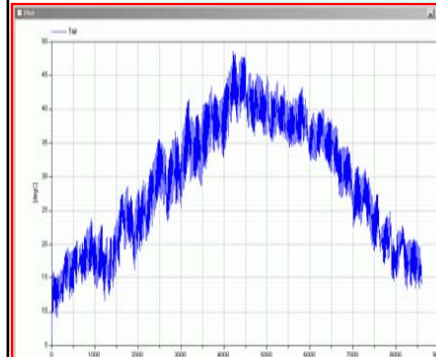
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## Simulation Results I



Air temperature inside Biosphere 2 without air-conditioning  
January 1 – December 31, 1995

- The program works with weather data that record temperature, radiation, humidity, wind velocity, and cloud cover for an entire year.
- Without climate control, the inside temperature follows essentially outside temperature patterns.
- There is some additional heat accumulation inside the structure because of reduced convection and higher humidity values.


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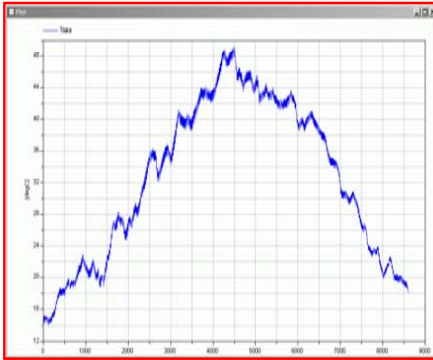


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## Simulation Results II



- Since water has a larger heat capacity than air, the daily variations in the pond temperature are smaller than in the air temperature.
- However, the overall (long-term) temperature patterns still follow those of the ambient temperature.


*Water temperature inside Biosphere 2 without air-conditioning January 1 – December 31, 1995*

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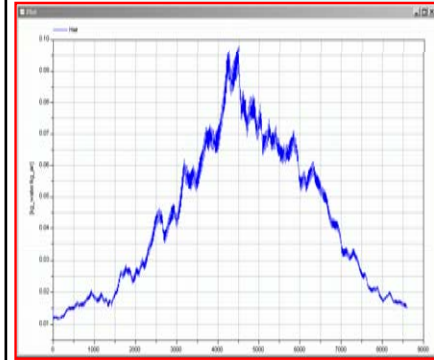


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## Simulation Results III



- The humidity is much higher during the summer months, since the saturation pressure is higher at higher temperature.
- Consequently, there is less condensation (fog) during the summer months.
- Indeed, it could be frequently observed during spring or fall evening hours that, after sun set, fog starts building up over the high savannah that then migrates to the rain forest, which eventually gets totally fogged in.


*Air humidity inside Biosphere 2 without air-conditioning January 1 – December 31, 1995*

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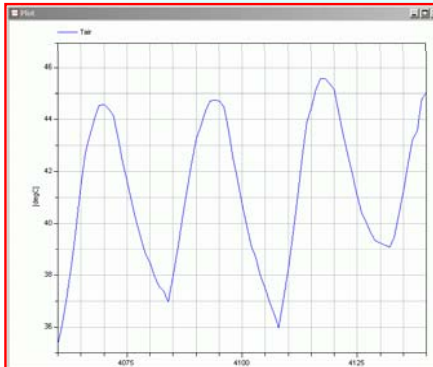


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## Simulation Results IV



- Daily temperature variations in the summer months.
- The air temperature inside *Biosphere 2* would vary by approximately 10°C over the duration of one day, if there were no climate control.


*Daily temperature variations during the summer months*

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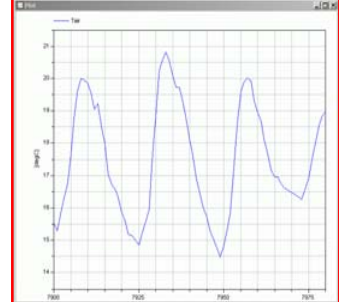
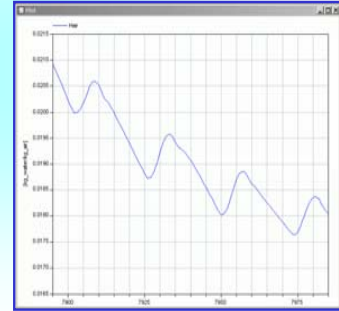


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## Simulation Results V


- Temperature variations during the winter months. Also in the winter, daily temperature variations would be close to 10°C.
- The humidity decreases as it gets colder. During day-time hours, it is higher than during the night.

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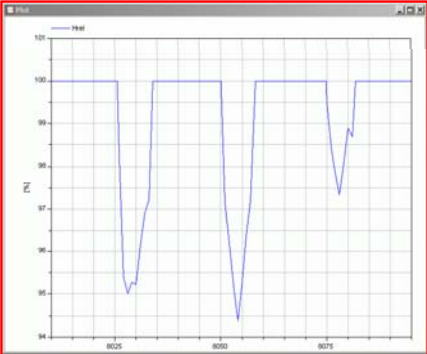


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Simulation Results VI




- The **relative humidity** is computed as the quotient of the true humidity and the humidity at saturation pressure.
- The atmosphere is almost always saturated. Only in the late morning hours, when the temperature rises rapidly, will the fog dissolve so that the sun may shine quickly.
- However, the relative humidity never decreases to a value below 94%.
- Only the climate control (not included in this model) makes life inside *Biosphere 2* bearable.


Relative humidity during three consecutive days in early winter.

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Simulation Results VII


- In a closed system, such as *Biosphere 2*, evaporation necessarily leads to an increase in humidity.
- However, the humid air has no mechanism to ever dry up again except by means of cooling. Consequently, the system operates almost entirely in the vicinity of 100% relative humidity.
- The climate control is accounting for this. The air extracted from the dome is first cooled down to let the water fall out, and only thereafter, it is reheated to the desired temperature value.
- However, the climate control was not simulated here.
- Modeling of the climate control of *Biosphere 2* is still being worked on.

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