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Eidgenössische Technische Hochschule Zürich
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Mathematical Modeling of Physical Systems

Convective Mass Flows II

- In today's lecture, we shall perform a number of thought experiments that will help us deepen our understanding of the interactions between *mass flow* and *heat flow* dynamics.
- We shall look at five different thought experiments, first from a conceptual point of view, then from a bond-graphic modeling perspective.

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Table of Contents

- [Thought experiments](#)
- [Capacitive fields](#)
- [More thought experiments](#)

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1st Experiment

- We shall perform a few thought experiments.


- Given 4 hot pellets. Three are located on the left side, whereas the fourth is located on the right side.




We transport one of the three pellets from the left to the right side.



All pellets still have the same temperature. The transported pellet has carried its stored heat with it.

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2nd Experiment

- Given a bottle of wine and a pneumatic cork screw. The cork screw contains a cartridge filled with compressed gas, as well as an injection needle. The needle is pushed through the cork, and a knob is pushed, to empty the compressed gas into the bottle. The bottle is now over-pressurized, and therefore, the cork is pushed out of the bottle.

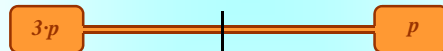
After the opening of the bottle, the gas cartridge is freezing cold!

Hmm ...

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3rd Experiment I

3. Given two gas cartridges of equal sizes. Both are at room temperature. One of the cartridges contains gas with a pressure of $3p$, the other with a pressure of p . The two cartridges are connected by a thin air tube that is clamped in the middle.



The clamp is now removed, and the pressures are being equalized.



After the pressure equilibration, the left cartridge is cold, whereas the right cartridge is hot.

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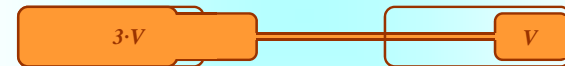
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4th Experiment I

4. Given two gas cartridges containing gas at an equal gas pressure. Both are at room temperature. One of the cartridges has a volume of $3 \cdot V$, the other one of V . The cartridges are connected by an air tube.



We now press against the cartridge on the left until the volumes have become identical.



After the volume equilibration, both cartridges still are at the same temperature.

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Questions

- In the 3rd and 4th experiment, the same mass flow from the left to the right gas cartridge takes place. How is it then possible that, in one case, the temperatures change, whereas in the other, they don't?
- Does the way, proposed in the last lecture, to deal with combined mass and heat flows still not cover all cases?

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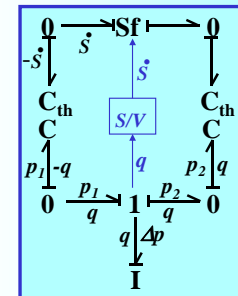
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3rd Experiment II

- Let us start with the analysis of the third thought experiment. This one is simpler, because no forced flows are present.



$p_2 < p_1$: A volume flow q takes place.

The volume flow q drains energy from the capacity on the left. Consequently, the pressure p_1 must decrease.

The volume flow q induces an entropy flow \dot{S} .

The entropy flow \dot{S} drains heat from the capacity on the left. The temperature decreases.

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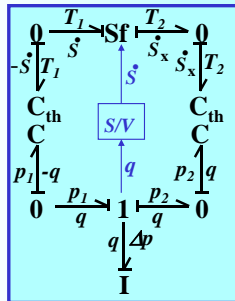
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3rd Experiment III

- We continue with the analysis of the third experiment.



The temperature T_1 on the left is now lower than the temperature T_2 on the right. An entropy current is produced on the right, so that energy conservation is satisfied at the Sf-element: $T_1 \cdot \dot{S} = T_2 \cdot \dot{S}_x$.

The generated entropy current \dot{S}_x flows into the capacity on the right, which leads to an increase in the temperature T_2 .

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4th Experiment II

- The situation is here considerably more complicated, since there is a **forced flow**. Without external interference, the system would be in steady state.
- The system behaves like an air balloon with a constriction at the center. When you press the balloon on the left, the balloon on the right increases in size, and vice-versa.
- The system is constantly in equilibrium with its environment, since the air pressures everywhere (in both balloons as well as in the environment) are the same, namely p .

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4th Experiment III

- For simplicity, we shall start with the cartridge to the right, i.e., the passive cartridge.
- A volume flow reaches the cartridge, and since it doesn't have any other alternative, flows into the capacity.
- Consequently, the pressure in that cartridge increases, and the temperature rises.
- The pressure inside the cartridge is now bigger than that of the surrounding air. Consequently, a resulting force pushes the piston to the right.
- As a consequence, the volume of the cartridge increases, and both pressure and temperature decrease again to their former values.

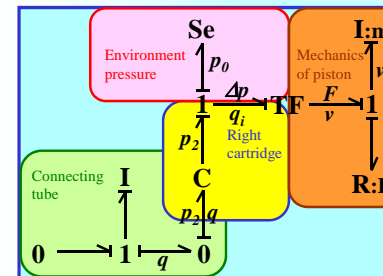
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4th Experiment IV



The flow q reaches the cartridge, and enters it.

The pressure p_2 increases, and produces a pressure difference Δp .

This generates a resulting force F on the piston.

The force generates a resulting velocity v .

The velocity induces a volume flow q_i , which re-equilibrates the pressure.

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Capacitive Fields I

- The capacity looks strange, since it has two ports.
- The general **Multiport-capacity**, also referred to as **capacitive field**, satisfies the following vector equation:

$$\underline{q} = C(\underline{e})$$

- In the linear case:

$$\underline{f} = \underline{C} \cdot \underline{de/dt}$$

↑
symmetric matrix

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Capacitive Fields II

- In the given situation:

$\frac{p_1}{q_1} \rightarrow C \frac{p_2}{q_2}$

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} = \begin{bmatrix} 1/C & 1/C \\ 1/C & 1/C \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

- Since we really have only one capacity and one pressure, we can save one of the two integrators, and write:

$$\begin{bmatrix} \dot{p}_1 \\ p_2 \end{bmatrix} = (q_1 - q_2) / C$$

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4th Experiment V

- Let us now consider the active cartridge. We shall use a DC-motor to drive the left piston.

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4th Experiment VI

- What happens on the thermal side?
- Whenever there exists a volume flow, a convective heat flow is induced.
- In this system, we have three volume flows:
 - the induced flow on the left
 - the flow through the connecting tube
 - the induced flow on the right.
- Consequently, we require three non-linear (modulated) Sf-elements on the thermal side.

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4th Experiment VII

$T_1 \dot{S}_1 = T_2 \dot{S}_2$

$q_1 = q_2$

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4th Experiment VIII

- In the given situation, the introduction of a **C-field** would actually not have been necessary. We would have found the same equations, had we taken the secondary flow of the capacity off the 0-Junction located below the capacity.

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5th Experiment I

- Given a straw. It is plunged into water and totally filled with water. Now, one of the two openings of the straw is closed with the index finger, the straw is removed from the water, and placed vertically with the closed opening at the top.

The water remains in the straw.

Now, the upper end is opened.

The water leaves the straw through the opening at the bottom.

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5th Experiment II

- Until now, we have considered only two properties of the water: its **volume** and its **heat**.
- However, every material also has **mass**.
- In the previous experiments, this property was not important, but it becomes important, as soon as we need to consider the effects of **gravity**.

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5th Experiment III

- The physics behind this experiment is very simple. As long as the index finger closes off the top opening of the straw, no volume can flow in. Consequently, a flow can only take place by reducing the pressure. However, this would have to happen against the pressure of the outside air, which previously was in equilibrium with the water pressure. Therefore, no water flows.

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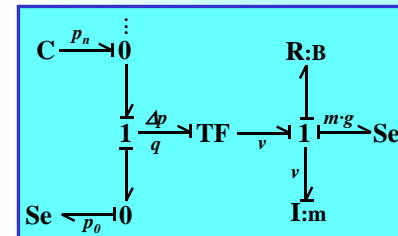
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5th Experiment IV

- The gravitational force can easily be modeled using a transformer with a mass, a friction, and an Se-element.



First, a flow q is produced. This flow reduces the pressure further up in the straw, which reduces Δp to 0, so that no water flows any longer.

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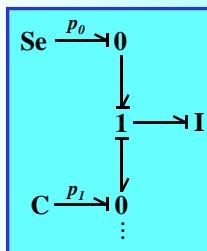
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5th Experiment V

- When the index finger is removed, the top C-element is replaced by a pressure source.



Through this source, an arbitrary amount of volume can flow in from the top.

One complication to be considered is that the mass gets constantly reduced, until no water is left in the straw. Therefore, the Se-element, that represents the gravitational force, must be modulated.

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