

## Multi-bond Graphs

- We shall today look at vectors of bonds, called multi-bonds.
- Especially when dealing with 2D and 3D mechanics, the d'Alembert principle must be applied to each degree of freedom separately.
- Each equation looks structurally the same.
- This leads naturally to a demand for multi-bond graphs.

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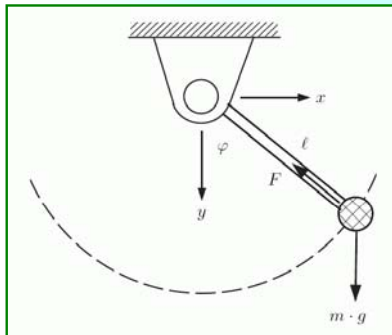
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## A Planar Pendulum

- Let us model the following planar pendulum:



$$\begin{aligned} m \cdot \frac{dv_x}{dt} &= -\frac{F \cdot x}{\ell} \\ m \cdot \frac{dv_y}{dt} &= m \cdot g - \frac{F \cdot y}{\ell} \\ \frac{dx}{dt} &= v_x \\ \frac{dy}{dt} &= v_y \end{aligned}$$

**Holonomic Constraint**

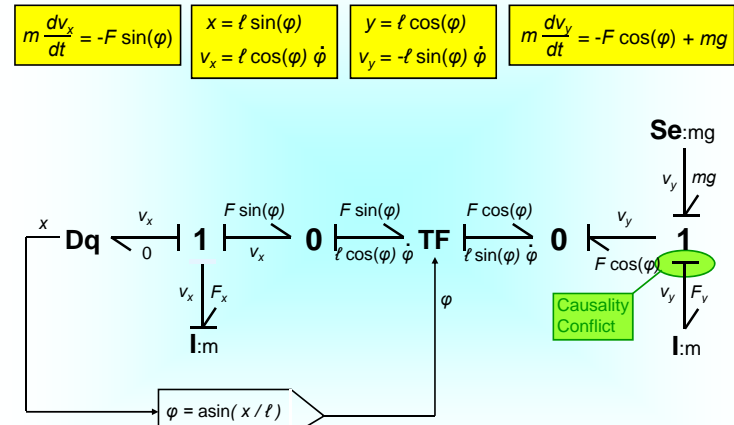
$$x^2 + y^2 = \ell^2$$

$$\begin{aligned} m \cdot \frac{dv_x}{dt} &= -\frac{F \cdot x}{\ell} \\ m \cdot \frac{dv_y}{dt} &= m \cdot g - \frac{F \cdot y}{\ell} \\ \frac{dx}{dt} &= v_x \\ \frac{dy}{dt} &= v_y \\ x &= \ell \cdot \sin(\varphi) \\ y &= \ell \cdot \cos(\varphi) \end{aligned}$$

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## Analysis

It has been possible to describe the motion of the planar pendulum by a bond graph enhanced by activated bonds for the description of the *holonomic constraint*. Unfortunately, the bond graph doesn't tell us much that we didn't know already.

- ❖ We shouldn't have to derive the equations first in order to be able to derive the bond graph from them.
- ❖ The resulting bond graph didn't preserve the topological properties of the system in any recognizable form.

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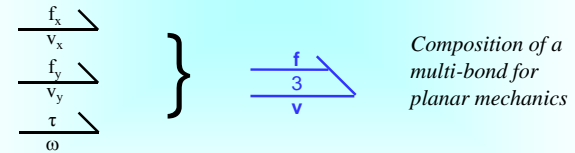
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## Multi-bonds

- Multi-bond graphs are a vector extension of the regular bond graphs.
- A multi-bond contains a freely selectable number of regular bonds of identical or similar domains.



- All bond graph component models are adjusted in a suitable fashion.

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## Multi-bond Graph Library

- A *Dymola* library for modeling systems by means of multi-bond graphs has been developed.
- The library has been designed with an interface that looks as much as possible like that of the original *BondLib* library.
- Just like the original library, also the new multi-bond graph library contains sub-libraries supporting modelers in modeling systems from particular application domains, especially from mechanics.

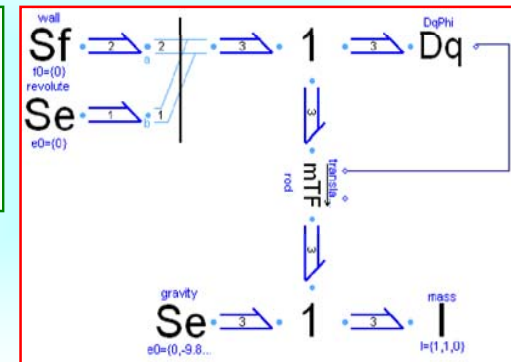
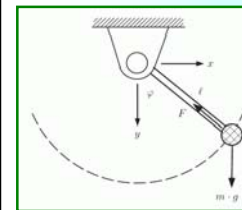
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## Planar Pendulum III




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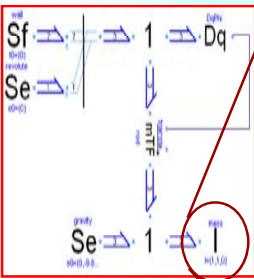
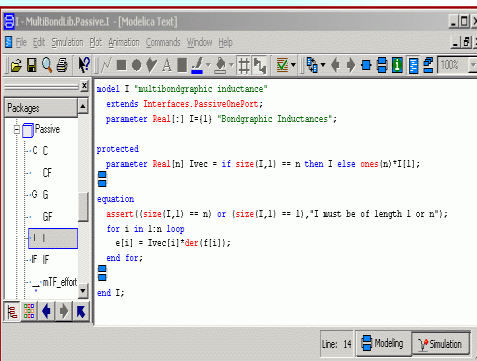




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## Planar Pendulum IV





```

model I "multibondgraphic inductance"
  extends Interfaces.PassiveOnePort;
  parameter Real I = 1 "Bondgraphic Inductances";

  protected
    parameter Real Ivec = if size(I,1) == n then I else ones(n)*I(1);
  equation
    assert((size(I,1) == n) or (size(I,1) == 1), "I must be of length 1 or n");
    for i in 1:n loop
      e[i] = Ivec[i]*der(f[i]);
    end for;
  end I;
  
```

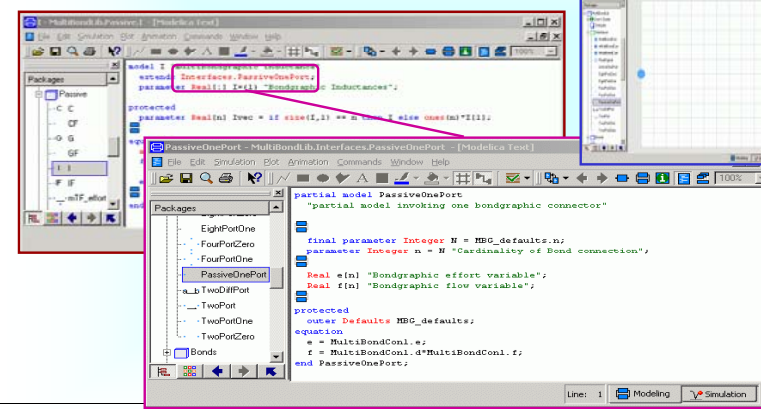
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## Planar Pendulum V




```

model I "multibondgraphic inductance"
  extends Interfaces.PassiveOnePort;
  parameter Real Ivec = if size(I,1) == n then I else ones(n)*I(1);

  protected
    parameter Real Ivec = if size(I,1) == n then I else ones(n)*I(1);
  equation
    assert((size(I,1) == n) or (size(I,1) == 1), "I must be of length 1 or n");
    for i in 1:n loop
      e[i] = Ivec[i]*der(f[i]);
    end for;
  end I;
  
```

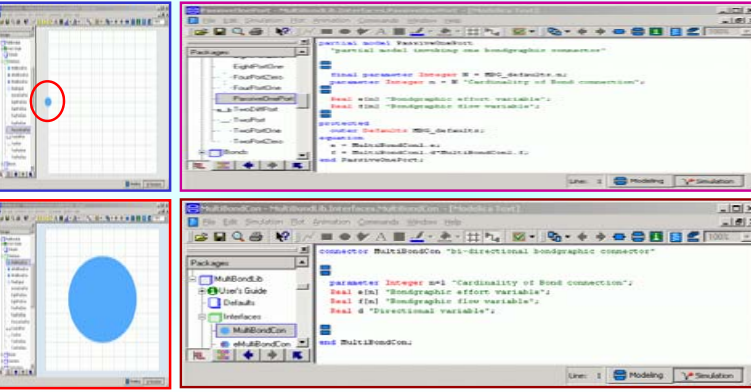
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## Planar Pendulum VI




```

model I "multibondgraphic inductance"
  extends Interfaces.PassiveOnePort;
  parameter Real Ivec = if size(I,1) == n then I else ones(n)*I(1);

  protected
    parameter Real Ivec = if size(I,1) == n then I else ones(n)*I(1);
  equation
    assert((size(I,1) == n) or (size(I,1) == 1), "I must be of length 1 or n");
    for i in 1:n loop
      e[i] = Ivec[i]*der(f[i]);
    end for;
  end I;
  
```

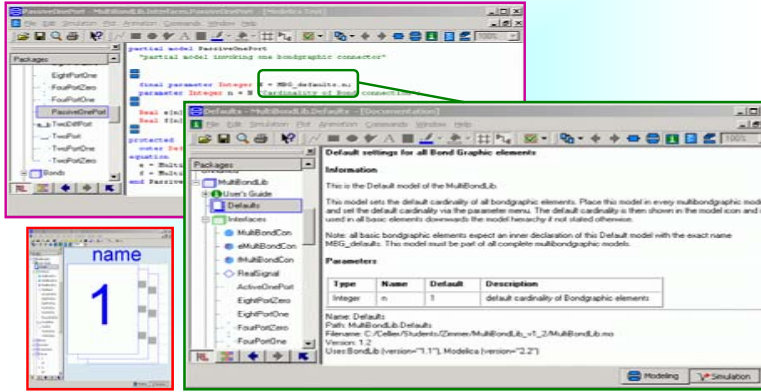
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## Planar Pendulum VII



```

model I "multibondgraphic inductance"
  extends Interfaces.PassiveOnePort;
  parameter Real Ivec = if size(I,1) == n then I else ones(n)*I(1);

  protected
    parameter Real Ivec = if size(I,1) == n then I else ones(n)*I(1);
  equation
    assert((size(I,1) == n) or (size(I,1) == 1), "I must be of length 1 or n");
    for i in 1:n loop
      e[i] = Ivec[i]*der(f[i]);
    end for;
  end I;
  
```

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## Planar Pendulum VIII

```

model Se "source of effort"
  extends Interfaces.ActiveOnePort;
  parameter Real[:1] e0 = {1} "Bondgraphic effort";
  protected
    parameter Real[n] e0vec = if size(e0,1) == n then e0 else ones(n)*e0[1];
  equation
    assert((size(e0,1) == n) or (size(e0,1) == 1), "e0 must be of length 1 or n");
    e = e0vec;
  end Se;

```

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## Planar Pendulum IX

```

model J1 "Model of a standard 1-junction with up to 4 ports"
  extends Interfaces.FourPortsOne;
  equation
    e1 + e2 + e3 + e4 = zeros(n);
    e1 = e2;
    e1 = e3;
    e1 = e4;
  end J1;

```

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## Planar Pendulum X

```

model J1 "Model of a standard 1-junction with"
  extends Interfaces.FourPortsOne;
  equation
    e1 + e2 + e3 + e4 = zeros(n);
    e1 = e2;
    e1 = e3;
    e1 = e4;
  end J1;

```

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## Planar Pendulum XI

```

model Dq "sensor of bondgraphic position"
  extends Interfaces.PassiveOnePort;
  parameter Boolean stateInitialCondition = true
    "state the initial bondgraphic position q";
  parameter Real[n] q_start = {0} "initial value for q";
  initial equation
    if stateInitialCondition then
      q = q_start;
    end if;
  equation
    e = f(q, n);
    d(e) = f2;
  end Dq;

```

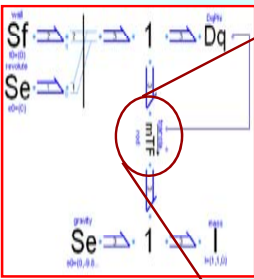
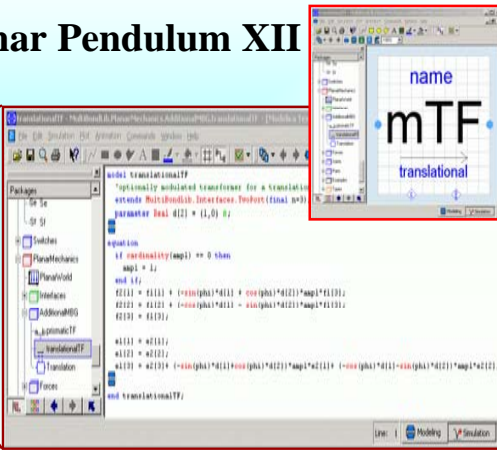
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## Planar Pendulum XII

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## Multi-bond Graph Basics

- The basic multi-bond graph models contain little that is surprising. They represent essentially natural extensions of the regular bond graph models.
- A few points are worth mentioning though. First, there is the *defaults model* that must be included in each multi-bond graph model. It contains only a single parameter, the dimensional parameter, *n*, that specifies, how many bonds each multi-bond contains by default.
- The defaults model must be referenced in each multi-bond graph model as an *outer model*.

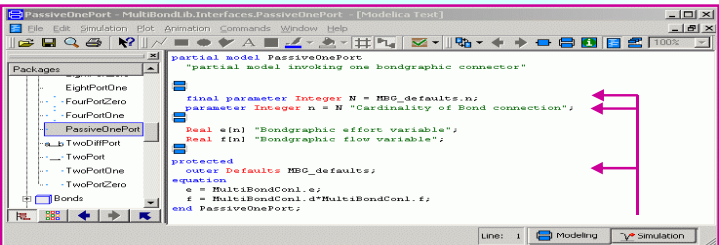
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## Multi-bond Graph Basics II



- If the multi-bond graph model inherits one of the partial models, this has already been taken care of.

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## Multi-bond Graph Basics III

- A second difference concerns the use of junctions. Whereas the general bond graph library provides separate junction models for 2, 6 bond connections, the multi-bond graph library offers only junctions with either 4 or 8 connectors. Yet, individual connectors may be left unconnected as needed.
- A third difference is in the use of transformers and gyrators. The multi-bond graph library offers a much larger variety of different transformer and gyrator models when compared to the regular bond graph library.

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## Multi-port Transformers

$$\begin{array}{c} e_1 \\ \hline f_1 \end{array} \xrightarrow[\text{M}]{\text{TF}} \begin{array}{c} e_2 \\ \hline f_2 \end{array}$$

**Transformation:**  $e_1 = M \cdot e_2$  (1)

**Energy Conservation:**  $e_1^T \cdot f_1 = e_2^T \cdot f_2$  (2)

$$\Rightarrow (M \cdot e_2)^T \cdot f_1 = e_2^T \cdot f_2$$
 (3)
$$\Rightarrow f_2 = M^T \cdot f_1$$
 (4)

⇒ The transformer may either be described by means of equations (1) and (2) or using equations (1) and (4).

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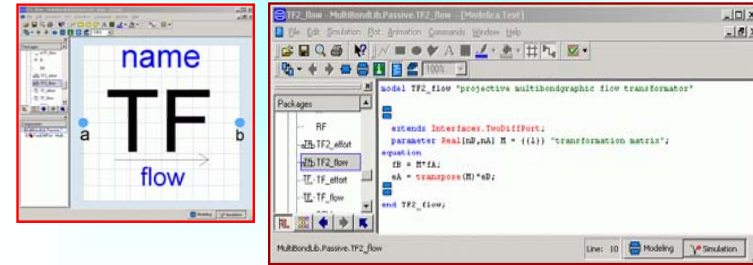
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## Multi-port Transformers II

- The transformer that looks most similar to the TF element of the regular bond graph library is the flow multi-port transformer. The cardinality of the bonds on the two sides doesn't have to be identical.



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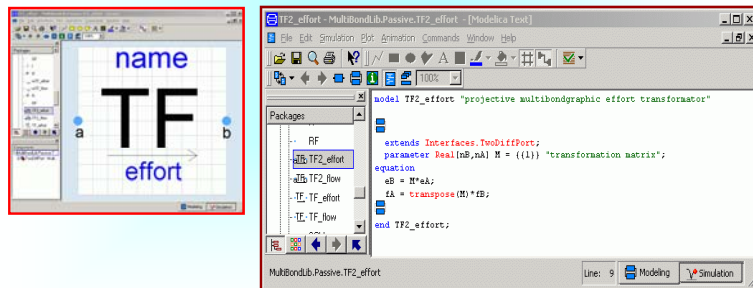
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## Multi-port Transformers III

- Yet, since  $M$  doesn't usually have an inverse, an effort transformer model must also be provided.



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## Multi-port Transformers IV

- Also offered are modulated versions of multi-port transformers and gyrators.
- Yet, this is still insufficient. Special transformers for particular purposes ought to be provided as well, since they are being used frequently in mechanics.
- We already met the *translational transformer*.
- Also provided is a *prismatic transformer*.
- The special transformers are contained in the 2D mechanics sub-library, since they are only useful in that context.

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Multi-port Transformers V

```

model prismaticTF "modulated transformer for a prismatic joint"
  extends MultiBondLib.Interfaces.TwoDiffPort(final nA=1, final nB=2);
  parameter Real d[2] = {1,0} s;

  equation
    tB[1] = (cos(phi)*d[1] + sin(phi)*d[2])*tA[1];
    tB[2] = (-sin(phi)*d[1] + cos(phi)*d[2])*tA[1];
    eA[1] = (cos(phi)*d[1] + sin(phi)*d[2])*eB[1] + (-sin(phi)*d[1] + cos(phi)*d[2])*eB[2];
  end prismaticTF;
  
```

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Multi-bond Graph Basics IV

- Finally, although the library offers causal multi-bonds, these are much less useful than the causal regular bonds, because many multi-bonds have mixed computational causality. Hence causal multi-bonds are rarely used in practice.

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- Zimmer, D. and F.E. Cellier (2006), "[\*The Modelica Multi-bond Graph Library\*](#)," *Proc. 5<sup>th</sup> Intl. Modelica Conference*, Vienna, Austria, Vol.2, pp. 559-568.

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