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ON THE EXTENSION OF THE BONDGRAPHIC POWER POSTULATE TO SOME RELATIVISTIC PHENOMENA

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ABSTRACT

Since their official birth on April 24, 1959, bondgraphs evolved to become one of the most effective and most elegant tools for modeling system dynamics. The unifying attitude of bondgraphs, or BG's for short, towards systems, enables modelbuilders to connect system components from different physical domains using the same set of bondgraphic elements. It was this unifying attitude that furnished the impetus for this work that investigates the possibility of applying BG's to Einstein's standard theories of relativity; thus extending the BG approach of modeling to relativistic dynamics. To this end a new formulation of the bondgraphic power postulate (that is built using four-vector efforts and flows) is developed. Then special relativistic (SR) particle-mechanics and electrodynamics (via a one particle system) are investigated, and the bondgraphic interpretation for both is also provided. Finally we provide Einstein's gravitational field equations, a bondgraphic interpretation of them, and the energetic problems facing standard general relativity (GR).

Key Words. bondgraphs, coenergy, contravariance, covariance, electromagnetic field-strength tensor, energy, energy-momentum tensor, Galilean transformations, general relativity, Lorentz transformations, the principle of relativity, special relativity, tensors.

1 INTRODUCTION

Since their official birth on April 24, 1959 **(26)**, bondgraphs evolved to become one of the most powerful designing, modeling and simulation tools. Their application to a growing diversified constellation of fields **(8)** is proving time and again their genuine contributions. From a variety of versions **(9, 5, 29, & 6)** that appeared after the classical work by their inventor Henry M. Paynter **(25)**, this work adopts the versions introduced by Breedveld **(7)**, and Fahrenthold and Wargo **(16)**. We still diverge from the tensorial treatment of Fahrenthold and Wargo, in the sense that we avail ourselves of the standard component notation, and the range and summation conventions of tensor calculus **(30)**, that become a necessity for treating general tensors **(21, 22, 14 & 4)**. (Note that the treatment of Fahrenthold and Wargo was directed towards Cartesian tensors only.)

In this paper we begin with the SR energy equation for a one-particle system. Then we expose the consequences of assuming a constant mass on the formulation of the SR energy equation. We also provide a BG interpretation for the Lorentz force equation in its covariant form. Finally we investigate the meaning of Einstein's gravitational field equations from a bondgraphic perspective. (For a detailed treatment of the subjects presented in this paper see **[17]**).

2 Einstein's SR Energy Equation

In its contravariant form, the four-vector velocity is given by **(23 & 2)**:

1a $$V^\alpha \equiv \frac{dx^\alpha}{d\tau} = \frac{dx^\alpha}{dt}\frac{dt}{d\tau} = \gamma \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix},$$

where $\mathbf{v} \equiv \dfrac{d\mathbf{x}}{dt}$ and $\gamma \equiv (1-\mathbf{v}^2)^{-1/2}$. The length of this vector is given by

1b $$V^\alpha V_\alpha = -1 .$$

Note that the equations are written in natural units for which $c = 1$. (In equation [1a], τ is the so-called proper time.) Similarly, the contravariant four-vector momentum is given by:

2 $$P^\alpha = m_0 V^\alpha = m_0 \gamma \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix}$$

$$= \begin{bmatrix} m_0\gamma \\ m_0\gamma\mathbf{v} \end{bmatrix} = \begin{bmatrix} m \\ m\mathbf{v} \end{bmatrix},$$

where m ($\equiv m_0\,\gamma$) is known as the relativistic mass. For a one-particle system we can proceed to define a four-vector force given by

$$F^\alpha \equiv \frac{dP^\alpha}{d\tau} = m_0\frac{dV^\alpha}{d\tau} = m_0\frac{dV^\alpha}{dt}\frac{dt}{d\tau} \tag{3}$$

$$= m_0\gamma\begin{bmatrix} \dot{\gamma} \\ \dfrac{d(\gamma\mathbf{v})}{dt} \end{bmatrix} = \begin{bmatrix} \gamma\dot{\gamma}m_0 \\ \gamma f_1 \\ \gamma f_2 \\ \gamma f_3 \end{bmatrix} = \begin{bmatrix} F^0 \\ F^1 \\ F^2 \\ F^3 \end{bmatrix}$$

where $\mathbf{f} \equiv \dfrac{d(\gamma m_0\mathbf{v})}{dt}$.

Now differentiating (1b) with respect to τ, (and using direct notation) we get

$$\mathbf{V}.\frac{d\mathbf{V}}{d\tau} = (m_0)^{-1}\mathbf{V}.\mathbf{F} = 0. \tag{4}$$

Thus the four-velocity and four-force are orthogonal. Substituting for $\mathbf{V}$ and $\mathbf{F}$ in (4) we get

$$\gamma^2(\mathbf{v}.\mathbf{f} - \dot{\gamma}m_0) = \gamma^2(\mathbf{v}.\mathbf{f} - \dot{m}) = 0 \tag{5a}$$

$$\therefore \mathbf{v}.\mathbf{f} = \dot{m} \tag{5b}$$

(Note that we have divided by γ^2 .) And since, by definition, the left hand side of (5b) is the rate of work, we can obtain the kinetic energy, T as

$$T = \int \dot{m}\,dt = m + \text{constant}\ .$$

For v = 0, $T = 0$ and we get

$$T = m - m_0 = E - m_0\ , \tag{6}$$

where

$$E = \gamma m_0\ , \tag{7}$$

is the Einstein energy equation for the system, and E is the total energy associated with it **(23)**. Please not that this result can be modeled by bondgraphs with velocity dependent inertance.

3 THE POWER POSTULATE IN BONDGRAPHS

In this section we consider what is known in bondgraphs as the power postulate **(7, 11 & 20)**. This postulate gives the power for any system as follows:

$$\Pi = \frac{dE}{d\tau} = e_\alpha f^\alpha \tag{8}$$

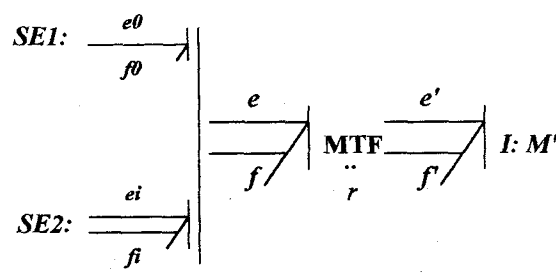
where e_α is the effort tensor (e.g. the four-vector force for a one-particle system in classical mechanics) and f^α is the flow tensor (e.g. the four-vector velocity of the one-particle system.) Note that since the right-hand side of equation (8) is a tensor of valence zero, the left-hand side must also be a tensor of valence zero. (The new symbol Π, is reserved for tensorial power.) Applied to our previous example, assuming non-relativistic flow (velocity), we can use Newton's second law to obtain

$$\frac{dE}{dt} = m_0\dot{\mathbf{v}}.\mathbf{v}\ . \tag{9}$$

Thus the energy is given by

$$E = T = \frac{1}{2}m_0\mathbf{v}.\mathbf{v} = \frac{1}{2}m_0\mathbf{v}^2. \tag{10}$$

In order to extend this formalism to SRT we need first to observe the following discrepancies between SR mechanics and classical mechanics. In the SR case we model the relativistic effects through a modulation of the mass of the particle; thus defining the relativistic mass m. In the classical case such effects would rather be obtained by attributing the modulation to the flow (velocity) and maintaining the parameter character of the mass. That is to say we can model the relativistic effects in the one-particle system as shown in Fig. 1,



(1)

Fig. 1: BG Representation for a Relativistic Particle.

where $M' = m_0 I_4$,

(note that I_4 is a 4×4 identity matrix) and

$$r = \gamma,$$
$$e = e_\alpha = F_\alpha = \gamma\dot{P}_\alpha,$$
$$e' = e'_\alpha = e/\gamma = \gamma^{-1}F_\alpha = \dot{P}_\alpha,$$
$$f' = f'^\alpha = V^\alpha,$$
$$f = f^\alpha = f'/\gamma = \gamma^{-1}V^\alpha.$$

Substituting (1a) and (3) in (8) gives the tensorial power as

$$\Pi = e'_\alpha f'^\alpha = F_\alpha \gamma^{-1} V^\alpha = \gamma^{-1}\eta_{\alpha\beta}F^\beta V^\alpha = -\gamma\dot{\gamma} m_0 + \gamma\mathbf{v}.\mathbf{f} = 0 \tag{11}$$

(compare with equation [5b]) which can be re-written as

$$e'_i f'^i = \gamma^{-1}V^1F^1 + \gamma^{-1}V^2F^2 + \gamma^{-1}V^3F^3 = \frac{dE}{dt} = m_0\gamma\dot{\gamma} \tag{12}$$

(Note that the left-hand side of equation [12] is not a tensor with respect to orthogonal [Lorentz] transformations. Thus the energy is not a tensor either. Fortunately, the power postulate still holds. One can also consider equation [12] as a tensor equation applicable only to rectangular, stationary axes in spacetime.) and since

$$\dot{\gamma} = (-1/2)(1-\mathrm{v}^2)^{-3/2}(-2\mathbf{v}).\dot{\mathbf{v}} = \gamma^3\mathbf{v}.\dot{\mathbf{v}} \tag{13}$$

we have

$$\frac{dE}{dt} = m_0\gamma^4\mathbf{v}.\dot{\mathbf{v}} \tag{14}$$

4 THE SRQ ENERGY EQUATION

Integrating (14) or (12), we get

$$E = (1/2)\gamma^2 m_0 + E_0 \tag{15}$$

(compare with [7]) where E_0 is the constant of integration. For any system with $m_0 = 0$, the total energy E must be equal to zero (unless $|\mathbf{v}| = 1$). Thus the constant of integration must be zero for zero mass. Still, in order to align this model with the Einsteinian one, we can use the rest energy formula as follows. For zero velocity, equation (15) becomes

$$E_{rest} = (1/2)m_0 + E_0 \; . \tag{16}$$

Now, using Einstein's formula $E_{rest} = m_0$, we can take the constant of integration to be equal to

$$E_0 = (1/2)m_0 \; .$$

The total energy can now be written as

$$E = (1/2)\gamma^2 m_0 + (1/2)m_0 = m_0 + (1/2)m_0\gamma^2\mathrm{v}^2 \tag{17}$$

(In the more familiar un-normalized units, equation [17] becomes

$$E = (1/2)m_0\gamma^2c^2 + (1/2)m_0c^2 = m_0c^2 + (1/2)m_0\gamma^2\mathrm{v}^2 = E_{rest} + T \tag{18}$$

where the first term is the rest energy and the second is the relativistic kinetic coenergy, which equals the relativistic kinetic energy.) We will call (18) the Special Relativistic Quadratic (SRQ) energy equation to distinguish it from (7).

Note that under this formalism, the equation for kinetic energy (and coenergy **[13]**, since both become numerically equal under this formalism), in both classic and relativistic mechanics, can be written as

$$T = (1/2)m_0(\mathit{flow})^2 \tag{19}$$

where the flow is given by $(\gamma\mathrm{v})$ in the relativistic case and by (v) in the classical one. (When $\mathrm{v} \ll c$, the factor modulating the flow can be set equal to one.) The concept of flow can be interpreted as the effect of spacetime on matter. Since the general theory of relativity forsakes the fixity of spacetime and shows that it is curved due to the existence of matter **(15)**, one can regard the flow as the effect of spacetime on matter (an established fact in general relativity is that spacetime acts on matter, telling it how to move and matter re-acts back on spacetime telling it how to curve). Thus the simple relation of change of position (in spacetime) to the change of time (i.e. velocity), is lost when matter (particles) travels with speeds commensurate to the speed of light.

Defining α as $T/(m_ec^2)$, where m_e is the rest mass of the electron, and β as (v/c); Einstein's energy equation then gives the following formula:

$$\beta^2 = 1-(1+\alpha)^{-2}. \tag{20}$$

On the other hand, the SRQ formula gives

$$\beta^2 = 1-(1+2\alpha)^{-1}. \tag{21}$$

It is easy to show that (20) and (21) are equal to the first order, which is actually the order used in classical mechanics. To support (21) we need to compare experimental results with analytical results from both equations. Using the results obtained by W. Bertozzi **(3)** shown in Table 1 (the values for $(\mathrm{v}/c)_{obs}$), we can see (Fig. 1) that (20) and (21) are quite close to the experimental results.

Other experimental data that can be shown to give similar results are abundant. For example the experiment by Perry and Chaffee at Harvard University, although used as an evidence on the contribution of the kinetic energy to the inertia – such dependence is also supported by the new model – can also be used to explicate the close relation between the experimental data and the proposed model **(12)**. Other promising results are those of the Guye, Ratnowsky and Lavanchy **(18)** experiment. For more on the experimental work on SRT, the reader is referred to the resource letter on special relativity **(27)**.

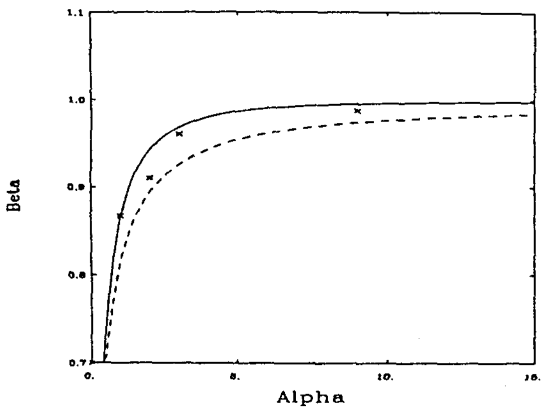


Fig. 1 : Beta vs. Alpha. The solid line corresponds to equation (20) where the dashed one corresponds to equation (21). The diagonal crosses correspond to the experimental data from table 1

T (MeV)	α	$(v/c)_{obs}$	$(v/c)^2_{obs}$	$(v/c)^2_{Ein}$	$(v/c)^2_{SRQ}$
0.5	1	0.867	0.752	0.750	0.667
1.0	2	0.910	0.828	0.889	0.800
1.5	3	0.960	0.922	0.937	0.857
4.5	9	0.987	0.974	0.990	0.947
15	30	1.0	1.0	0.999	0.984

Table 1: The experimental results of the Bertozzi experiment. The table also provides the analytical data from equations (20) and (21).

At this juncture we need to remind ourselves of the objective of this investigation. According to Karl Popper "... science is not in the business of validating models at all, but rather should be trying to falsify them." **(10)**. The work by Parker **(24)** seems to falsify the new model in favor of the Einsteinian one. He still declares his data as circular at the end of his paper. The best approach is probably the direct measurement of the time-of-flight of electrons within the range $\alpha < 5$ and comparing the experimental results to the analytical ones. Such an experiment is yet to be conducted. (Note that although the experiment by Bertozzi is structured as proposed, it lacks focusing on the suggested range where the difference between the competing models is maximum.)

Finally it is worth mentioning that our effort of building a model that competes with the Einsteinian one for special relativity is not an unprecedented one. Abraham **(1)** proposed a rigid model for the electron that produced a competing relation between mass and velocity. Actually most of the early experiments where carried out to support the validity of either the Abraham or Einstein mass-velocity equation (or rather to falsify one of them.) The prevailing of the Einstein equation over Abraham's should not deter inquiring minds from building new models (equations) that might bring about a more unified scientific structure.

5 A BG Interpretation of The Covariant Lorentz Force Equation

This section is designed in a very simple fashion that introduces an interpretation of the Lorentz force equation based on BG's. This interpretation leads to a generalized statement concerning the power composed of tensorial efforts and flows.

Let us define the tensor $\Phi^{\alpha\beta}$ as follows:

22 $$\Phi^{\alpha\beta} \equiv \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{bmatrix},$$

which is usually referred to as the (electromagnetic) field-strength tensor **(19)**. Using this tensor, we can provide the invariant form of the Lorentz force equation (giving the electromagnetic force on a charged particle) as follows:

23 $$F_\alpha = e\Phi_{\alpha\gamma}U^\gamma.$$

Recalling the power postulate, we can formulate the power for a particle traveling in the electromagnetic field as follows:

24 $$\Pi = e_\alpha f^\alpha = F_\alpha U^\alpha = \eta_{\alpha\beta} e \eta_{\delta\gamma} \Phi^{\beta\delta} U^\gamma U^\alpha.$$

(Where e is the charge of the particle and $\eta_{\beta\gamma}$ is the so-called Minkowski metric tensor used here to raise the indices of $\Phi_{\alpha\gamma}$ **[32]**.) Note that the (tensorial relativistic) power is identically equal to zero. The reader might have already noticed the resemblance between this result and the one for the one-particle system treated earlier. Actually one can even postulate that this is a general result for any tensorial (relativistic) power built from four-vector efforts and flows, since by definition the length of a four-vector is unchanged under rotation of axes (that is by a Lorentz transformation) **(28)**. Of course a rigorous study of other physical domains is first necessary before one can claim such a generalization.

The reader can easily see the gyrative character (as a 1-MP **GY**) of the tensor $e\Phi_{\alpha\gamma}$ – from the antisymmetric nature of the matrix in equation (22) – and since gyrators are non-energic elements, the result in (24) becomes natural (see Fig. 2). This of course is explained by noting that the function of the magnetic field is to influence the direction of the particle rather than influencing its transverse motion (this can be related to the Larmor theory which explains the role of the magnetic field as generating angular velocity.)

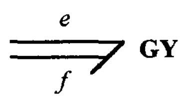


Fig. 2: The 1-MP GY

6 Einstein's Gravitational Field Equations

In this section we provide a BG interpretation for Einstein's gravitational field equations (EGFE's for short). For an (arbitrarily) strong gravitational field, EGFE's are written as follows **(32)**:

25 $$G_{\mu\nu} = -8\pi G T_{\mu\nu}.$$

Where $G_{\mu\nu}$ is the so-called Einstein tensor; a symmetric, conserved tensor (properties necessary to salvage the equality in [25]), and G is the Newtonian gravitational constant. The tensor on the right hand side $T_{\mu\nu}$ is the so-called energy-momentum tensor, and is also a symmetric tensor. Note that since the Einstein tensor is conserved, the right hand side must also be conserved.

It can be seen that EGFE's represent a 2-MP **TF** that transforms the power between two domains; the gravitational domain (associated with the Einstein tensor) and the material domain (associated with the energy-momentum tensor) (see Fig. 3). (Although both domains are not classically defined, the coining of such domains can be accomplished following the definition of physical domains provided by Breedveld [7].) Note that the effort tensors for the above mentioned domains are obtained by taking the divergence of the tensor associated with the domain. This is done by taking the so-called covariant derivative with respect to one of the indices of the tensor. (Here it does not matter which index we choose since the tensors are symmetric.)

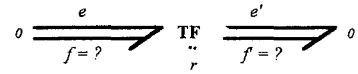


Fig. 3: The MP TF for EGFE's

Unfortunately, due to the fact that both, the Einstein tensor and the energy-momentum tensor are conserved, the divergence of both equals zero. This means that the effort equation of the BG MP **TF** collapses. Another problem one faces on this front is the choice of the flow tensors for the gravitational and material domains. From the authors research on this subject, the divergence of the so-called metric tensor seemed to be the natural choice for the flow tensor for both domains. Of course such a choice would not fit into the flow constitutive equation of MP **TF**'s. This problem is related to the absence of a known way by which a definition of the local energy density (of gravitational fields) can be found. Wald **(31)** suggests that the reason for this is related to the absence of a natural way by which the metric tensor can be decomposed into "background" and "dynamical" parts.

7 CONCLUSIONS

The SRQ equation is a model for the dynamics of a relativistic one-particle system, which unifies the formulae for kinetic energy (and coenergy) in both, classical and relativistic mechanics. In this work the authors do not present an approximation to the Einstein formula, but rather provide a model for SR mechanics (and not a theory).

A simple bondgraphic interpretation was given to the Lorentz force equation (in its covariant form). This interpretation shows explicitly the gyrative action of the electromagnetic field on the charged particle traveling through it.

Finally we interpreted the Einstein gravitational field equations as a transformation between two physical domains. Unfortunately due to intrinsic problems in the general theory of relativity, the flow variables for the domains involved were not identified. Another problem pertains to the conservation of the efforts of the domains. This conservation leads to the collapse of the effort equation of the identified transformer. We believe that a thorough study on other theories of gravitation might provide a better insight via BG's.

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