

Convective Mass Flows III

- In this lecture, we shall concern ourselves once more with convective mass and heat flows, as we still have not gained a comprehensive understanding of the physics behind such phenomena.
- We shall start by looking once more at the *capacitive field*.
- We shall then study the *internal energy* of matter.
- Finally, we shall look at *general energy transport phenomena*, which by now include mass flows as an integral aspect of general energy flows.

November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



Table of Contents

- Capacitive Fields
- Internal energy of matter
- Bus-bonds and bus-junctions
- Heat conduction
- Volume work
- General mass transport
- Multi-phase systems
- Evaporation and condensation
- Thermodynamics of mixtures
- Multi-element systems

November 22, 2012

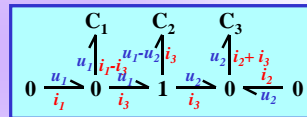
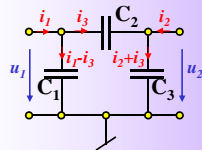
© Prof. Dr. François E. Cellier

Start Presentation



Capacitive Fields III

- Let us briefly consider the following electrical circuit:



$$\begin{aligned} i_1 - i_3 &= C_1 \cdot du_1/dt \\ i_2 + i_3 &= C_3 \cdot du_2/dt \\ i_3 &= C_2 \cdot (du_1/dt - du_2/dt) \end{aligned}$$

$$\Rightarrow \begin{aligned} i_1 &= (C_1 + C_2) \cdot du_1/dt - C_2 \cdot du_2/dt \\ i_2 &= -C_2 \cdot du_1/dt + (C_2 + C_3) \cdot du_2/dt \end{aligned}$$

November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



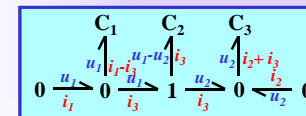
Capacitive Fields IV

$$\begin{aligned} i_1 &= (C_1 + C_2) \cdot du_1/dt - C_2 \cdot du_2/dt \\ i_2 &= -C_2 \cdot du_1/dt + (C_2 + C_3) \cdot du_2/dt \end{aligned}$$

Symmetric
capacity matrix

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} (C_1 + C_2) & -C_2 \\ -C_2 & (C_2 + C_3) \end{bmatrix} \cdot \begin{bmatrix} du_1/dt \\ du_2/dt \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} du_1/dt \\ du_2/dt \end{bmatrix} = \frac{1}{C_1 C_2 + C_1 C_3 + C_2 C_3} \begin{bmatrix} (C_2 + C_3) & C_2 \\ C_2 & (C_1 + C_2) \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$



$$\Rightarrow \begin{bmatrix} 0 & u_1 \\ i_1 & \end{bmatrix} \text{ CF } \begin{bmatrix} i_2 & \\ & 0 \end{bmatrix} \begin{bmatrix} u_2 \end{bmatrix}$$

November 22, 2012

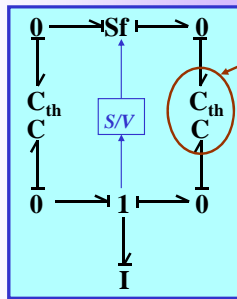
© Prof. Dr. François E. Cellier

Start Presentation



Volume and Entropy Storage

- Let us consider once more the situation discussed in the previous lecture.



It was no accident that I drew the two capacitors so close to each other. In reality, the two capacitors together form a two-port capacitive field. After all, heat and volume are only two different properties of one and the same material.

November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



The Internal Energy of Matter I

- As we have already seen, there are three different (though inseparable) storages of matter:
 - Mass
 - Volume
 - Heat
- These three storage elements represent different storage properties of one and the same material.
- Consequently, we are dealing with a **storage field**.
- This storage field is of a capacitive nature.
- The capacitive field stores the **internal energy of matter**.

November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



The Internal Energy of Matter II

- Change of the internal energy in a system, i.e. the total power flow into or out of the capacitive field, can be described as follows :

$$\dot{U} = T \cdot \dot{S} - p \cdot \dot{V} + \sum_i \mu_i \cdot \dot{N}_i$$

Flow of internal energy (pink arrow pointing to \dot{U})
 Heat flow (red arrow pointing to $T \cdot \dot{S}$)
 Volume flow (green arrow pointing to $-p \cdot \dot{V}$)
 Mass flow (orange arrow pointing to $\sum_i \mu_i \cdot \dot{N}_i$)
 Chemical potential (grey arrow pointing to μ_i)
 Molar mass flow (orange arrow pointing to \dot{N}_i)

- This is the **Gibbs equation**.

November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



The Internal Energy of Matter III

- The internal energy is proportional to the total mass n .
- By normalizing with n , all extensive variables can be made intensive.

$$u = \frac{U}{n} \quad s = \frac{S}{n} \quad v = \frac{V}{n} \quad n_i = \frac{N_i}{n}$$

- Therefore:

$$\frac{d}{dt}(n \cdot u) = T \cdot \frac{d}{dt}(n \cdot s) - p \cdot \frac{d}{dt}(n \cdot v) + \sum_i \mu_i \cdot \frac{d}{dt}(n \cdot n_i)$$

$$\Rightarrow \frac{d}{dt}(n \cdot u) - T \cdot \frac{d}{dt}(n \cdot s) + p \cdot \frac{d}{dt}(n \cdot v) - \sum_i \mu_i \cdot \frac{d}{dt}(n \cdot n_i) = 0$$

November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems

The Internal Energy of Matter IV

$$\frac{d}{dt}(n \cdot u) - T \cdot \frac{d}{dt}(n \cdot s) + p \cdot \frac{d}{dt}(n \cdot v) - \sum_i \mu_i \cdot \frac{d}{dt}(n \cdot n_i) = 0$$

$$\Rightarrow n \cdot \left[\frac{du}{dt} - T \cdot \frac{ds}{dt} + p \cdot \frac{dv}{dt} - \sum_i \mu_i \cdot \frac{dn_i}{dt} \right] + \frac{dn}{dt} \cdot \left[u - T \cdot s + p \cdot v - \sum_i \mu_i \cdot n_i \right] = 0$$

This equation must be valid independently of the amount n , therefore:

Finally, here is an explanation, why it was okay to compute with funny derivatives.

$$\frac{du}{dt} - T \cdot \frac{ds}{dt} + p \cdot \frac{dv}{dt} - \sum_i \mu_i \cdot \frac{dn_i}{dt} = 0 \quad \text{Flow of internal energy}$$

$$u - T \cdot s + p \cdot v - \sum_i \mu_i \cdot n_i = 0 \quad \text{Internal energy}$$

November 22, 2012 © Prof. Dr. François E. Cellier Start Presentation

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems

The Internal Energy of Matter V

$$U = T \cdot S - p \cdot V + \sum_i \mu_i \cdot N_i$$

$$\Rightarrow \dot{U} = T \cdot \dot{S} - p \cdot \dot{V} + \sum_i \mu_i \cdot \dot{N}_i + T \cdot \dot{S} - p \cdot \dot{V} + \sum_i \mu_i \cdot \dot{N}_i$$

$$= T \cdot \dot{S} - p \cdot \dot{V} + \sum_i \mu_i \cdot \dot{N}_i$$

$$\Rightarrow T \cdot \dot{S} - p \cdot \dot{V} + \sum_i \mu_i \cdot \dot{N}_i = 0$$

- This is the *Gibbs-Duhem equation*.

November 22, 2012 © Prof. Dr. François E. Cellier Start Presentation

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems

The Capacitive Field of Matter

November 22, 2012 © Prof. Dr. François E. Cellier Start Presentation

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems

Simplifications

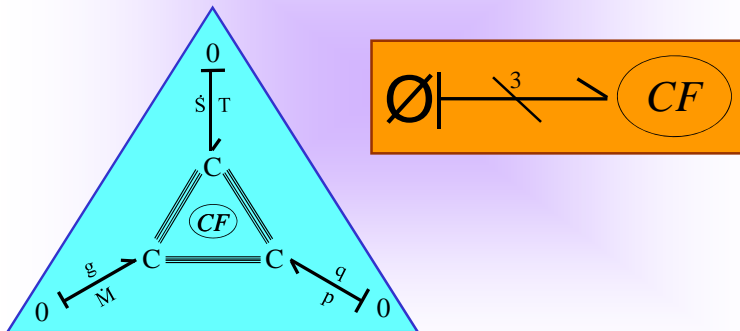
- In the case that no chemical reactions take place, it is possible to replace the *molar mass flows* by conventional *mass flows*.
- In this case, the *chemical potential* is replaced by the *Gibbs potential*.

$$\frac{dU}{dt} = T \cdot \dot{S} - p \cdot \dot{V} + g \cdot \dot{M}$$

November 22, 2012 © Prof. Dr. François E. Cellier Start Presentation

Bus-Bond and Bus-0-Junction

- The three outer legs of the CF-element can be grouped together.



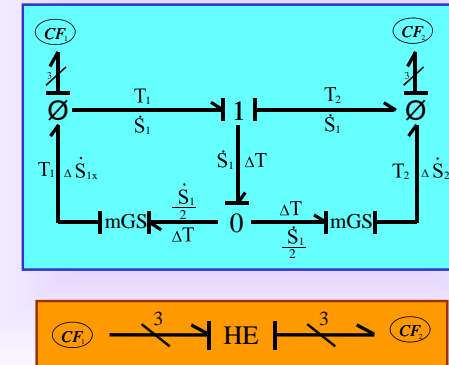
November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



Once Again Heat Conduction



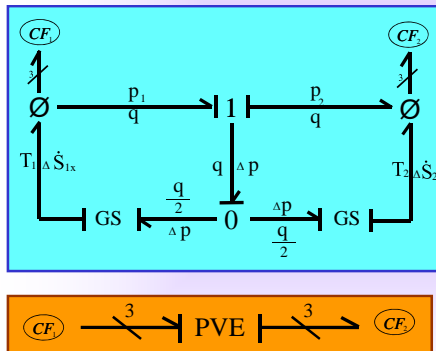
November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



Volume Pressure Exchange



Pressure is being equilibrated just like temperature. It is assumed that the inertia of the mass may be neglected (relatively small masses and/or velocities), and that the equilibration occurs without friction.

The model makes sense if the exchange occurs locally, and if not too large masses get moved in the process.

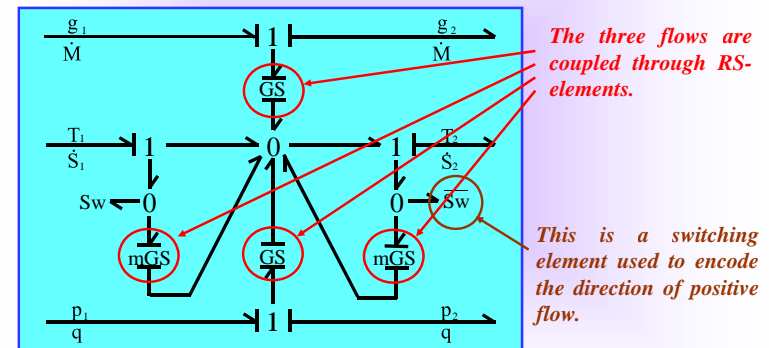
November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



General Exchange Element I



The three flows are coupled through RS-elements.

This is a switching element used to encode the direction of positive flow.

November 22, 2012

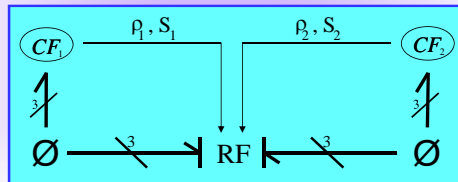
© Prof. Dr. François E. Cellier

Start Presentation



General Exchange Element II

- In the general exchange element, the temperatures, the pressures, and the Gibbs potentials of neighboring media are being equilibrated.
- This process can be interpreted as a **resistive field**.



November 22, 2012

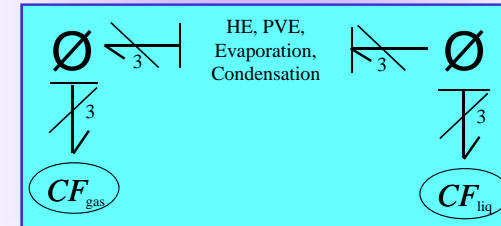
© Prof. Dr. François E. Cellier

Start Presentation



Multi-phase Systems

- We may also wish to study phenomena such as **evaporation** and **condensation**.



November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



Evaporation (Boiling)

- Mass and energy exchange between capacitive storages of matter (**CF-elements**) representing different **phases** is accomplished by means of special resistive fields (**RF-elements**).
- The mass flows are calculated as functions of the pressure and the corresponding saturation pressure.
- The volume flows are computed as the product of the mass flows with the saturation volume at the given temperature.
- The entropy flows are superposed with the enthalpy of evaporation (in the process of evaporation, the thermal domain loses heat \rightarrow **latent heat**).

November 22, 2012

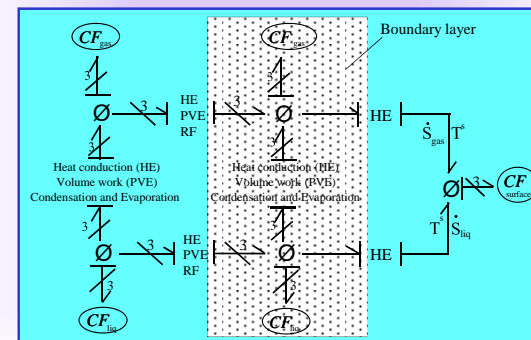
© Prof. Dr. François E. Cellier

Start Presentation



Condensation On Cold Surfaces

- Here, a boundary layer must be introduced.



November 22, 2012

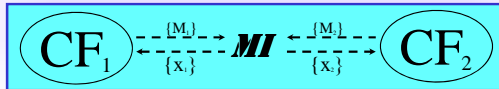
© Prof. Dr. François E. Cellier

Start Presentation



Thermodynamics of Mixtures

- When fluids (gases or liquids) are being mixed, additional entropy is generated.
- This **mixing entropy** must be distributed among the participating component fluids.
- The distribution is a function of the **partial masses**.
- Usually, neighboring **CF-elements** are not supposed to know anything about each other. In the process of mixing, this rule cannot be maintained. The necessary information is being exchanged.



November 22, 2012

© Prof. Dr. François E. Cellier

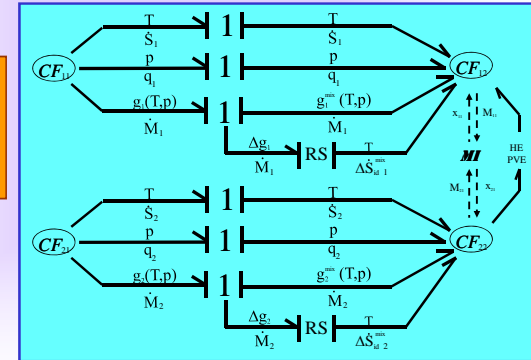
Start Presentation



Entropy of Mixing

- The mixing entropy is taken out of the Gibbs potential.

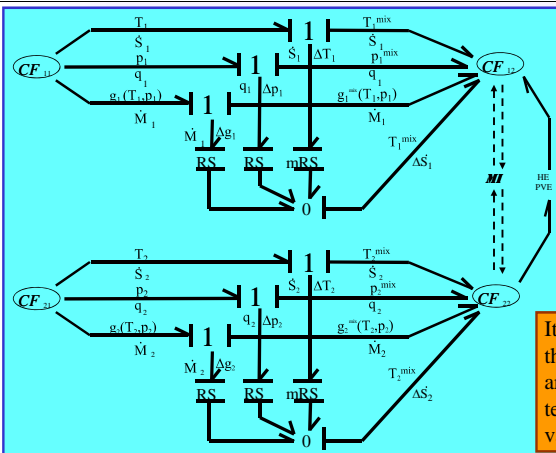
It was assumed here that the fluids to be mixed are at the same temperature and pressure.



November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



It is also possible that the fluids to be mixed are initially at different temperature or pressure values.

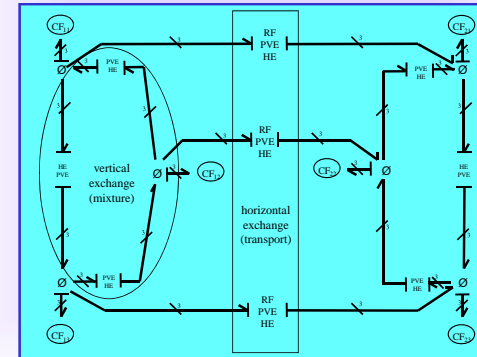
November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation



Convection in Multi-element Systems




November 22, 2012

© Prof. Dr. François E. Cellier

Start Presentation

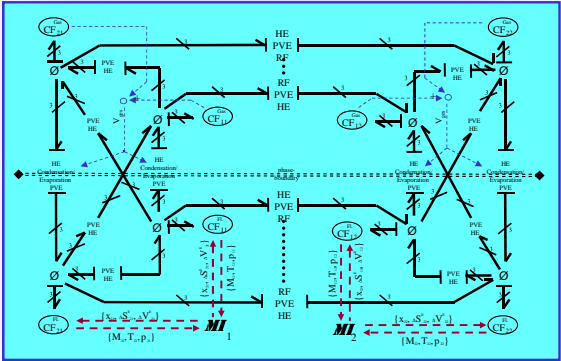





Eidgenössische Technische Hochschule Zürich
 Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems

Two-element, Two-phase, Two-compartment Convective System



November 22, 2012
 © Prof. Dr. François E. Cellier
 Start Presentation

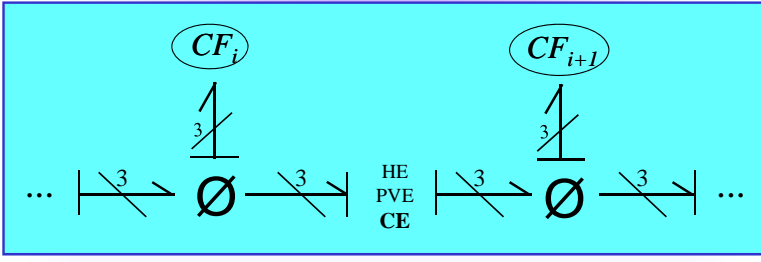


Eidgenössische Technische Hochschule Zürich
 Swiss Federal Institute of Technology Zurich


Mathematical Modeling of Physical Systems

Concentration Exchange

- It may happen that neighboring compartments are not completely homogeneous. In that case, also the concentrations must be exchanged.



November 22, 2012
 © Prof. Dr. François E. Cellier
 Start Presentation




Eidgenössische Technische Hochschule Zürich
 Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems

References I

- Cellier, F.E. (1991), *Continuous System Modeling*, Springer-Verlag, New York, [Chapter 9](#).
- Greifeneder, J. and F.E. Cellier (2001), “[Modeling convective flows using bond graphs](#),” *Proc. ICBGM'01, Intl. Conference on Bond Graph Modeling and Simulation*, Phoenix, Arizona, pp. 276 – 284.
- Greifeneder, J. and F.E. Cellier (2001), “[Modeling multi-phase systems using bond graphs](#),” *Proc. ICBGM'01, Intl. Conference on Bond Graph Modeling and Simulation*, Phoenix, Arizona, pp. 285 – 291.

November 22, 2012
 © Prof. Dr. François E. Cellier
 Start Presentation



Eidgenössische Technische Hochschule Zürich
 Swiss Federal Institute of Technology Zurich

Mathematical Modeling of Physical Systems

References II

- Greifeneder, J. and F.E. Cellier (2001), “[Modeling multi-element systems using bond graphs](#),” *Proc. ESS'01, European Simulation Symposium*, Marseille, France, pp. 758 – 766.
- Greifeneder, J. (2001), [Modellierung thermodynamischer Phänomene mittels Bondgraphen](#), Diploma Project, Institut für Systemdynamik und Regelungstechnik, University of Stuttgart, Germany.

November 22, 2012
 © Prof. Dr. François E. Cellier
 Start Presentation