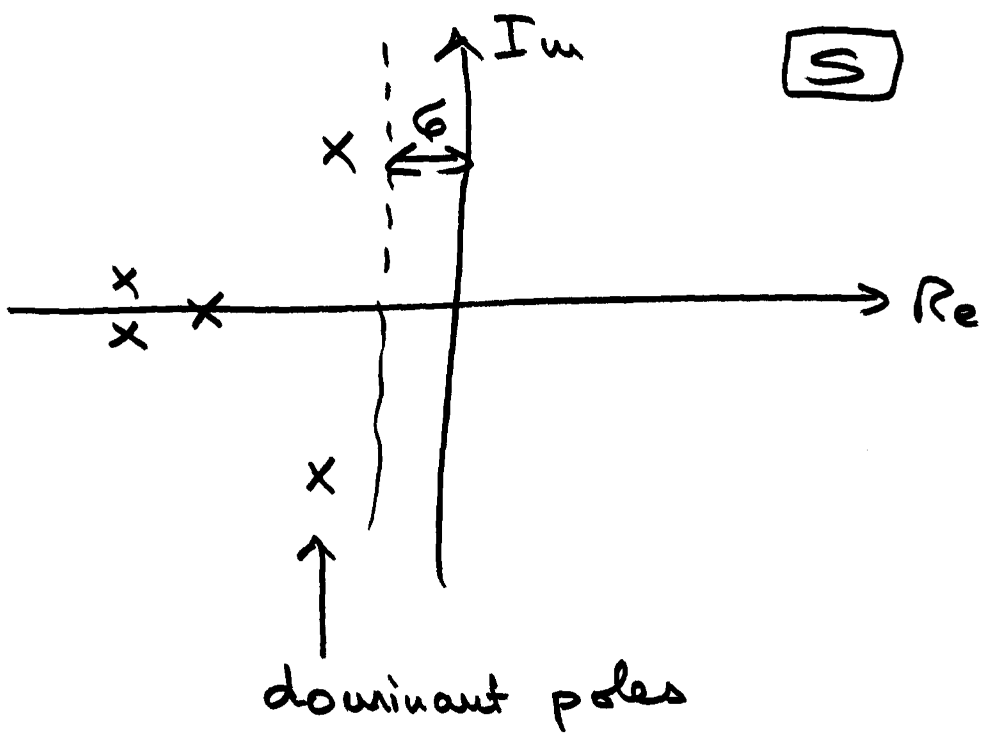
Exponential Stability:

Sometimes, we would like to ensure that the dominant poles (i.e., the poles most to the right) are at least σ away from the imaginary axis:



We want:

$$\mathrm{Re}\{\text{dominant pole}\} < -\sigma \quad .$$

Given:

$$D(s) = (s+2)(s-1+4j)(s-1-4j)$$

with:

$$s_1 = -2$$
$$s_2 = +1-4j$$
$$s_3 = +1+4j$$

Let us replace $s \longrightarrow s-1$:

$$\hat{D}(s) = ((s-1)+2)((s-1)-1+4j)((s-1)-1-4j)$$
$$= (s+1)(s-2+4j)(s-2-4j)$$

with:

$$\hat{s}_1 = -1$$
$$\hat{s}_2 = +2-4j$$
$$\hat{s}_3 = +2+4j$$

In the transformation, each pole has moved by 1 to the right.

In order to guarantee a margin of stability, we move all poles by σ to the right, then apply the Routh scheme. If $\hat{D}(s)$ is stable, $D(s)$ has a margin of stability of at least σ.

Example:

$$D(s) = s^4 + 11s^3 + 45s^2 + 83s + 60$$

s^4	1	45	60
s^3	11	83	
s^2	37.4545	60	
s^1	29.5027		
s^0	60		

$\Rightarrow$ stable.

Check, whether the margin of stability is at least 1:

$$\hat{D}(s) = (s-1)^4 + 11(s-1)^3 + 45(s-1)^2 + 83(s-1) + 60$$

$$\begin{array}{rrrrrr}
= & s^4 & -4s^3 & +6s^2 & -4s & +1 \\
 & & +11s^3 & -33s^2 & +33s & -11 \\
 & & & +45s^2 & -90s & +45 \\
 & & & & +83s & -83 \\
 & & & & & +60 \\
\hline
\end{array}$$

$$= s^4 + 7s^3 + 18s^2 + 22s + 12$$

s^4	1	18	12
s^3	7	22	
s^2	14.857	12	
s^1	16.346		
s^0	12		

$\Longrightarrow$ **still stable**

Check, whether the margin of stability is at least 2:

$$\hat{D}(s) = (s-2)^4 + 11(s-2)^3 + 45(s-2)^2 + 83(s-2) + 60$$

$$\begin{array}{rrrrrr}
= & s^4 & -8s^3 & +24s^2 & -32s & +16 \\
 & & +11s^3 & -66s^2 & +132s & -88 \\
 & & & +45s^2 & -180s & +180 \\
 & & & & +83s & -166 \\
 & & & & & +60 \\
\hline
\end{array}$$

$$= s^4 + 3s^3 + 3s^2 + 3s + 2$$

s^4	1	3	2
s^3	3	3	
s^2	2	2	
s^1	0		

$\Rightarrow H(s) = 2s^2 + 2$ is a true divides.

$$\Rightarrow H(s) = 2(s^2+1) = 2(s+j)(s-j)$$

$$
\begin{array}{rl}
 & (s^4 + 3s^3 + 3s^2 + 3s + 2) : (s^2+1) = s^2 + 3s + 2 \\
- & s^4 \qquad\qquad + s^2 \\
\hline
 & \diagdown \quad 3s^3 + 2s^2 + 3s + 2 \\
- & \qquad 3s^3 \qquad\quad + 3s \\
\hline
 & \qquad \diagdown \quad 2s^2 + \diagdown + 2 \\
- & \qquad\qquad 2s^2 \quad\; + 2 \\
\hline
 & \qquad\qquad \diagdown \qquad \diagdown
\end{array}
$$

$$s^2 + 3s + 2 = 0$$

$$\Rightarrow s_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2}$$

$$\Rightarrow \hat{\hat{s}}_3 = -1$$
$$\hat{\hat{s}}_4 = -2$$

$$\Rightarrow \hat{D}(s) = (s+1)(s+2)(s+j)(s-j)$$

$$\Rrightarrow D(s) = (s+3)(s+4)(s+2+j)(s+2-j)$$

$$\Rightarrow s_1 = -3$$
$$s_2 = -4$$
$$s_3 = -2 - j$$
$$s_4 = -2 + j$$

Parametric Stability:

Sometimes, the polynomial may contain one or several unknown design parameters, and the question may be, for which range of these design parameters will the system remain stable.

Example:

$$D(s) = s^3 + 3s^2 + 3s + (1+k)$$

s^3	1	3
s^2	3	$(1+k)$
s^1	$\frac{8-k}{3}$	
s^0	$(1+k)$	

$$\frac{8-k}{3} > 0 \Rightarrow k < 8$$

$$1+k > 0 \Rightarrow k > -1$$

$$\Rightarrow \underline{\underline{k \in (-1, +8)}}$$