Sublogarithmic Distributed Algorithms for Lovász Local Lemma, and the Complexity Hierarchy

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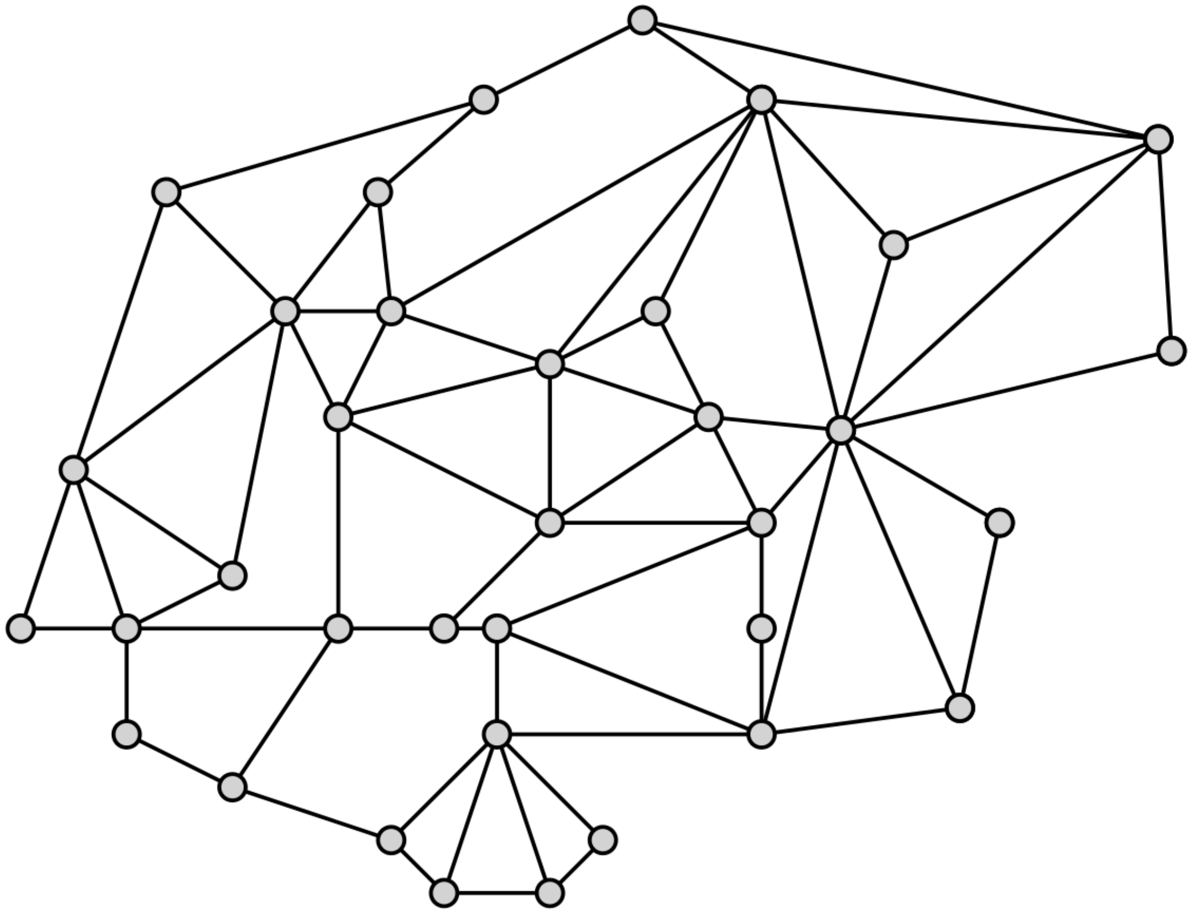
LOCAL Model *Linial* [FOCS'87]

- undirected graph $G = (V, E)$,
  n nodes, maximum degree Δ
- synchronous message-passing rounds
- unbounded message size
- unbounded computation
- **Round Complexity**:
  number of rounds to solve the problem



Chang, Pettie [FOCS'17]

Completeness of Lovász Local Lemma for Sublogarithmic Problems

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Any algorithm for an LCL problem on bounded-degree graphs with round complexity $\boldsymbol{o}(\log \mathrm{n})$ can be **automatically sped up** to run in $\boldsymbol{O}(\mathrm{T}_{LLL}(\boldsymbol{n}))$ rounds.

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$\boldsymbol{T_{LLL}(n)}$ = round complexity of Lovász Local Lemma
on n events/nodes
under $pd^c < 1$ for some constant c,
for $d = O(1)$

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Naor, Stockmeyer '95
Locally Checkable Labeling (LCL)
solution checkable in $O(1)$ rounds

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Lovász Local Lemma (LLL) *Erdős, Lovász '75*

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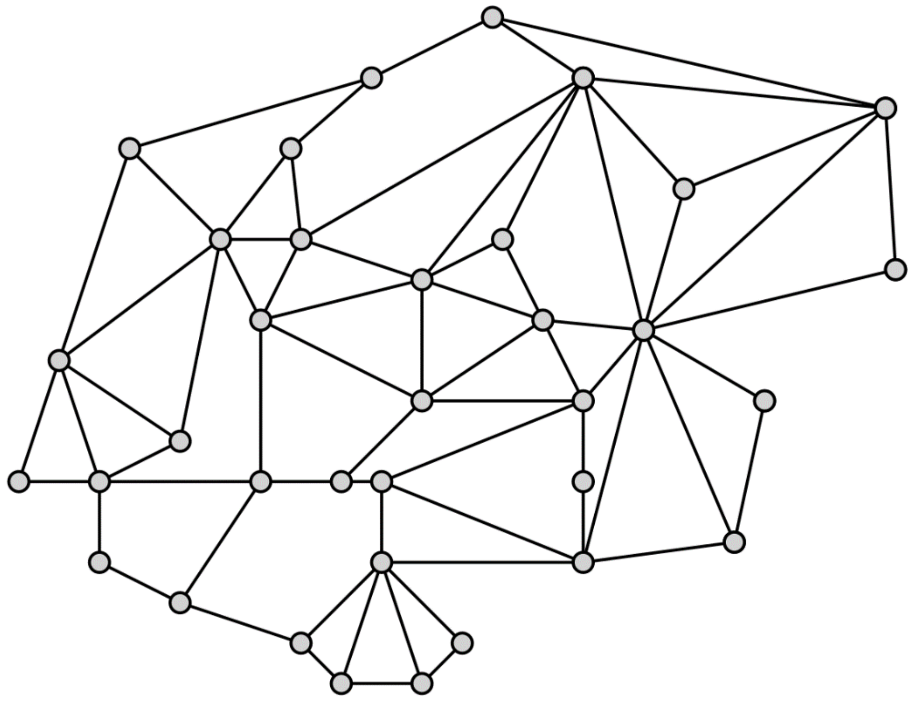
independent variables $\mathcal{V}$ (w.l.o.g. fair coins)



Not too likely bad events:

n bad events $\mathcal{X}$ with $vbl(A) \subseteq \mathcal{V}$ for all $A \in \mathcal{X}$

$\Pr[A] \leq p$ for all $A \in \mathcal{X}$

Not too many dependencies:

dependency graph $G = (\mathcal{X}, E)$

$E = \{(A, B) : vbl(A) \cap vbl(B) \neq \emptyset\}$

maximum degree d

If local union bound (with some slack) is satisfied, then all bad events can be avoided!

If $\mathrm{epd} \leq 1$, then $\Pr[\bigcap_{A \in \mathcal{X}} \bar{A}\,] > 0$.

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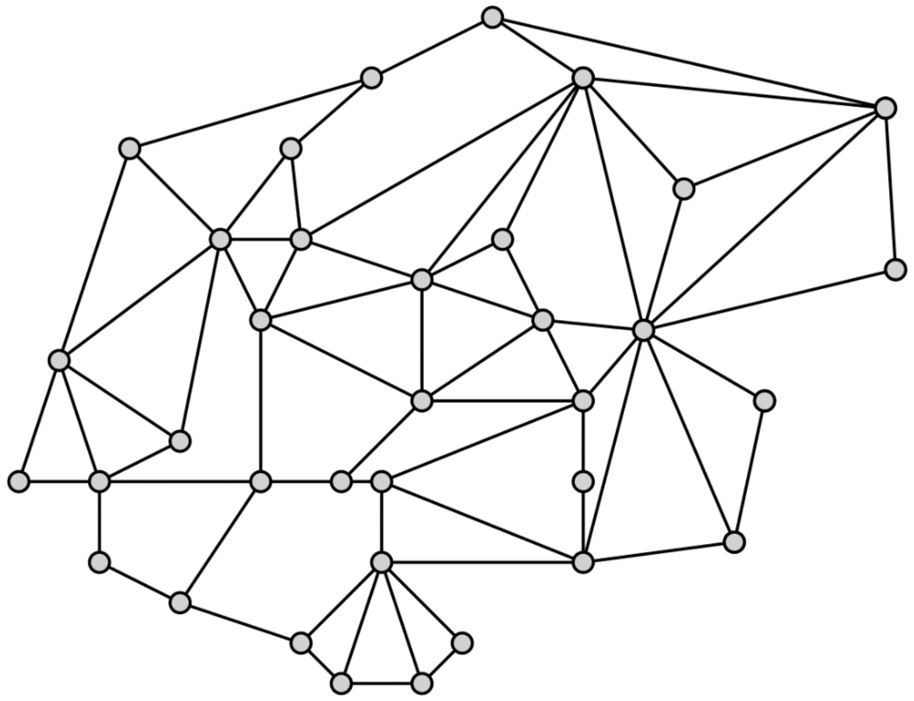

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standard criterion

LOCAL Complexity of the Lovász Local Lemma

$\boldsymbol{O}(\log^2 \text{n})$ for $\boldsymbol{epd \le 1}$ **(STANDARD)** *Moser, Tardos* [JACM'10]	$\boldsymbol{O}(\log \text{n})$ for $\boldsymbol{epd^2 < 1}$ **(POLYNOMIAL)** *Chung, Pettie, Su* [PODC'14]	$\boldsymbol{O}(\log \text{n}/\text{loglog n})$ for $\boldsymbol{p2^d < 1}$ **(EXPONENTIAL)** *Chung, Pettie, Su* [PODC'14]
	CONJECTURE: $\boldsymbol{T_{LLL}(n) = O(\log \log \text{n})}$ *Chang, Pettie* [FOCS'16]	$\boldsymbol{\Omega}(\log \log \text{n})$ for $\boldsymbol{p2^d < 1}$ **(EXPONENTIAL)** *Brandt et al.* [STOC'16]

$\boldsymbol{T_{LLL}(n)}$ = round complexity of Lovász Local Lemma
on n events/nodes
under $pd^c < 1$ for some constant c,
for $d = O(1)$

Our Results

$$\boldsymbol{T_{LLL}(n) = 2^{O\left(\sqrt{\log\log n}\right)}}$$

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$$T_{LLL}(n) = 2^{O(\sqrt{\log\log n})}$$

Automatic Speedup

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Our Results

$T_{LLL}(n) = 2^{O(\sqrt{\log\log n})}$

Automatic Speedup ⇒

Gap in Distributed Complexity Hierarchy

for LCLs on bounded-degree graphs

$o(\log n) \rightarrow 2^{O(\sqrt{\log\log n})}$

$o(\log\log n) \rightarrow O(\log^* n)$

$T_{LLL}(n)$ = round complexity of Lovász Local Lemma
on n events/nodes
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Other Applications

$2^{O(\sqrt{\log\log n})}$ - round Graph Coloring Algorithms

- f-defective $O\left(\frac{\Delta}{f}\right)$-coloring
- β-frugal $120\,\Delta^{1+\frac{1}{\beta}}$-coloring
- List-vertex-coloring

Previously best known $O(\log n)$ by *Chung, Pettie, Su* [PODC'14]

Algorithm for Lovász Local Lemma

BASE ALGORITHM

$\mathrm{O}(\mathrm{d}^2) + \lambda \cdot \log^{\frac{1}{\lambda}} n \cdot 2^{O\left(\sqrt{\log\log n}\right)}$ rounds

under $p(e \cdot d)^{4\lambda} < 1$

under polynomial criterion $\boldsymbol{\lambda = O(1)}$:

$\gg 2^{O\left(\sqrt{\log\log n}\right)}$ rounds

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BOOTSTRAPPING:
Automatic
Speedup

under polynomial criterion $\lambda = O(1)$:
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BOOTSTRAPPING:
Automatic Speedup

$\Rightarrow$

$$T_{LLL}(n) = 2^{O(\sqrt{loglogn})}$$

$2^{O(\sqrt{\log\log n})}$ - round algorithm
under $p(e \cdot d)^{32} < 1$ (POLYNOMIAL)
for $d = O(\log^{1/5} \log n)$

under polynomial criterion $\lambda = O(1)$:
$\gg$ **$2^{O(\sqrt{\log\log n})}$ rounds**

BASE ALGORITHM $O(\mathrm{d}^2) + \lambda \cdot \log^{\frac{1}{\lambda}} n \cdot 2^{O\left(\sqrt{\log \log n}\right)}$ rounds under $p(e \cdot d)^{4\lambda} < 1$

Shattering Technique, rooted in breakthrough LLL algorithm of Beck [RSA'91]

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I. PARTIAL SAMPLING

assign values to subset of variables
such that remainder graph
1) satisifes polynomial criterion
2) consists of "small" components

in $O(d^2 + \log^* n)$ under $p(e \cdot d)^{4\lambda} < 1$

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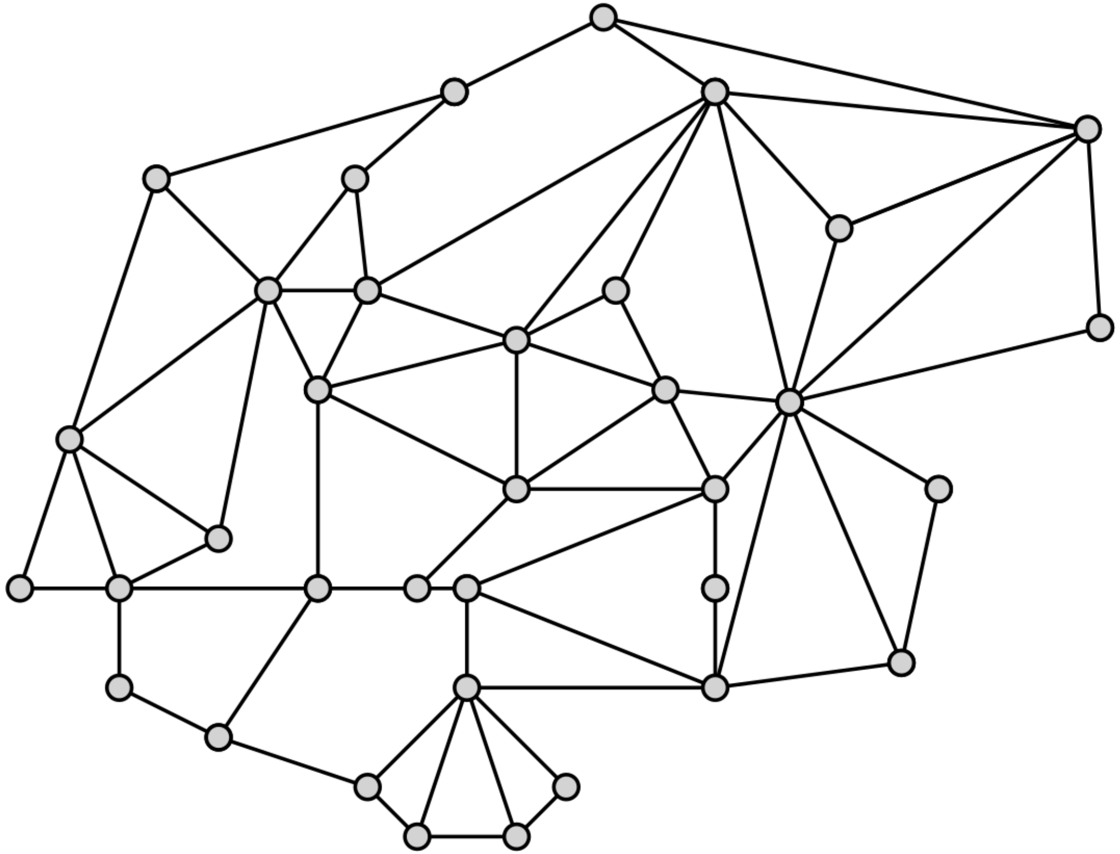

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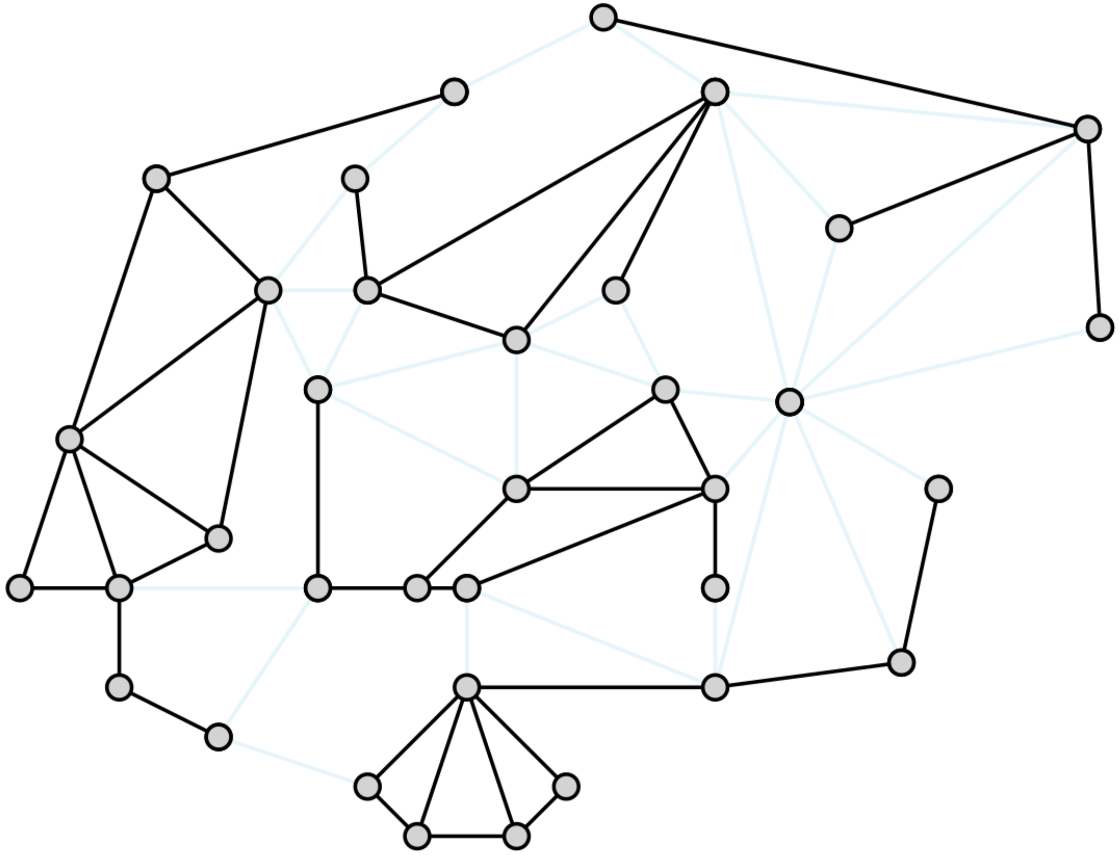

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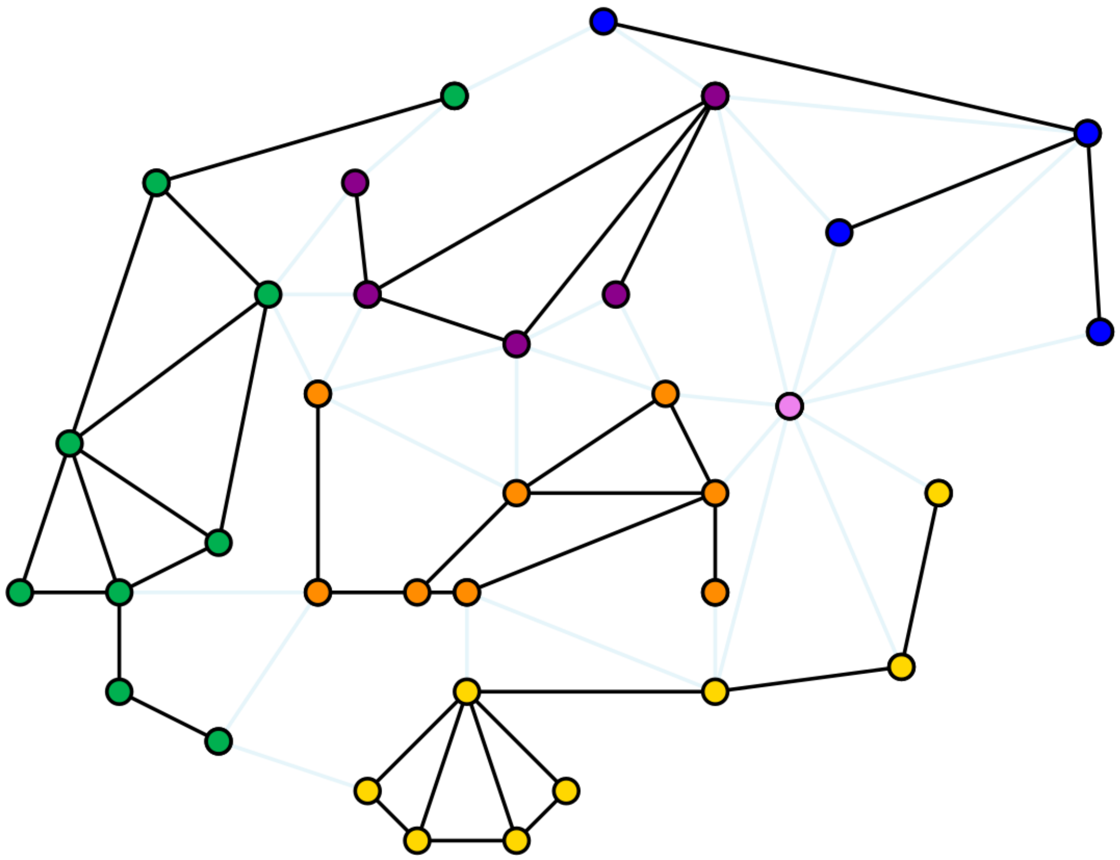

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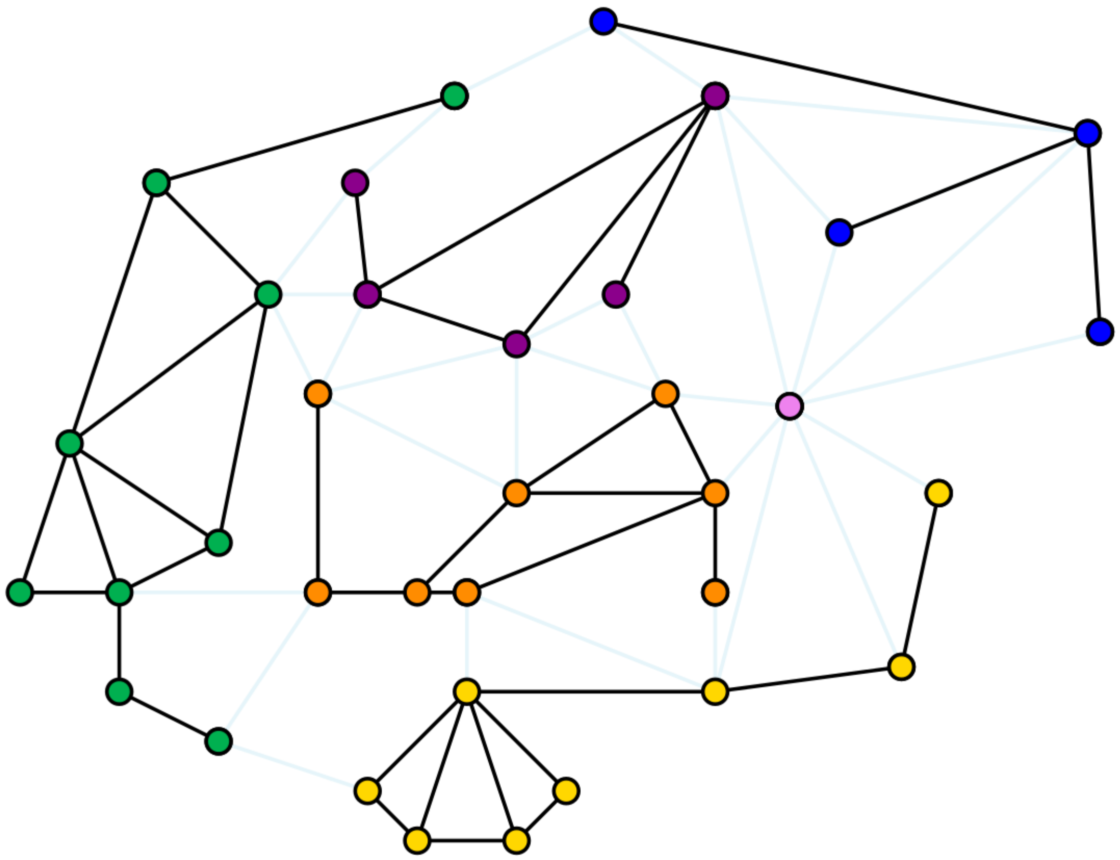

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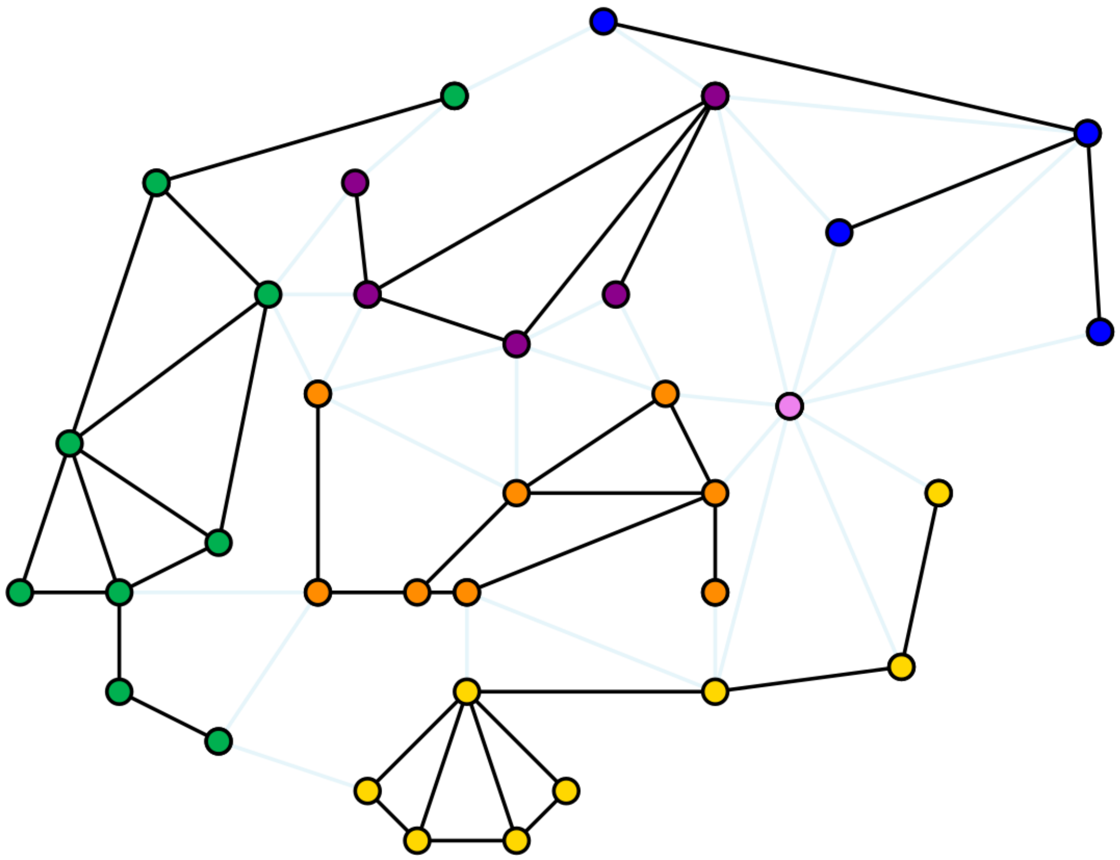

I. PARTIAL SAMPLING in $O(d^2 + \log^* n)$ under $p(e \cdot d)^{4\lambda} < 1$ s.t. remainder
1) satisifes polynomial LLL
2) has “small“ components

Inspired by Molloy and Reed’s
Sequential Partial Sampling [STOC’98]

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iteratively, for unblocked $\mathrm{v} \in \mathcal{V}$

flip coin for v

if $\boldsymbol{P[A \mid \text{flipped coins}] > \frac{\sqrt{p}}{2}}$

block $\mathrm{vbl(A)}$

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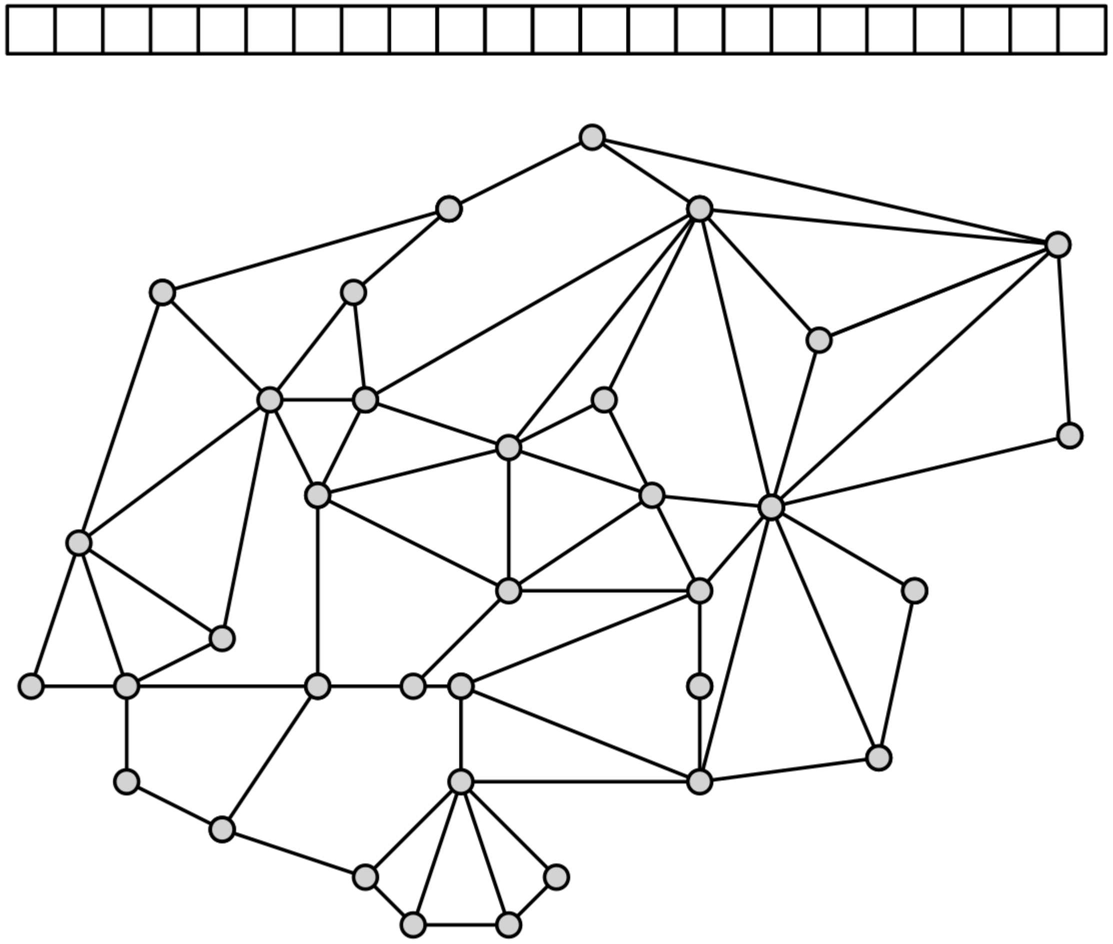

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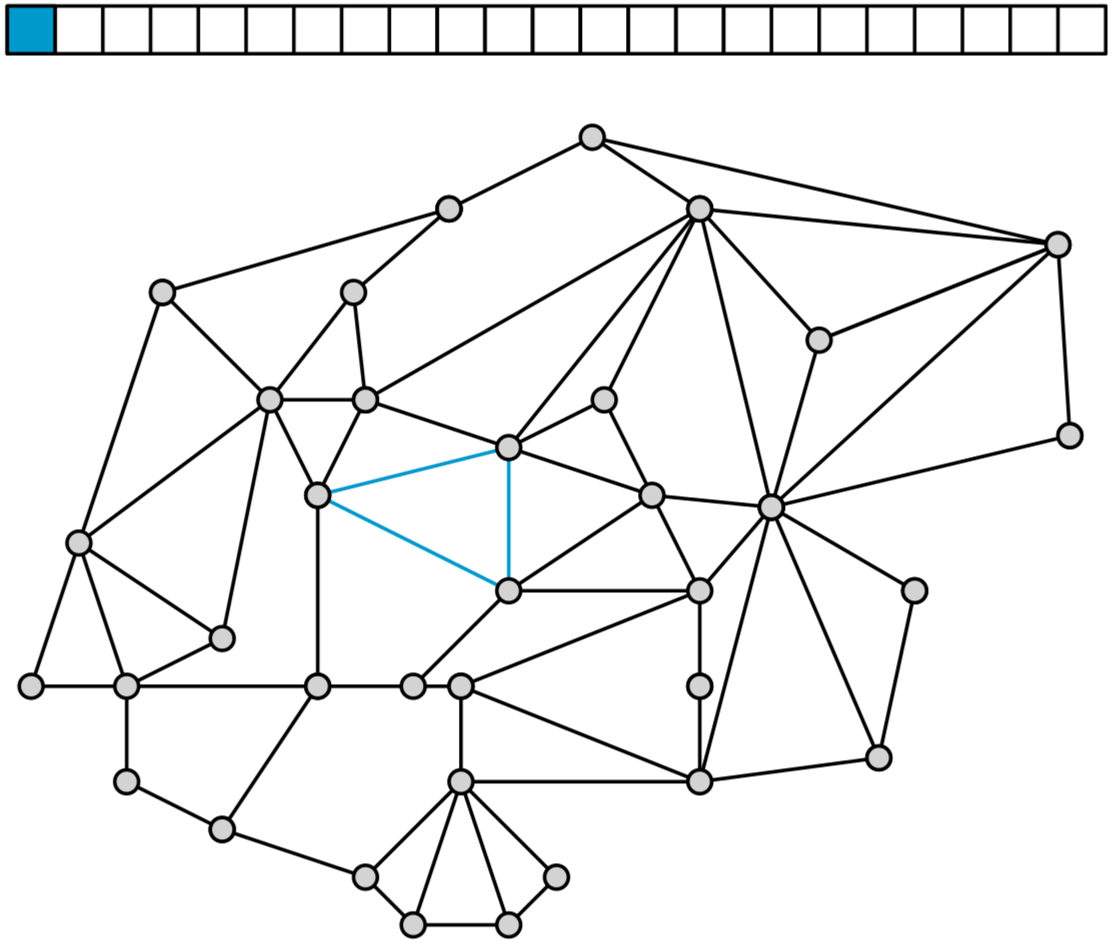

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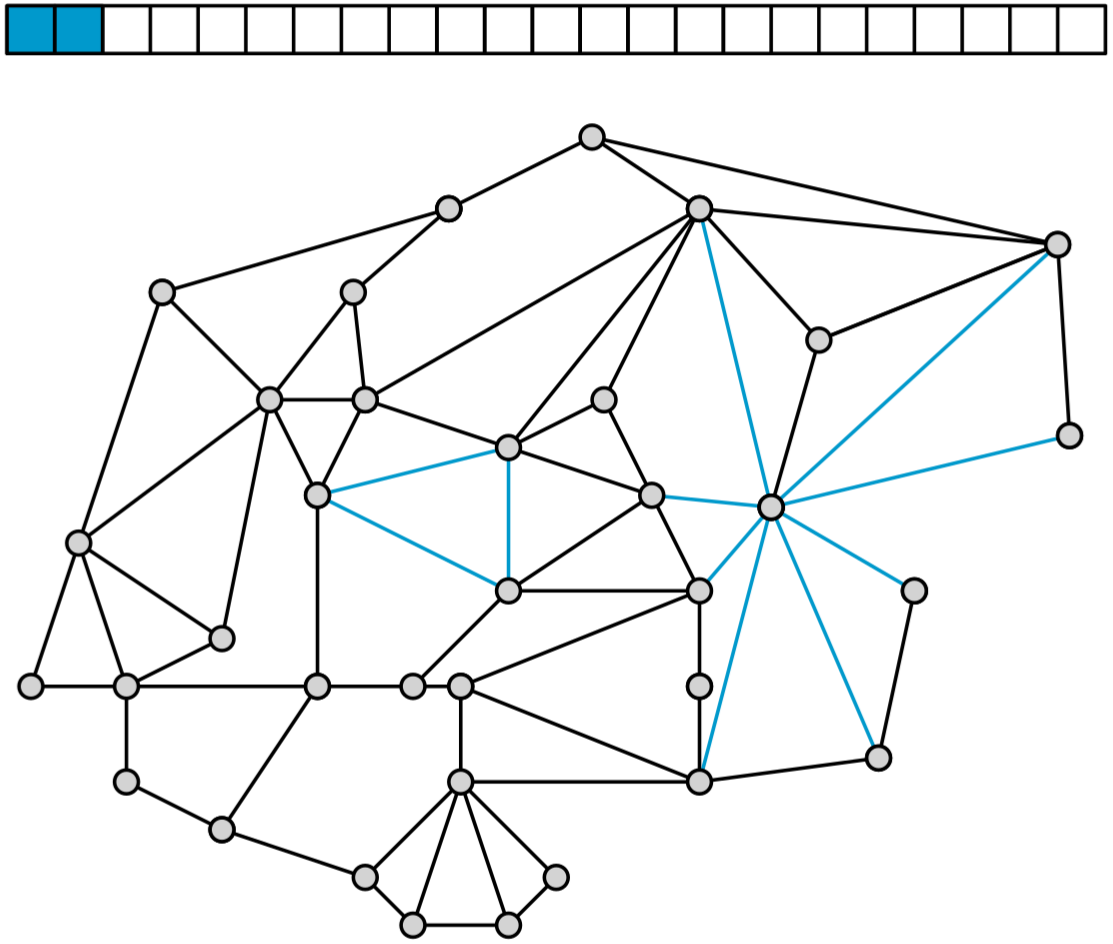

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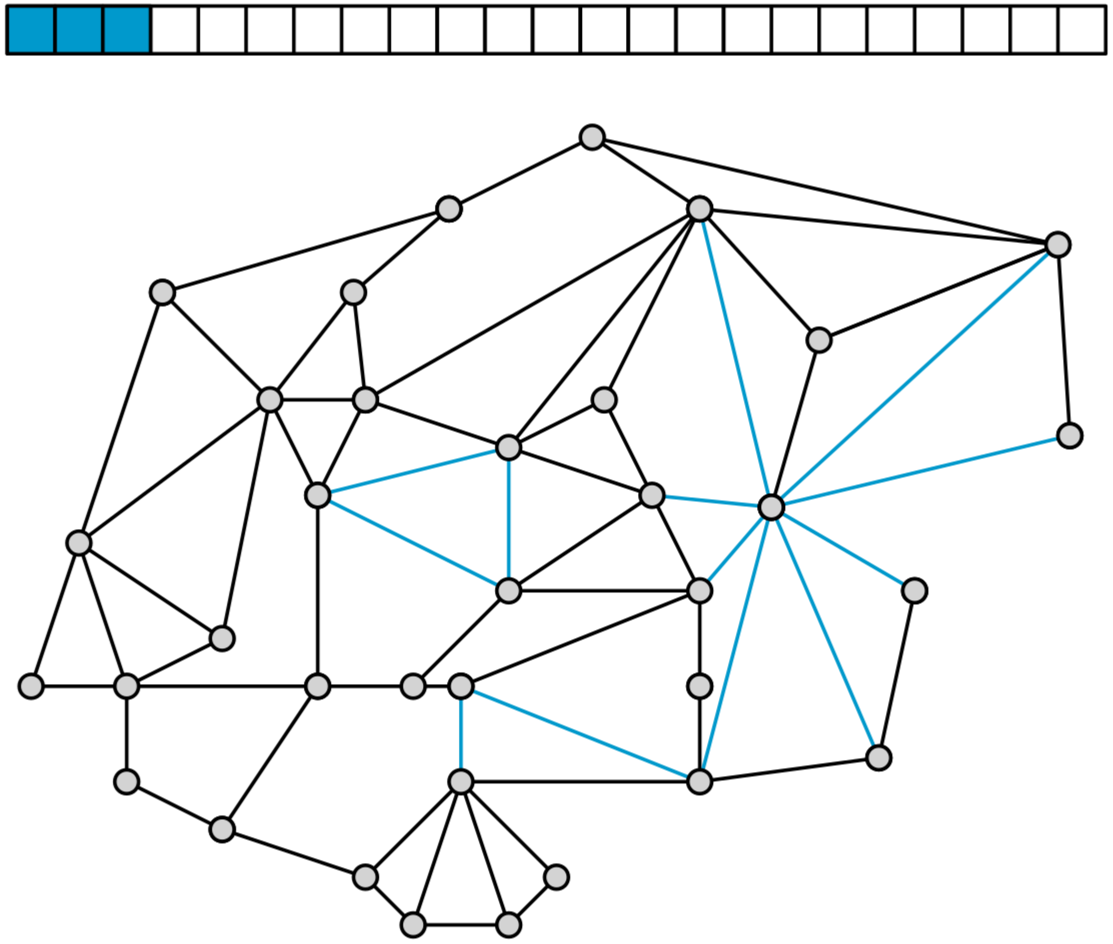

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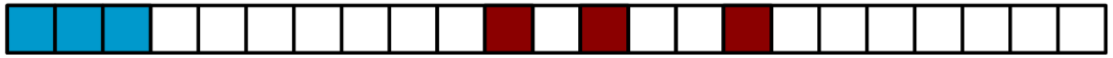

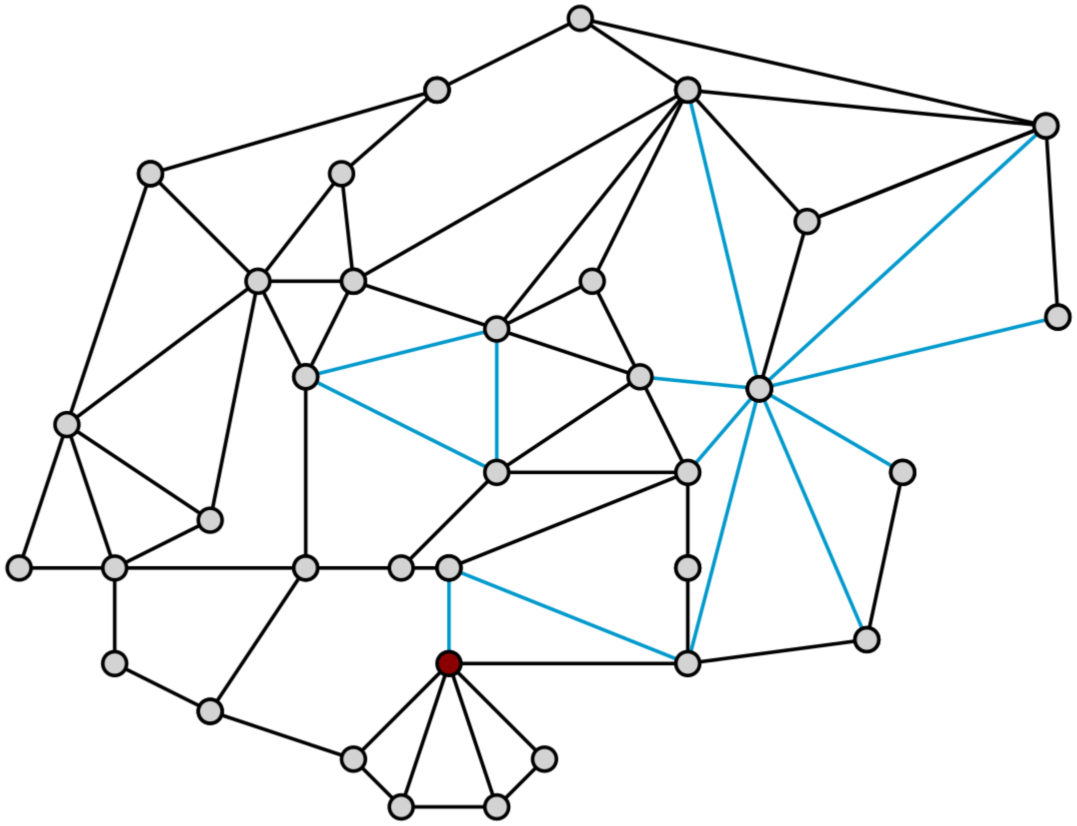

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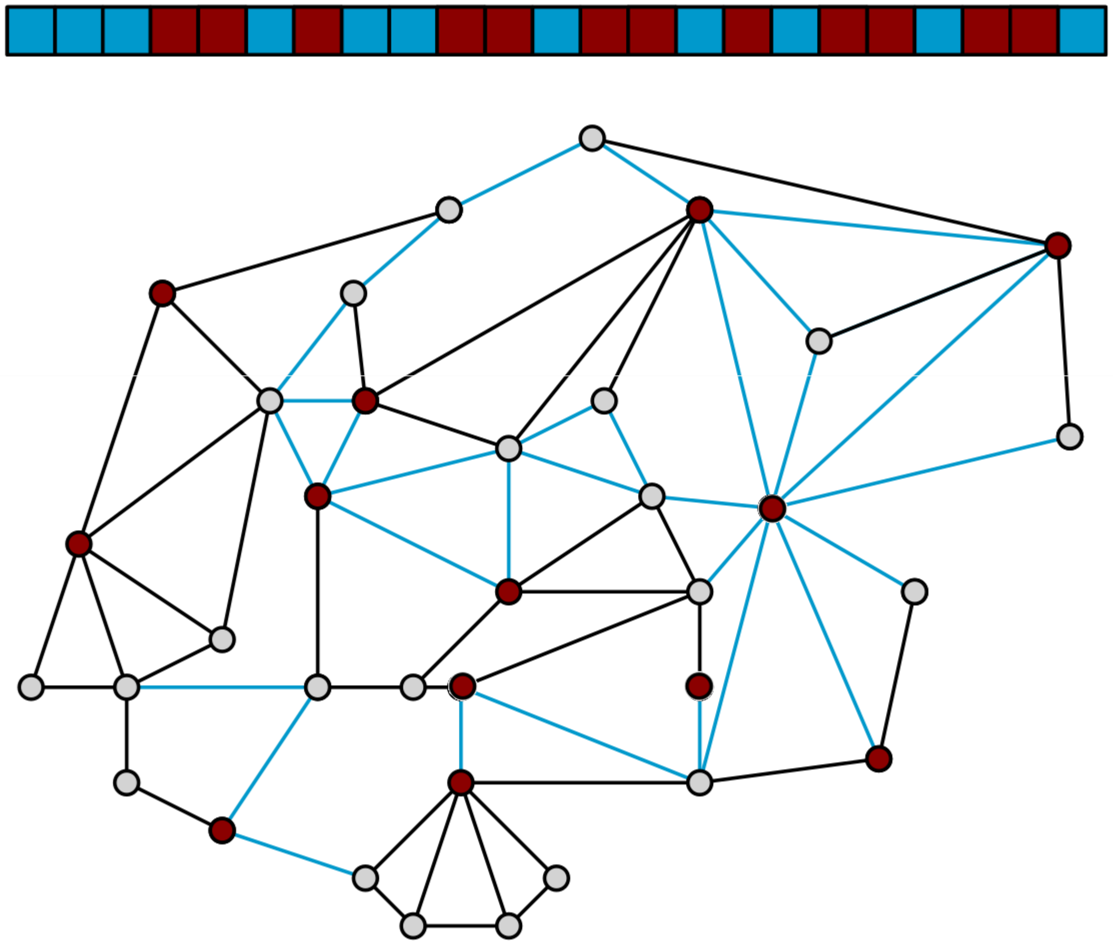

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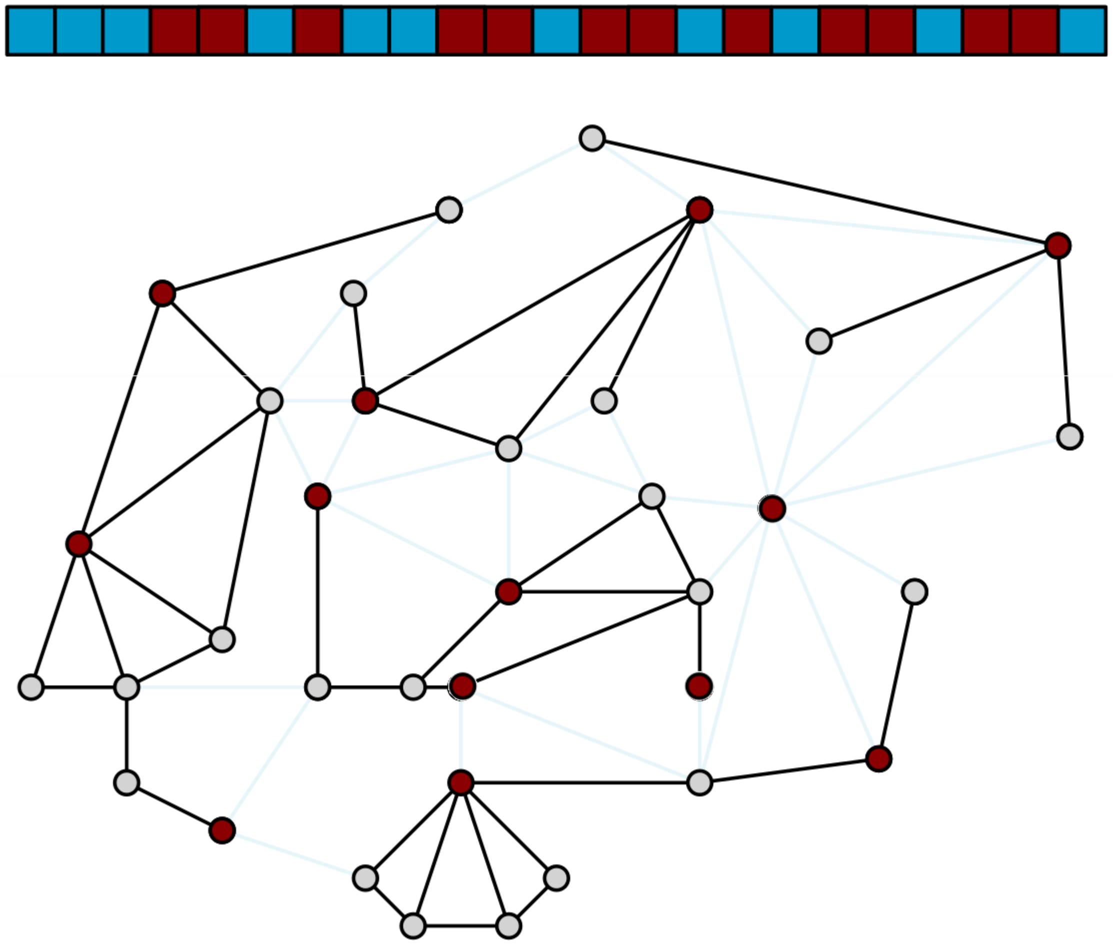

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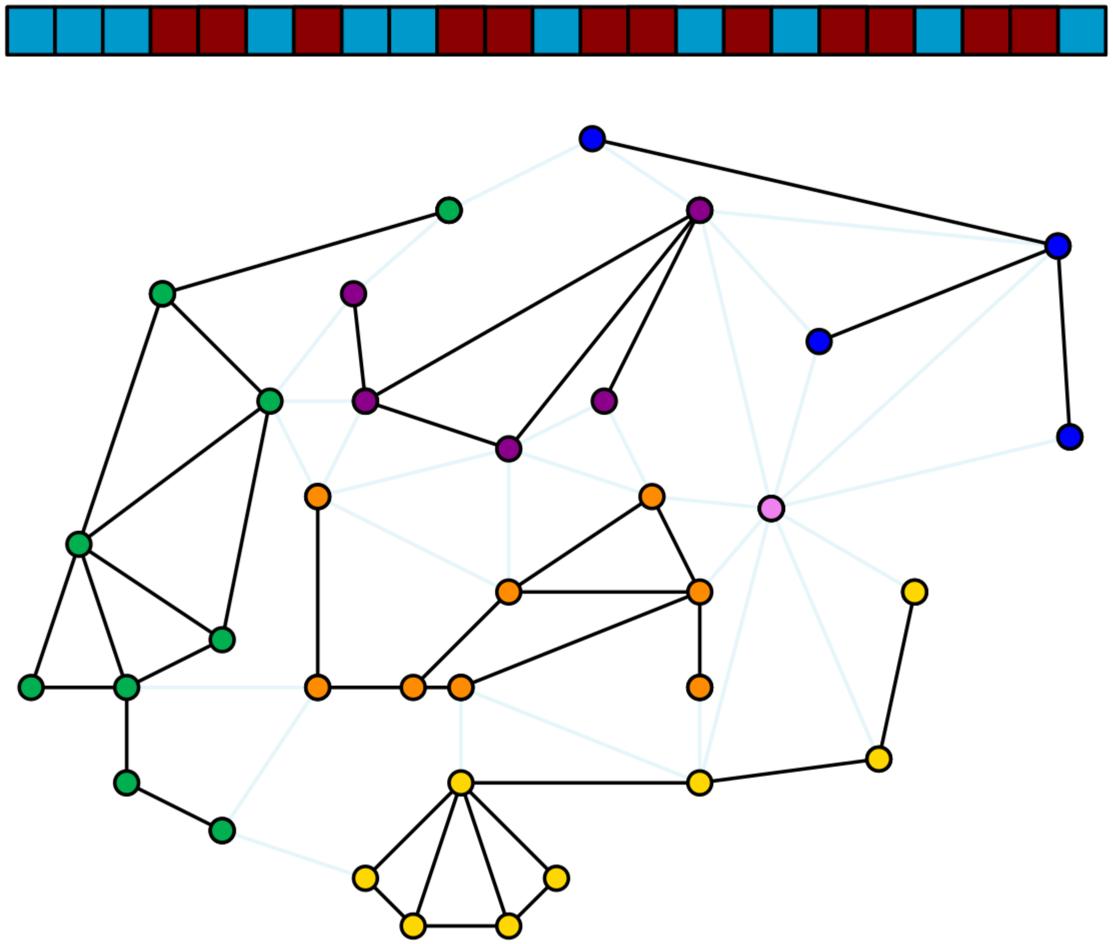

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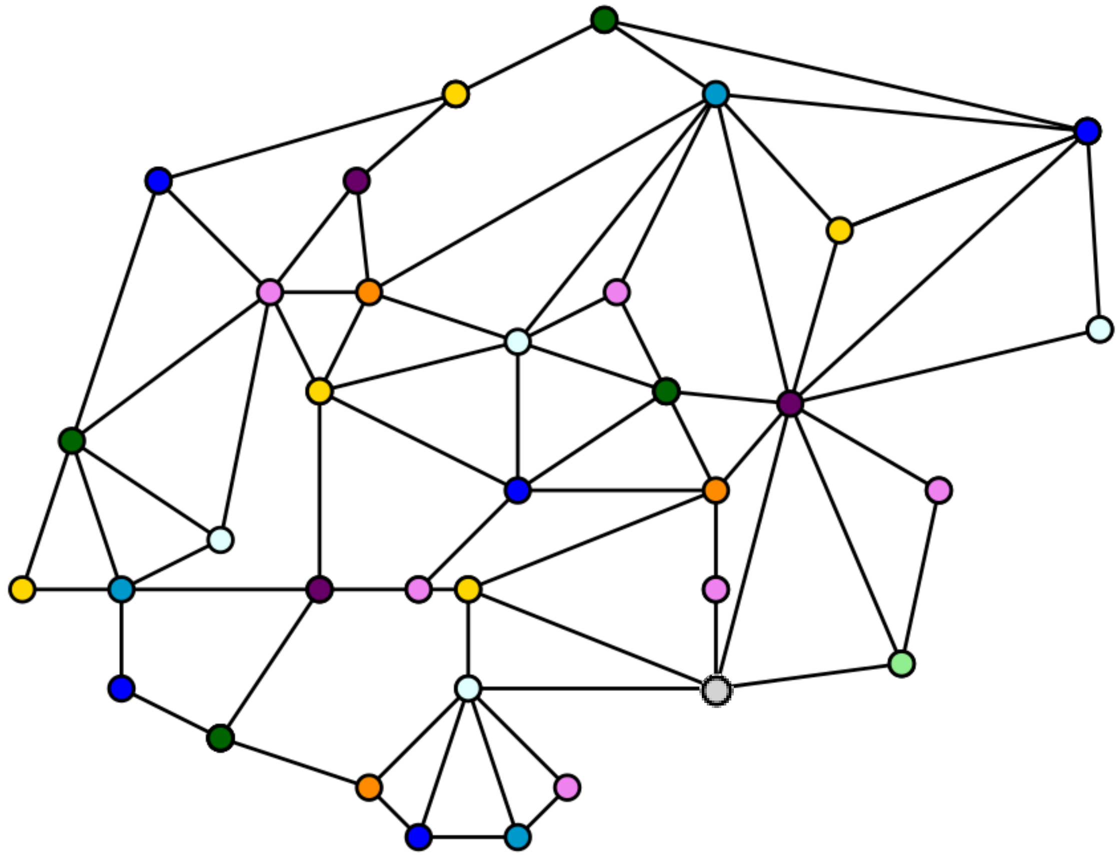


OBSERVATIONS:

- $P[A \mid \text{flipped coins}\,] \leq \sqrt{p}$

  satisfies polynomial LLL

- $P[A \text{ dangerous}\,] \leq \frac{p}{\frac{\sqrt{p}}{2}} = 2\sqrt{p}$
- $P[A \text{ remains}] = O(d)P[A \text{ dangerous}] \leq O\left(d\sqrt{p}\right) = \frac{1}{poly\ d}$
- A and B at distance > 2 remain independently

  by Shattering Lemma: small components
  (see, e.g., Barenboim, Elkin, Pettie, Schneider [FOCS'12])

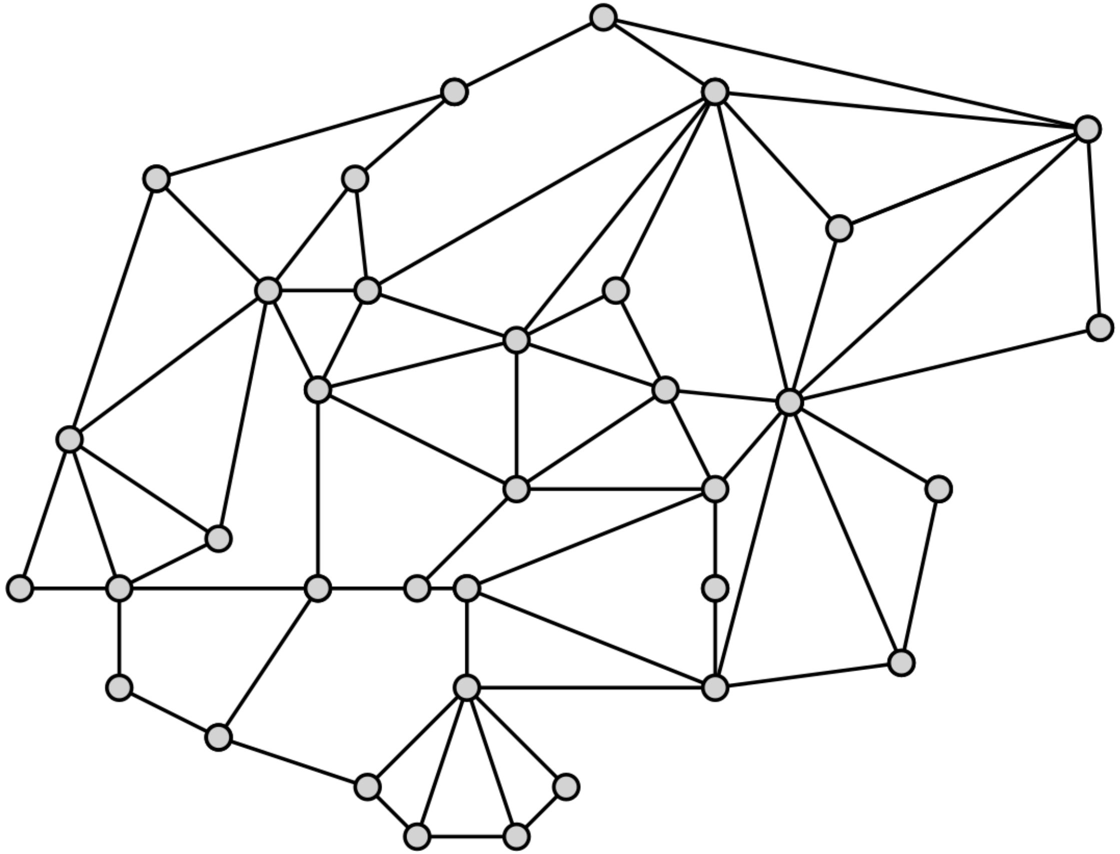
- can be parallelized using a $(\mathrm{d}^2 + 1)$ - coloring of G^2

  consistent with one sequential global order

BASE ALGORITHM

I. PARTIAL SAMPLING

II. DETERMINISTIC LLL ALGORITHM



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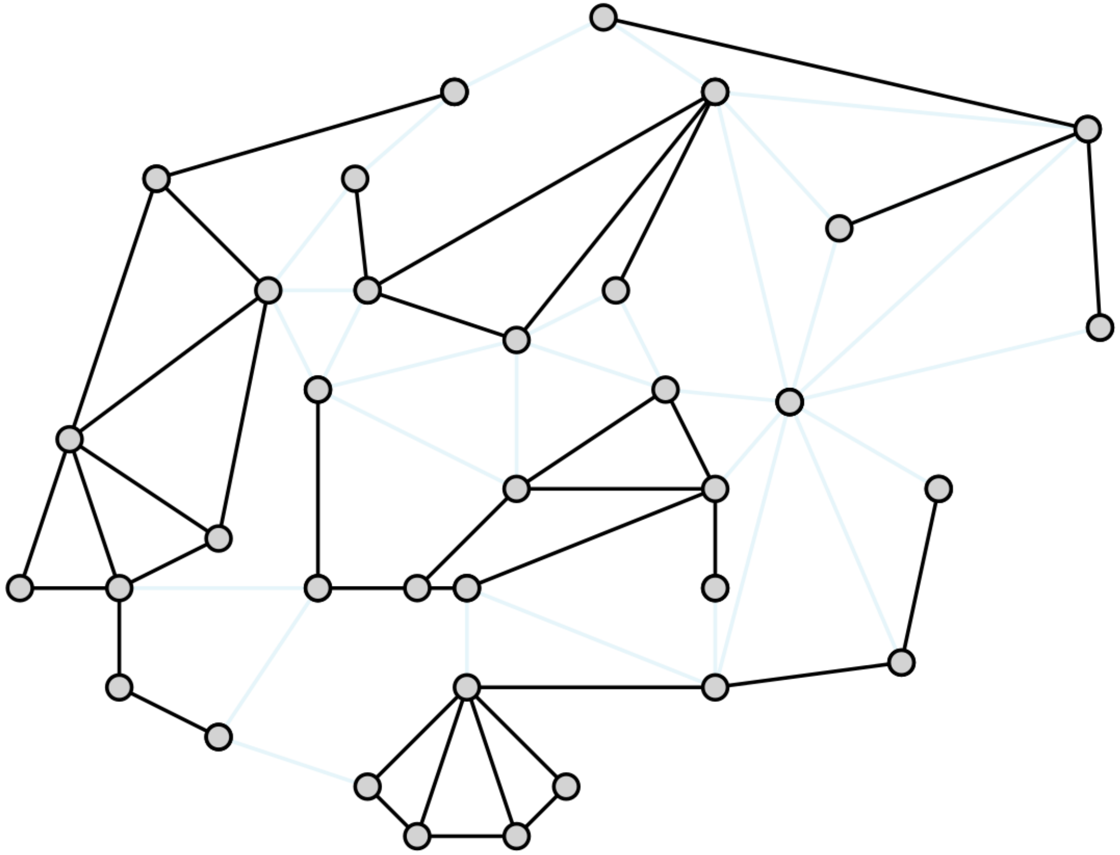

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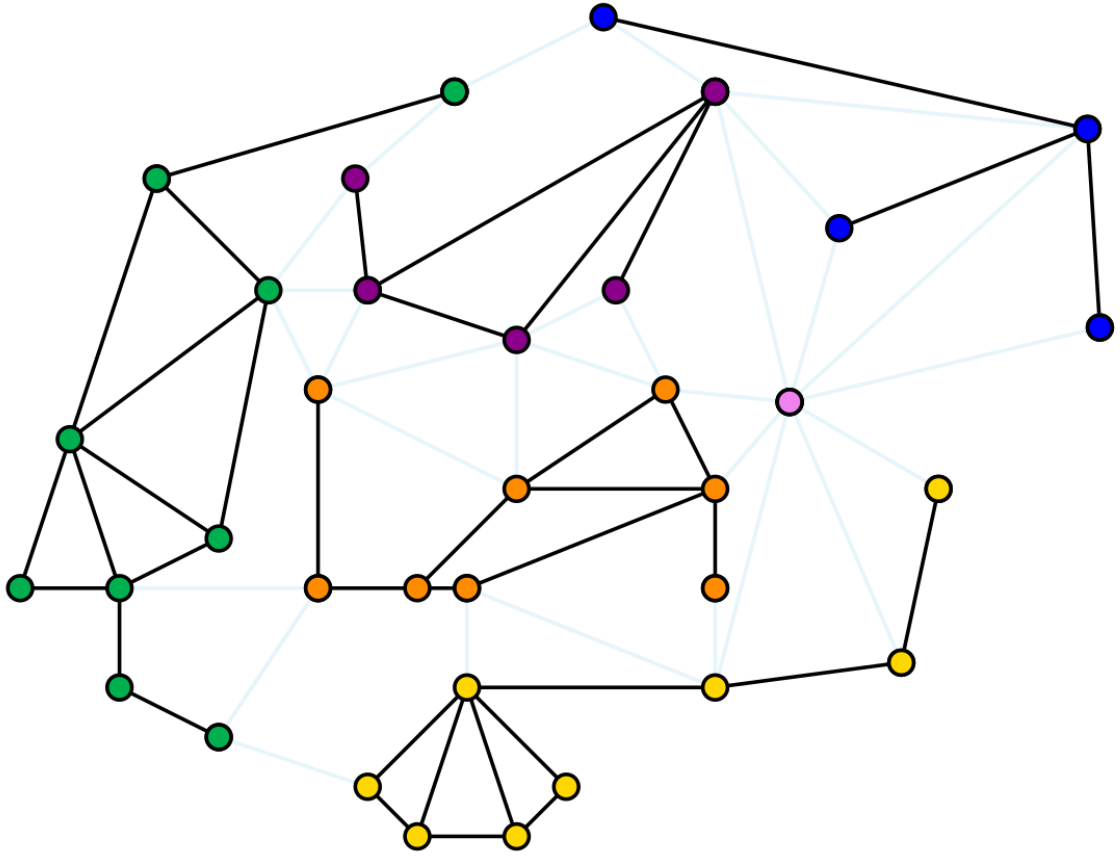

II. DETERMINISTIC LLL ALGORITHM

$\lambda \cdot n^{\frac{1}{\lambda}} \cdot 2^{O(\sqrt{\log n})}$ rounds under $p(e \cdot d)^{4\lambda} < 1$

1. $\left(\boldsymbol{\lambda}, \widetilde{\mathbf{O}}\left(\mathbf{n}^{1/\lambda}\right)\right)$**-network decomposition of** $\mathbf{G^2}$ in $\lambda \cdot \mathrm{n}^{\frac{1}{\lambda}} \cdot 2^{\mathrm{O}(\sqrt{\log \mathrm{n}})}$

combining approaches by *Awerbuch and Peleg* [FOCS'90*], Panconesi and Srinivasan* [STOC'92], and *Awerbuch, Luby, Goldberg, Plotkin* [FOCS'89]

2. Iterative Assignment

Inductively, $P[\, A \mid \mathcal{V}_{\leq i} \,] \leq p(e \cdot d)^i$

Eventually, $P[\, A \mid \mathcal{V}_{\leq \lambda} \,] = P[\, A \mid \mathcal{V} \,] \leq p(e \cdot d)^{\lambda} < 1$

II. DETERMINISTIC LLL ALGORITHM

$\lambda \cdot n^{\frac{1}{\lambda}} \cdot 2^{O(\sqrt{\log n})}$ rounds under $p(e \cdot d)^{4\lambda} < 1$

1. **$\left(\lambda, \tilde{O}\left(n^{1/\lambda}\right)\right)$-network decomposition of G^2** in $\lambda \cdot n^{\frac{1}{\lambda}} \cdot 2^{O(\sqrt{\log n})}$
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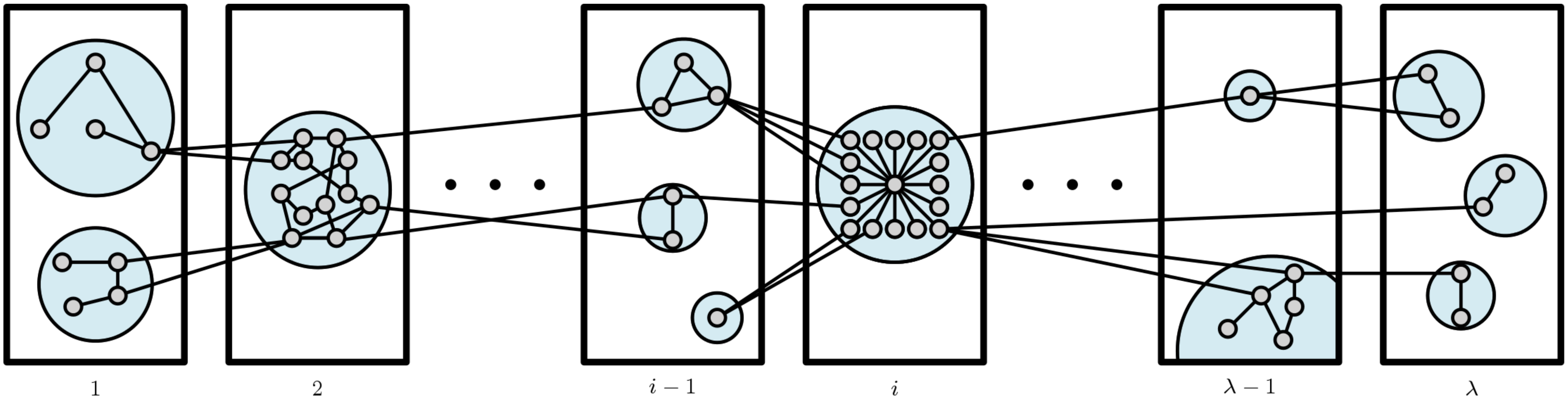

II. DETERMINISTIC LLL ALGORITHM

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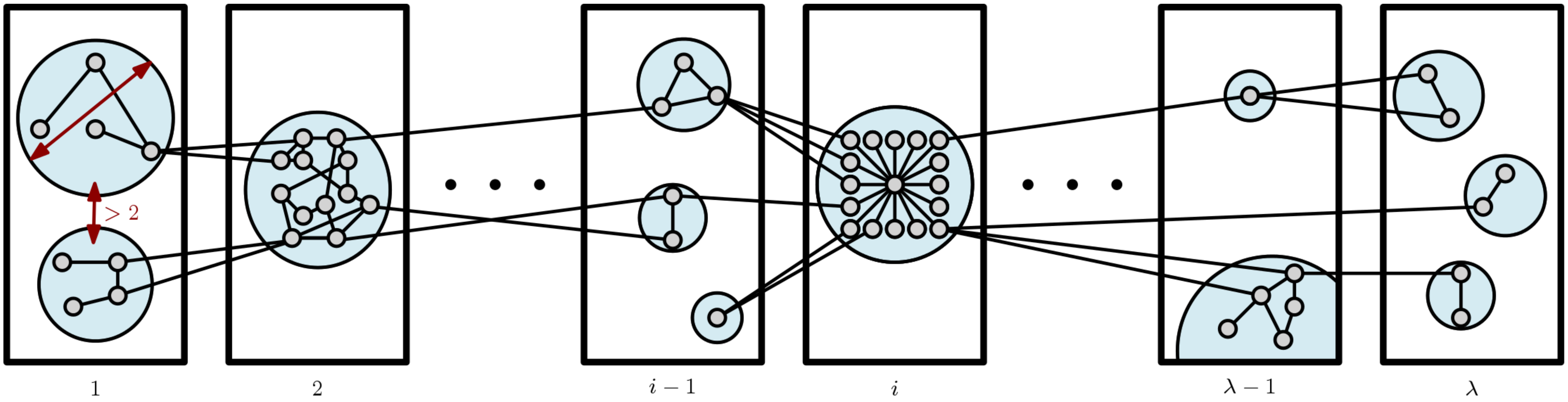

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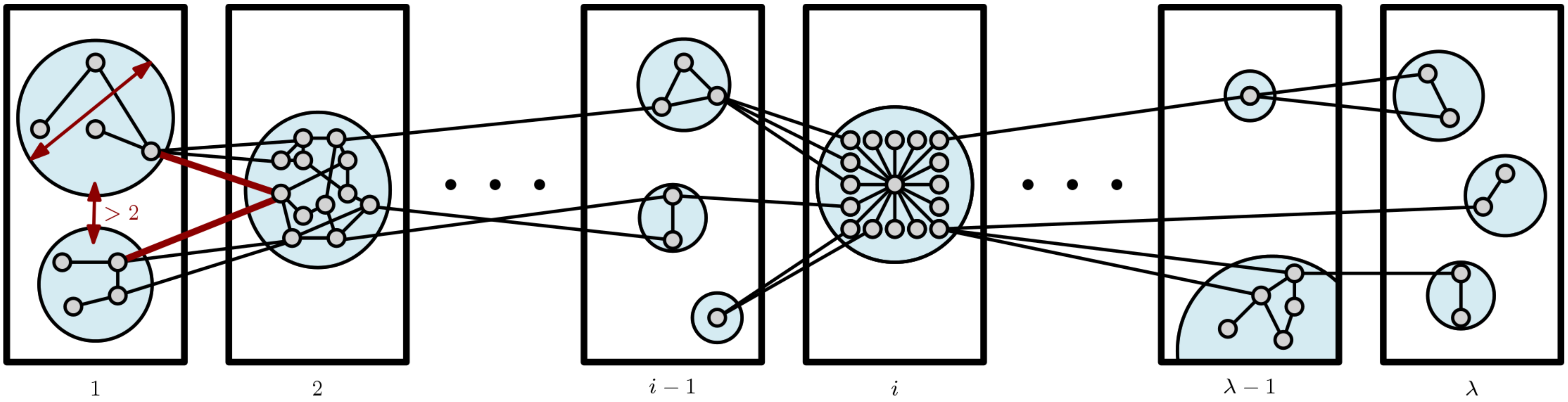

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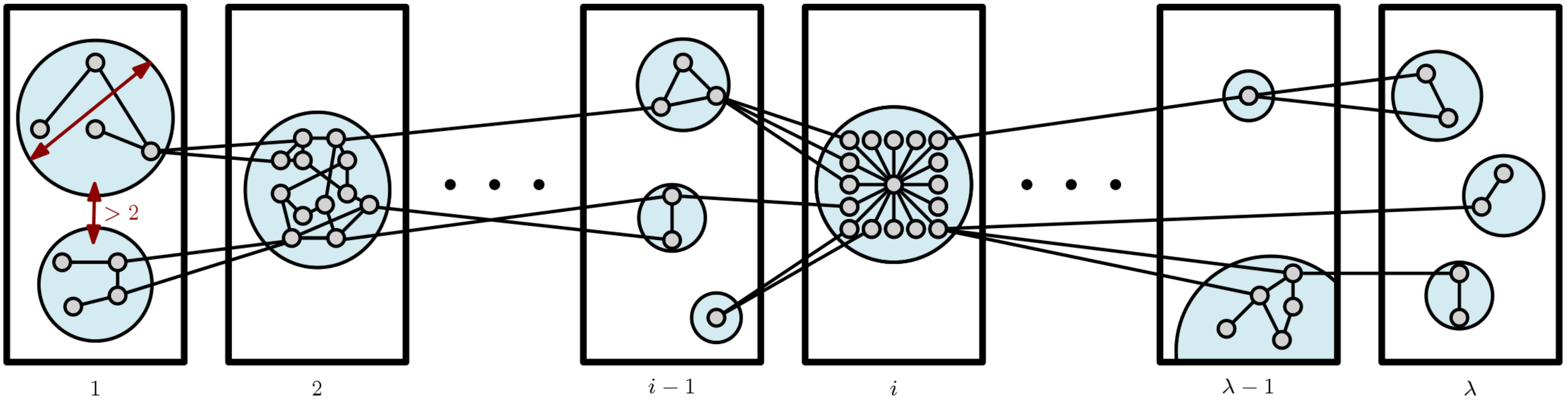

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LLL for step i

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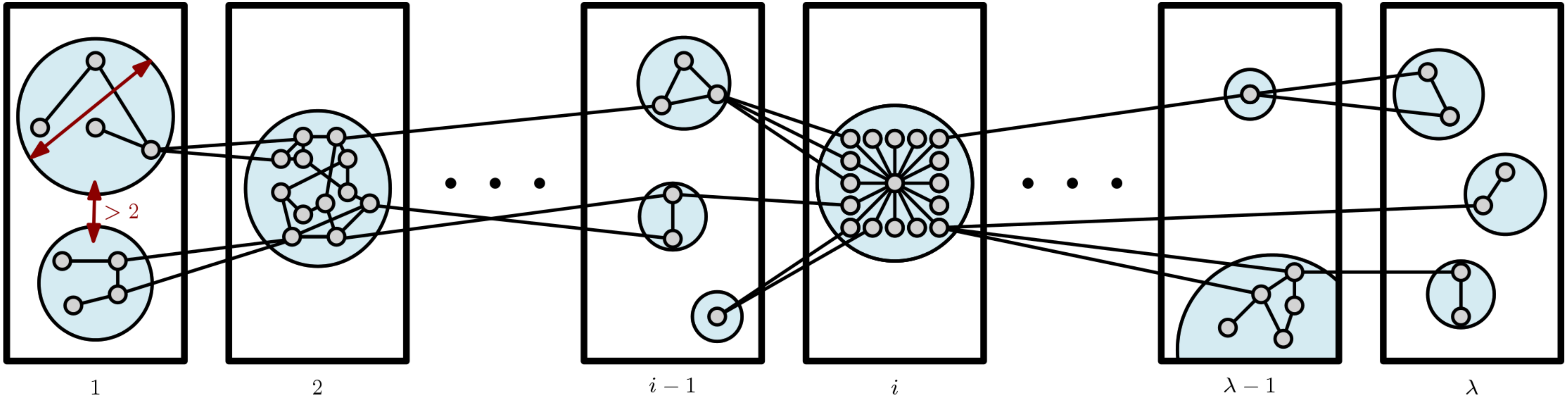
probability: $\le \frac{1}{e \cdot d}$



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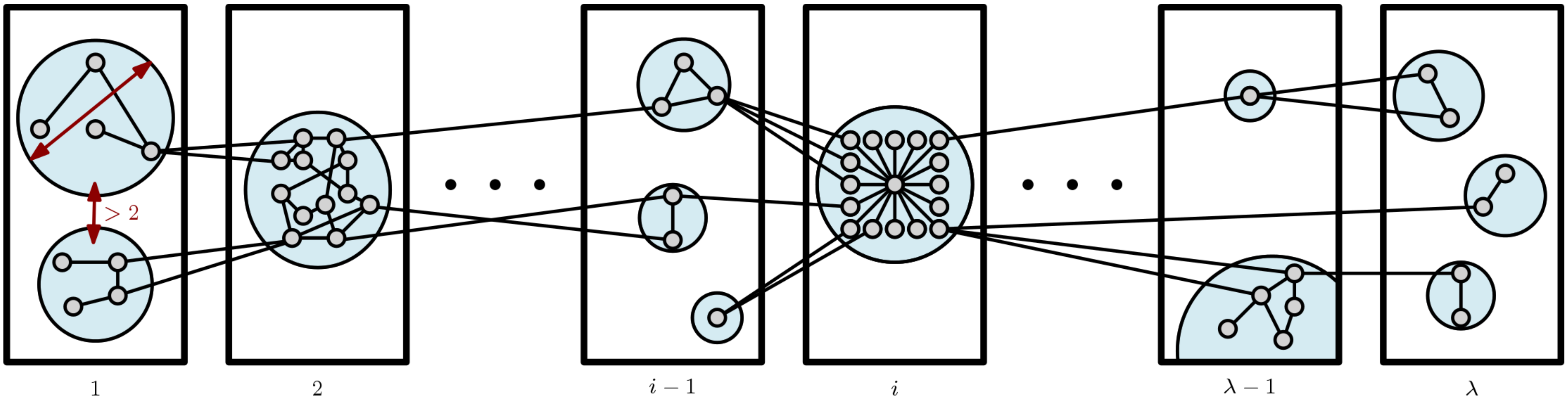

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LLL for step i satisfies standard LLL criterion

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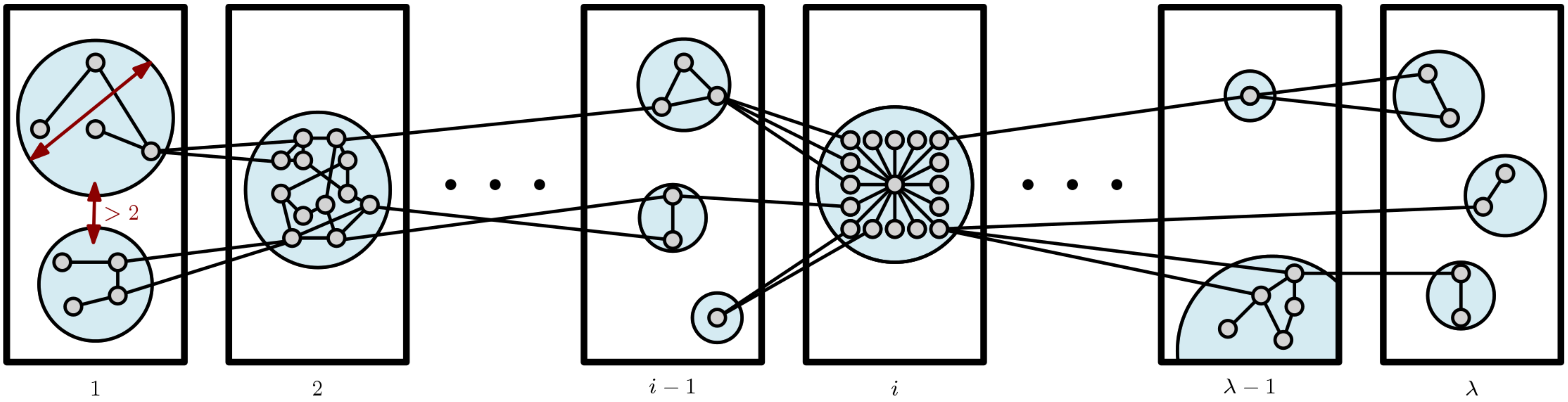
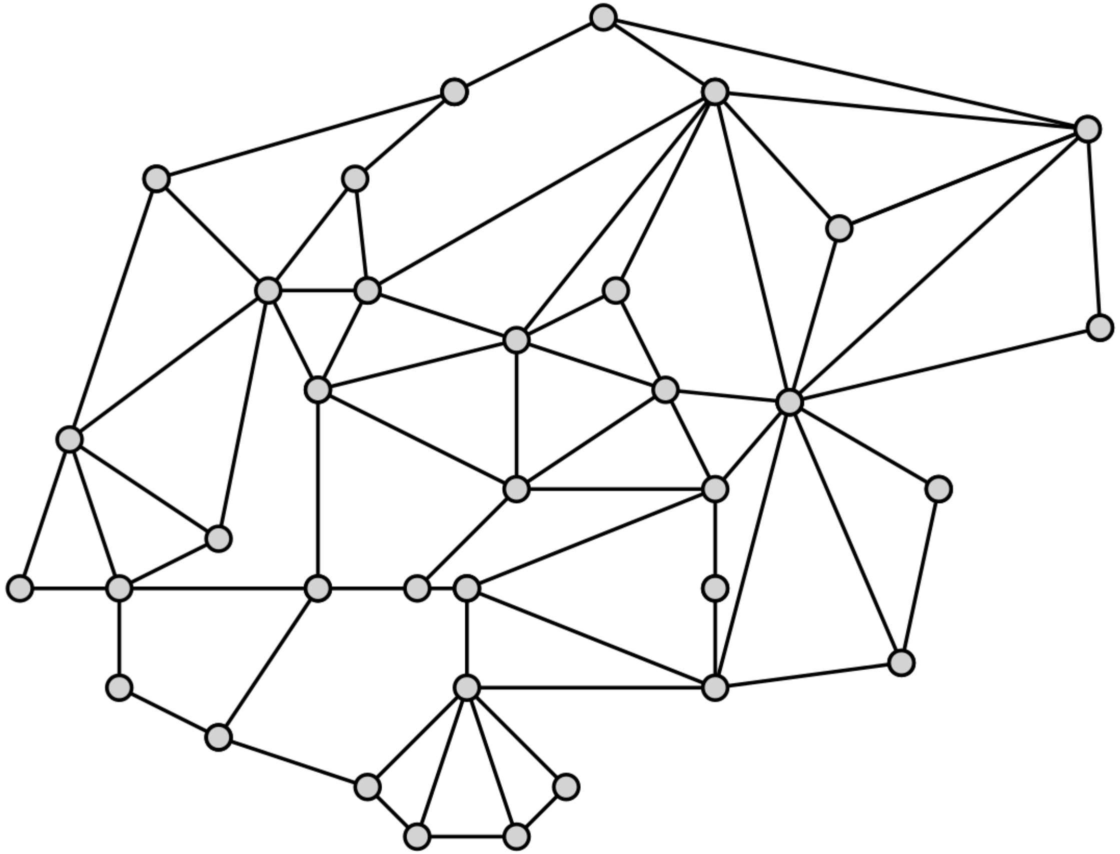


BASE ALGORITHM

I. PARTIAL SAMPLING

II. DETERMINISTIC LLL ALGORITHM



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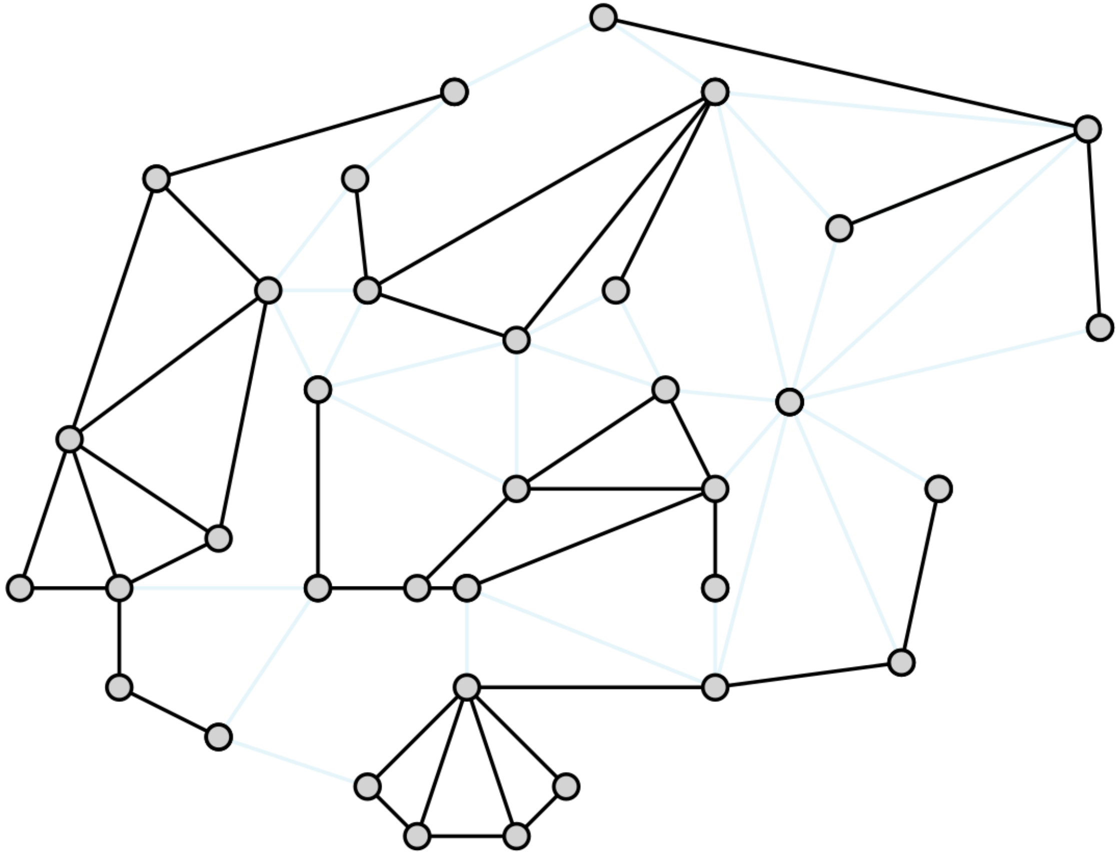

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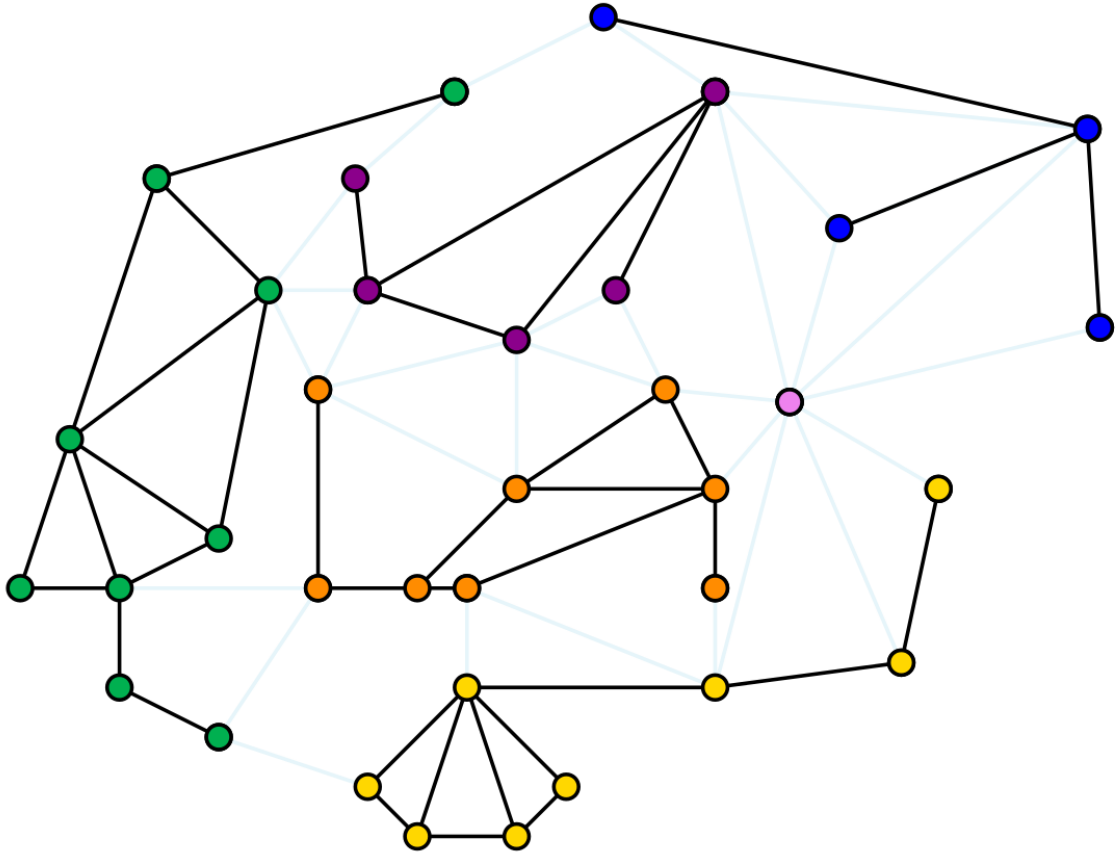

Summary and Open Problems

BASE ALGORITHM

BOOTSTRAPPING (Speed-Up) →

$$T_{LLL}(n) = 2^{O(\sqrt{\log\log n})}$$

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Chang, Pettie [FOCS'16]

Summary and Open Problems

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BOOTSTRAPPING
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$(\text{poly} \log n, \text{poly} \log n)$ - network decomposition

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$(\text{poly} \log n, \text{poly} \log n)$ - network decomposition

- **Devise a faster deterministic algorithm.**
- **Devise a faster algorithm under weaker LLL condition.**

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