Tight Analysis of Parallel Randomized Greedy MIS

Manuela Fischer and **Andreas Noever**
ETH Zurich

Maximal Independent Set (MIS)

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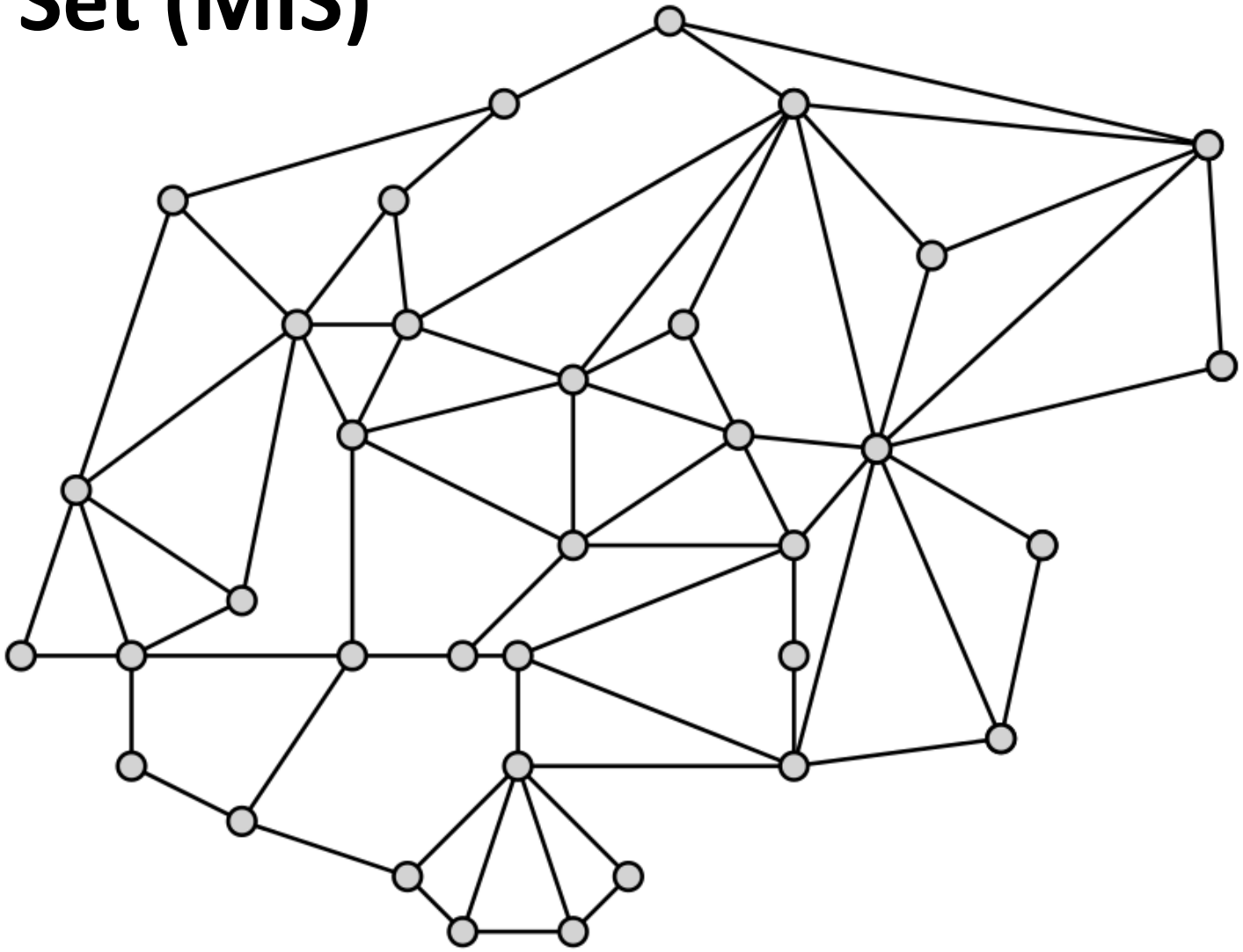
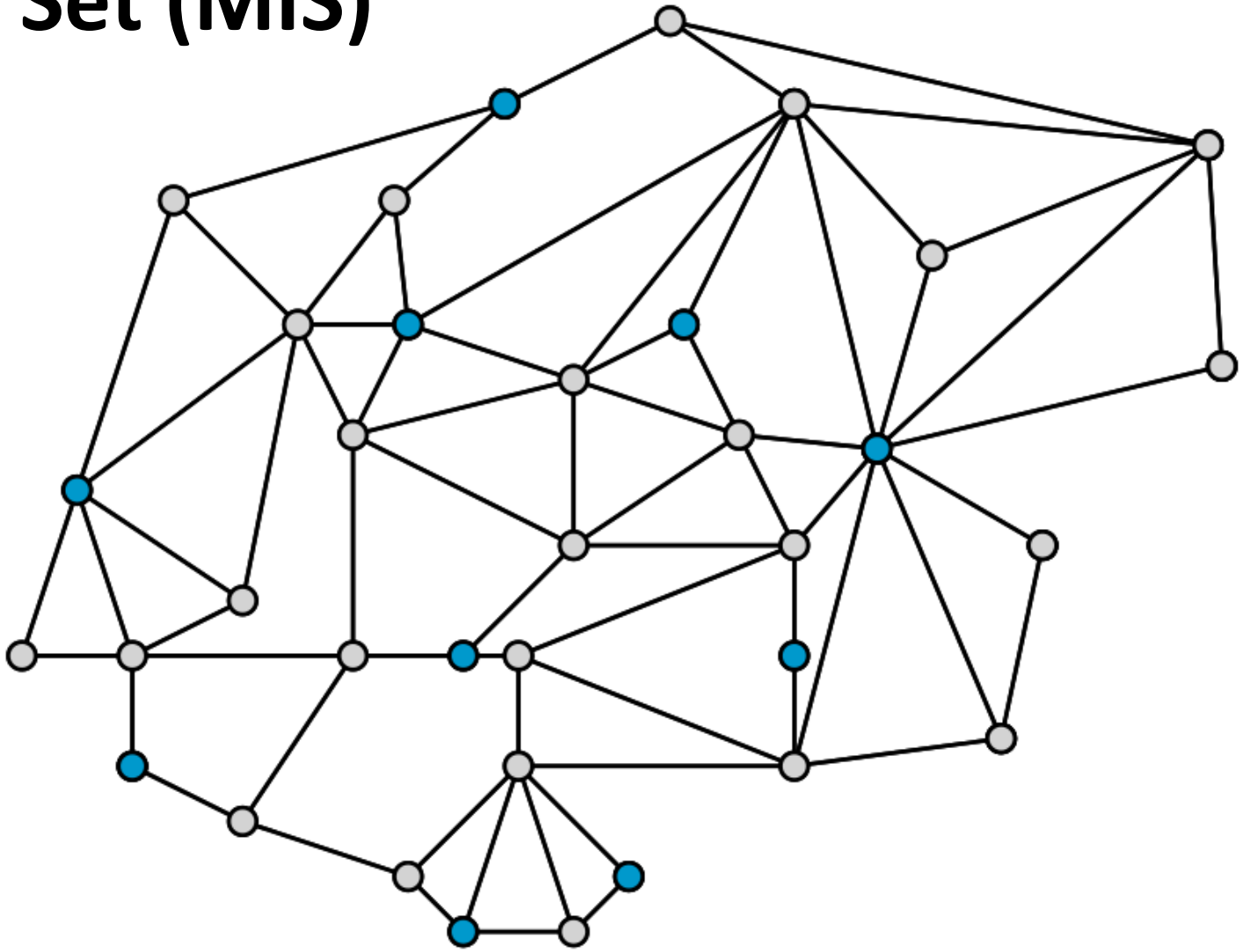
arbitrary graph G on n nodes



Maximal Independent Set (MIS)

arbitrary graph G on n nodes

<u>Independent:</u>
set of non-adjacent nodes



Maximal Independent Set (MIS)

arbitrary graph G on n nodes

<u>Independent:</u>
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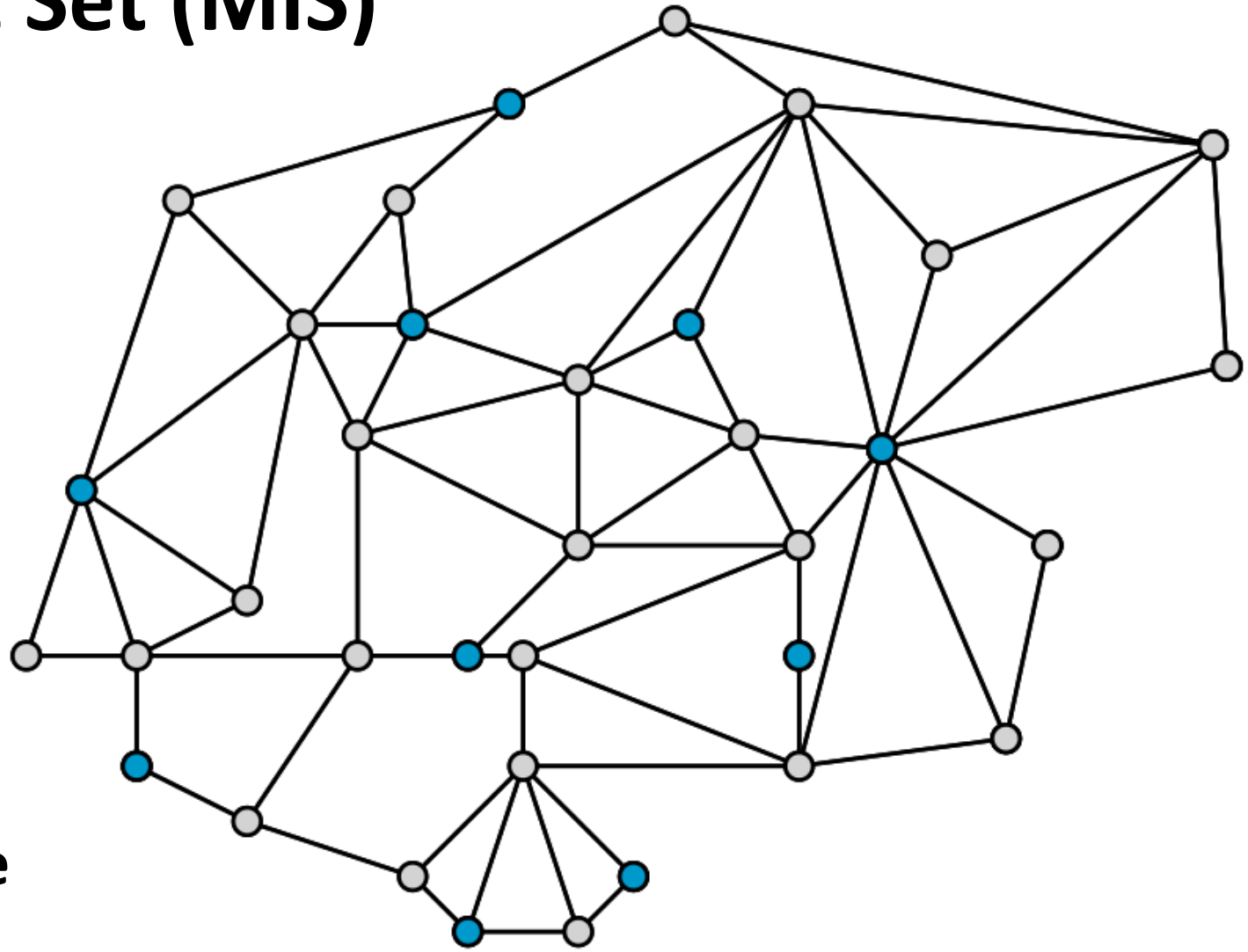
<u>Maximal:</u>
no node can be added
without violating independence



Maximal Independent Set (MIS)

arbitrary graph G on n nodes

<u>Independent:</u>
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<u>Maximal:</u> not maximum!
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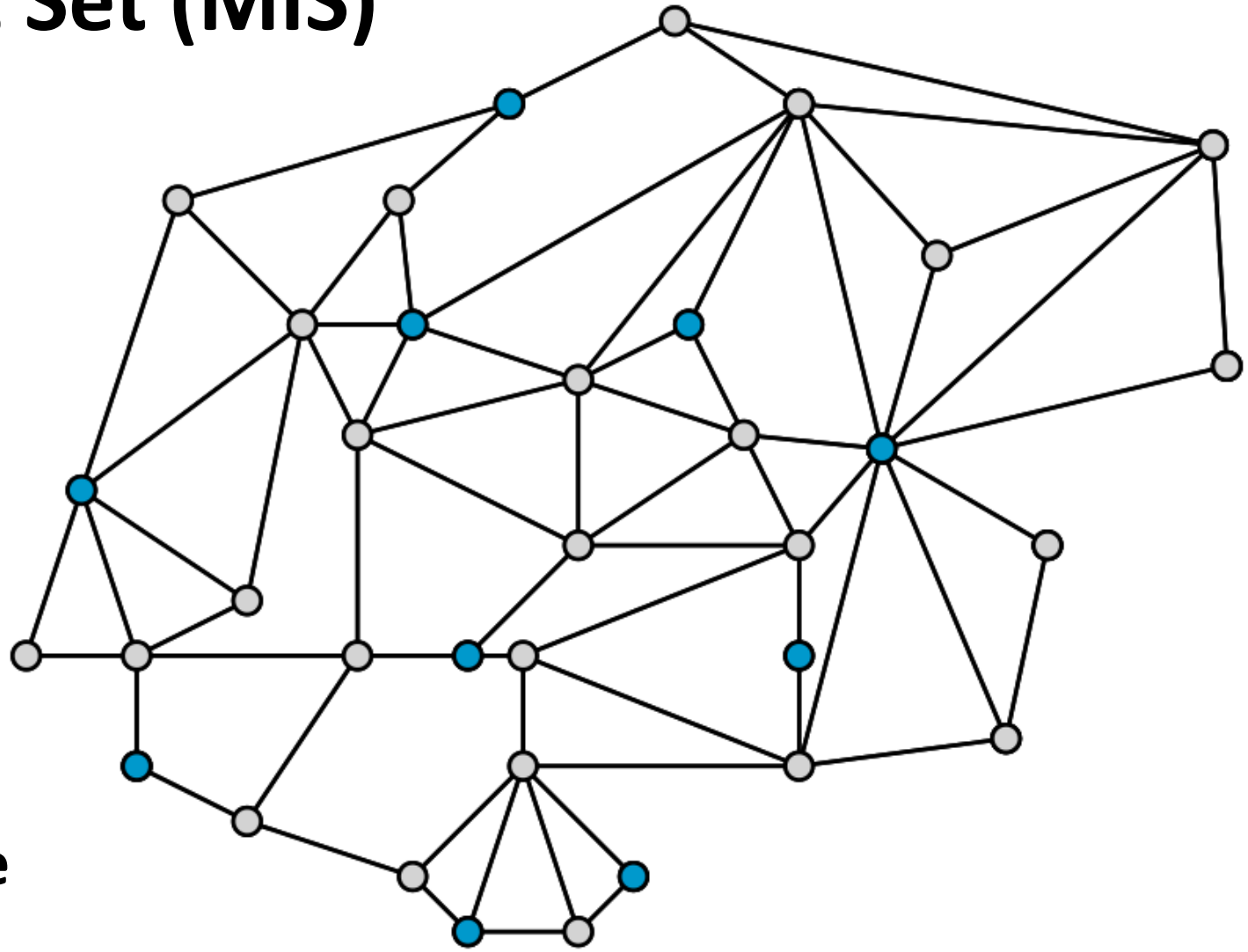

Sequential Greedy Algorithm

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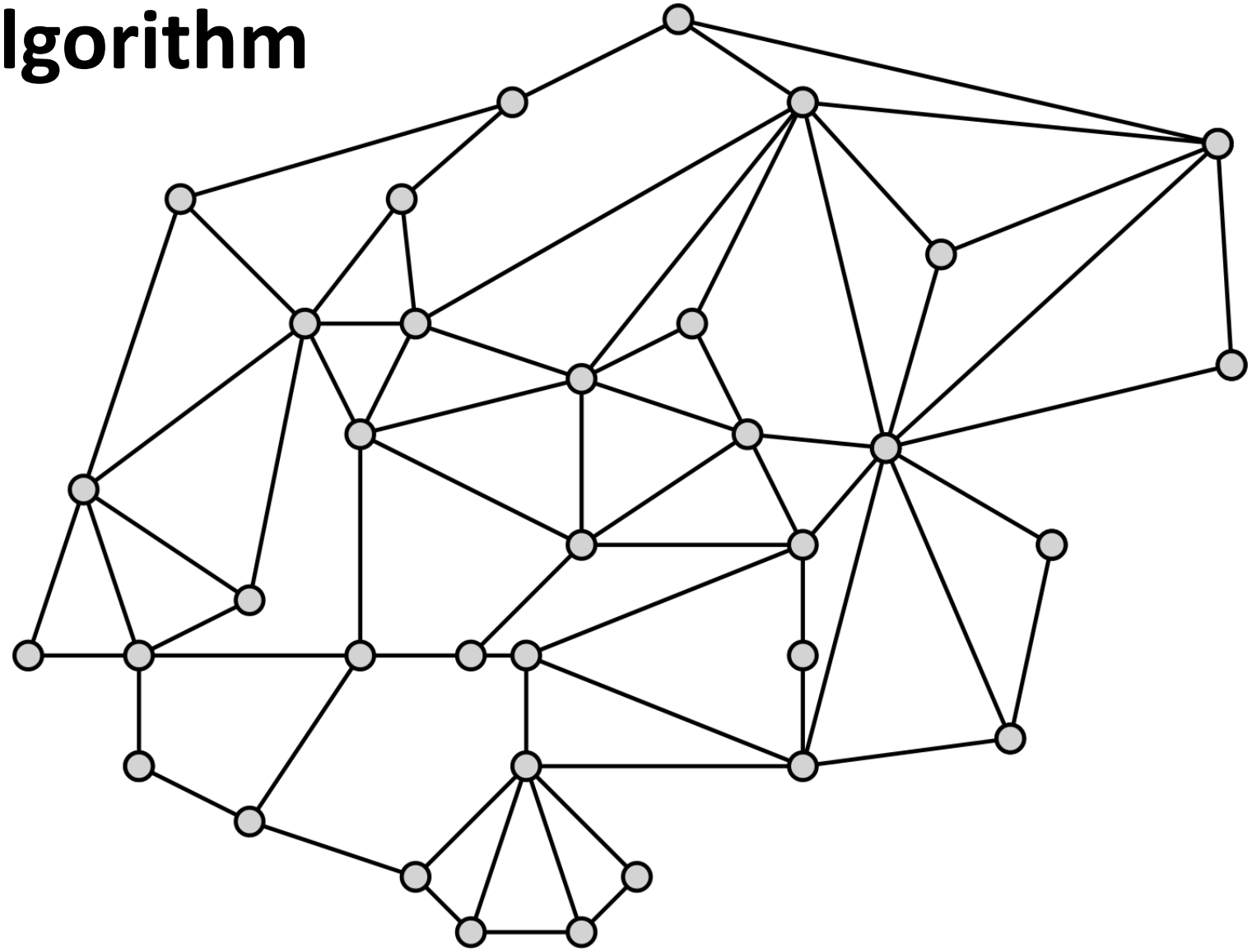

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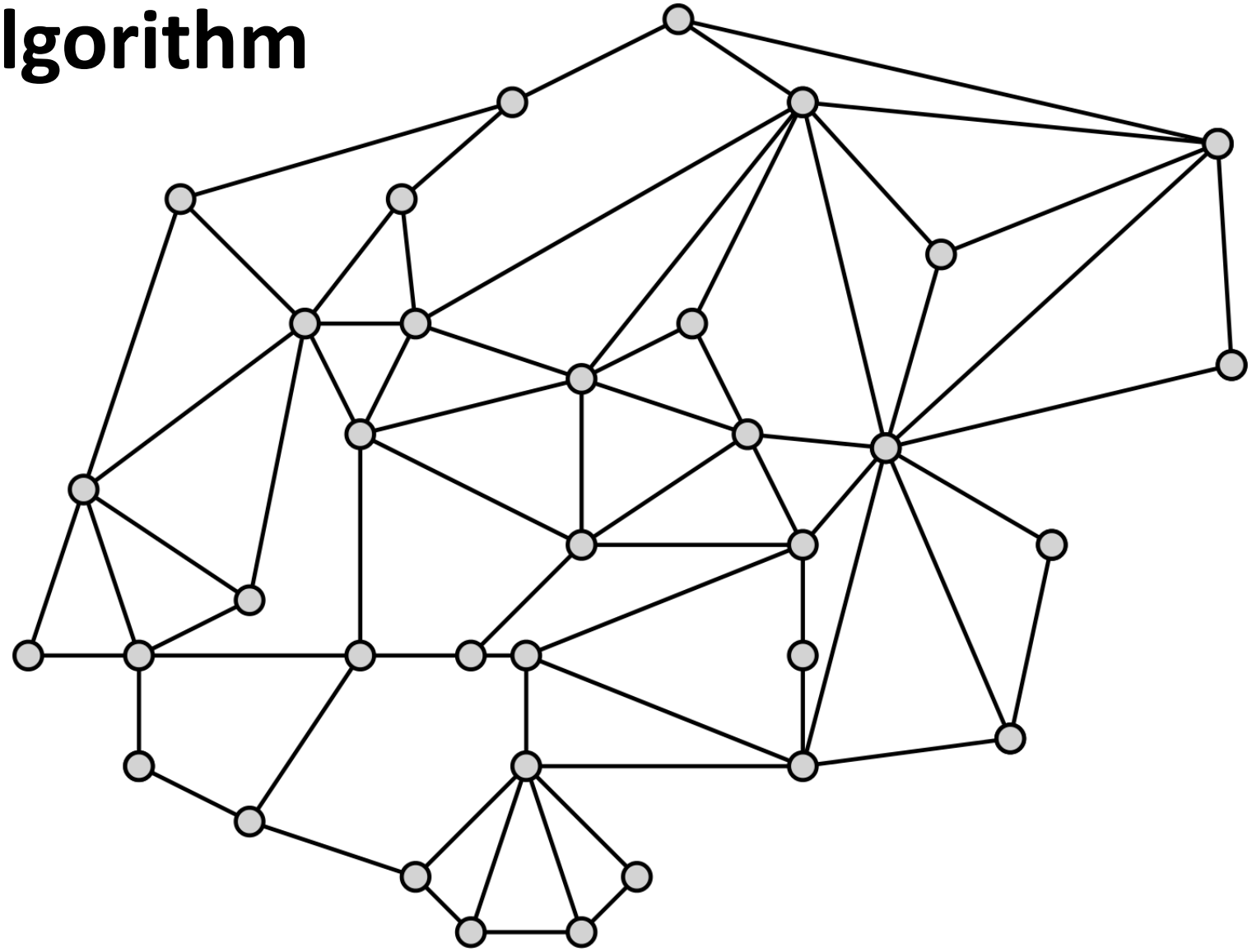

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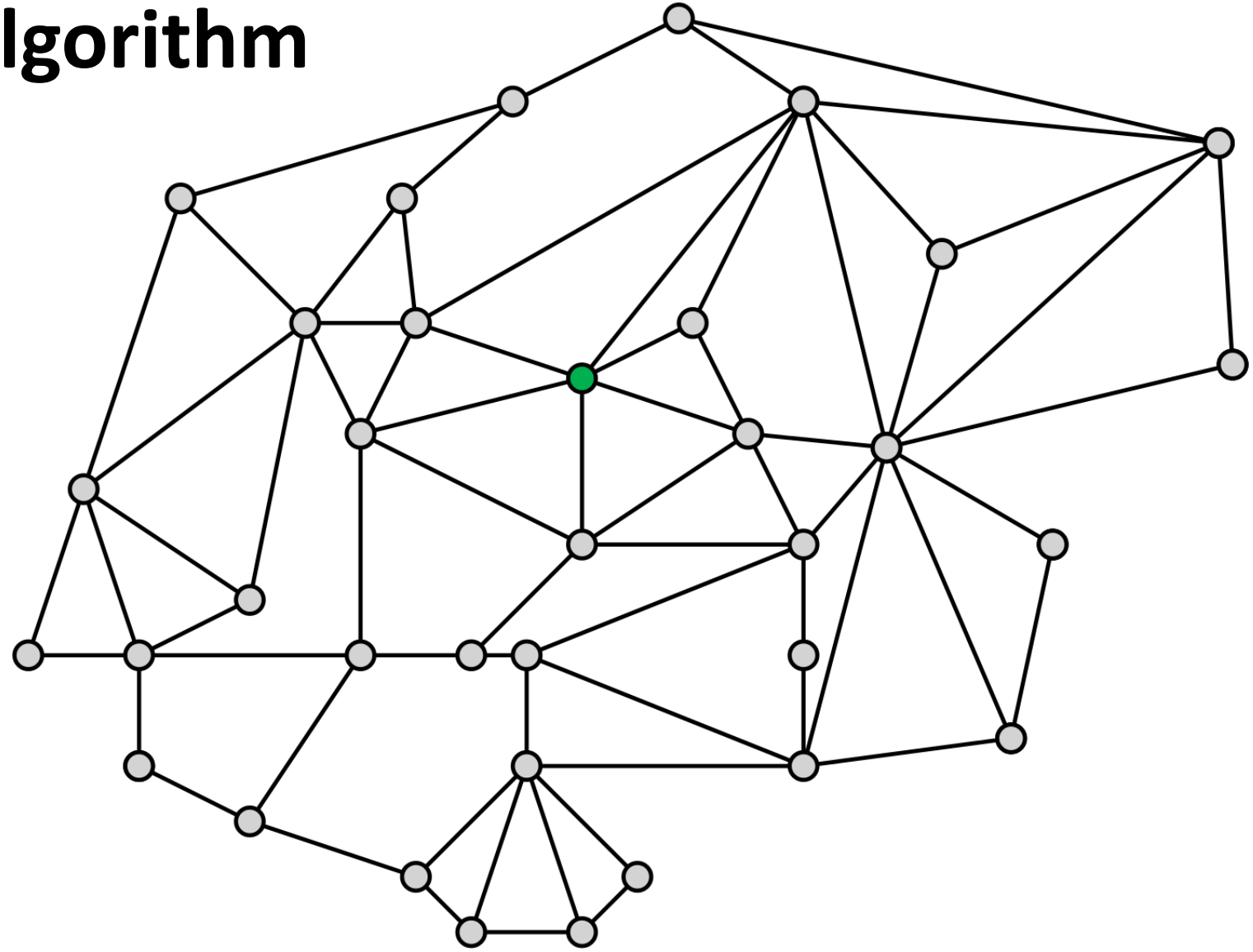

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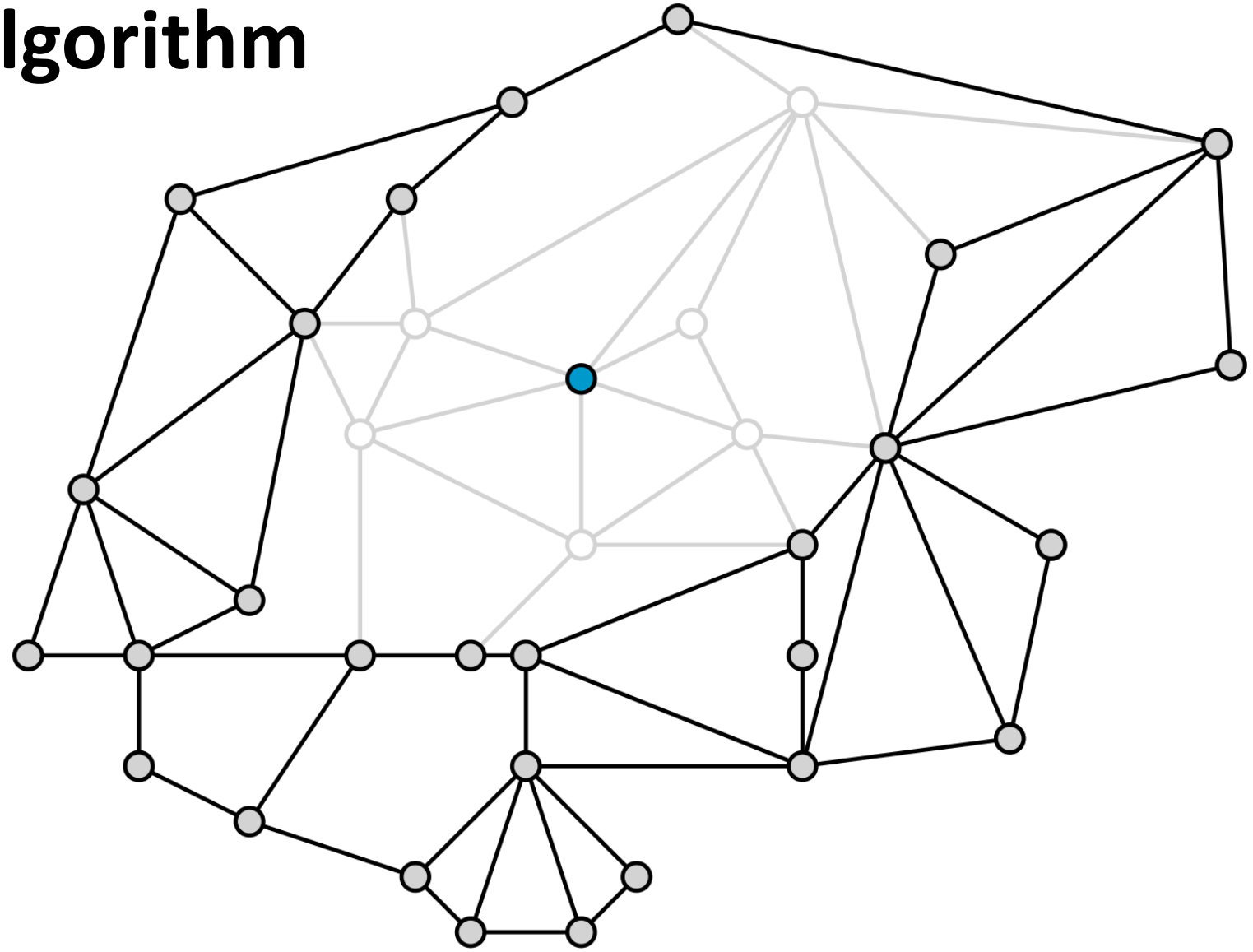

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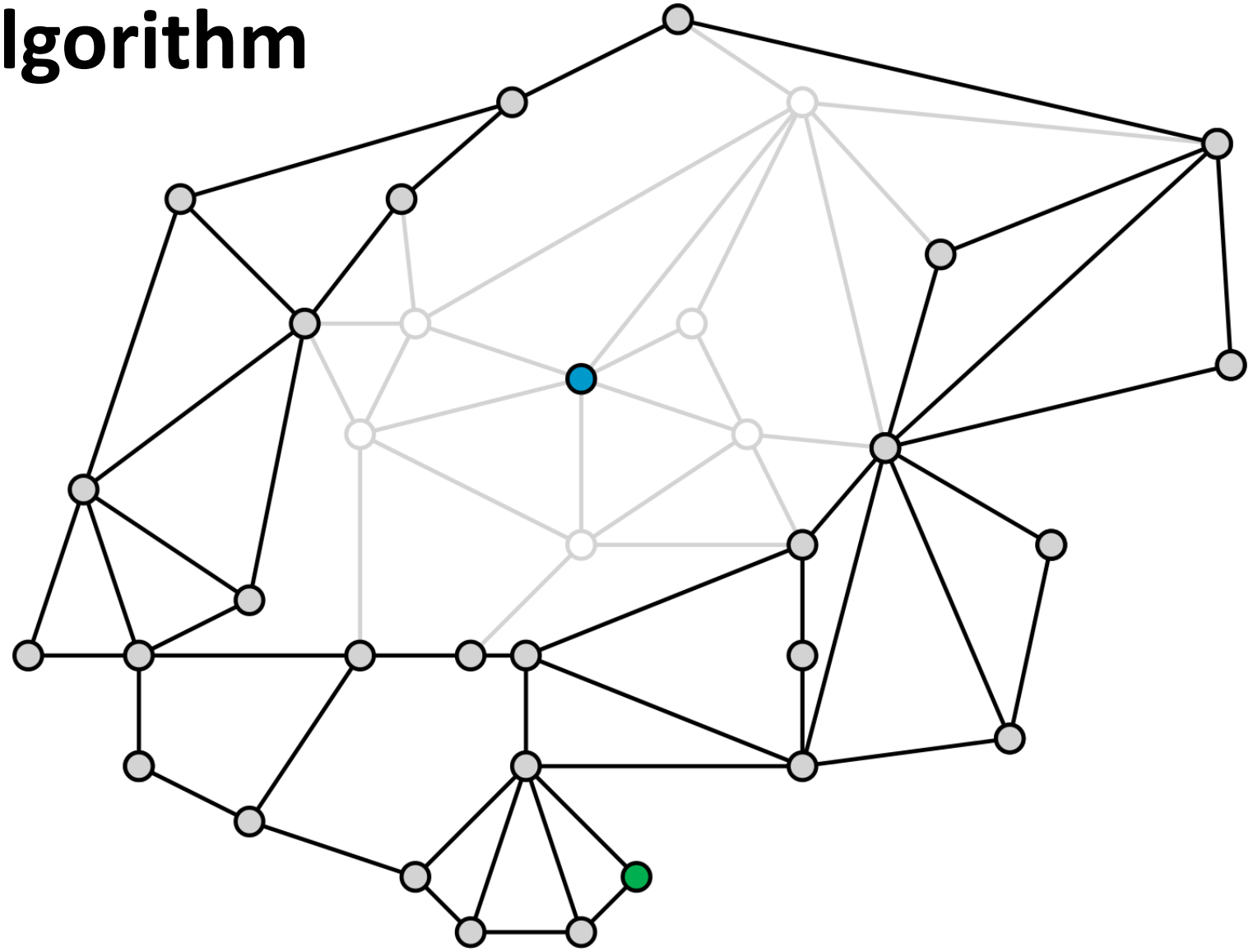

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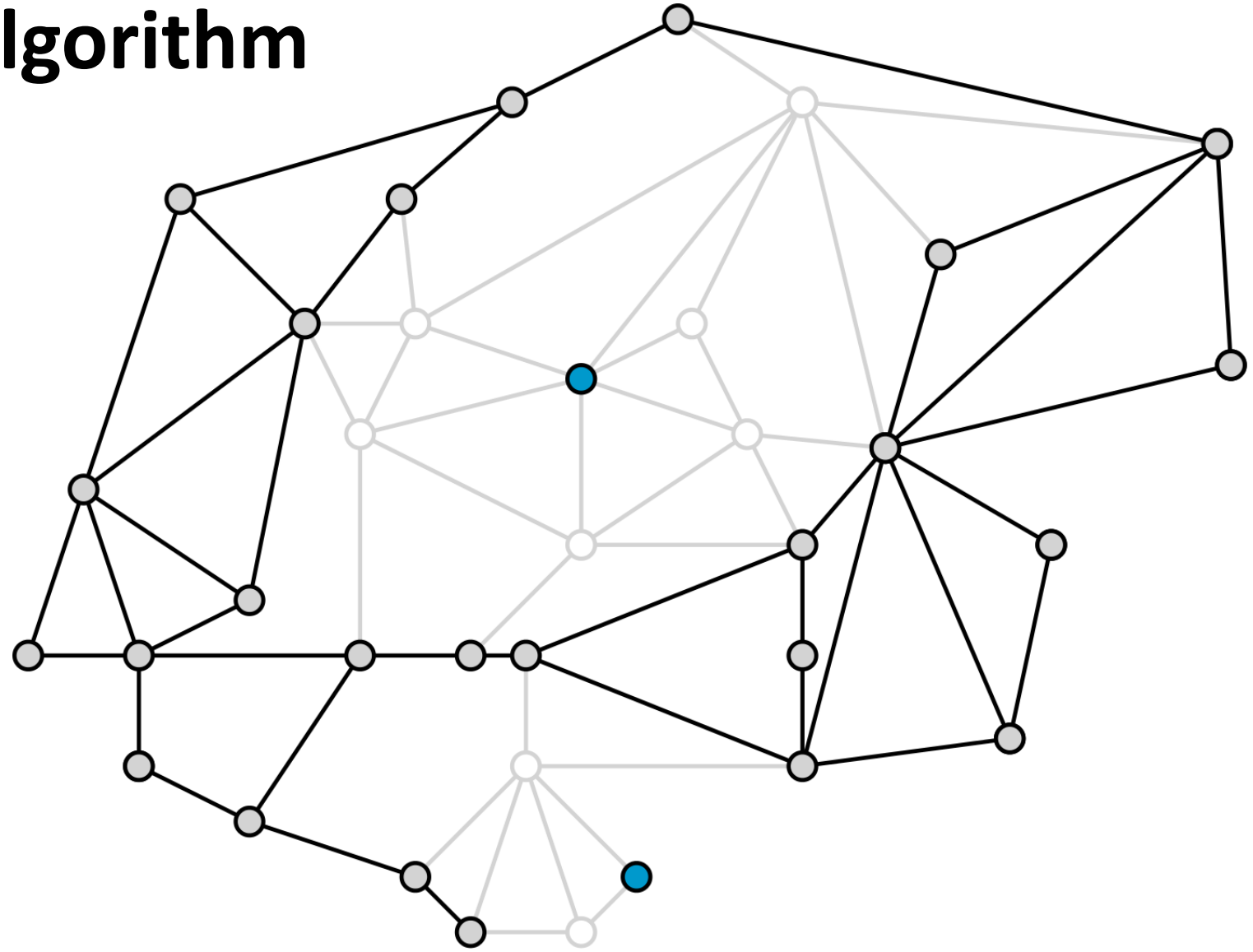

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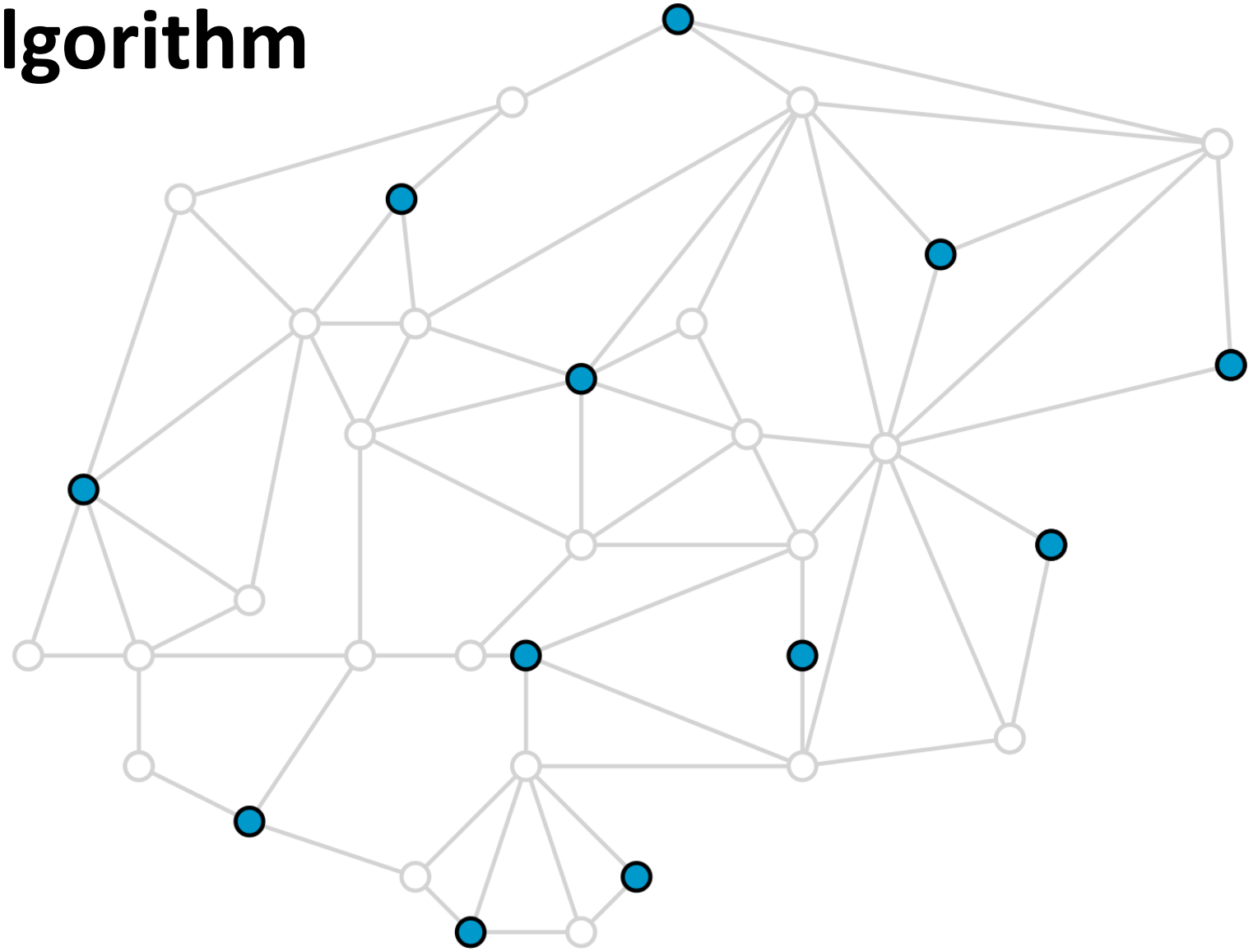

Parallel Greedy Algorithm

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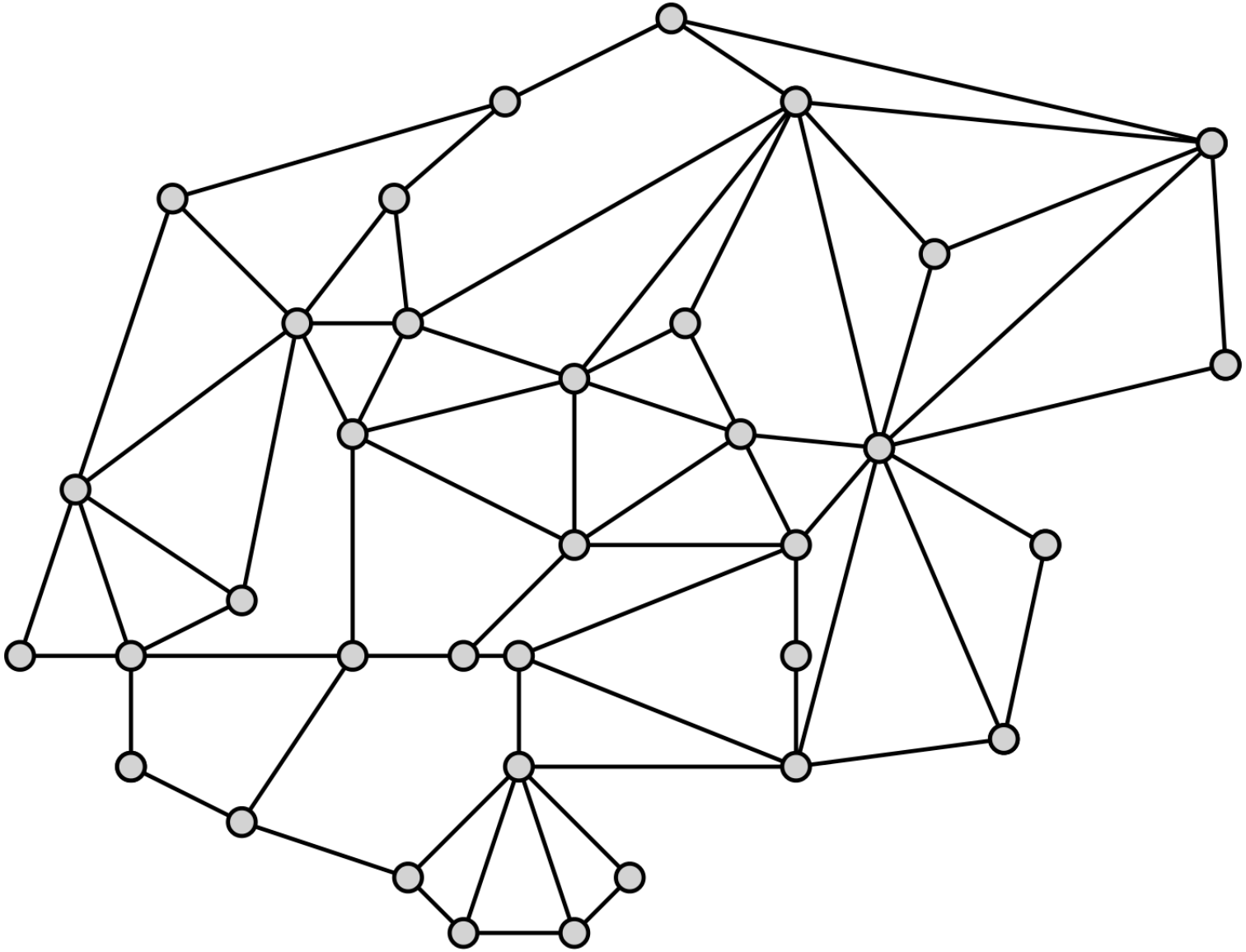

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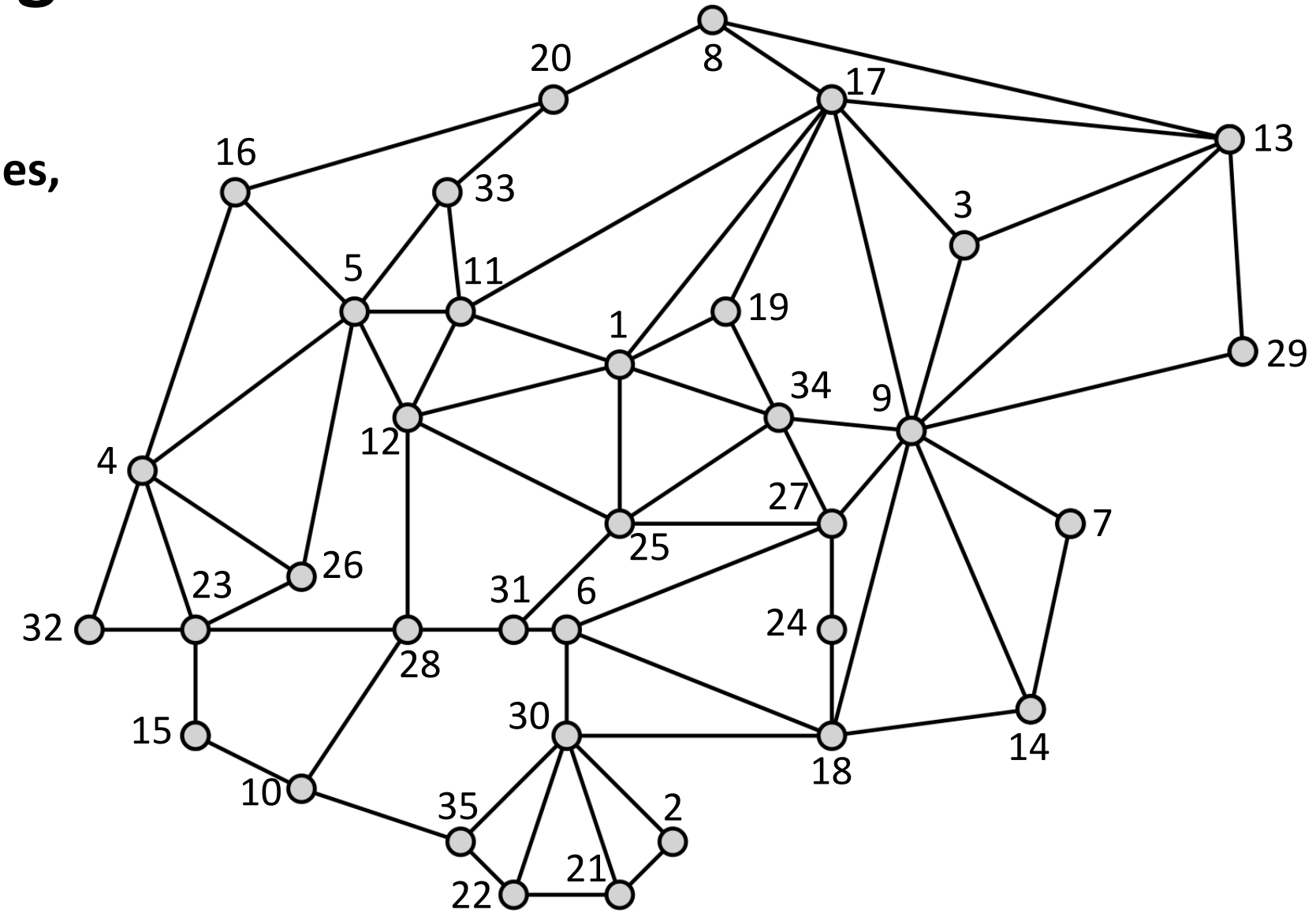

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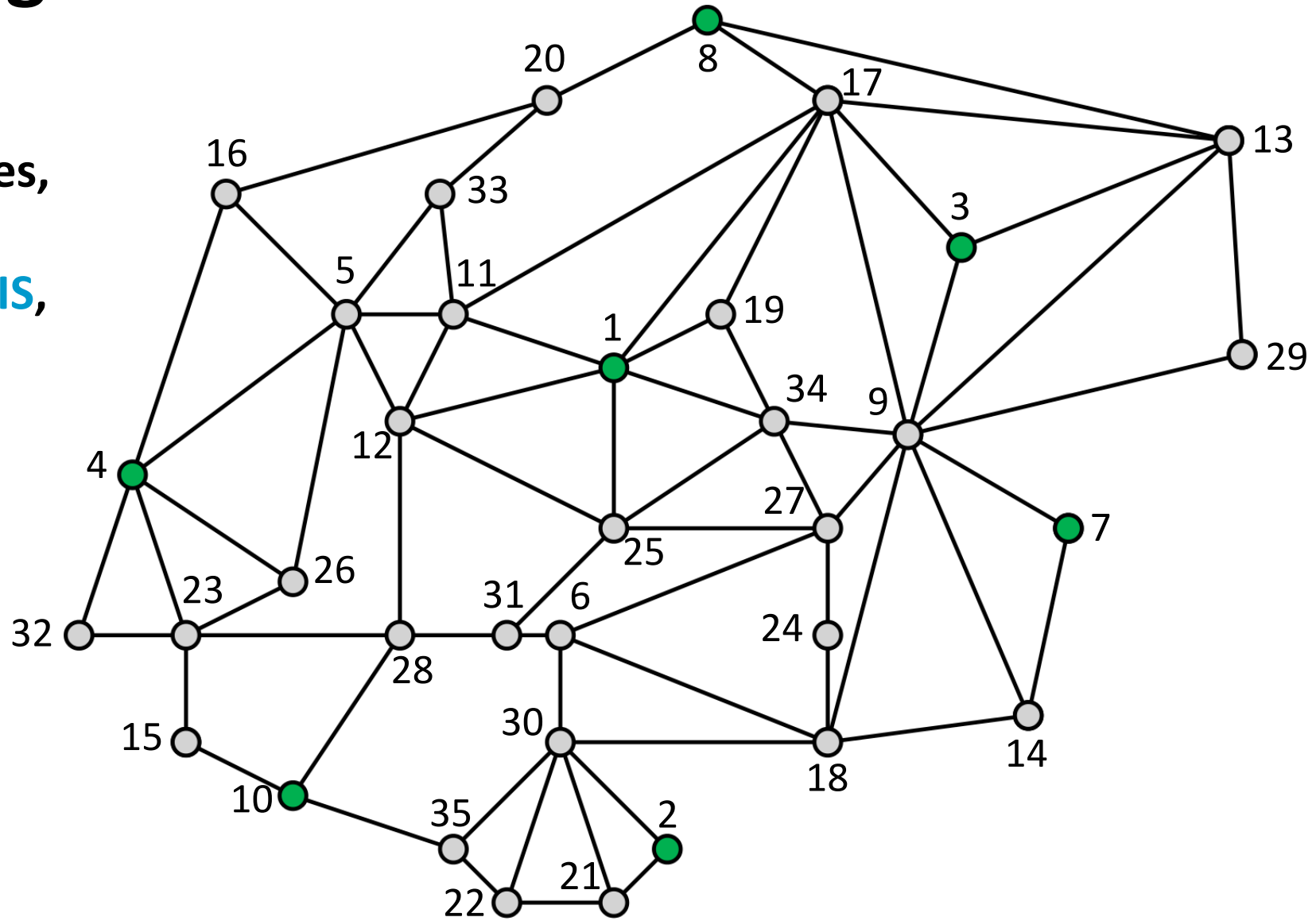

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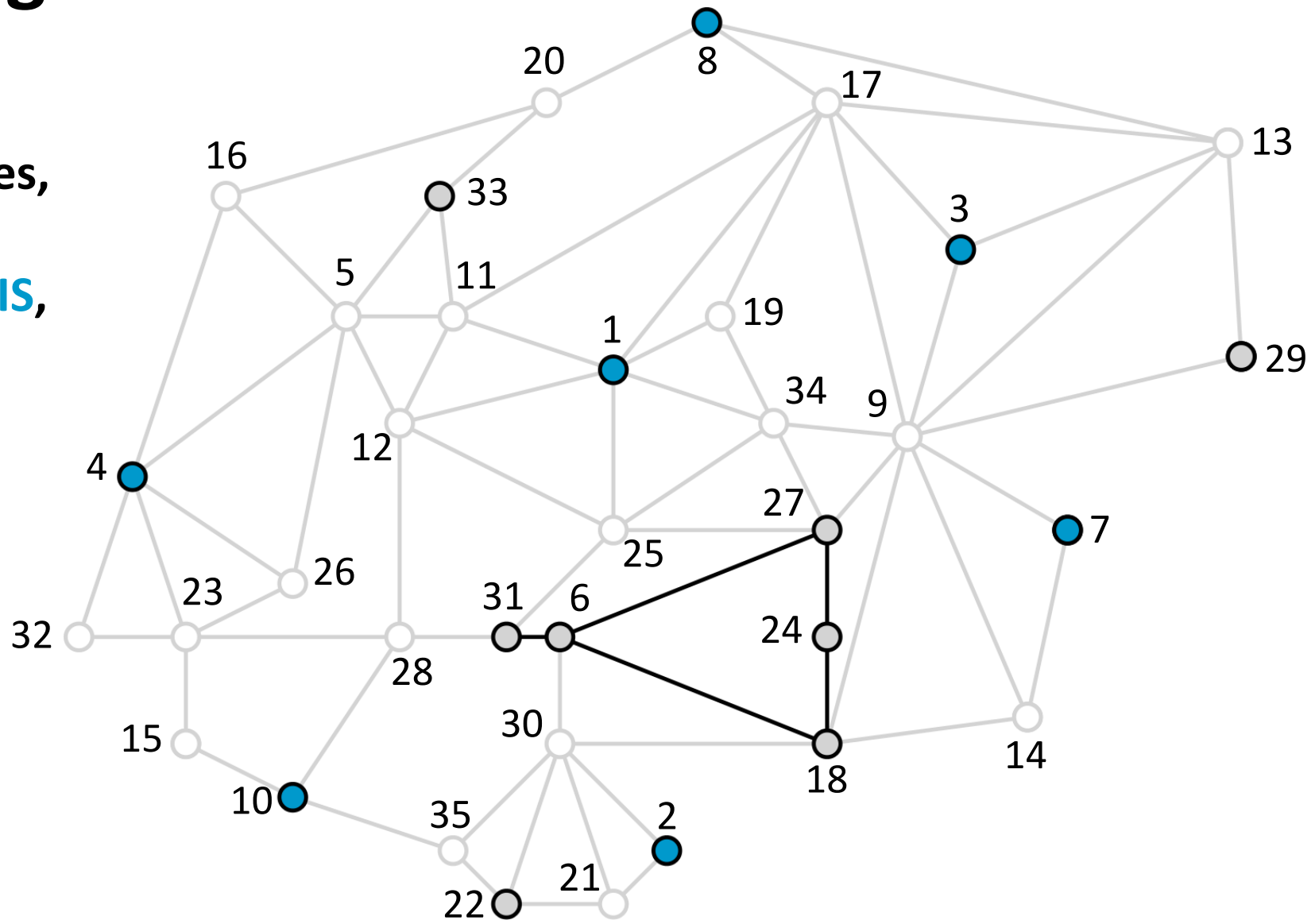

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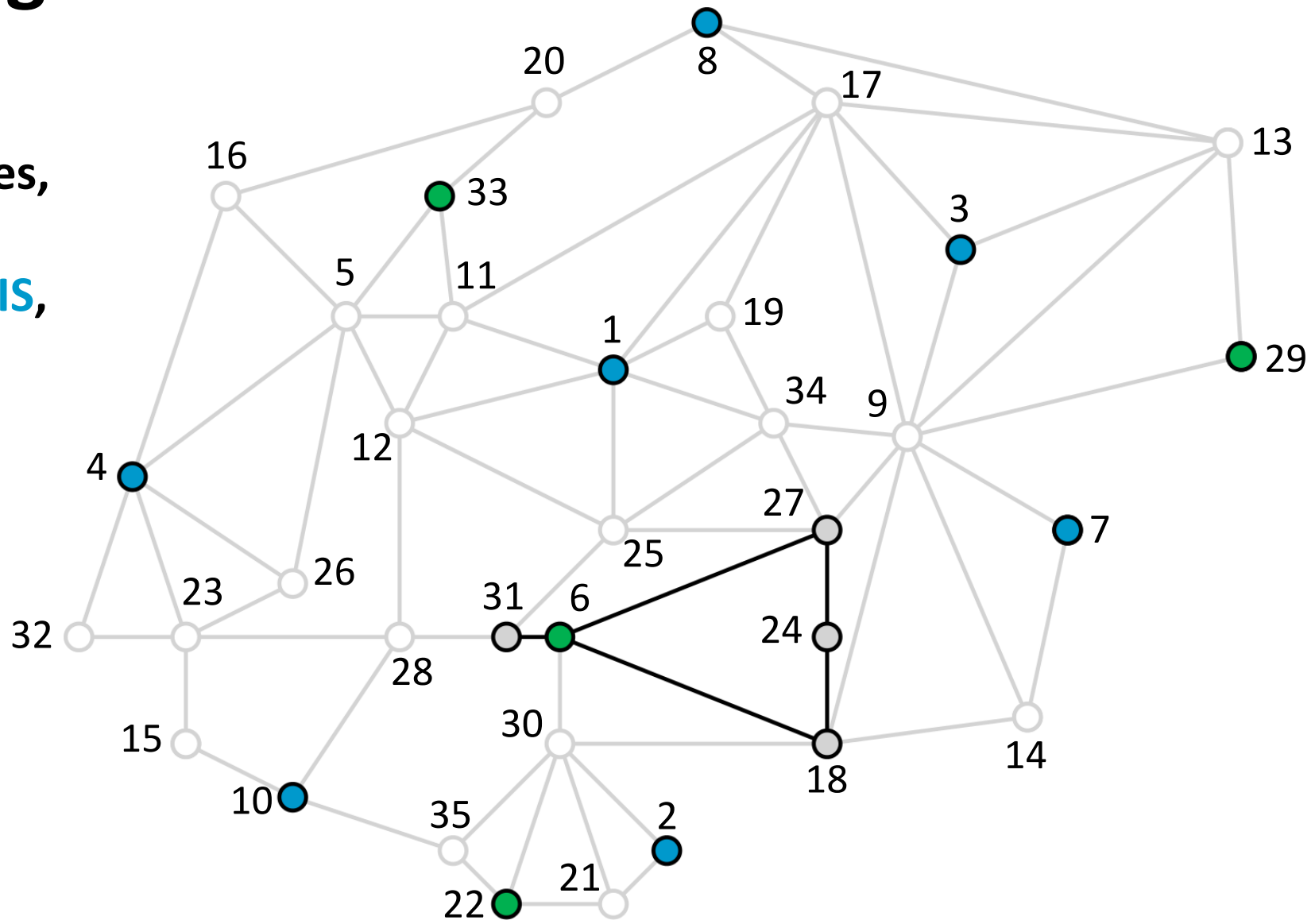

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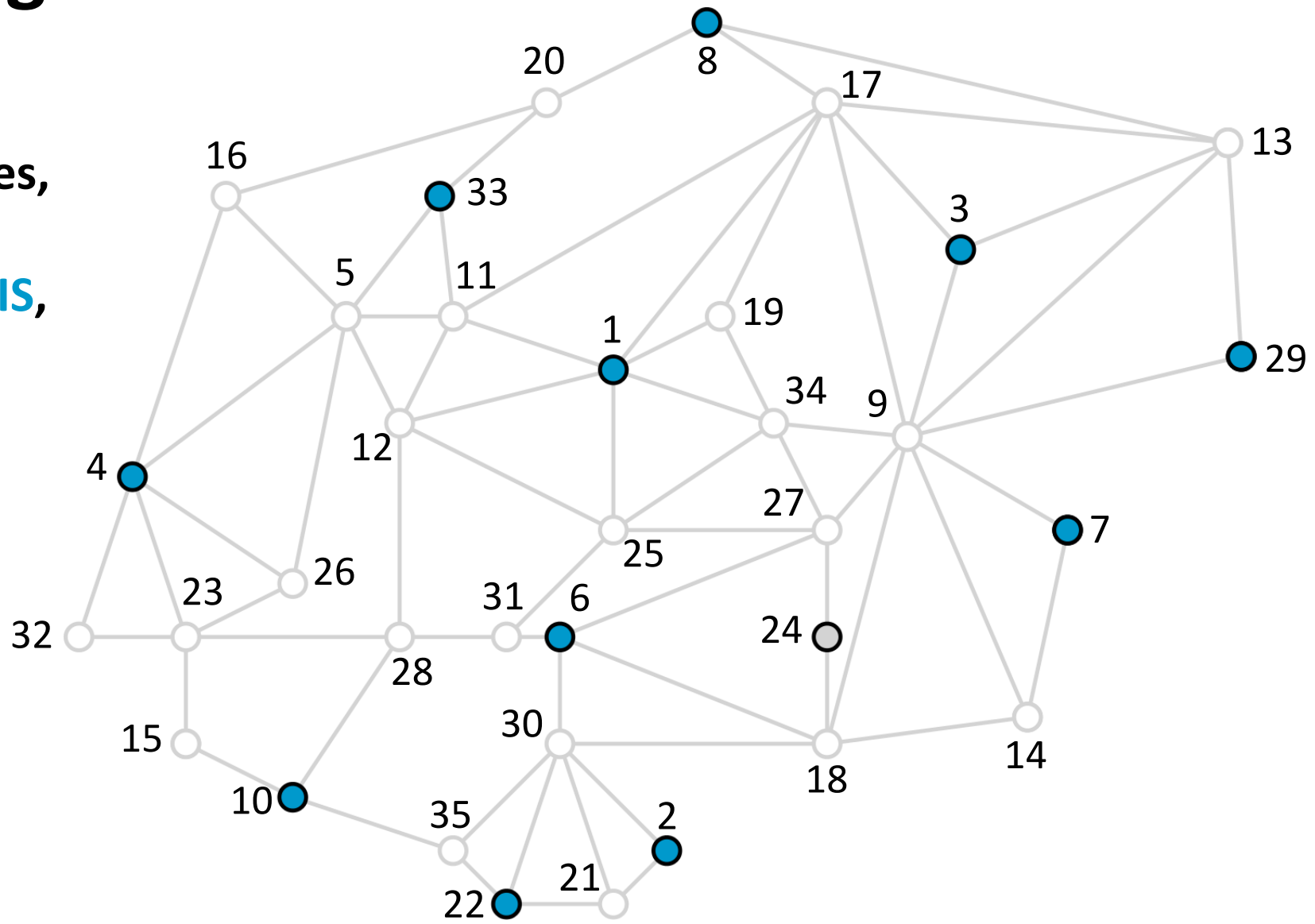

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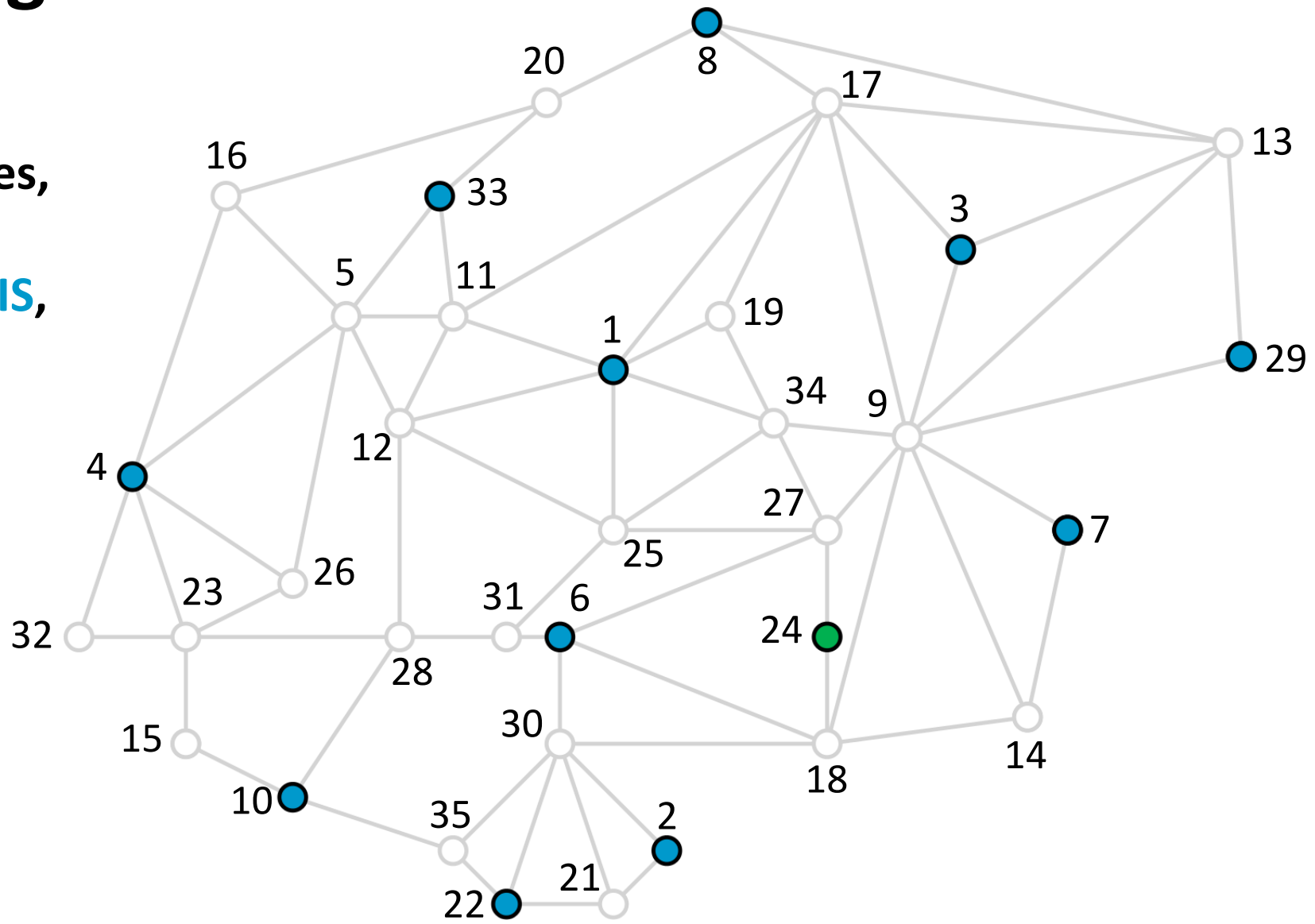

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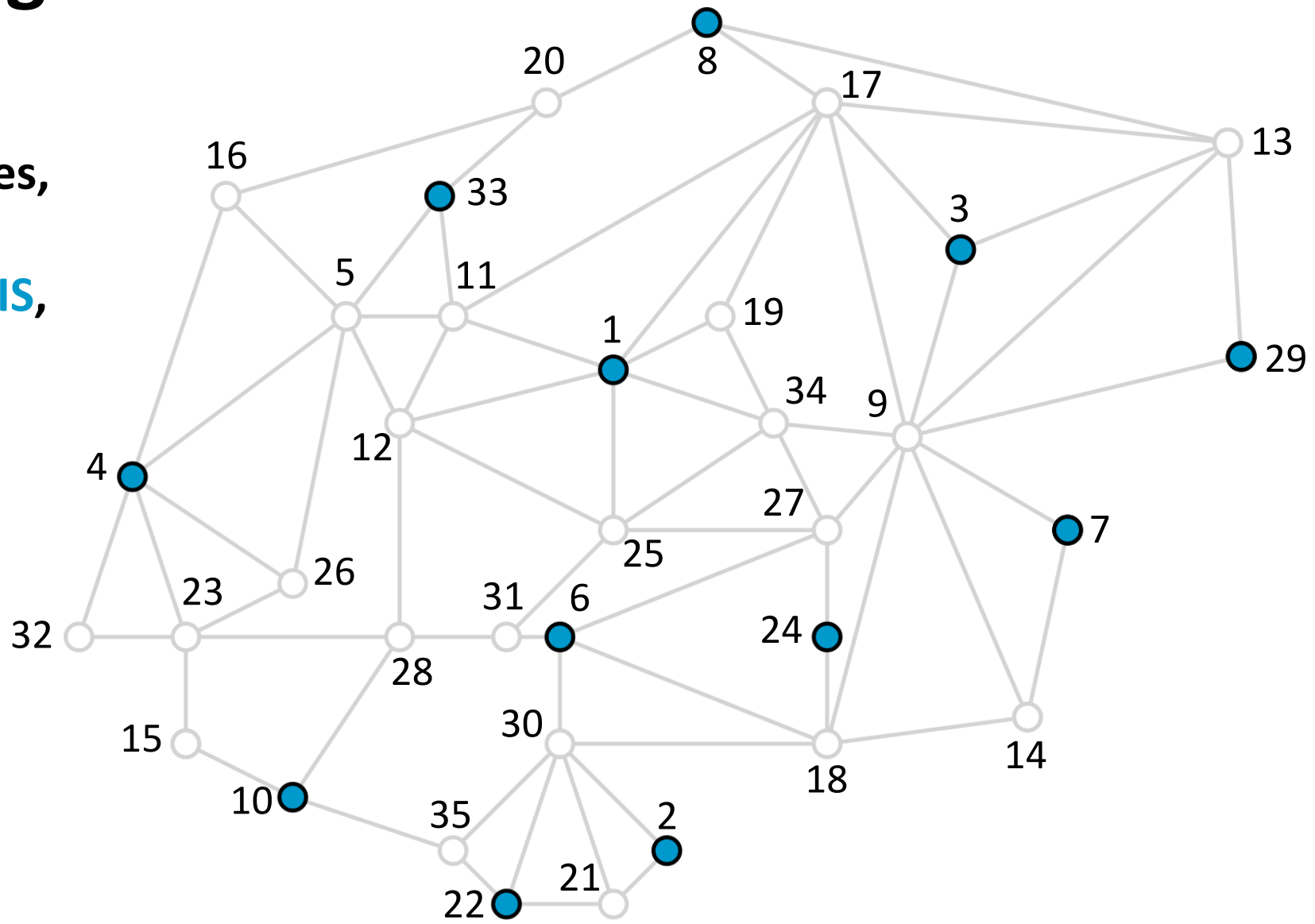

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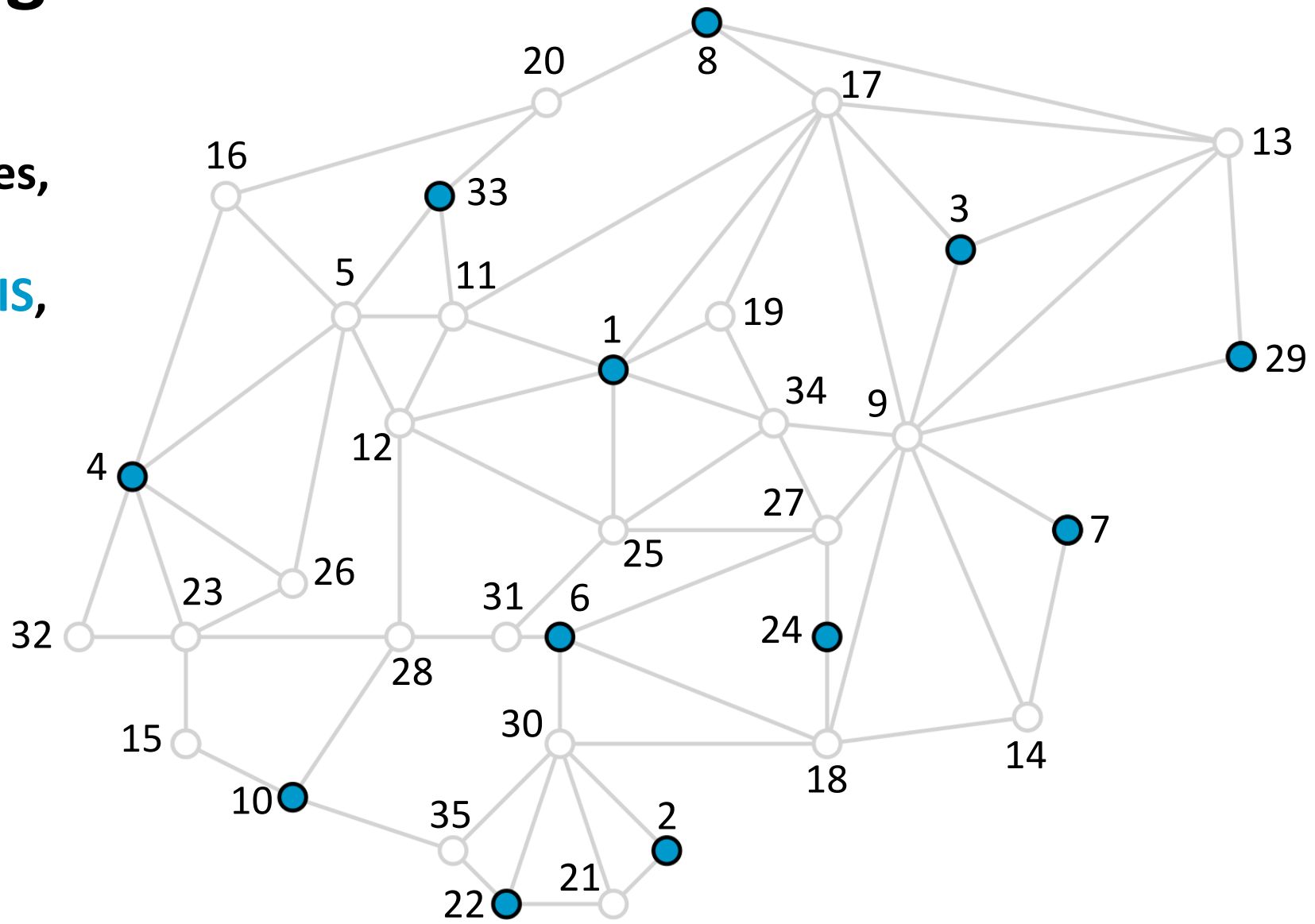

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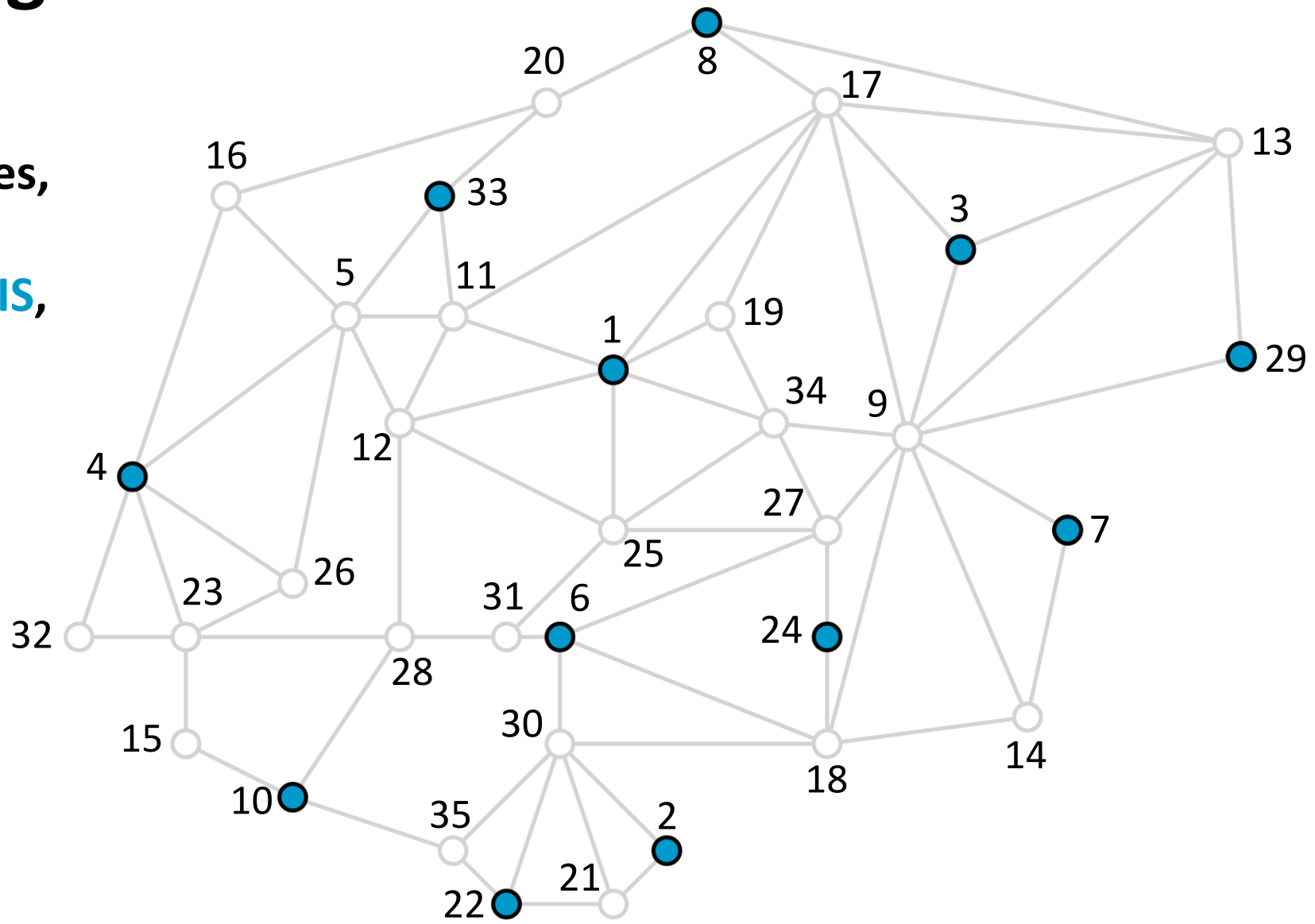

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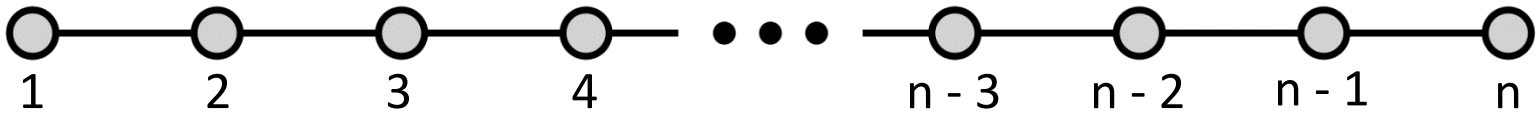

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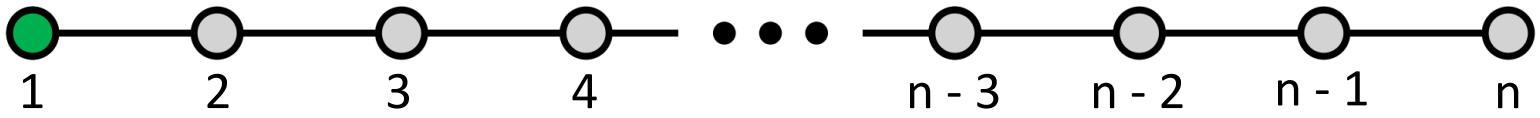

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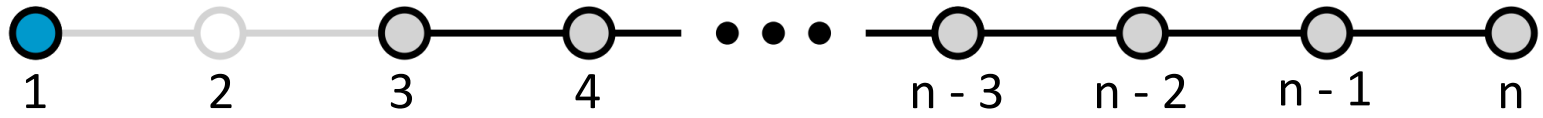

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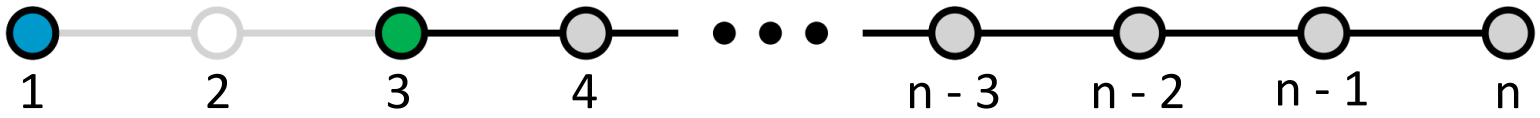

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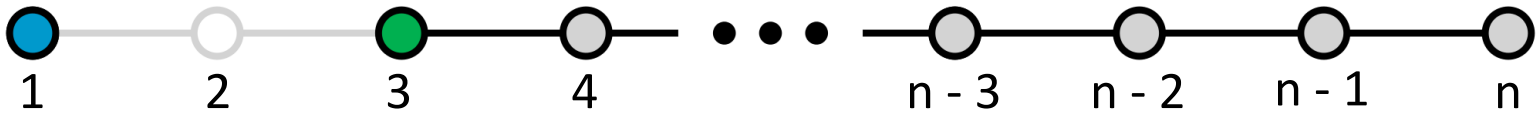

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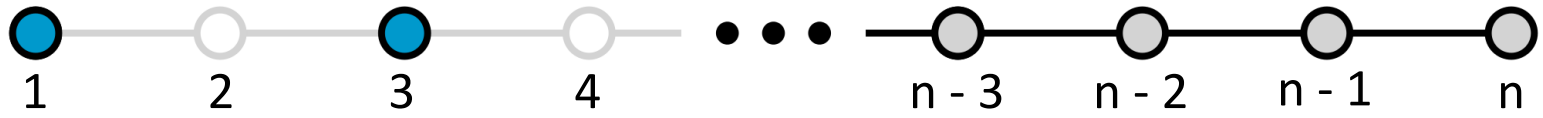

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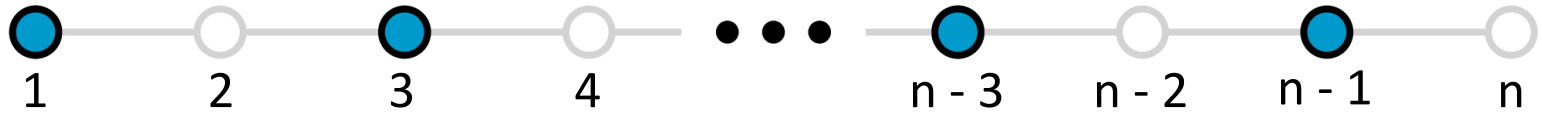

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Randomized
Parallel Greedy Algorithm

random
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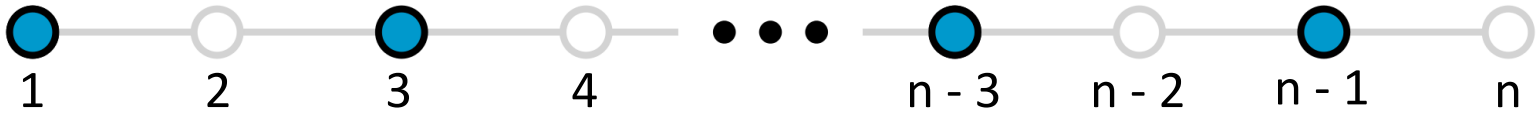

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Randomized Parallel Greedy Algorithm

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Random Graphs

Randomized Parallel Greedy Algorithm

Random Graphs $\mathbf{O}\left(\frac{\log^2 n}{\log \log n}\right)$ *Coppersmith, Raghavan, Tompa* [FOCS'87]

Randomized Parallel Greedy Algorithm

Random Graphs

$$\mathbf{O}\left(\frac{\log^2 n}{\log\log n}\right)$$ *Coppersmith, Raghavan, Tompa* [FOCS'87]

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General Graphs

Randomized Parallel Greedy Algorithm

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Randomized Parallel Greedy Algorithm

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Also applies to **$(\Delta + 1)$-Vertex-Coloring, Maximal Matching,** and **$(2\Delta - 1)$-Edge-Coloring**

Proof Sketch

Proof Sketch

 # rounds $\leq$ dependency chain length $= \mathbf{O}(\log n)$

Dependency Chains

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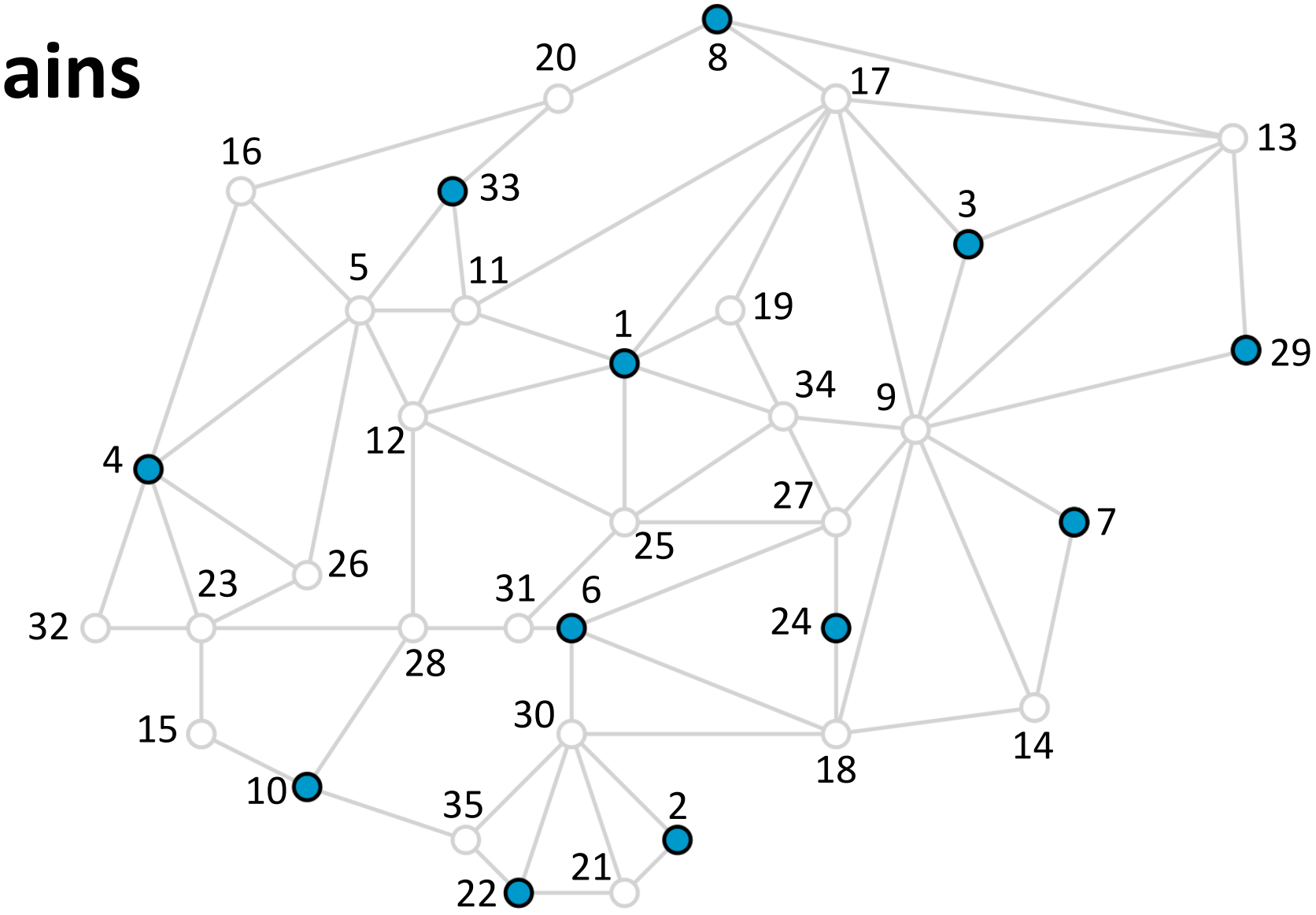

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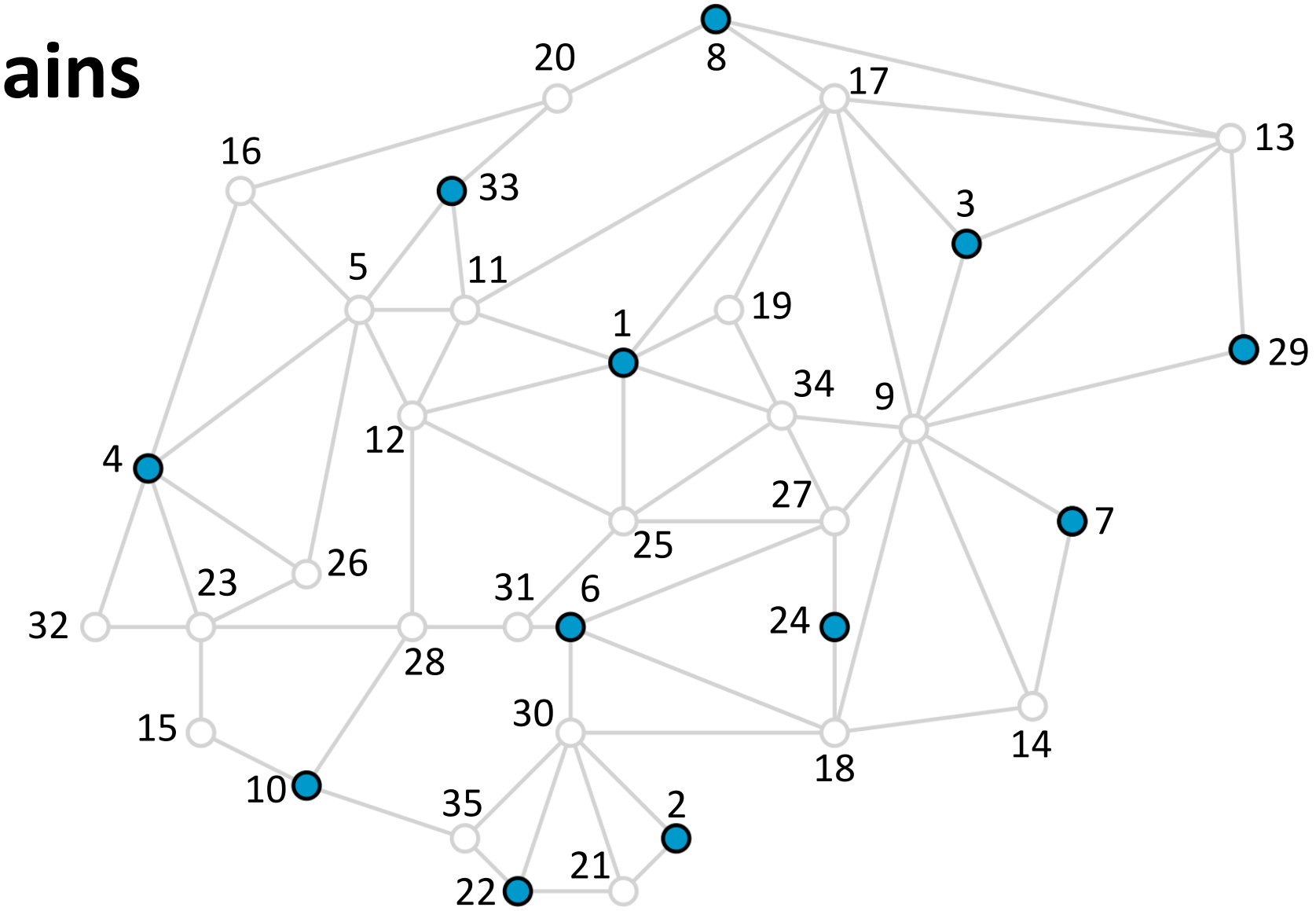

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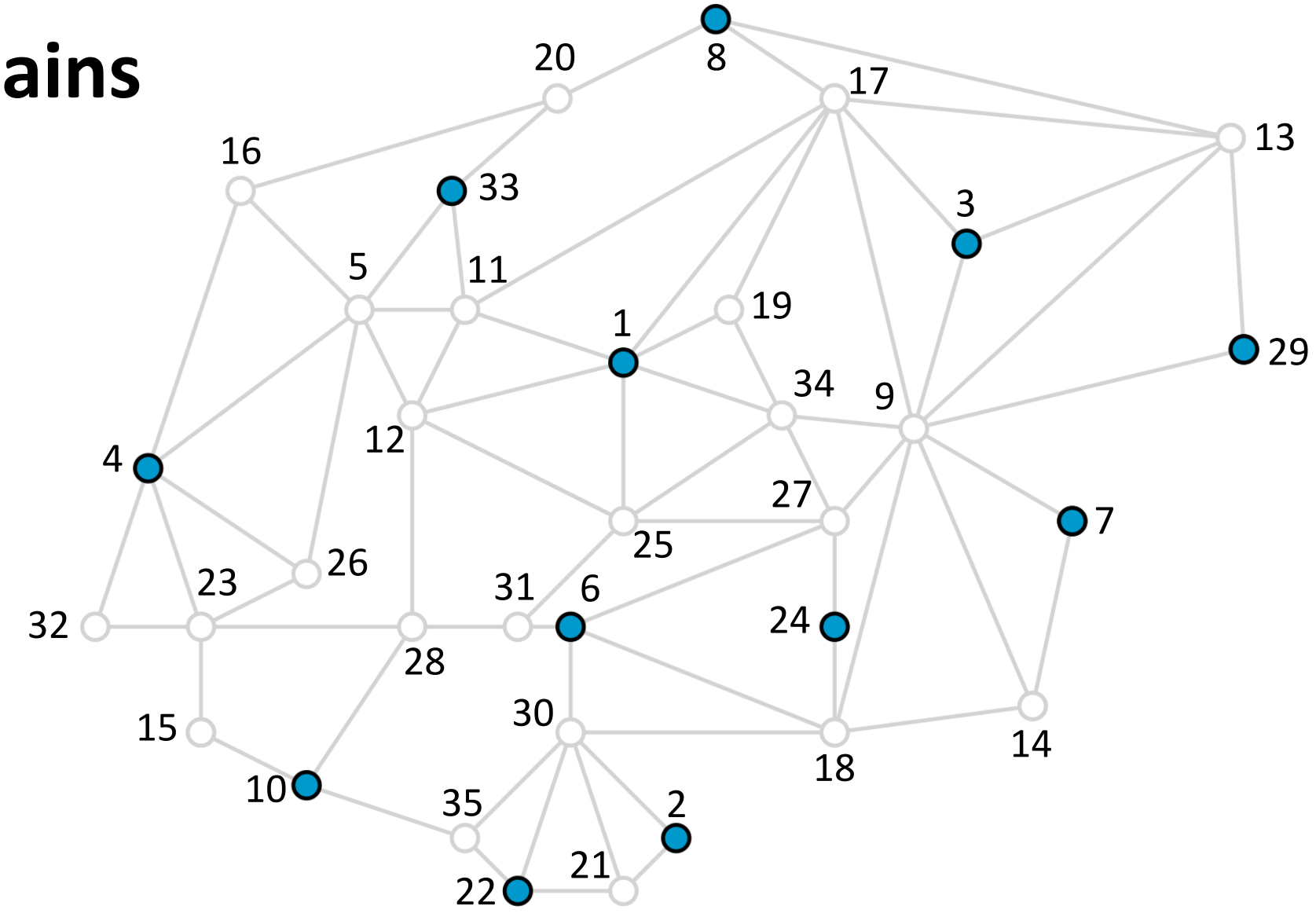

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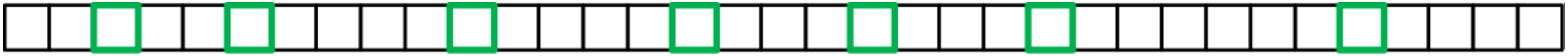

positions form dependency chain if

Dependency Chains

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assigned nodes form a (monotonically increasing) path

alternating between MIS nodes and non-MIS nodes

MIS node is inhibitor of next non-MIS node

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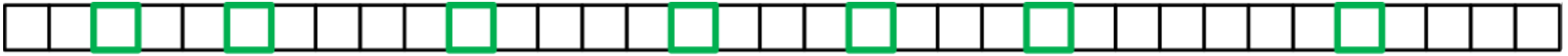

bound probability that $\Omega(\log n)$ positions form dependency chain

Dependency Chain Length

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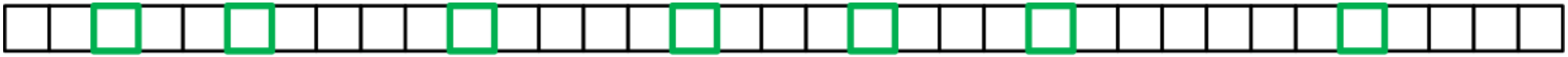
union bound over all choices of positions

Probability of Positions Forming Dependency Chain



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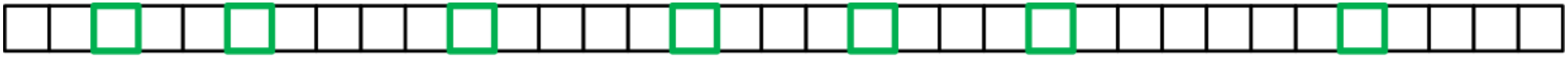

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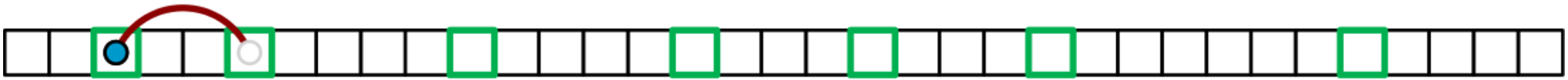

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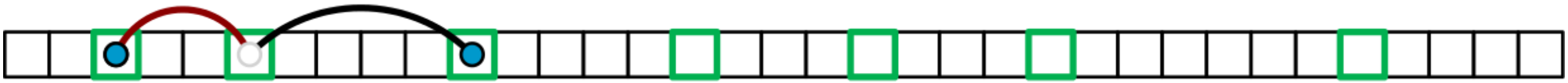

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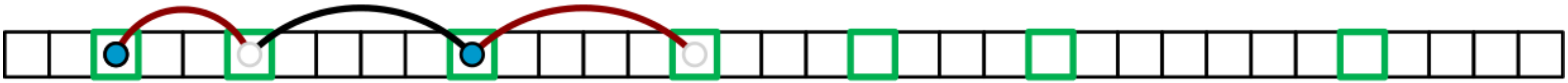

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Probability of Continuing Dependency Chain

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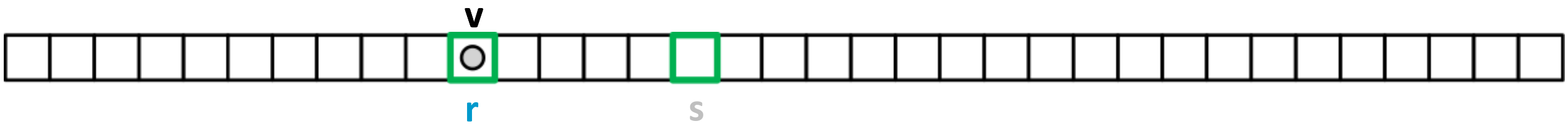
alternating MIS and non-MIS nodes

MIS is inhibitor of next non-MIS node



analyze probability dependency chain continues through positions r and s

Probability of Continuing Dependency Chain

positions form dependency chain if

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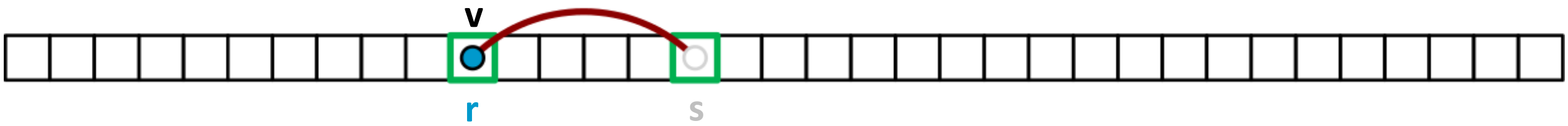
alternating MIS and non-MIS nodes

MIS is inhibitor of next non-MIS node



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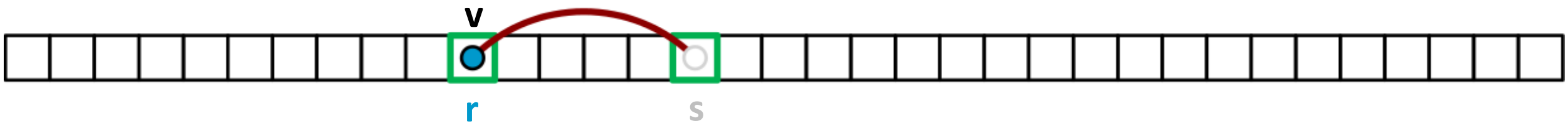

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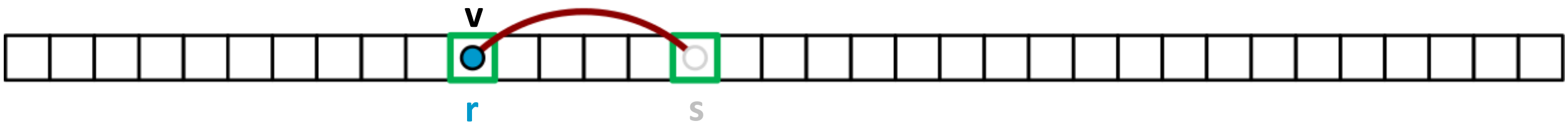

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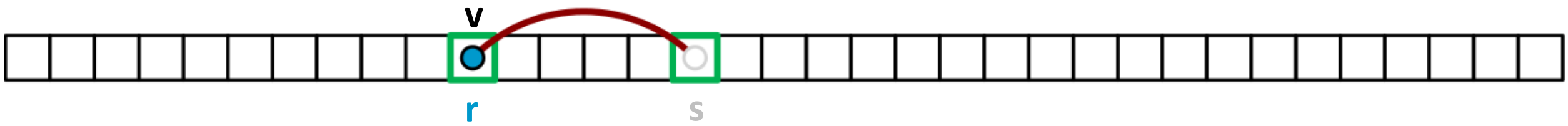

analyze probability dependency chain continues through positions r and s

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at step step $t \leq r$,

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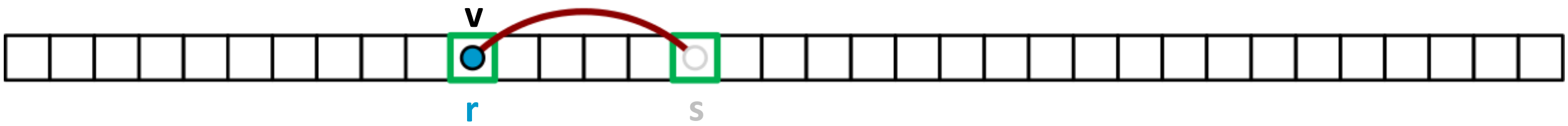

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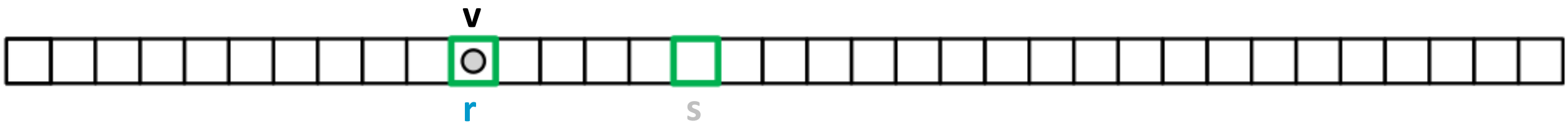

Probability of Continuing Dependency Chain

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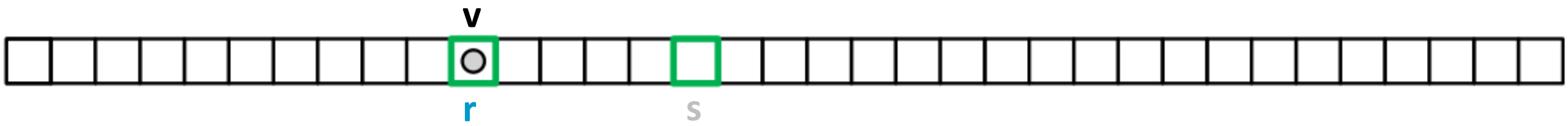
degree sequence of v: $d_0, d_1, \ldots, d_t$



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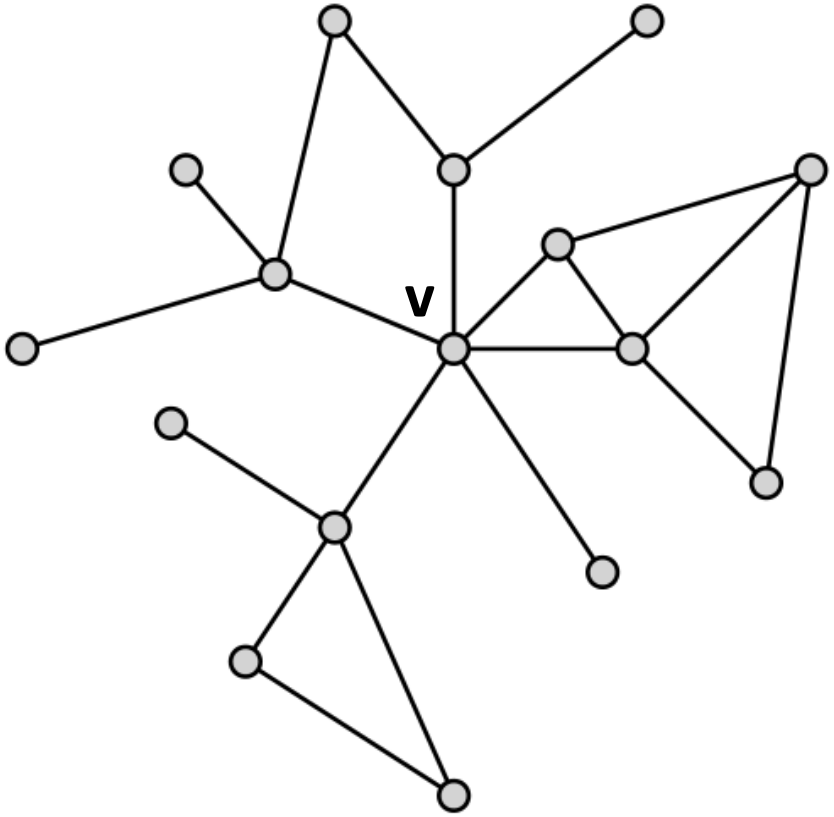

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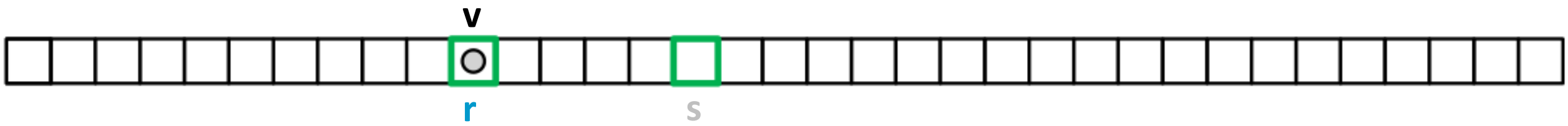

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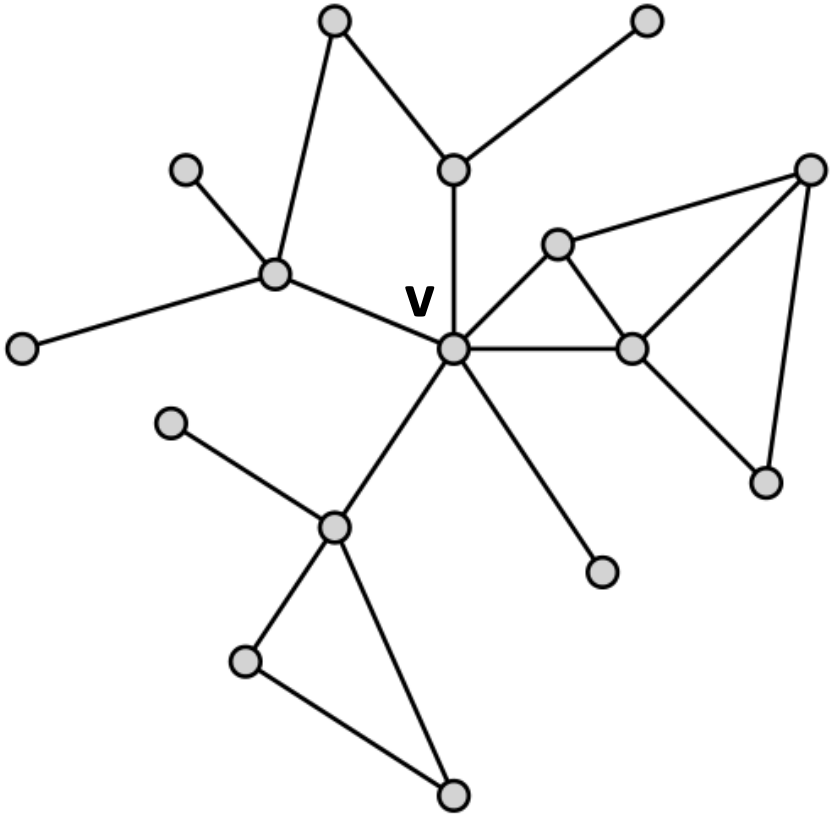

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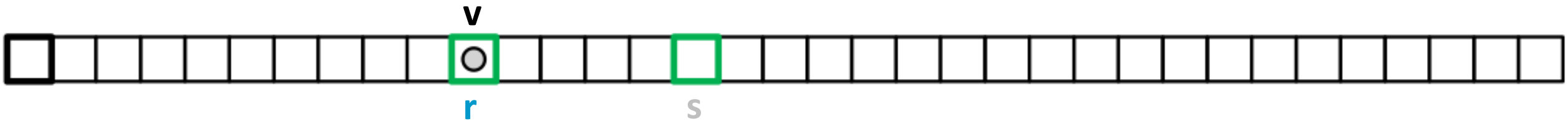

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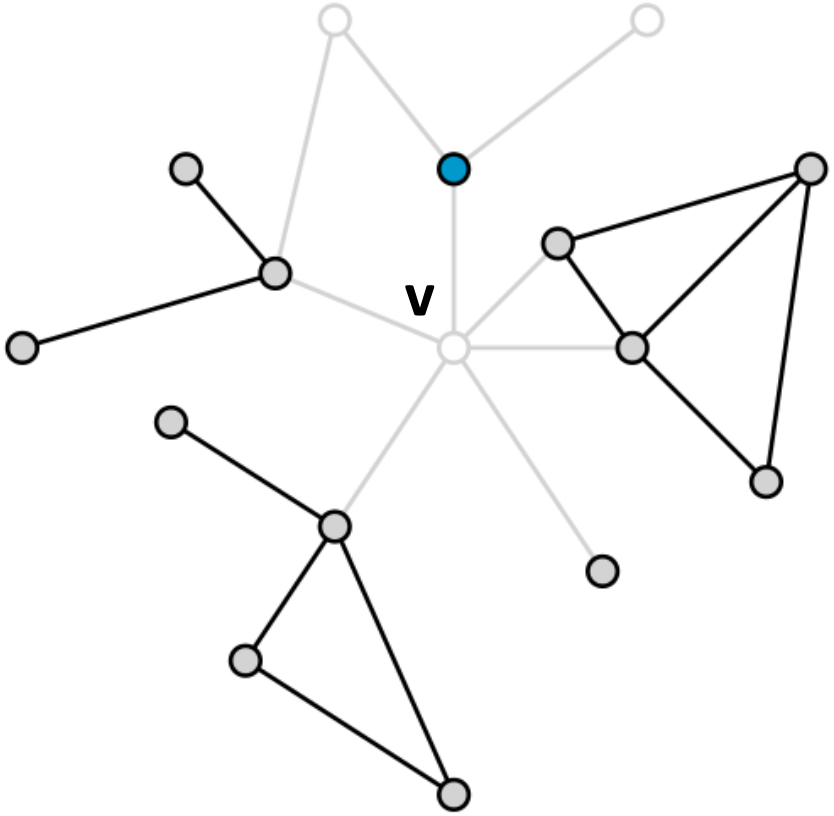

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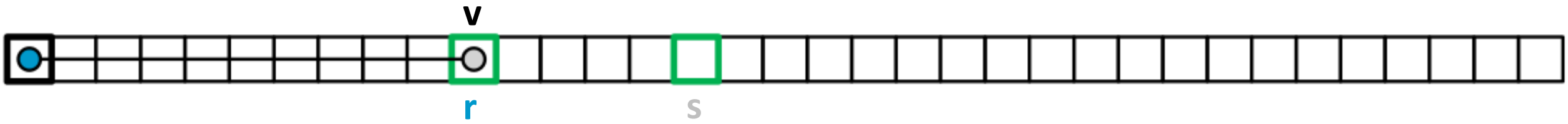

Probability of Continuing Dependency Chain

at step $t \leq r$,

A) v must be alive

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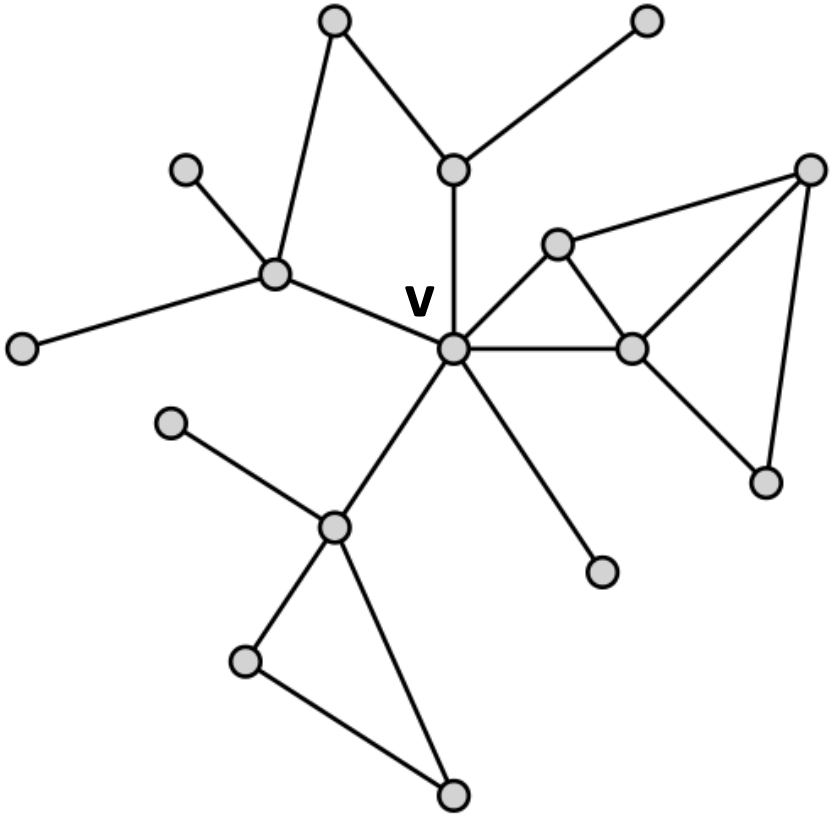
$$\mathbf{P}[\,\mathbf{A}\,] \approx \left(1 - \frac{d_0}{n}\right)$$



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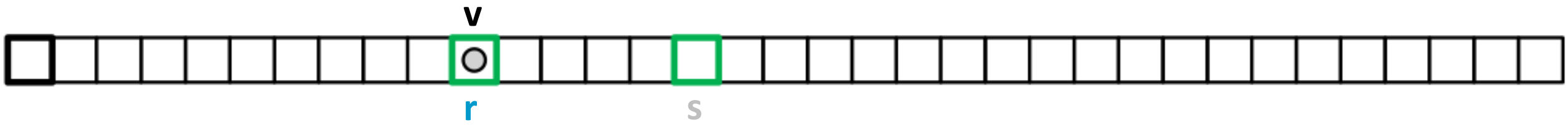

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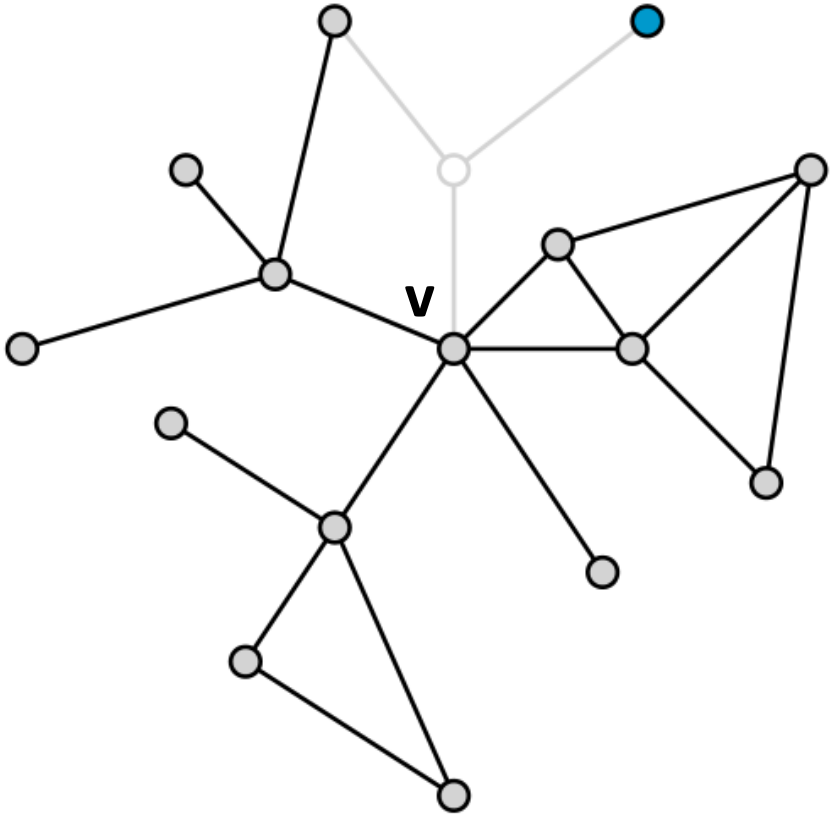

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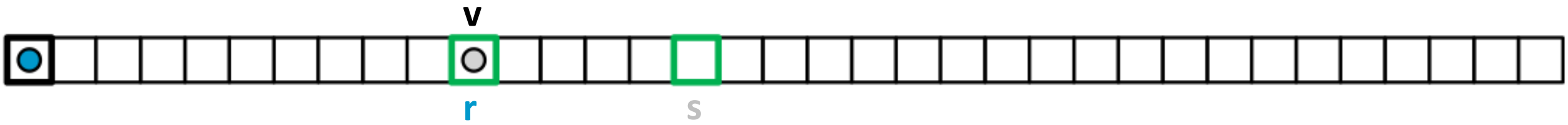

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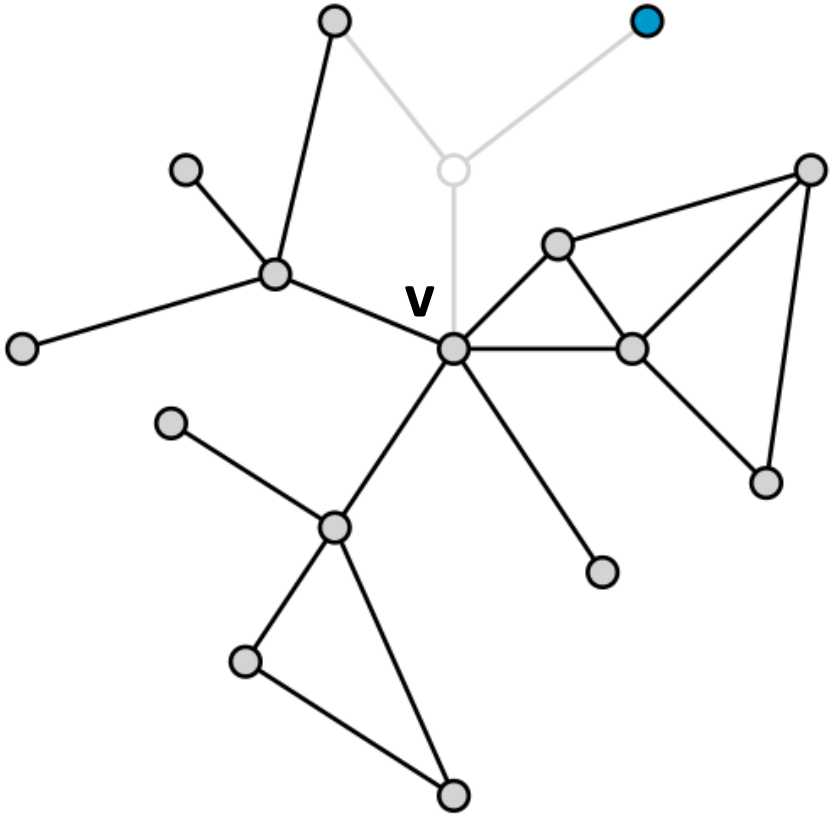

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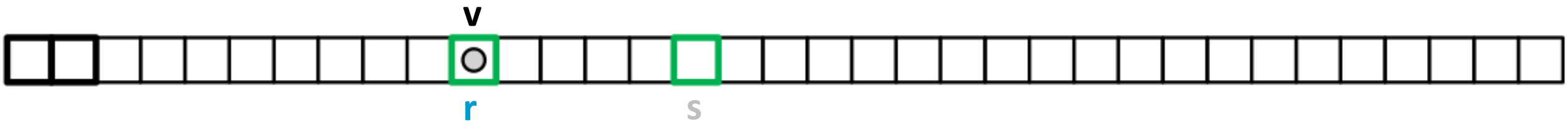

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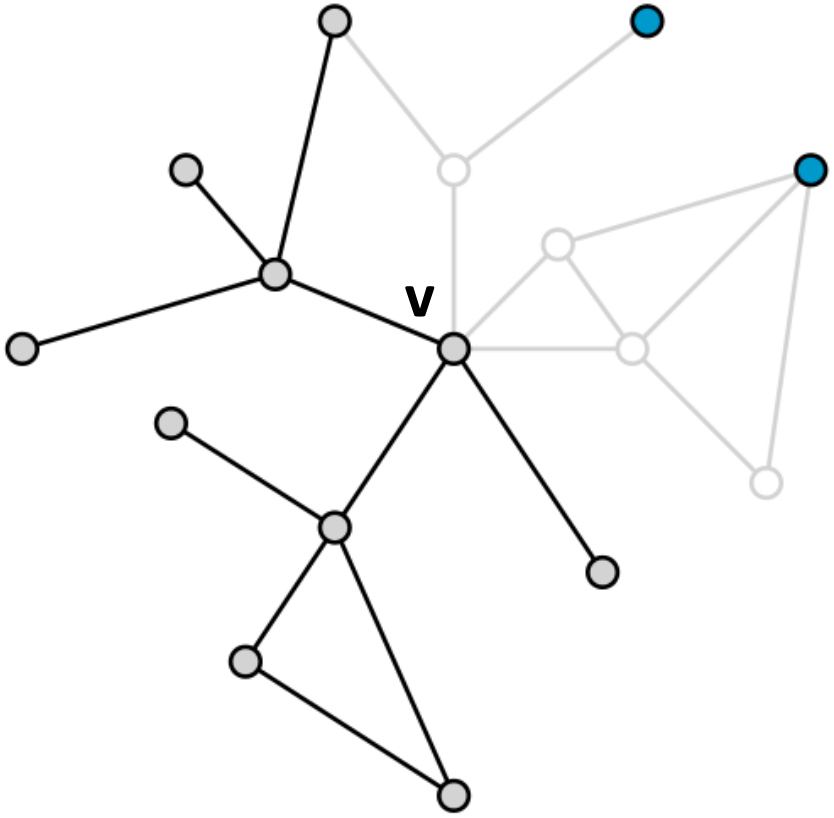

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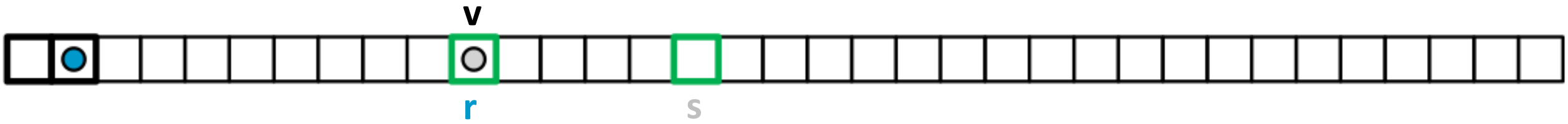

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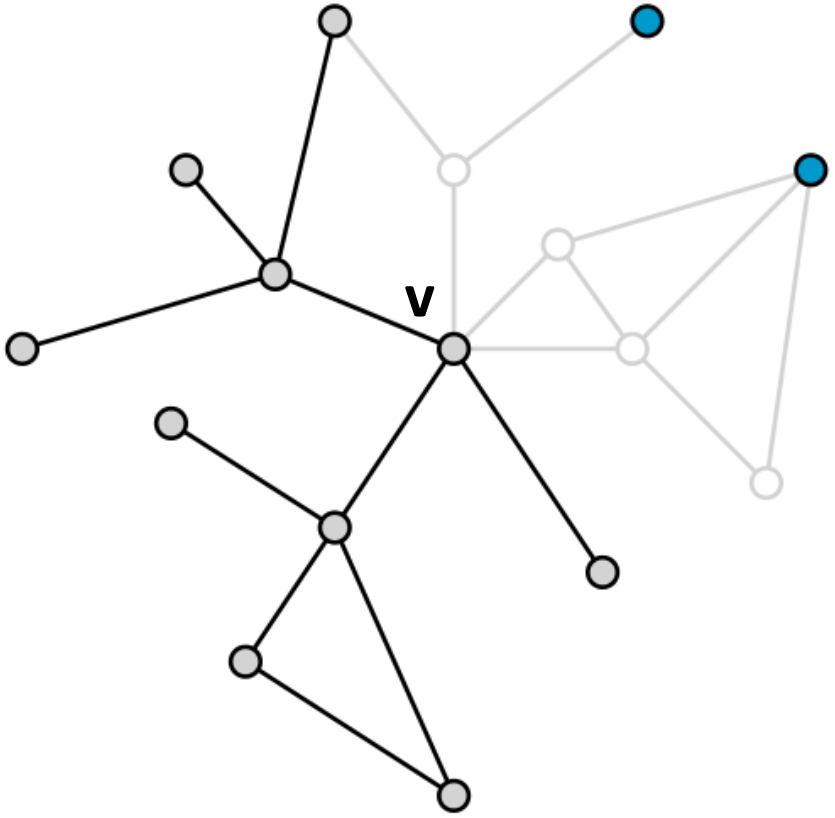

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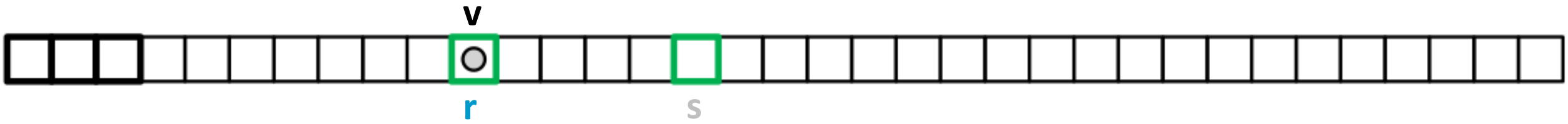

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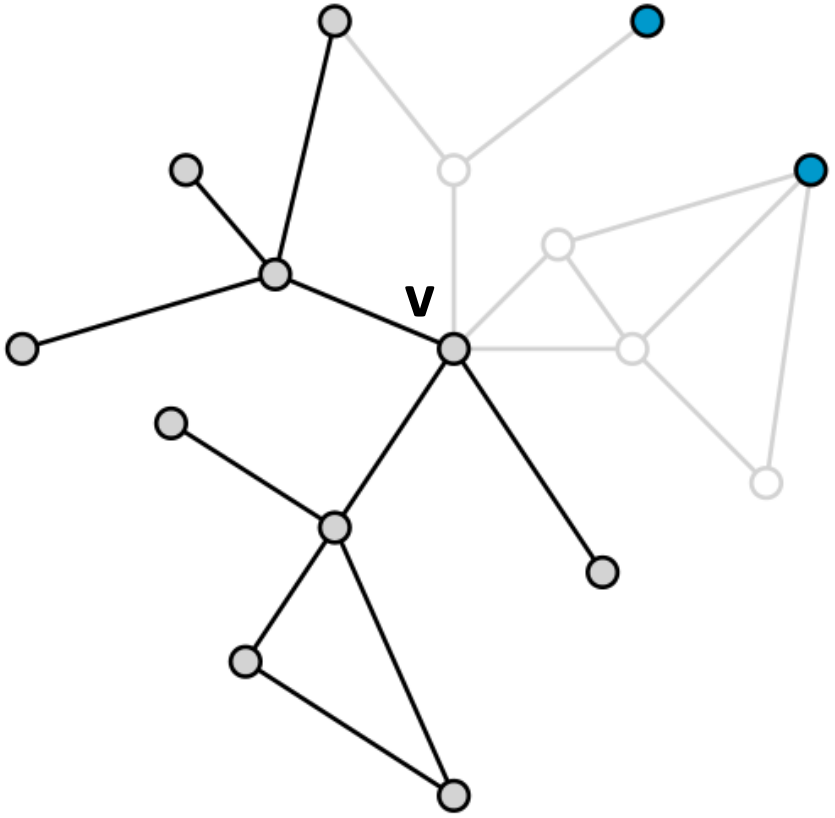

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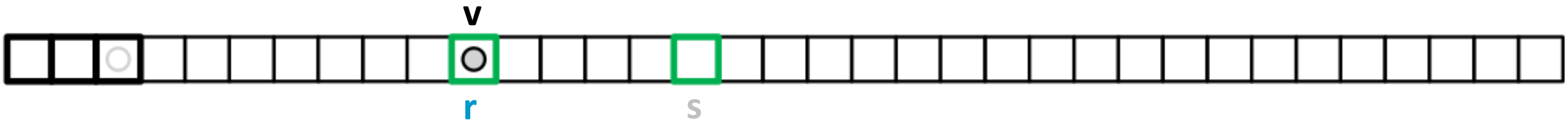

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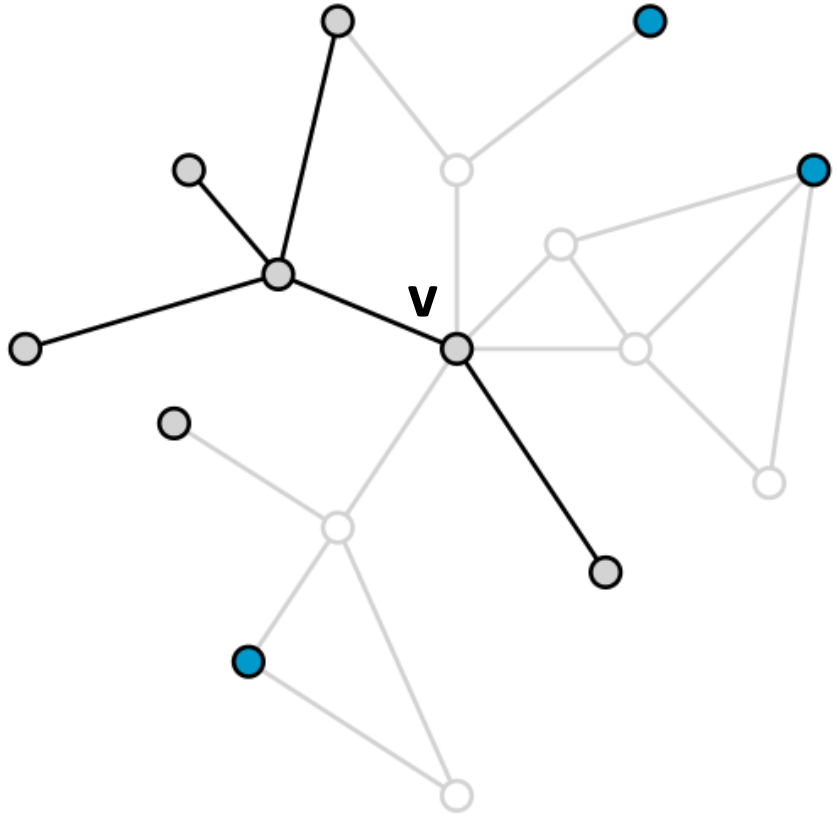

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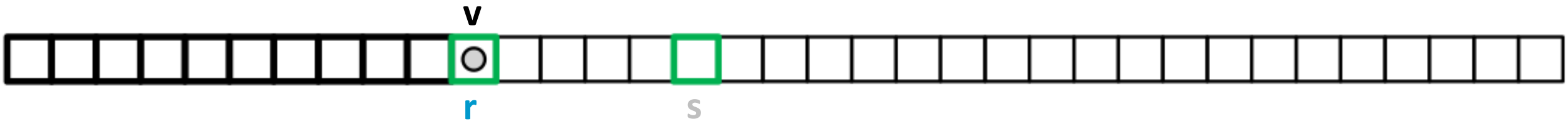

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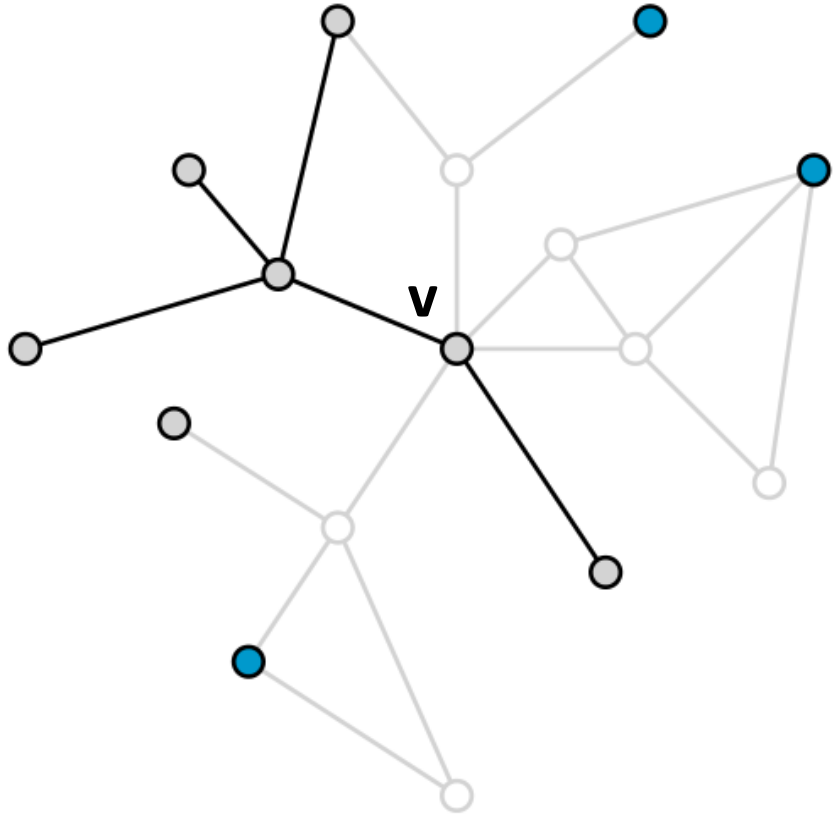

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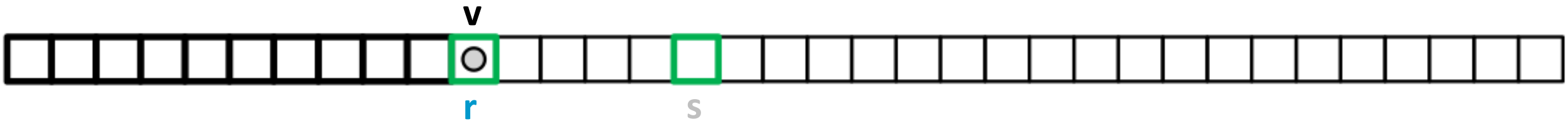

Probability of Continuing Dependency Chain

at step $t \leq r$,

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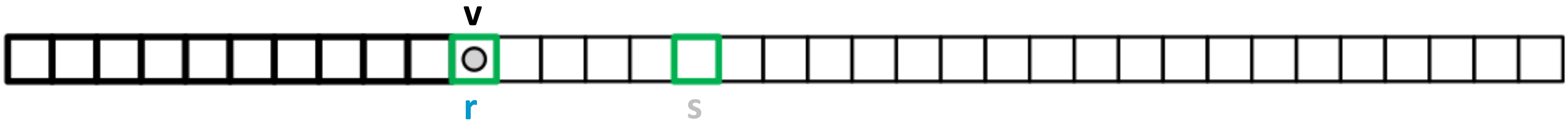
$$\mathbf{P}[\ \mathbf{continue}\ ] \approx \prod_{i=0}^{t-1} \left(1 - \frac{d_i}{n}\right) \cdot \frac{d_t}{n}$$

degree sequence of v: $d_0, d_1, \ldots, d_t$



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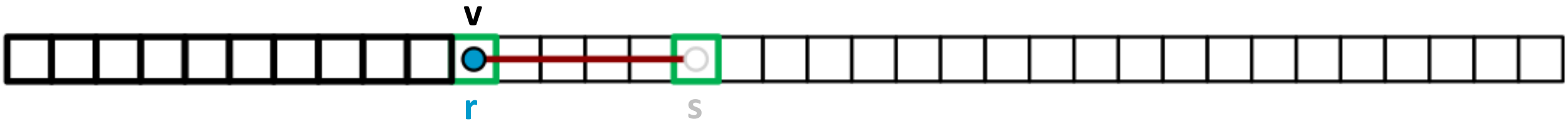

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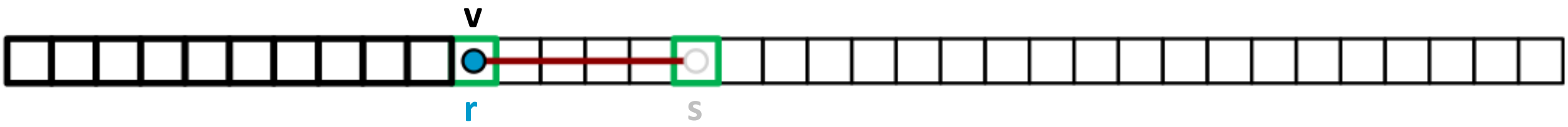
small enough for any degree sequence



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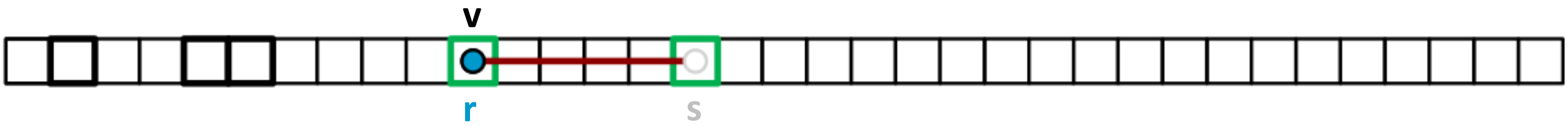
$$\mathbf{P}[\ \mathbf{continue}\ ] \approx \prod_{i=0}^{t-1} \left(1 - \frac{d_i}{n}\right) \cdot \frac{d_t}{n}$$

degree sequence of v: $d_0, d_1, \ldots, d_t$

small enough for any degree sequence, even if we only look at (any) t positions



Recap of Proof

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 # rounds $\leq$ dependency chain length $= \mathbf{O}(\log n)$

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bound probability that $\Omega(\log n)$ positions form dependency chain

union bound over all choices of positions

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reserve disjoint set of positions for each position

use these positions to bound continuation probability of position

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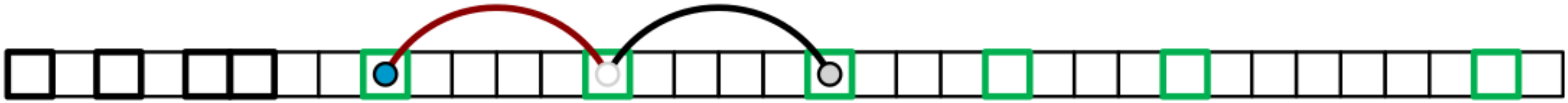

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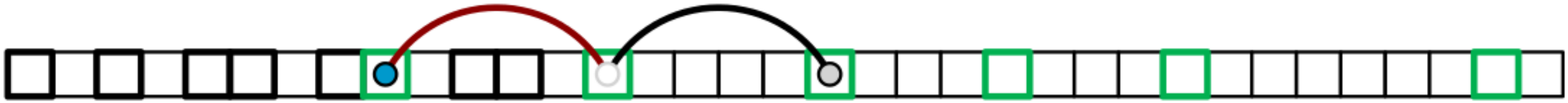

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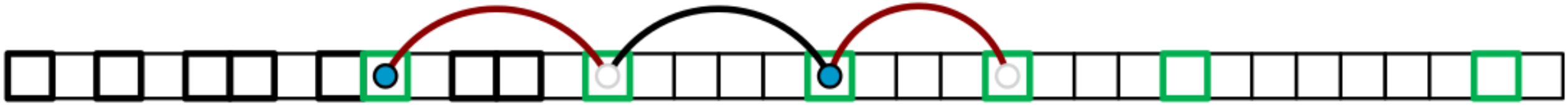

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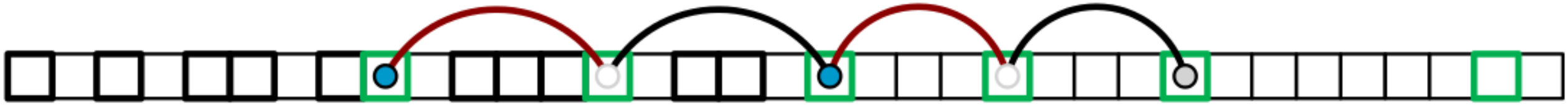

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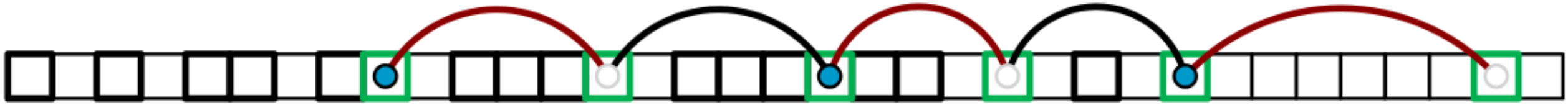

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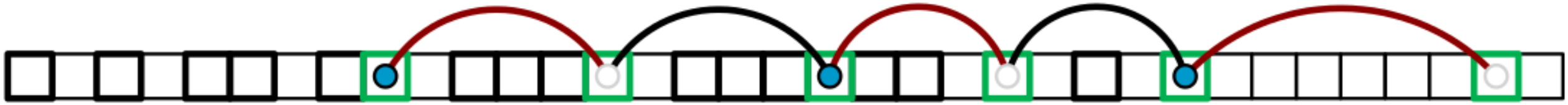
 # rounds $\leq$ dependency chain length $= O(\log n)$

bound probability that $\Omega(\log n)$ positions form dependency chain



reserve disjoint set of positions for each position

use these positions to bound continuation probability of position

probability of dependency chain is product of continuation probabilities

union bound over all choices of positions

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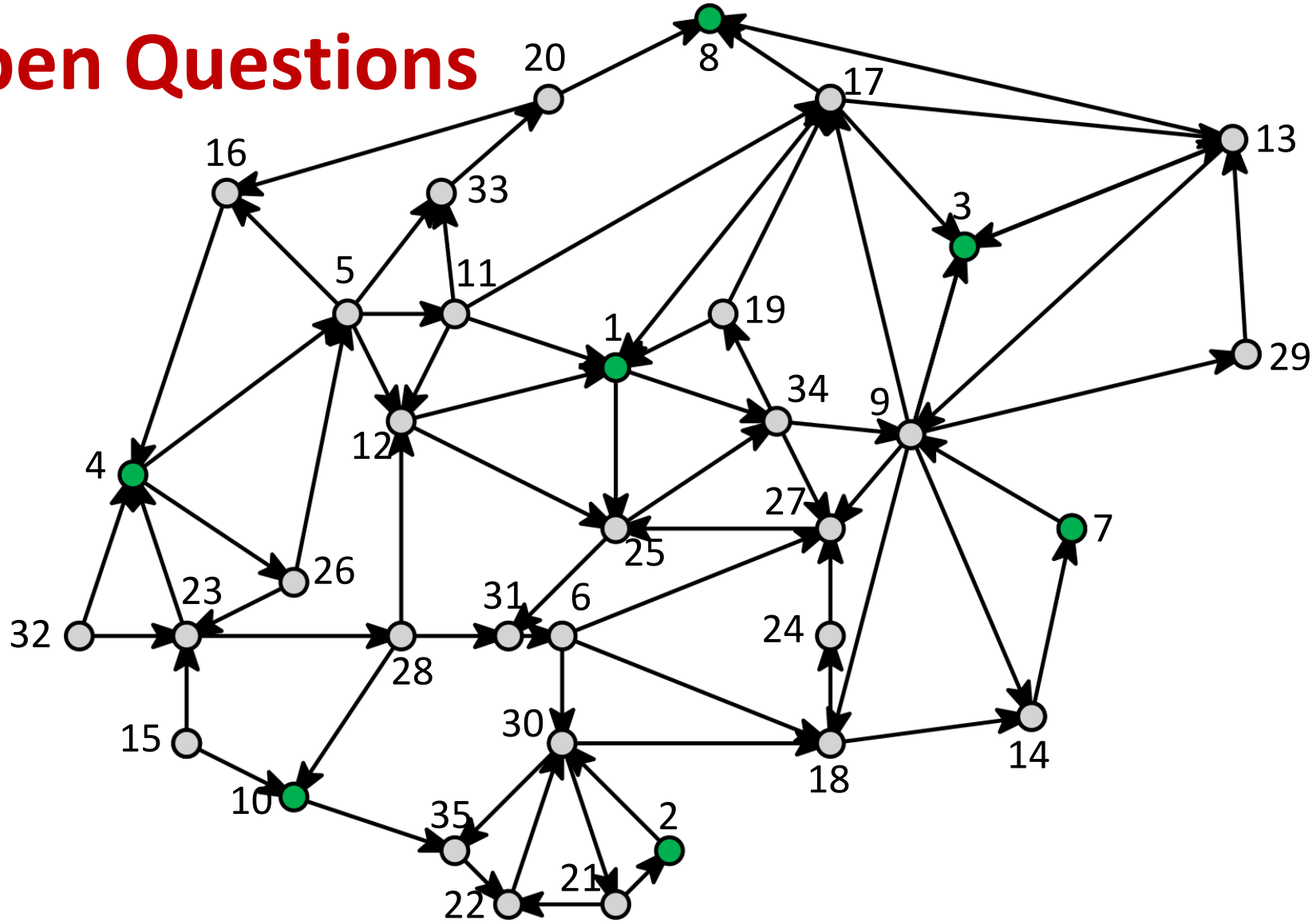

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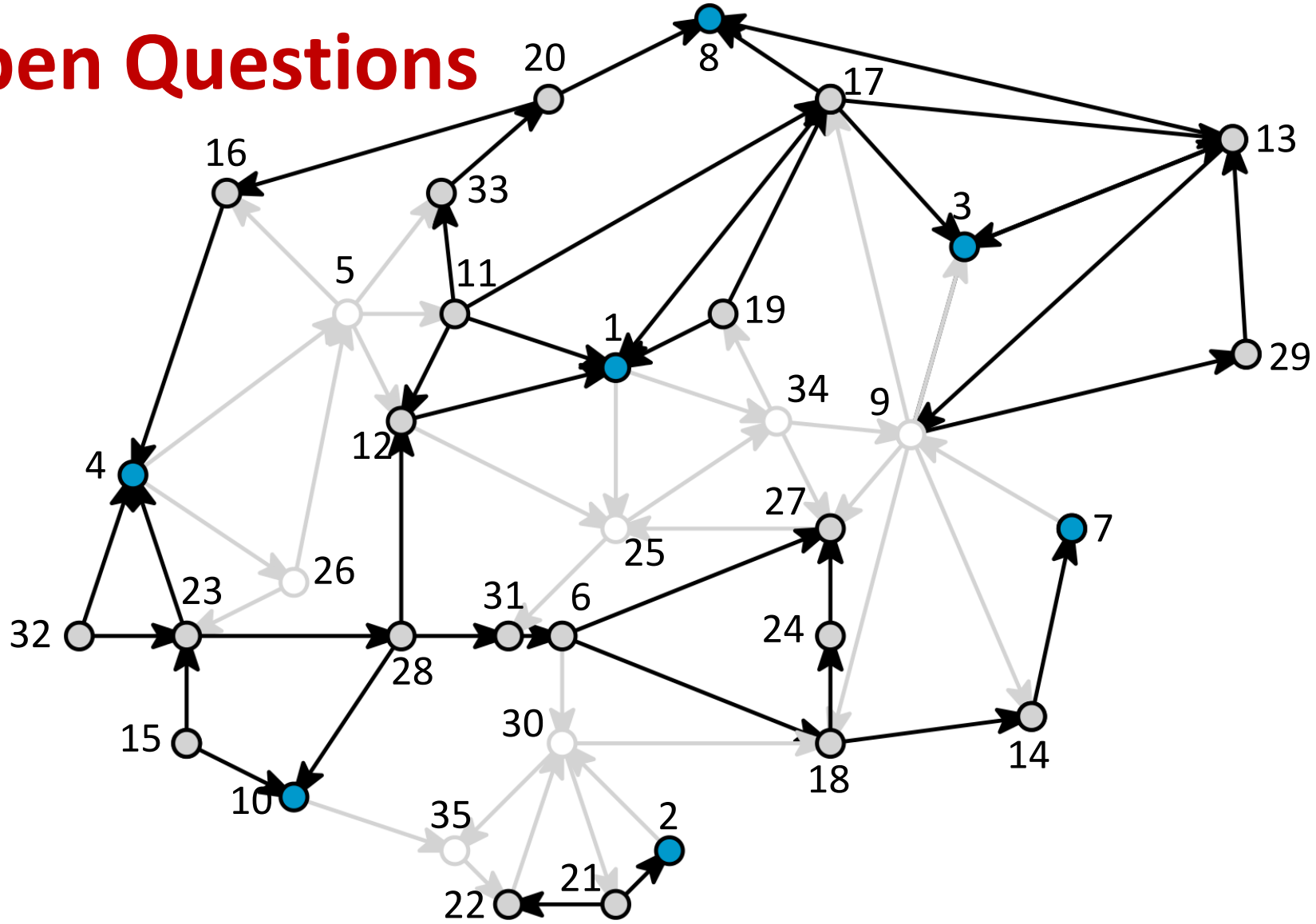

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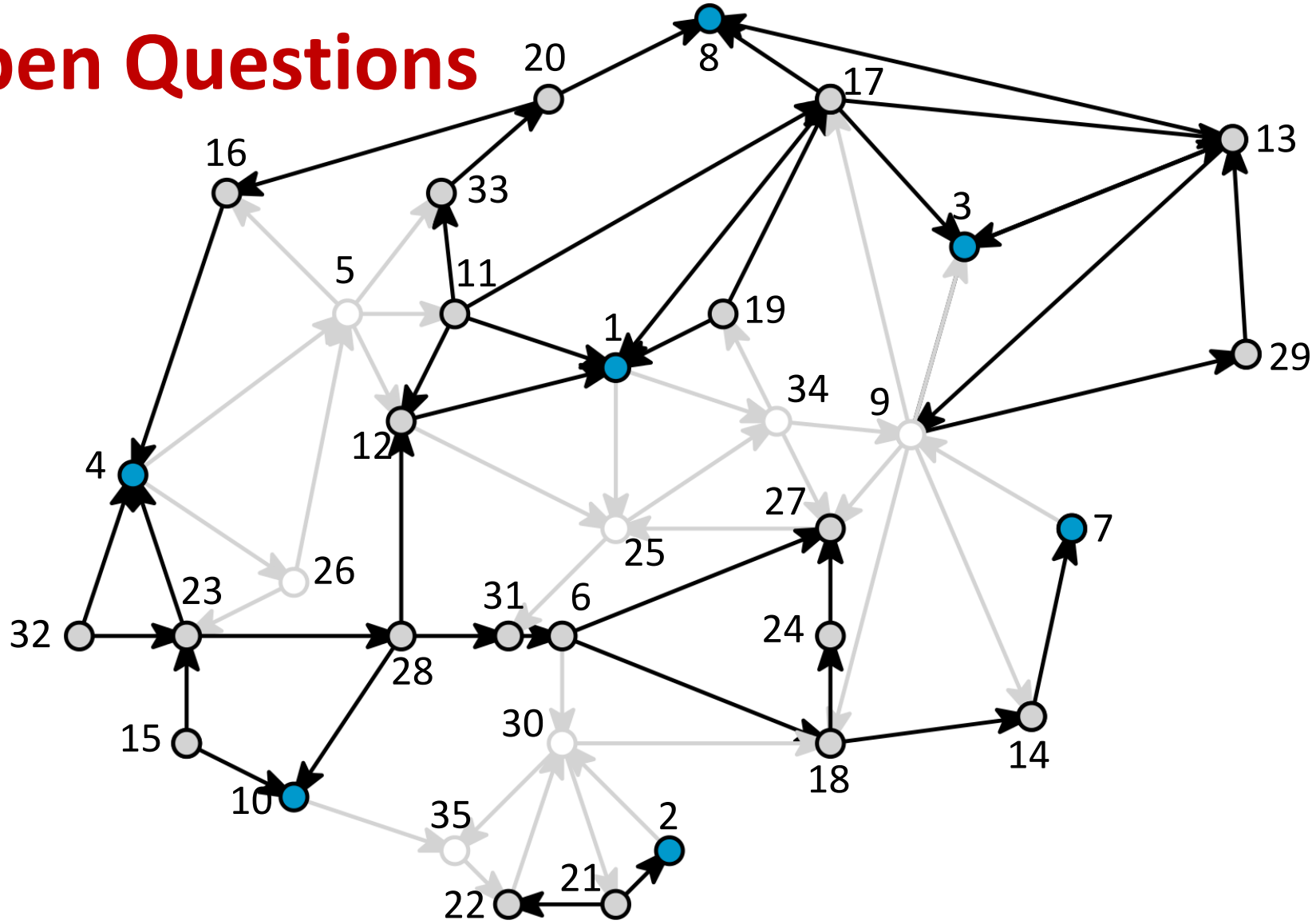

Thank you!

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