Improved Deterministic Distributed Matching via Rounding

Manuela Fischer
ETH Zurich

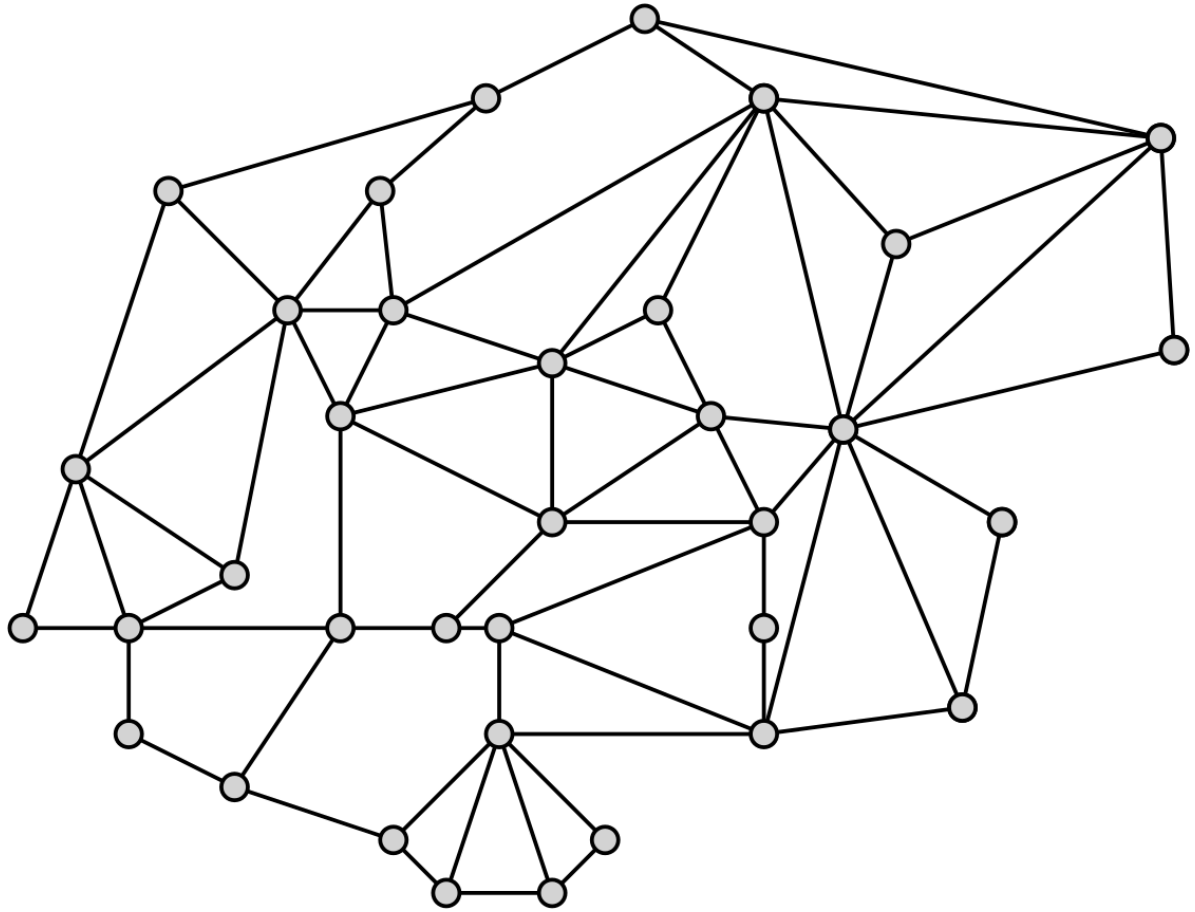
LOCAL Model *Linial* [FOCS'87]

- undirected graph $G = (V, E)$,
  n nodes, maximum degree Δ

- synchronous message-passing rounds
- unbounded message size

- unbounded computation

- **Round Complexity**:
  number of rounds to solve the problem



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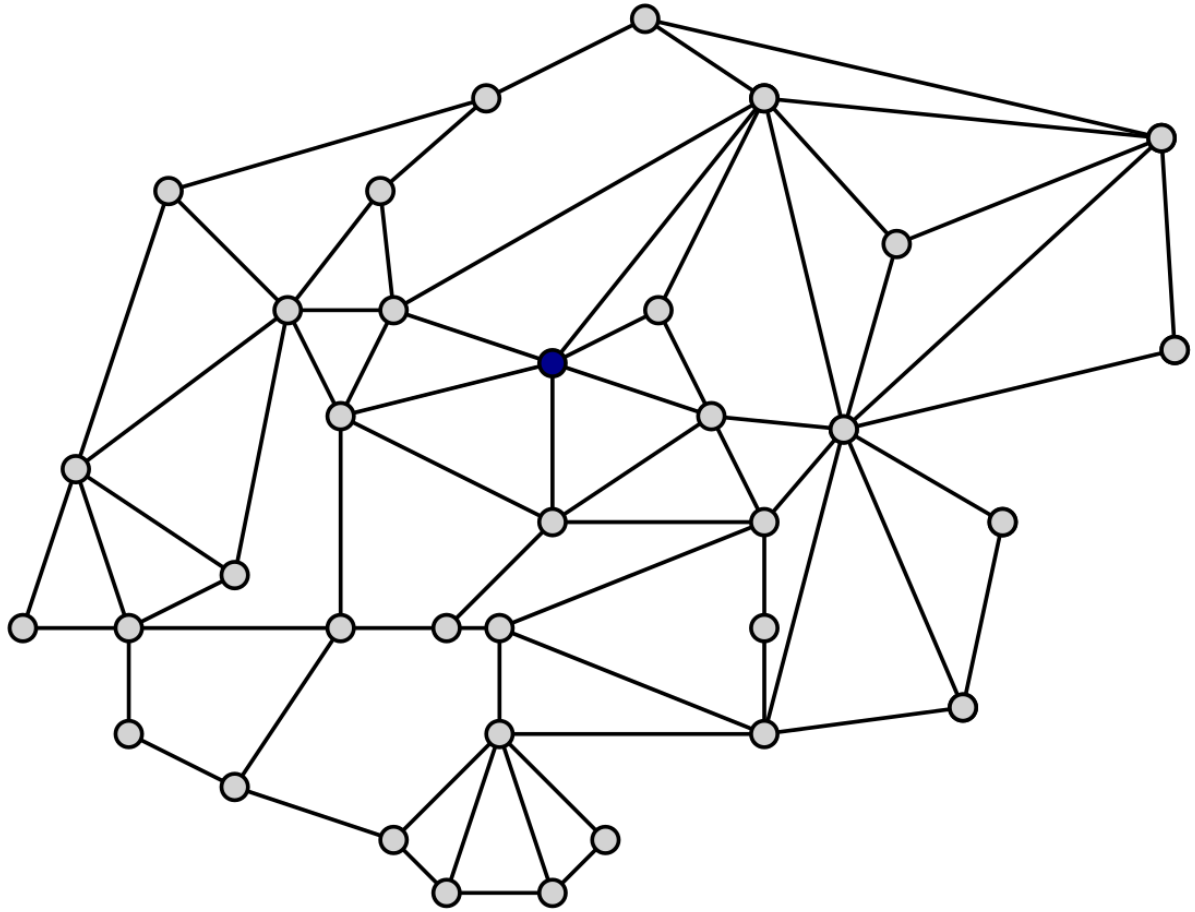

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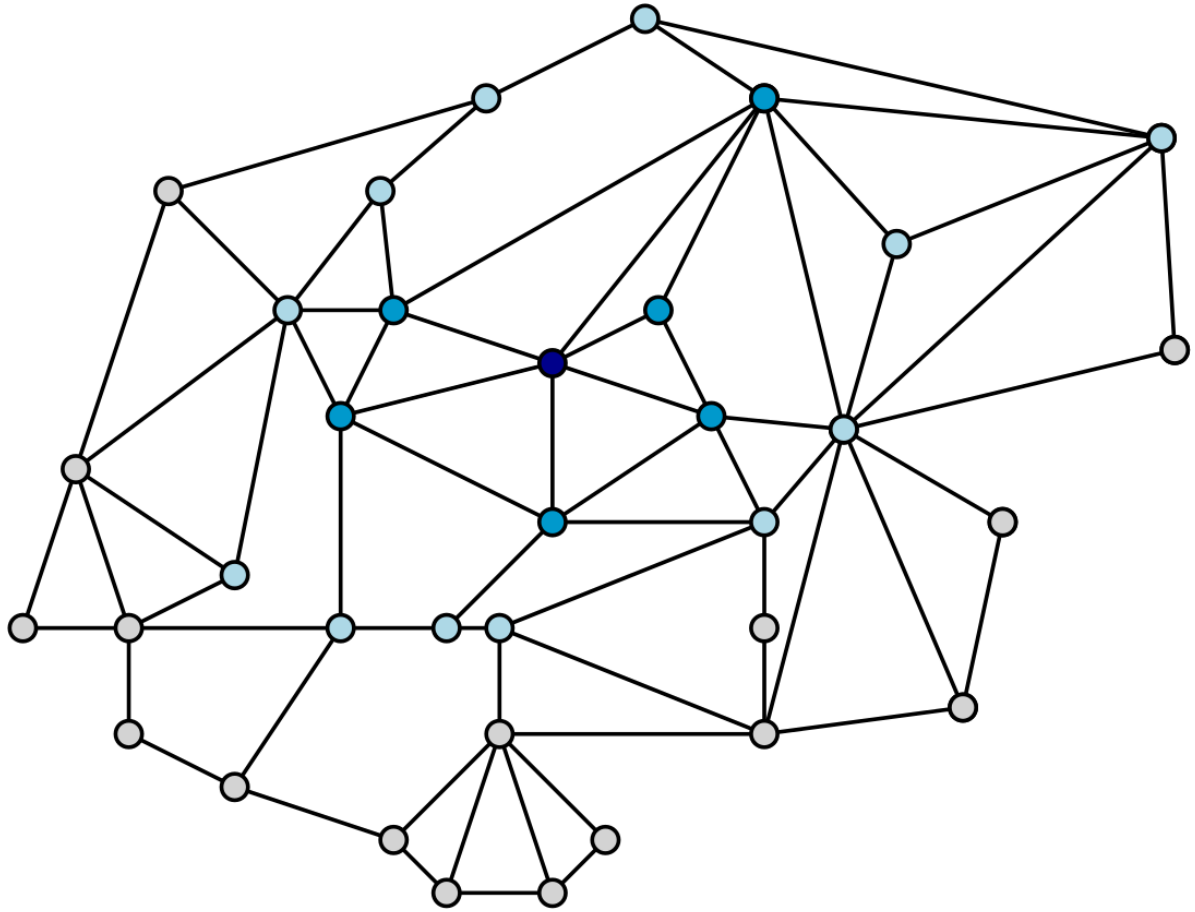
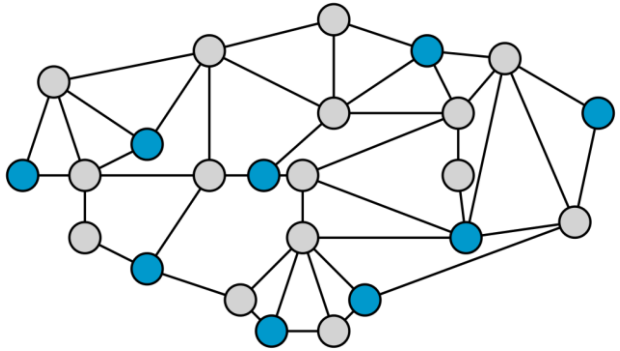


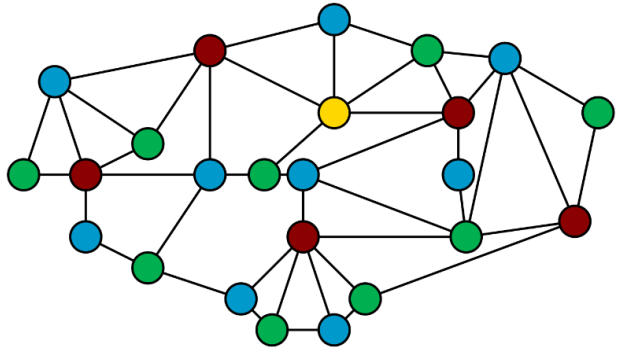
round complexity of a problem characterizes its locality

Classic LOCAL Graph Problems

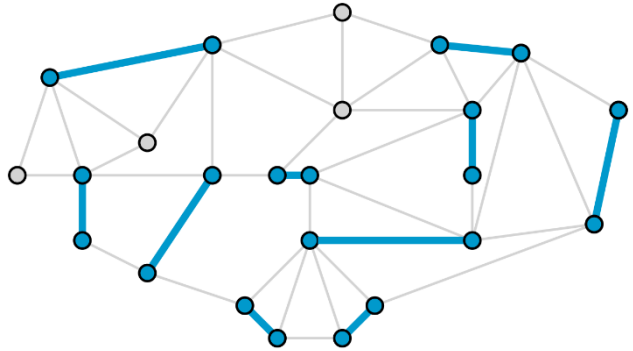
Maximal Independent Set



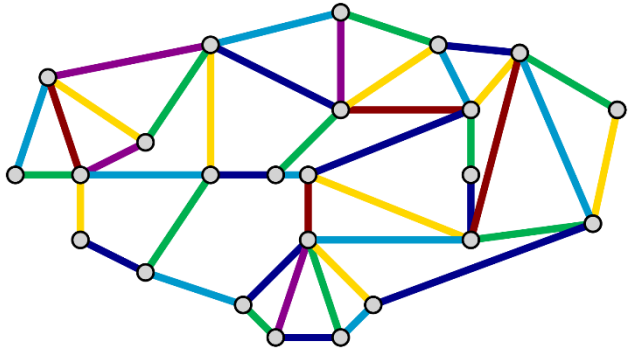
$(\Delta + 1)$-Vertex-Coloring



Maximal Matching



$(2\Delta - 1)$-Edge-Coloring



<u>Question from the 1980s</u> by Linial:

Are there efficient ($\text{poly}\log n$ rounds) **deterministic** algorithms for these problems?

Ghaffari, Kuhn, Maus [STOC'17]

Completeness of Deterministic Rounding

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Completeness of Deterministic Rounding

Efficient **deterministic rounding** is **the only obstacle** for efficient deterministic LOCAL graph algorithms.

Rounding:
Turning fractional values into integral ones
while approximately preserving some linear constraints.

Our Contribution:

Deterministic Rounding for Matching

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Deterministic Rounding for Matching

Matching admits an **efficient deterministic algorithm** because matching admits **efficient deterministic rounding**.

- first deterministic LOCAL rounding method
- not only improving over state of the art of matching algorithms, but (hopefully) step in direction of solving all classic LOCAL problems
- extension led to first efficient $(2\Delta - 1)$-edge-coloring algorithm *F., Ghaffari, Kuhn* [FOCS'17]

Our Results

- **Maximal Matching** $O(\log^2 \Delta \cdot \log n)$
- **Randomized** Maximal Matching $O(\log^3 \log\ n + \log \Delta)$

- $(\mathbf{2+\varepsilon})$ **- Approximate** Maximum Matching $O\left(\log^2 \Delta \cdot \log \frac{1}{\varepsilon} + \log^* n\right)$
- $(2+\varepsilon)$ - Approximate Maximum **Weighted Matching** $O\left(\log^2 \Delta \cdot \log \frac{1}{\varepsilon} + \log^* n\right)$

- $(2+\varepsilon)$ - Approximate Maximum (Weighted) **B-Matching** $O\left(\log^2 \Delta \cdot \log \frac{1}{\varepsilon} + \log^* n\right)$

- $\boldsymbol{\varepsilon}$ **- Maximal Matching** $O\left(\log^2 \Delta \cdot \log \frac{1}{\varepsilon}\right)$
- $(2+\varepsilon)$ - Approximate Minimum **Edge Dominating Set** $O\left(\log^2 \Delta \cdot \log \frac{1}{\varepsilon}\right)$

Our Results

$O(\log^4 n)$
Hańćkowiak, Karoński, Panconesi [SODA'98, PODC'99]

$O(\Delta + \log^* n)$
Panconesi, Rizzi [DIST'01]

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I) 4 - Approximate Fractional Matching

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$O(\log^2 \Delta)$ rounds, $O(1)$ loss

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$$\max \sum_{e \in E} x_e$$

$$\text{s.t.} \sum_{e \in E(v)} x_e \leq 1 \quad \text{for all } v \in V$$

$$x_e \in [0,1] \quad \text{for all } e \in E$$

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repeat until all edges are blocked

mark half-tight nodes

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double value of unblocked edges

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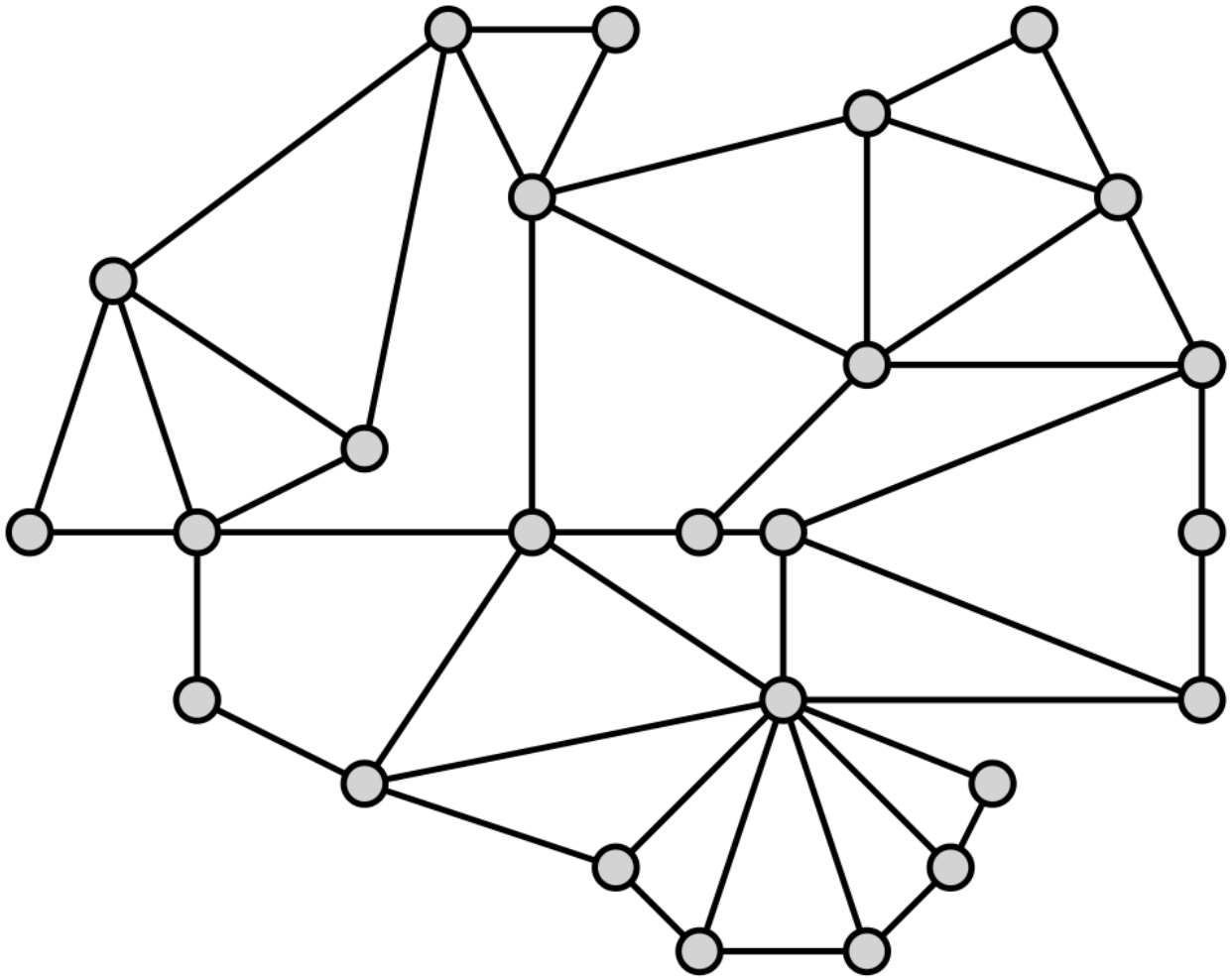

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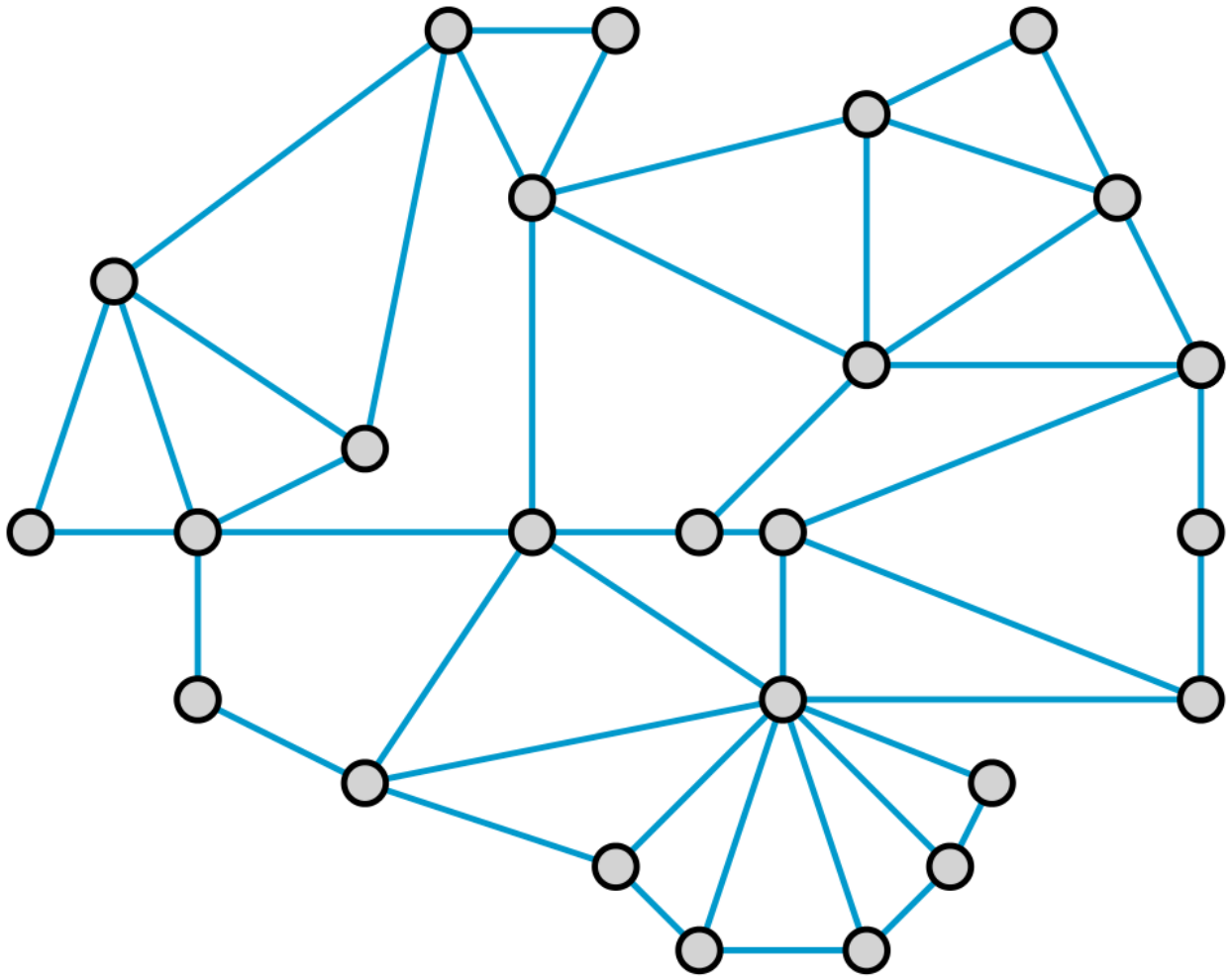

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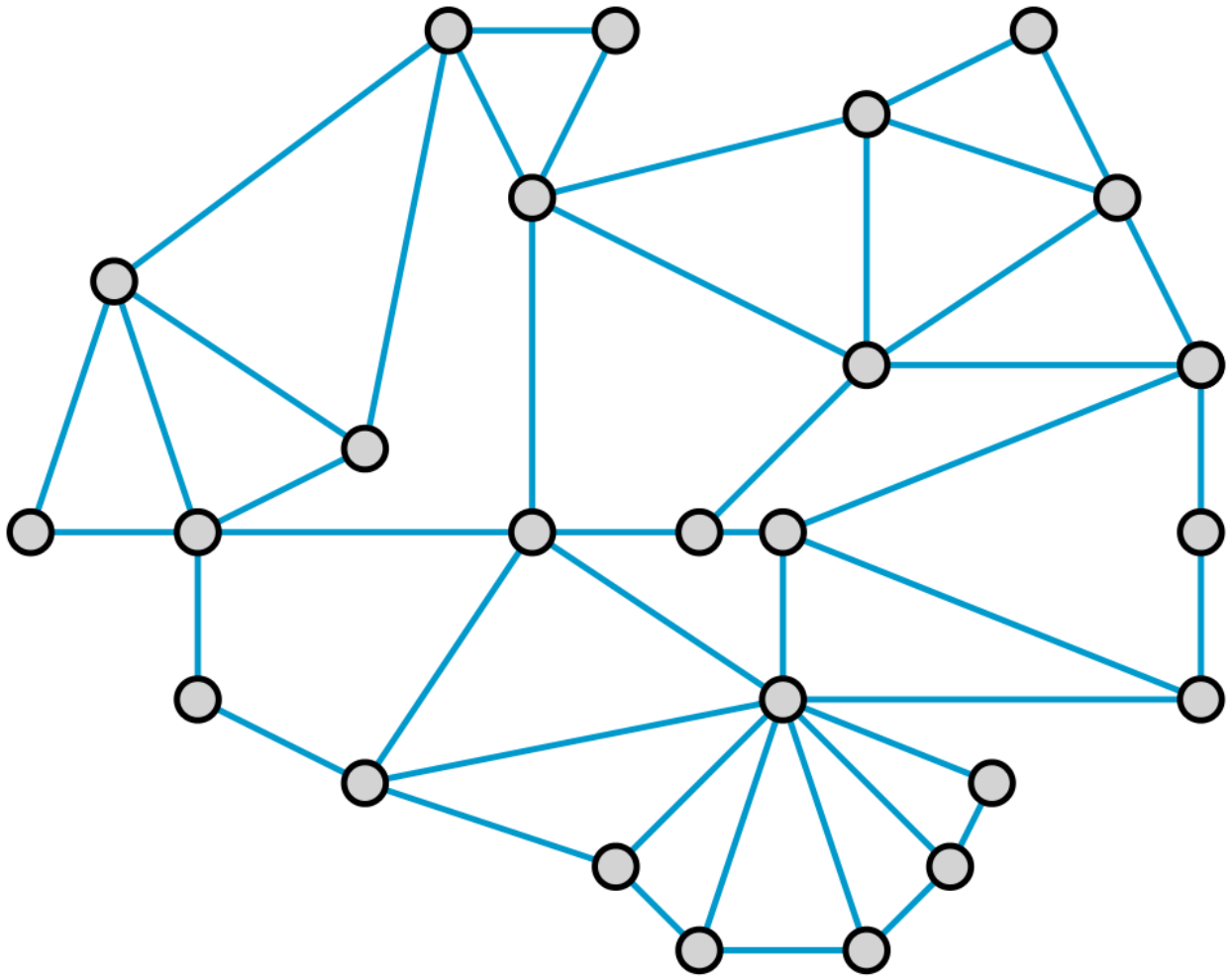

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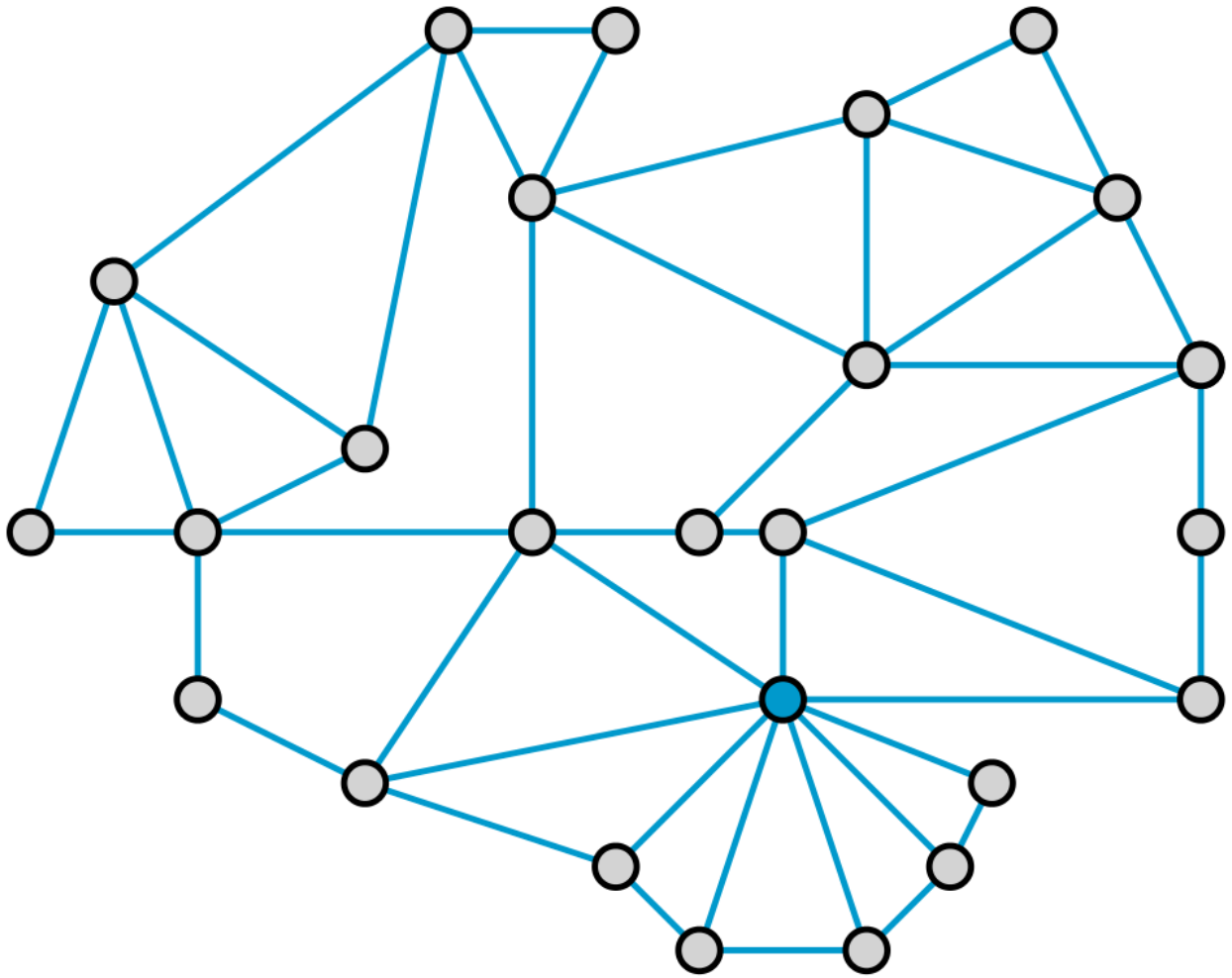

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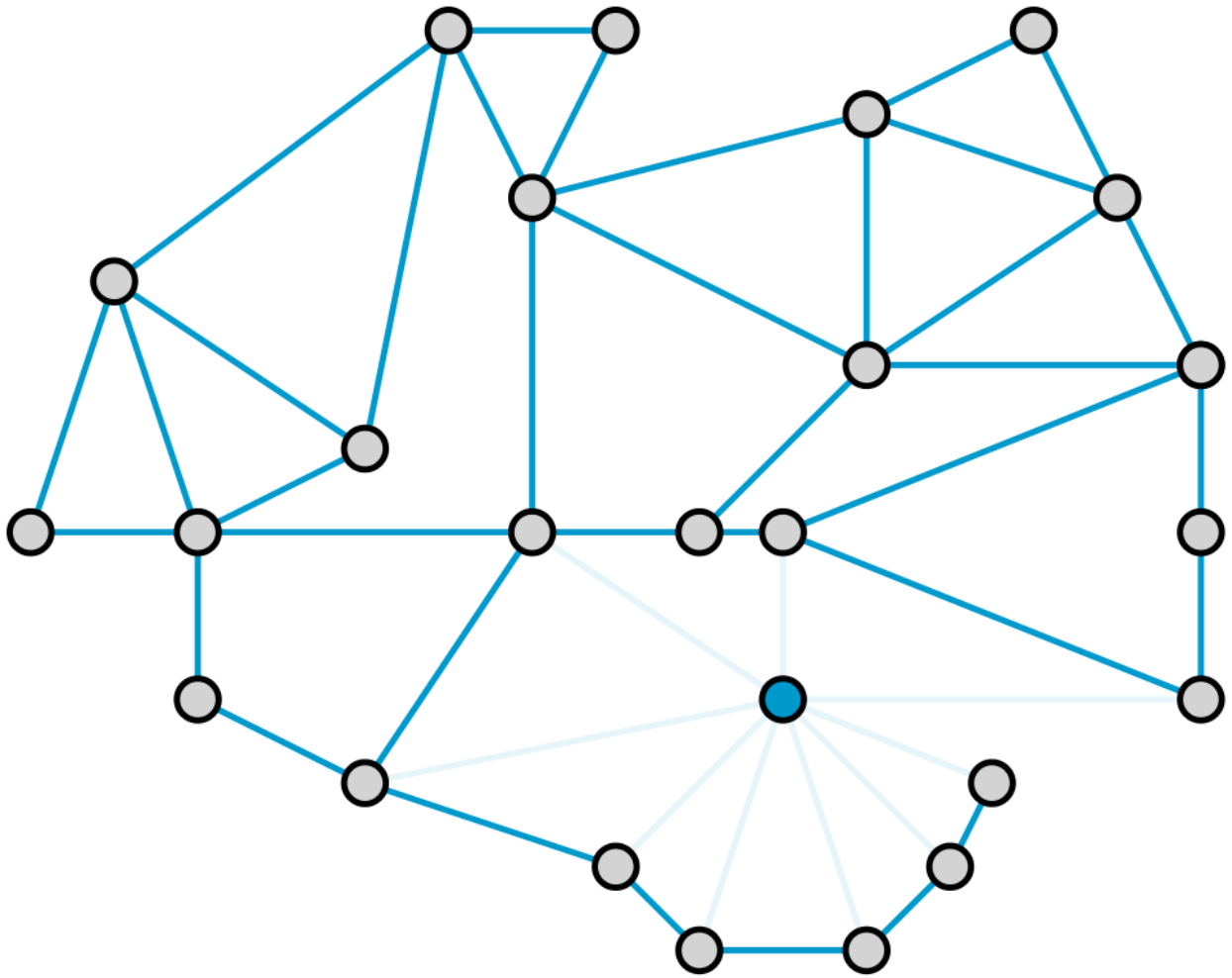

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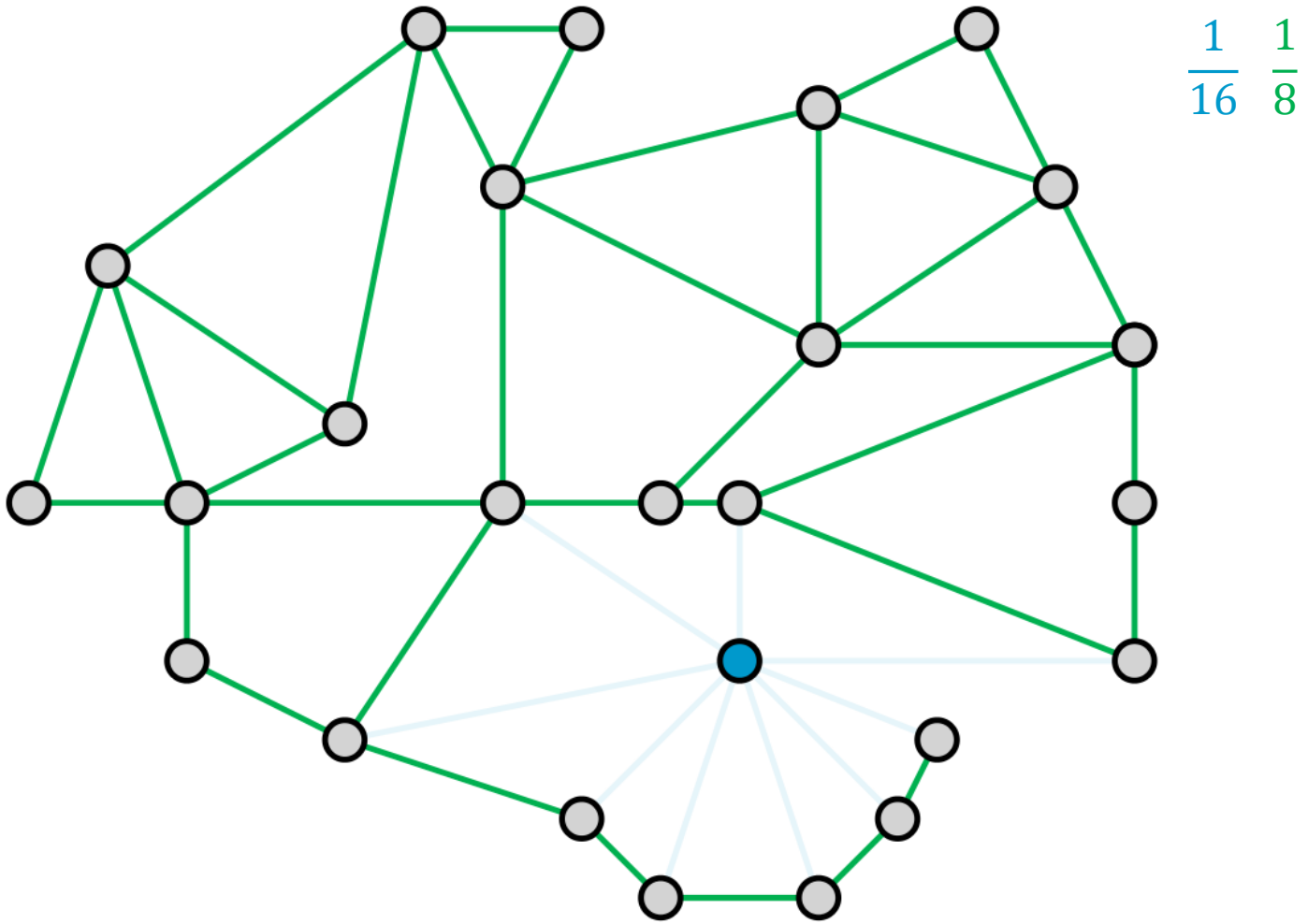

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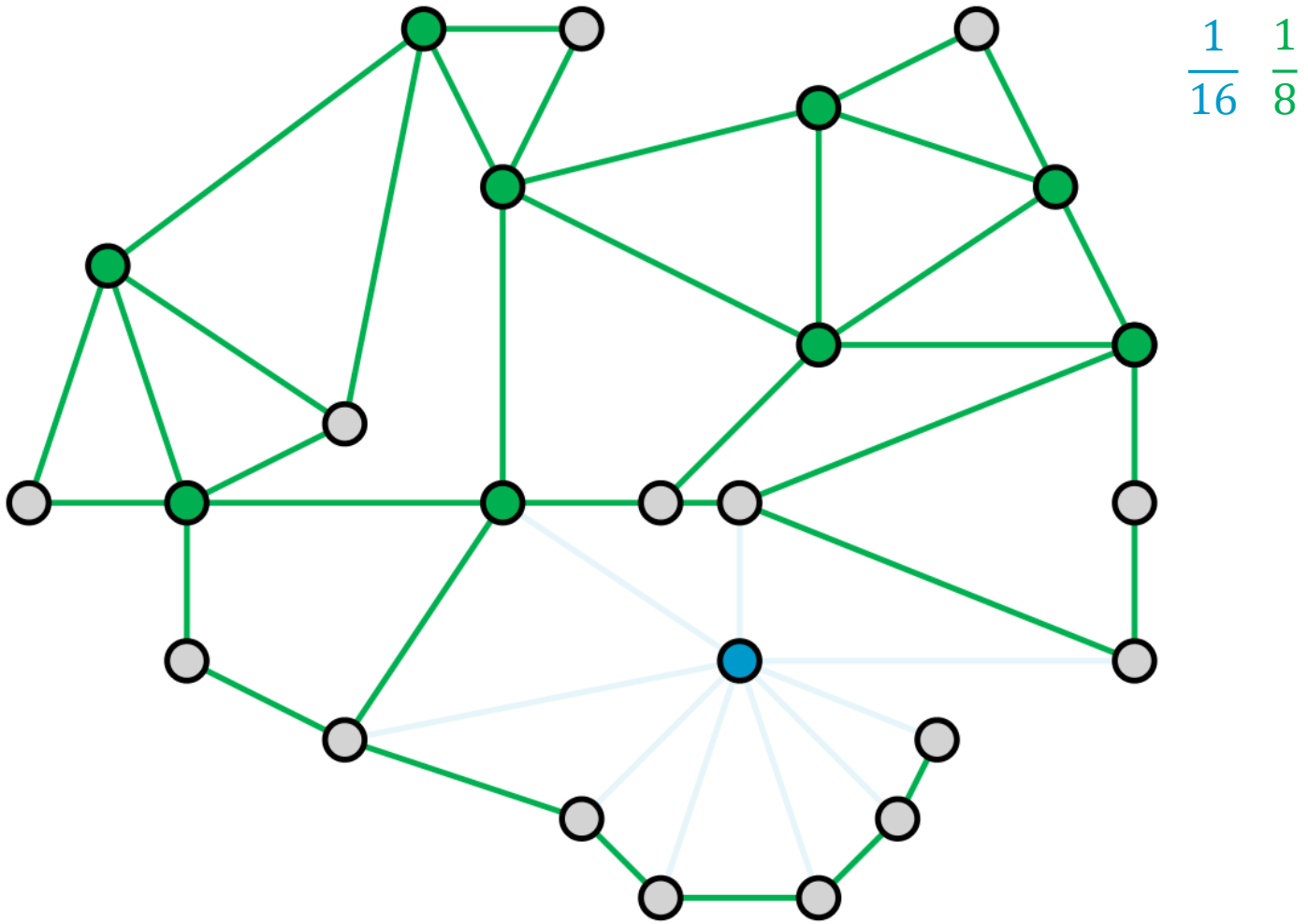

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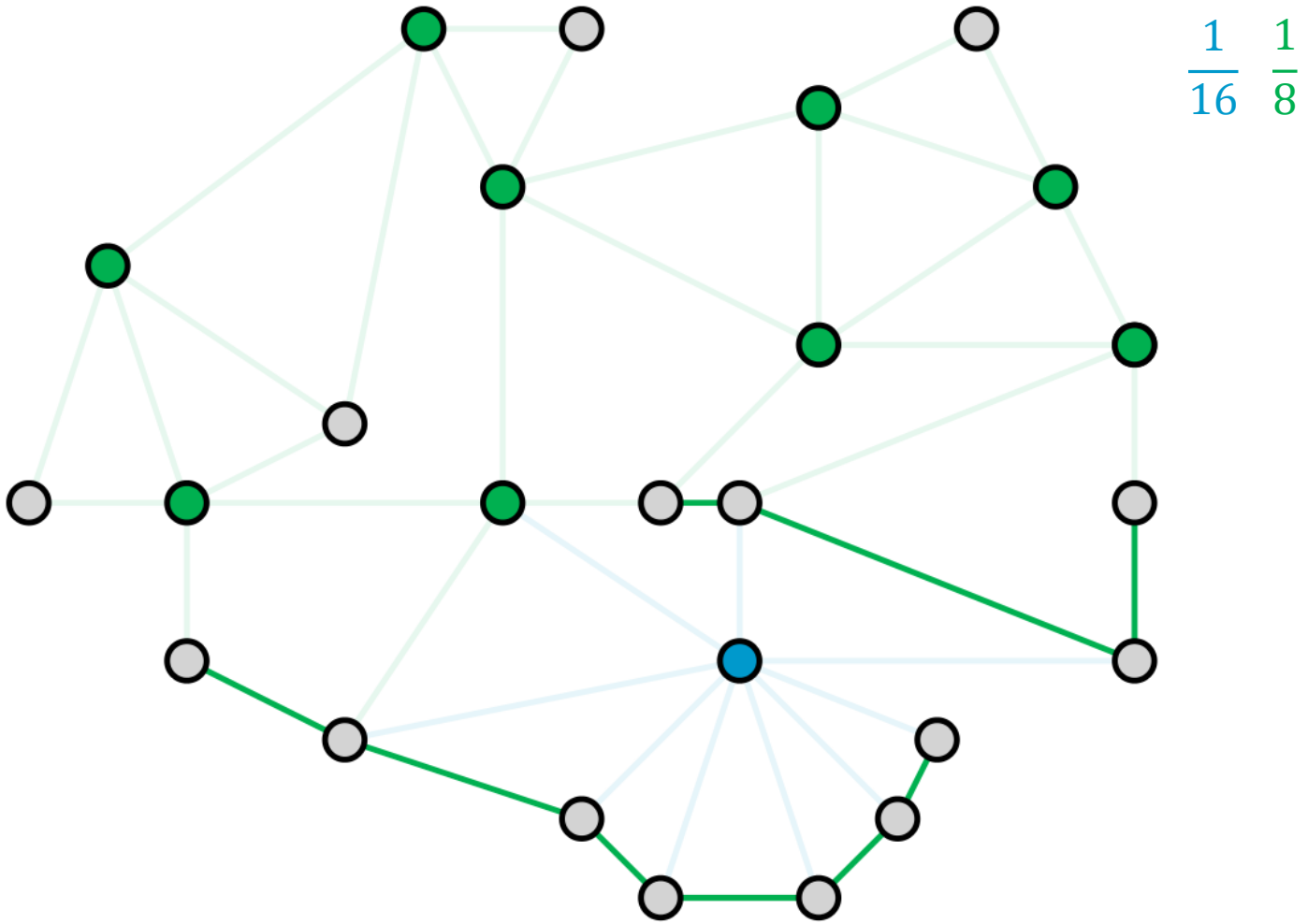

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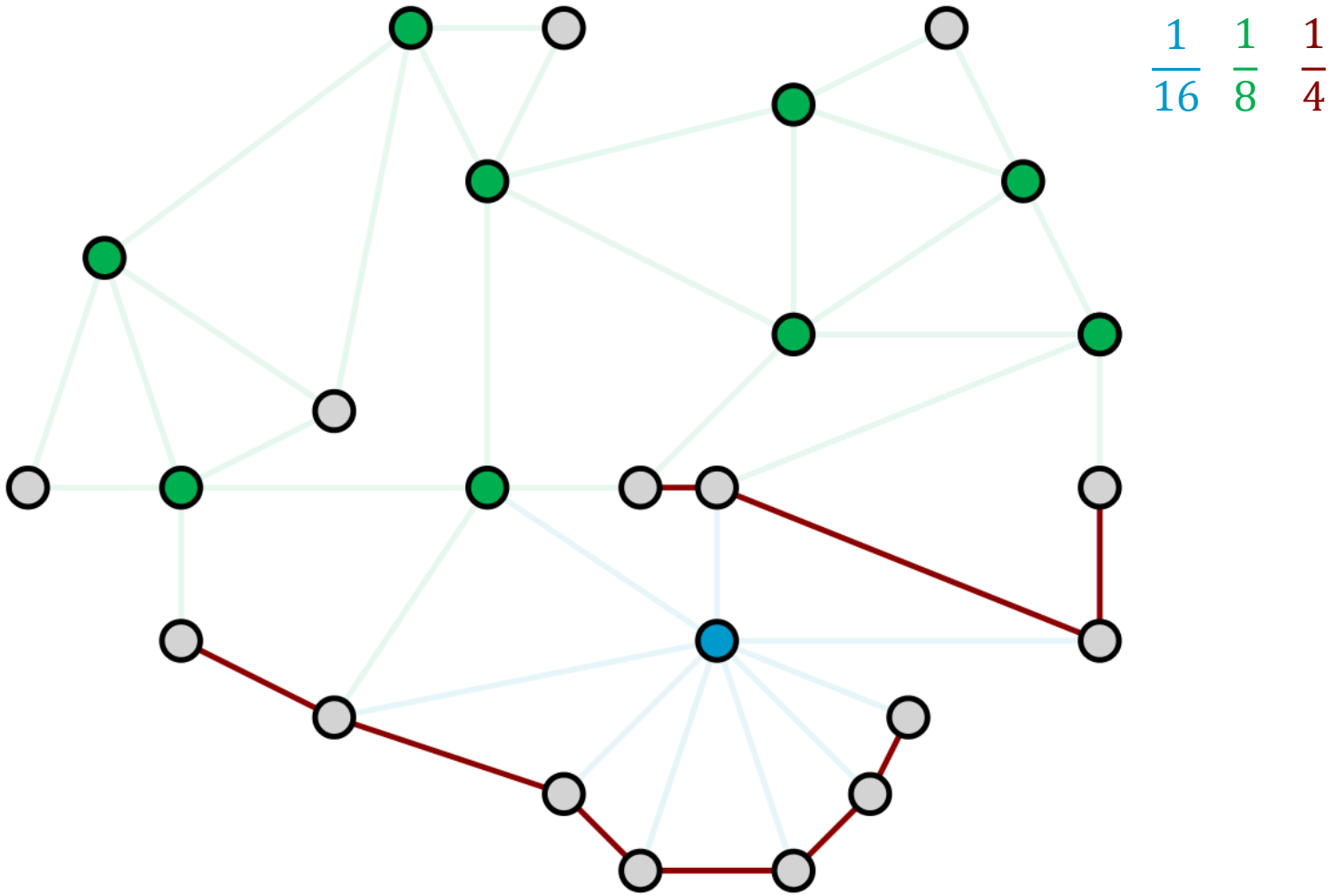

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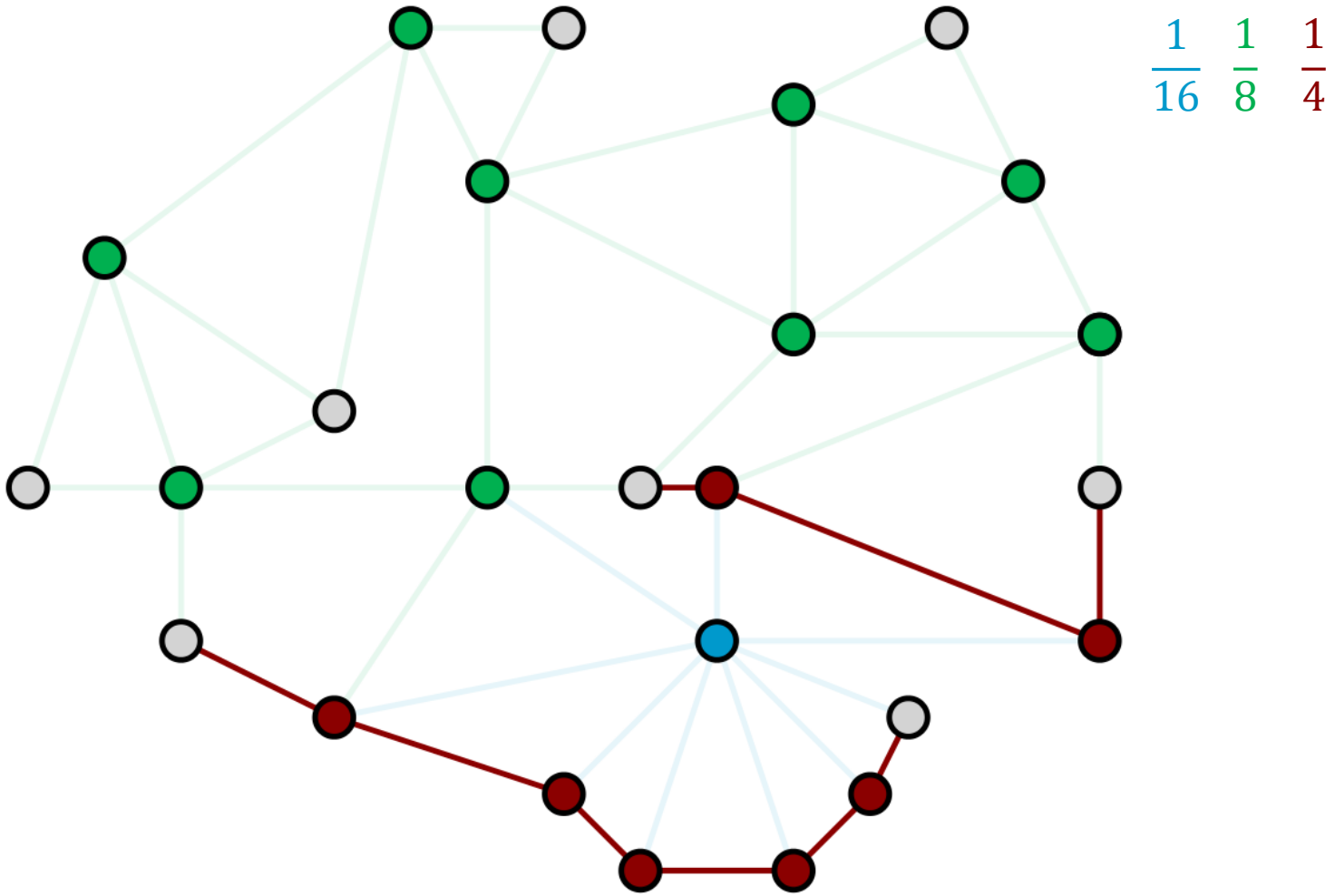

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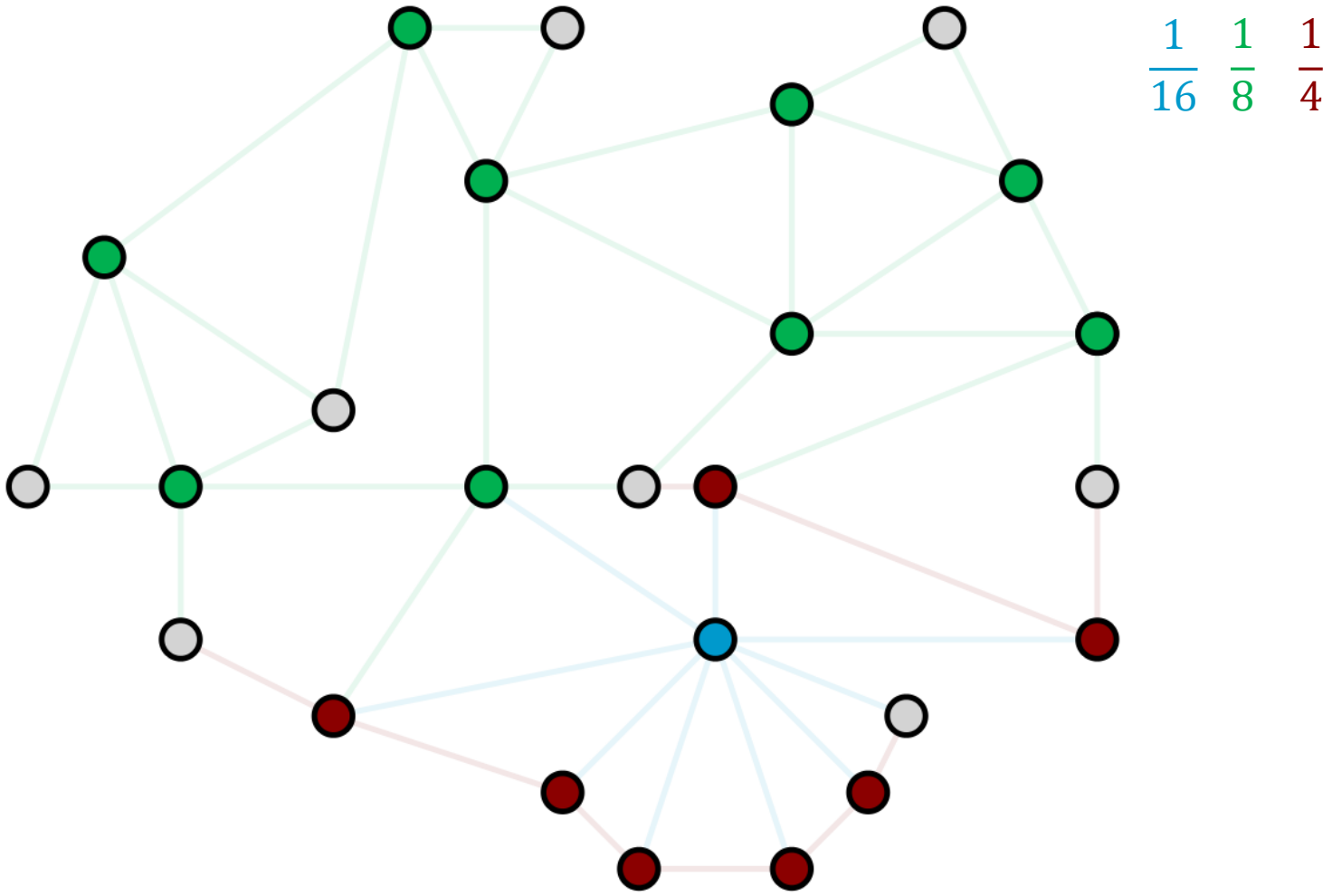

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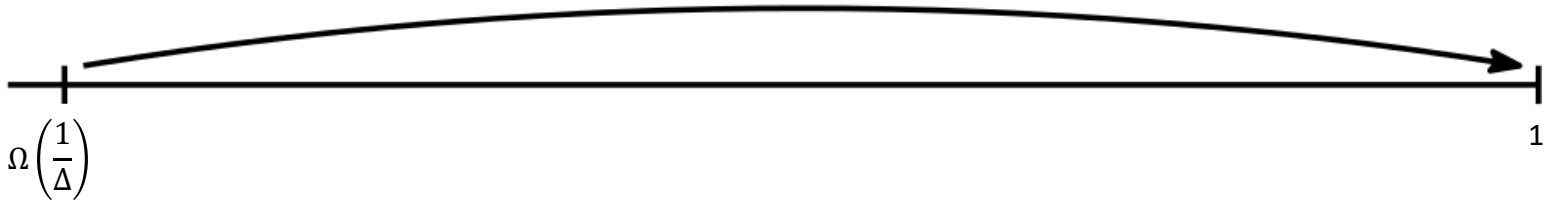
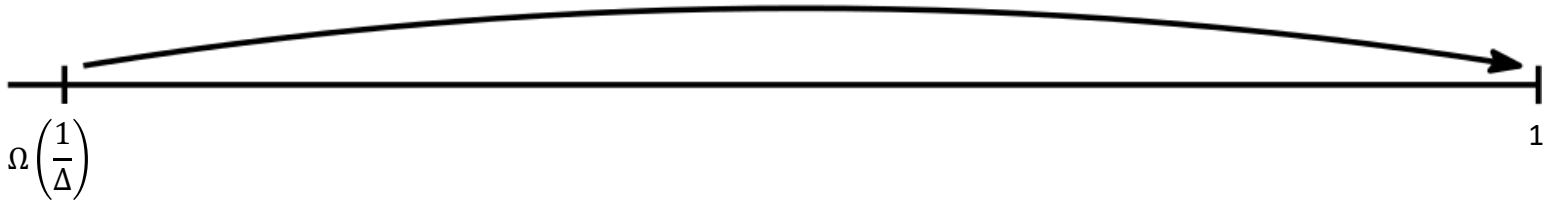


II) Rounding Fractional Bipartite Matching

$O(\log^2 \Delta)$ rounds, $O(1)$ loss

Direct Rounding



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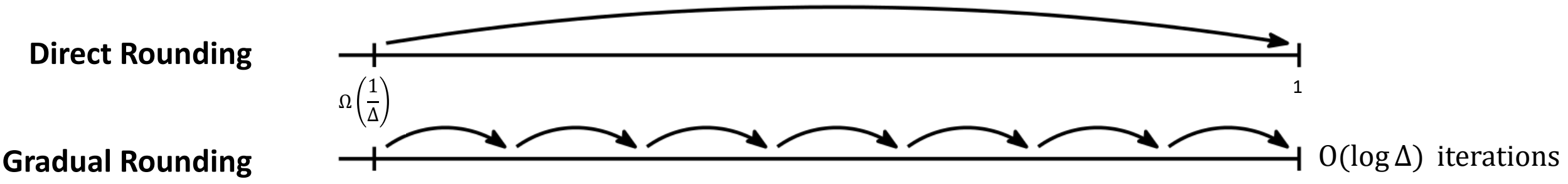

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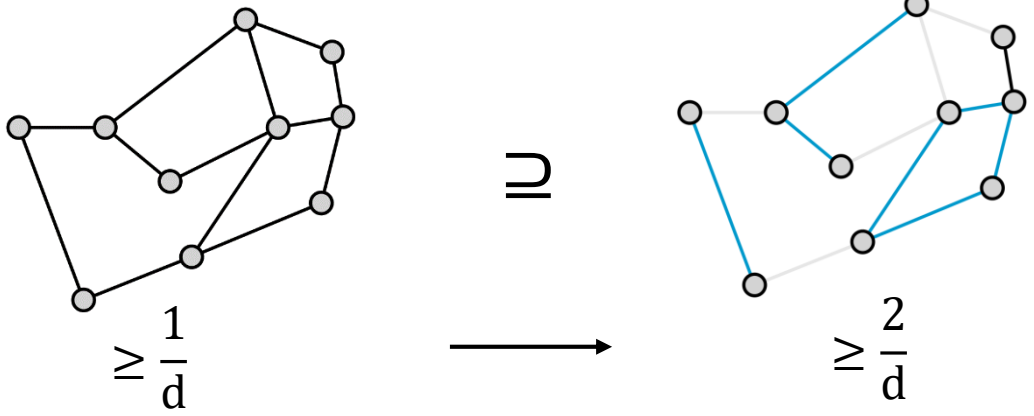
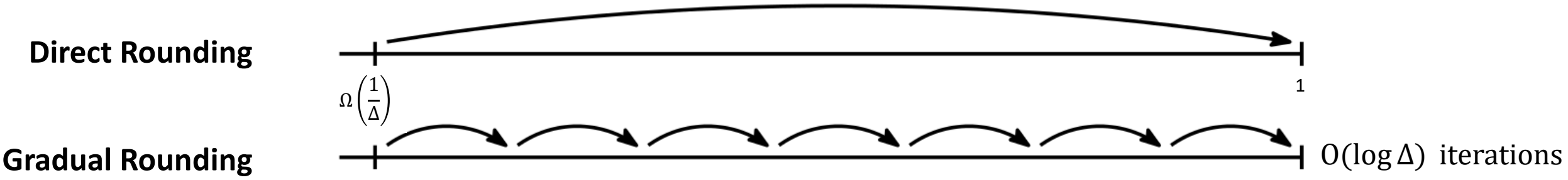
Factor-2-Rounding





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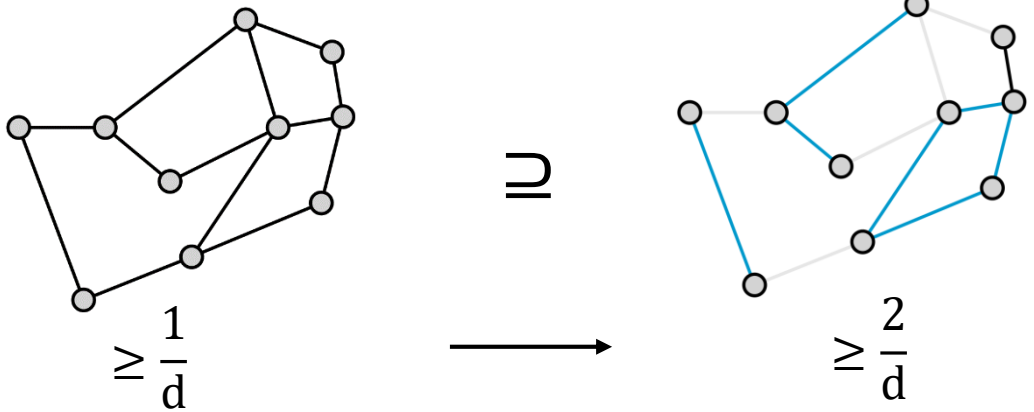
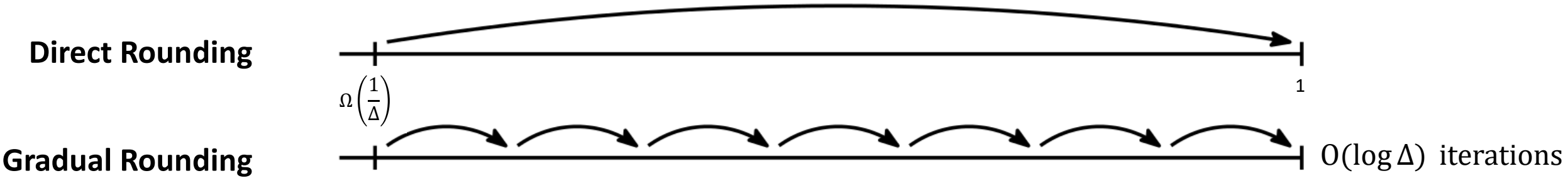


using Locally Balanced Splitting,
inspired by
Hańćkowiak, Karoński, Panconesi [SODA'98,PODC'99]



II) Rounding Fractional Bipartite Matching

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Iterated Factor-2-Rounding using Locally Balanced Splitting

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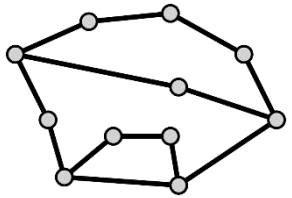
Iterated Factor-2-Rounding using Locally Balanced Splitting

Locally Balanced Splitting:
2-edge-coloring so that
every node roughly balanced



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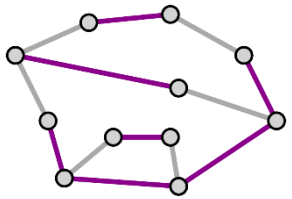

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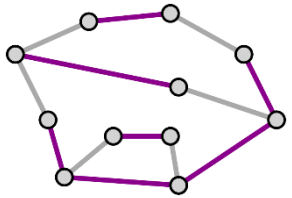

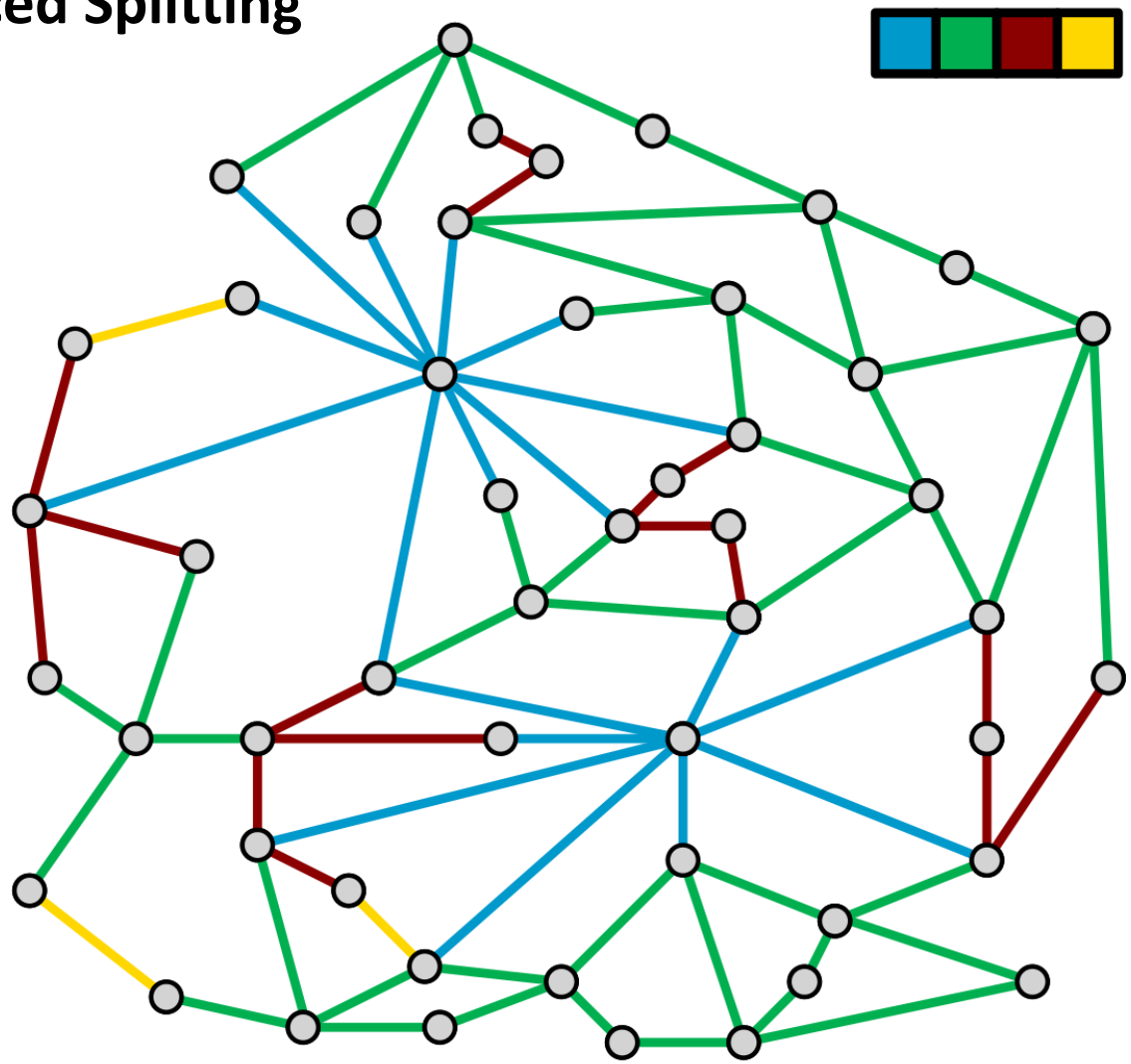

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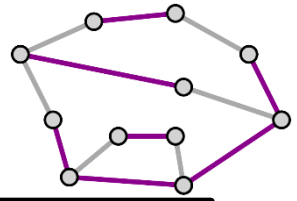

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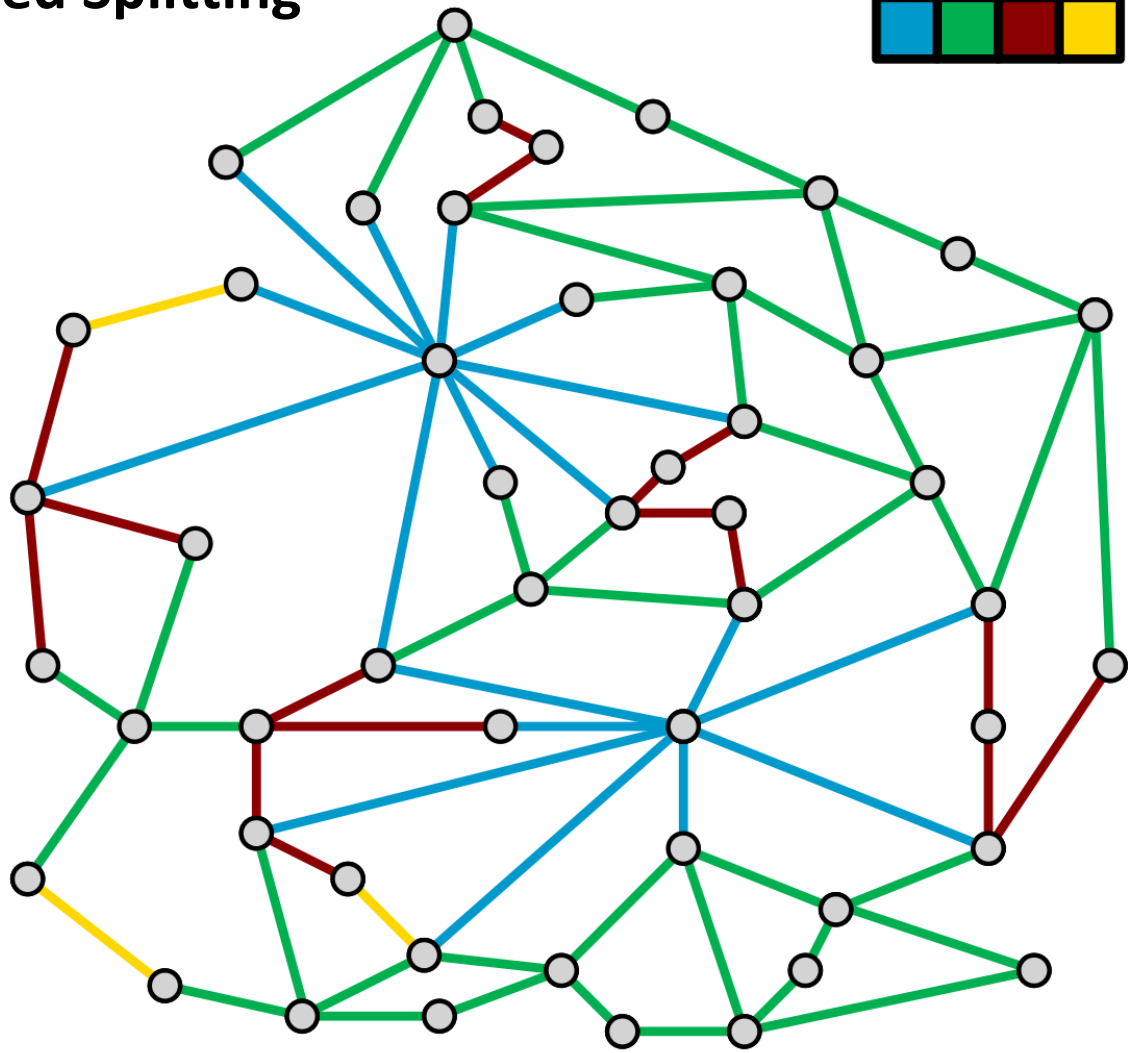

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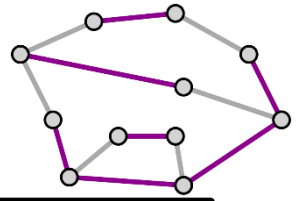

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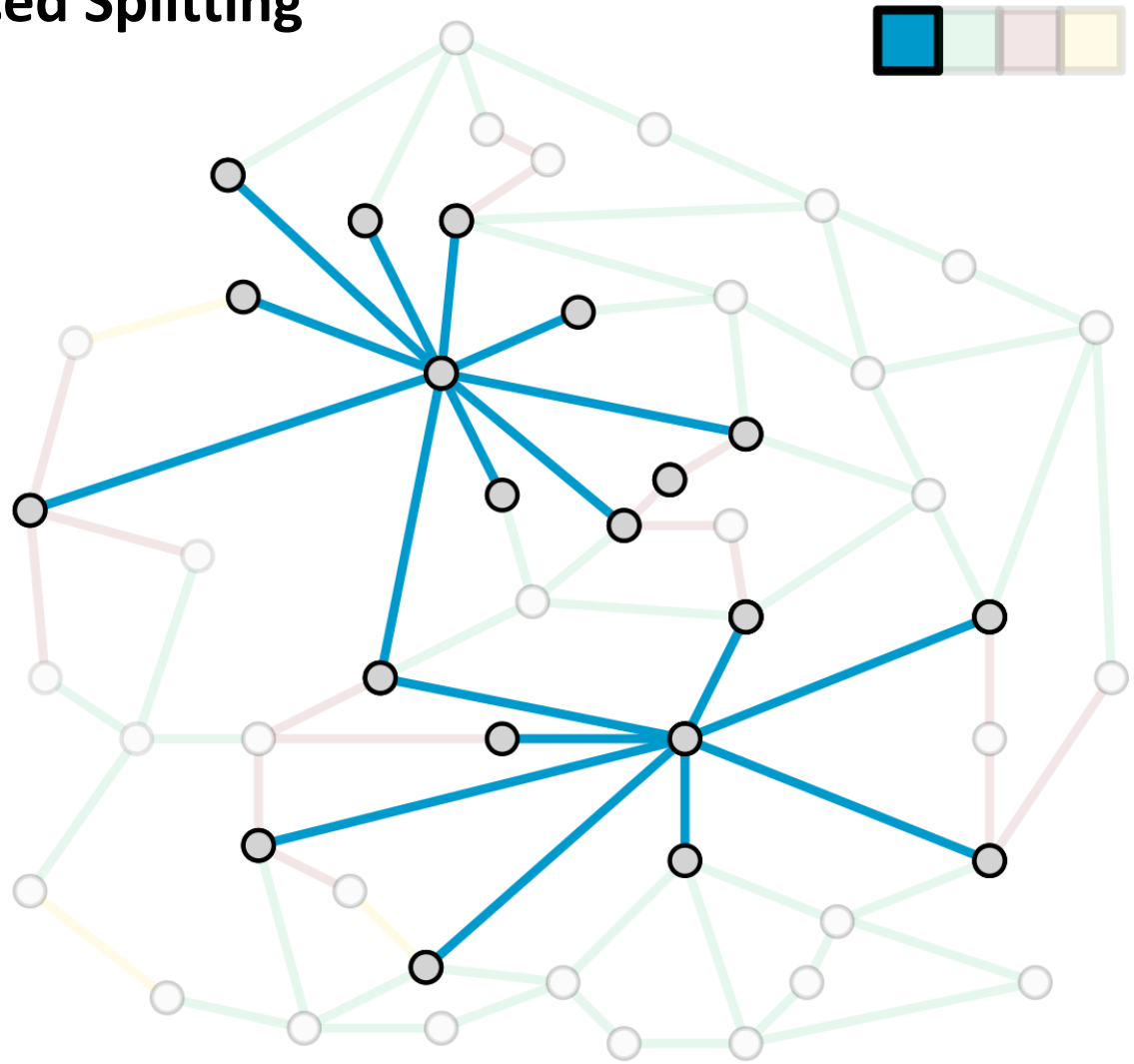

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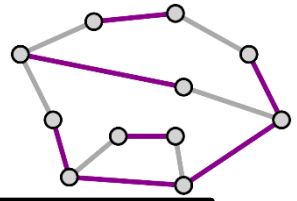

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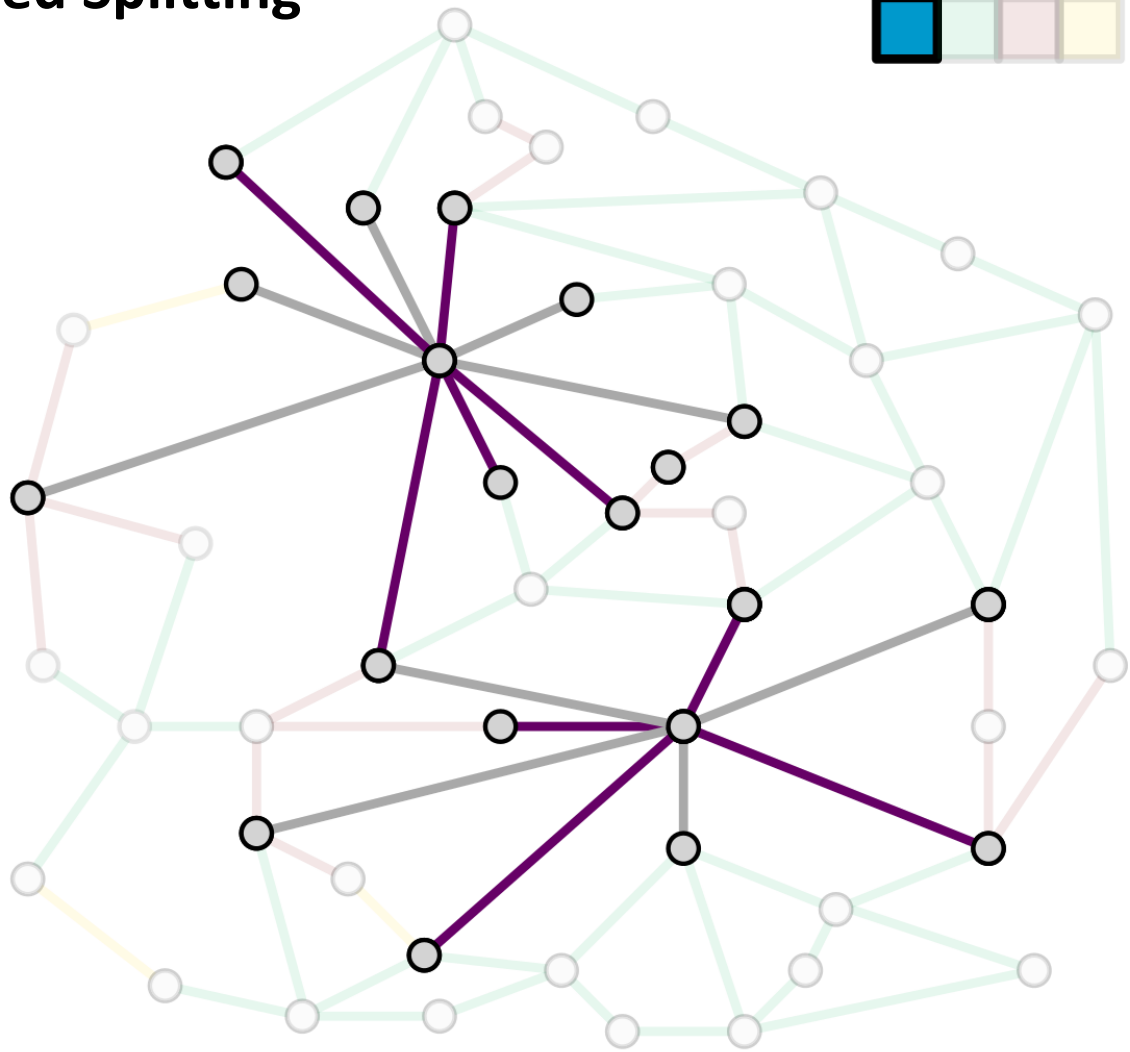

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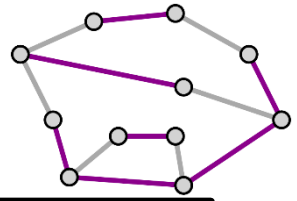

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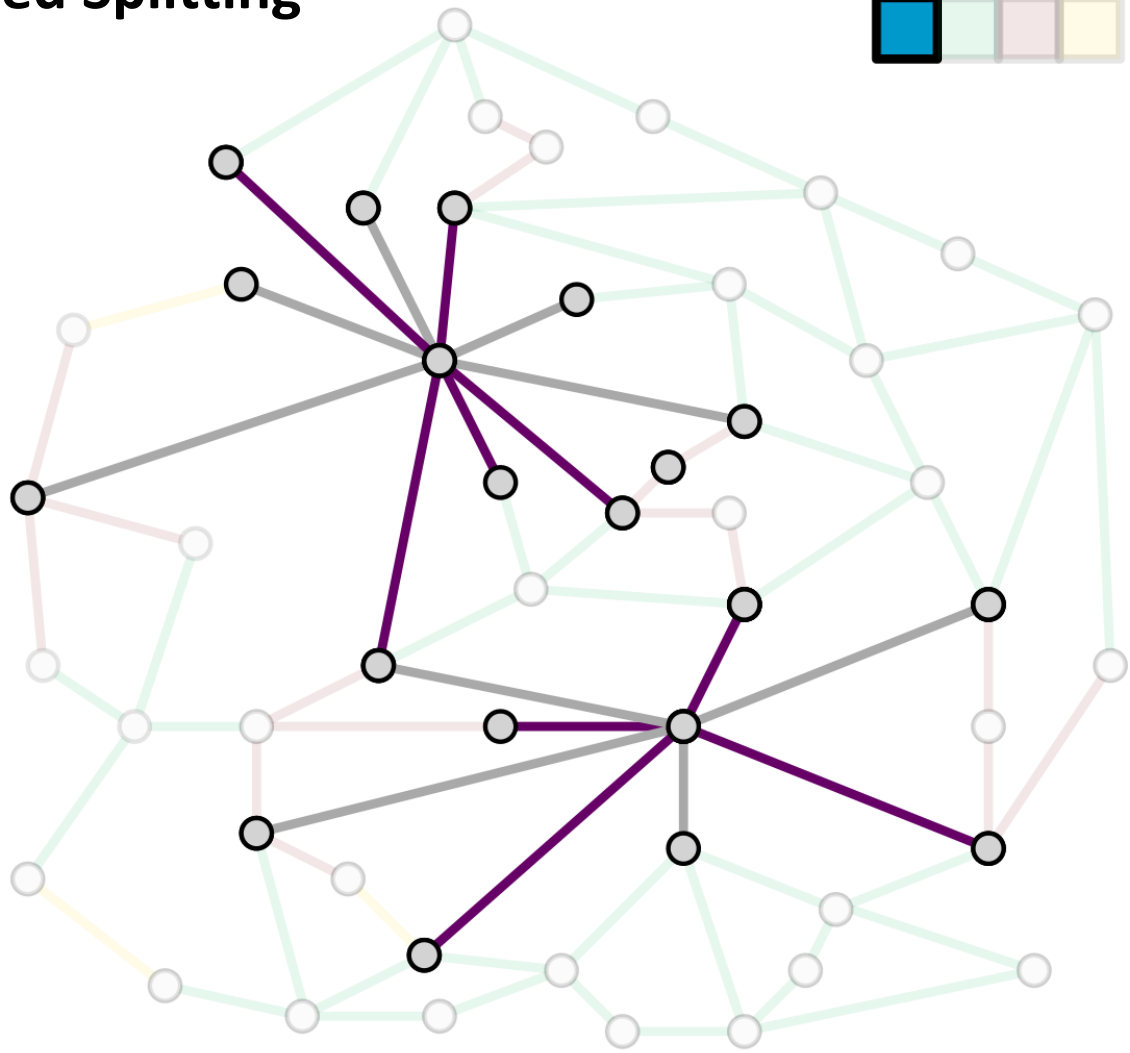

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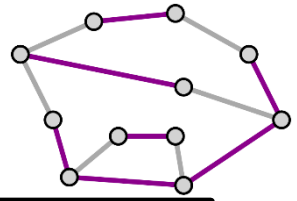

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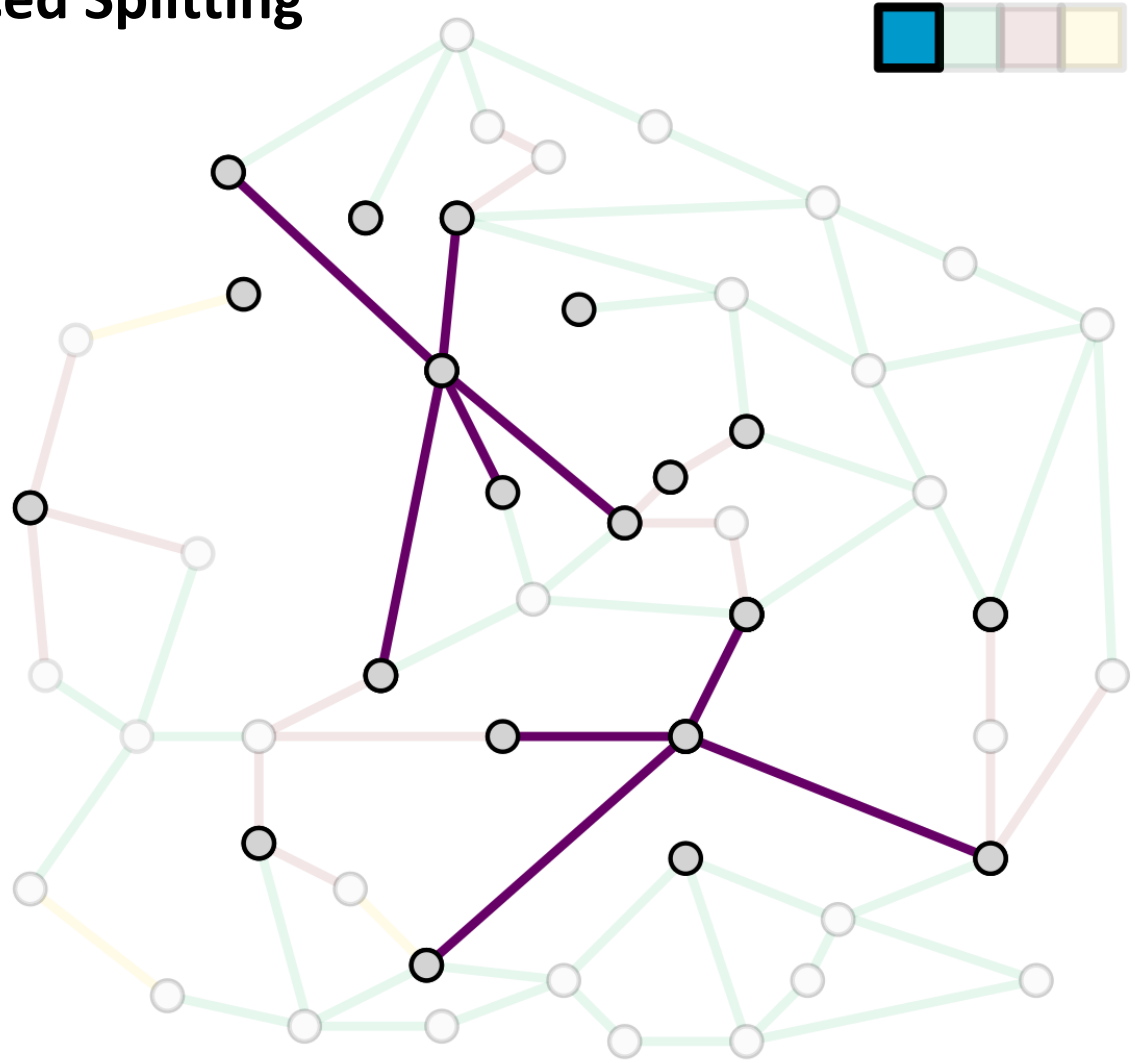

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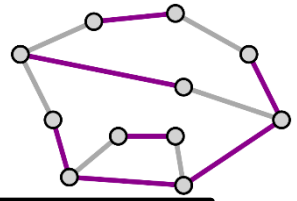

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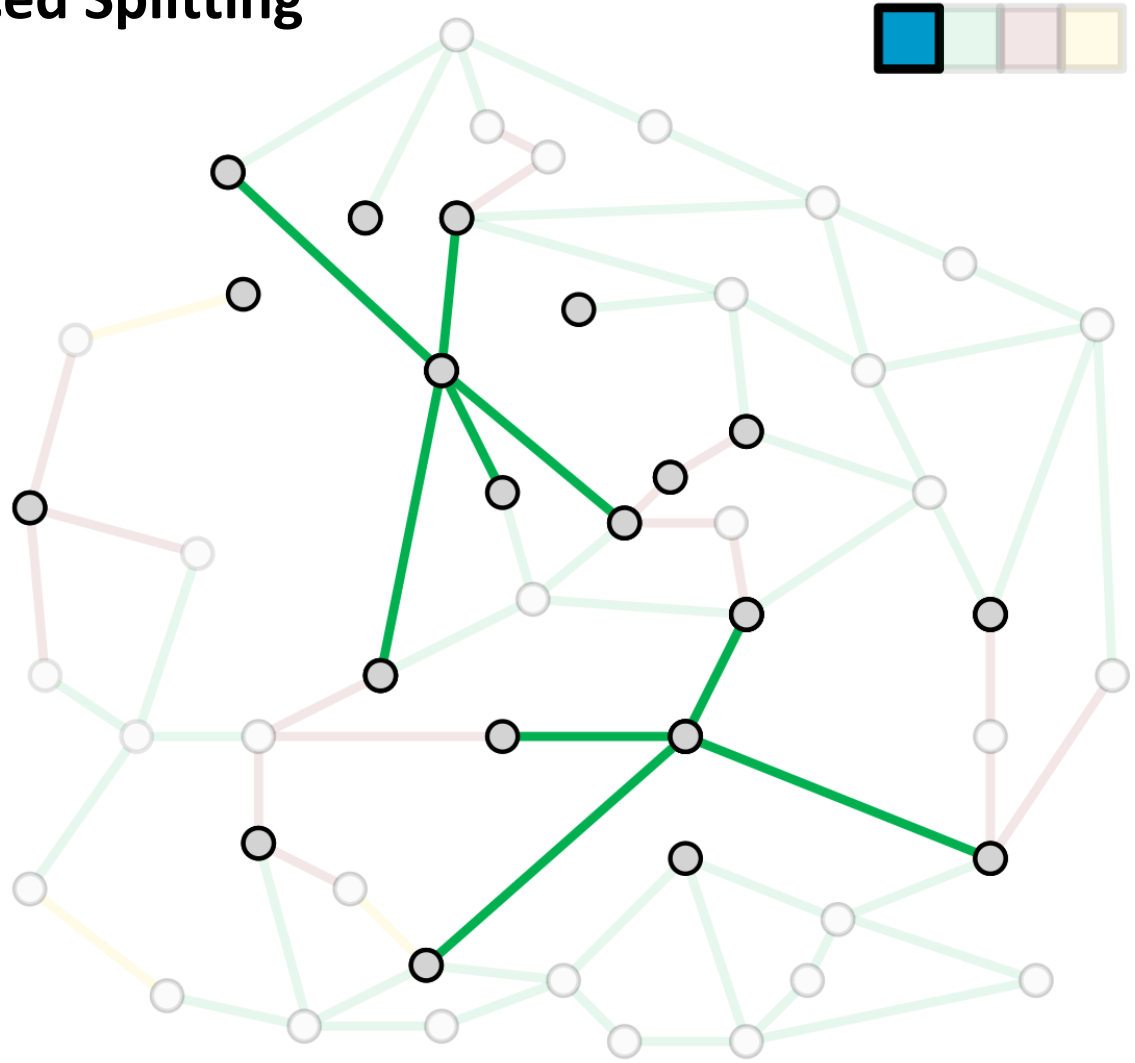

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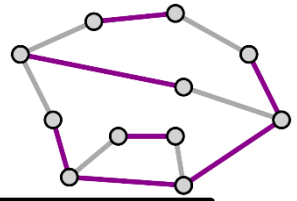

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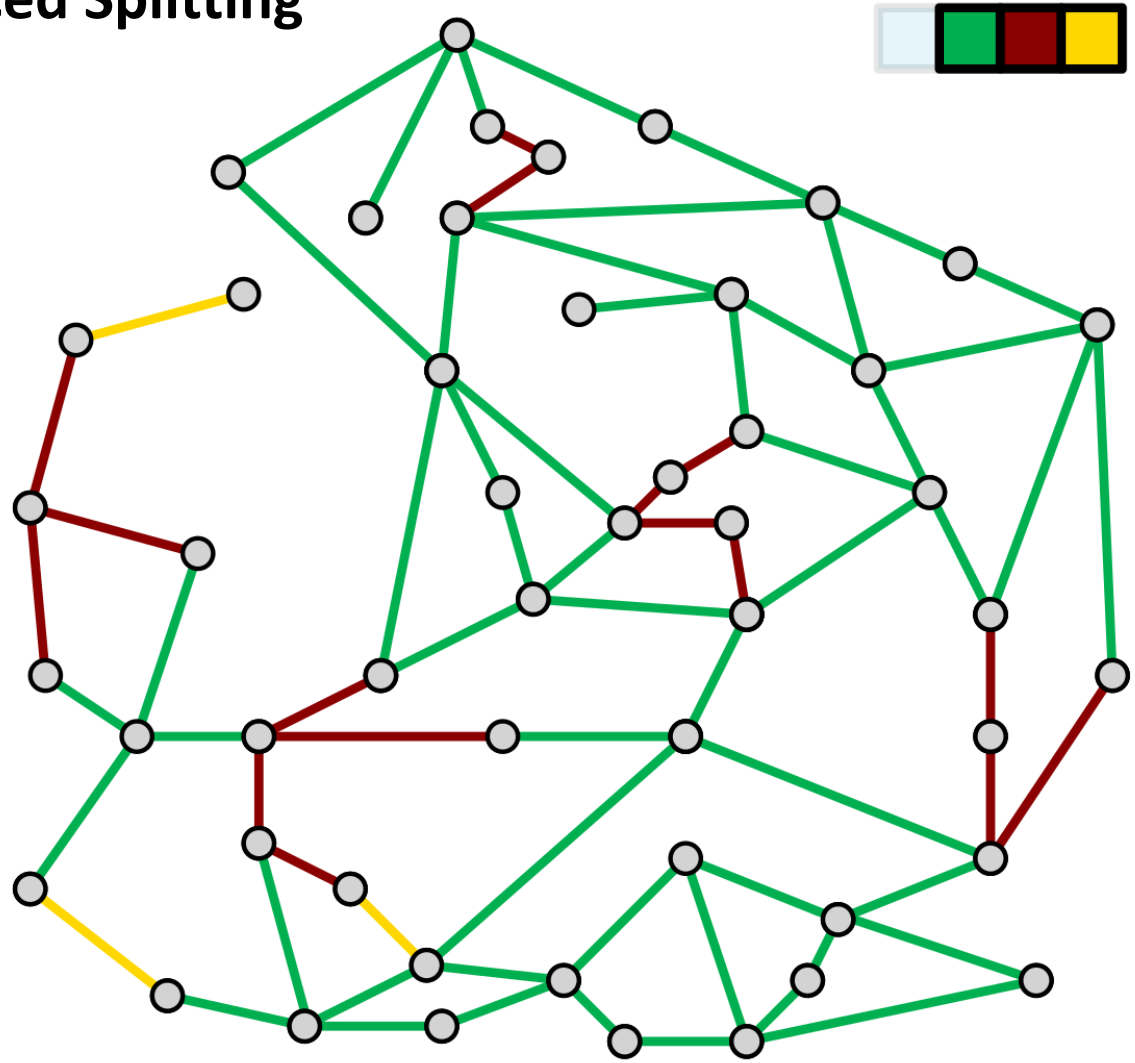

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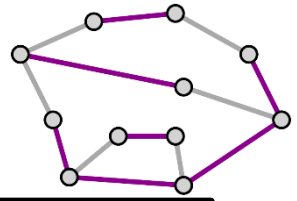

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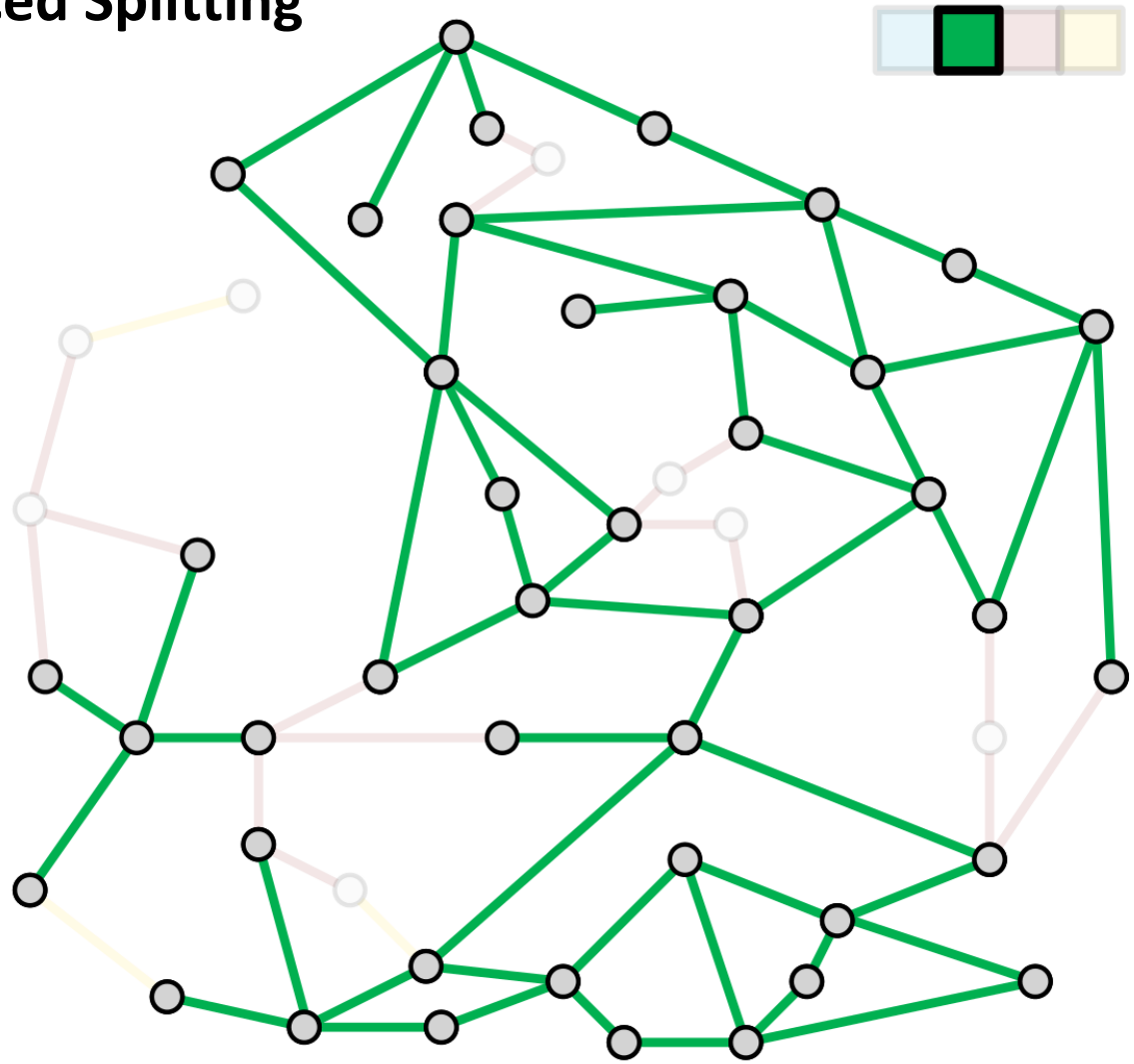

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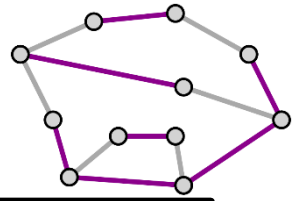

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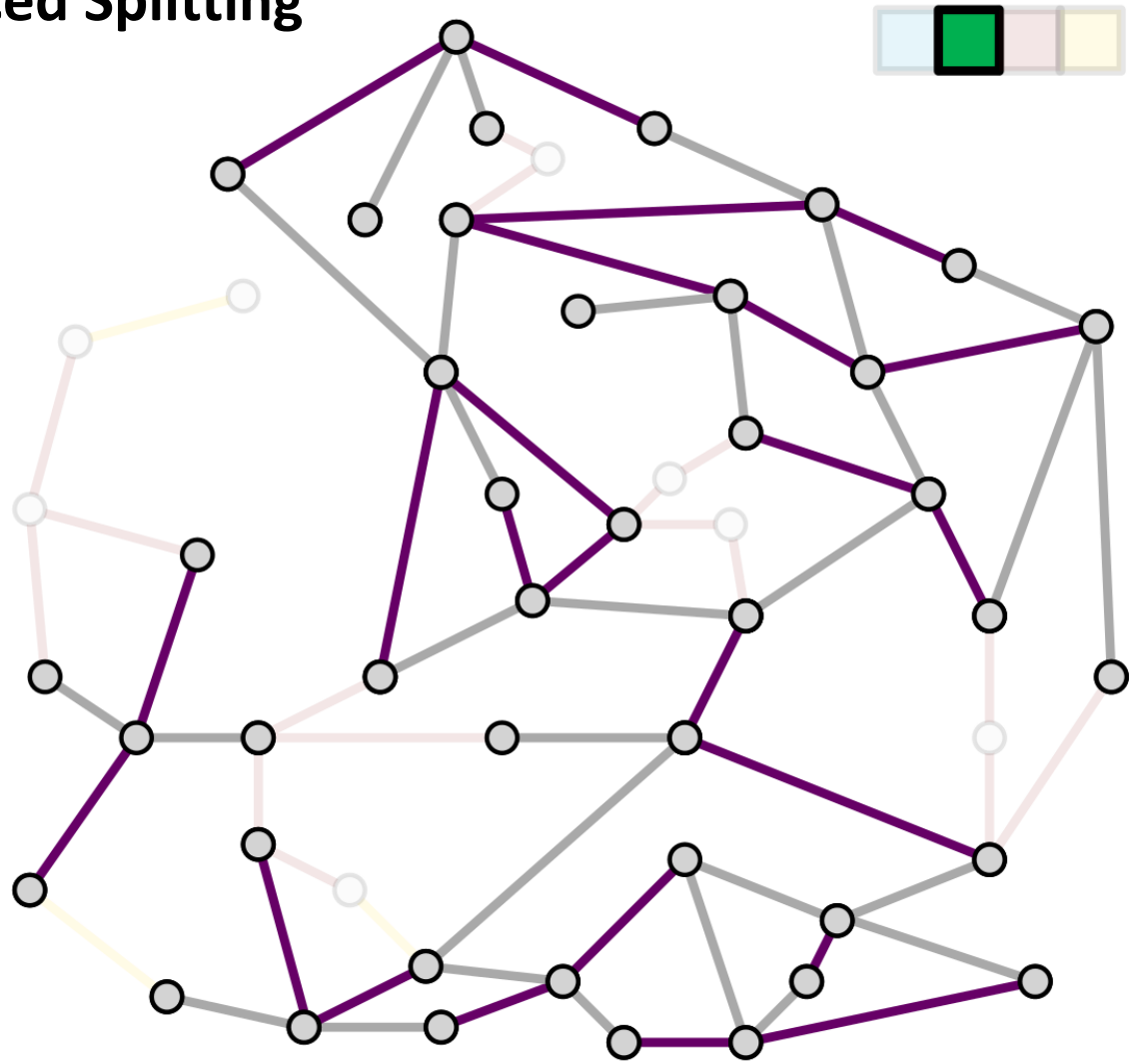

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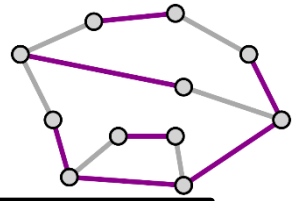

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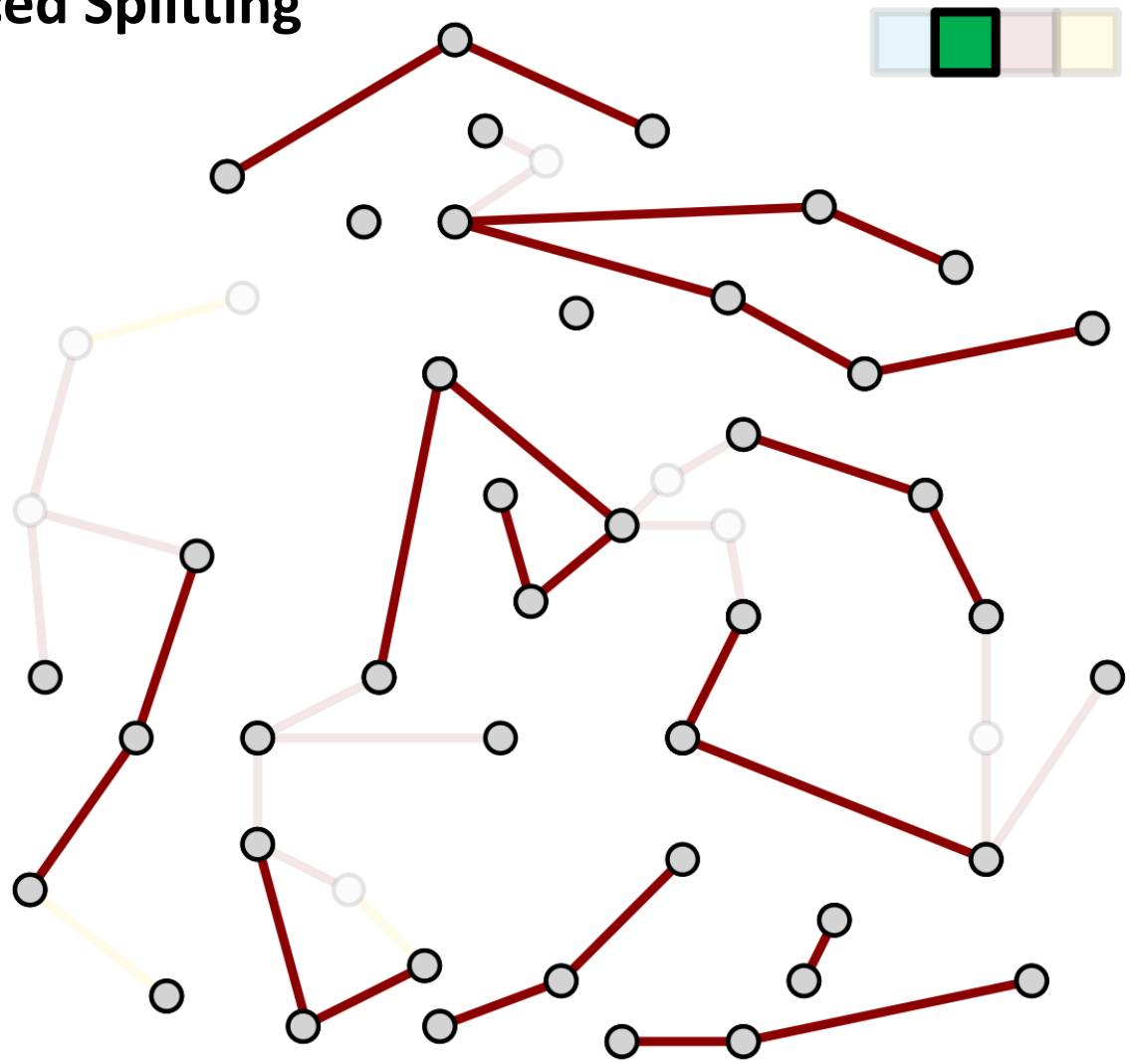

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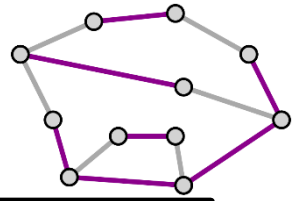

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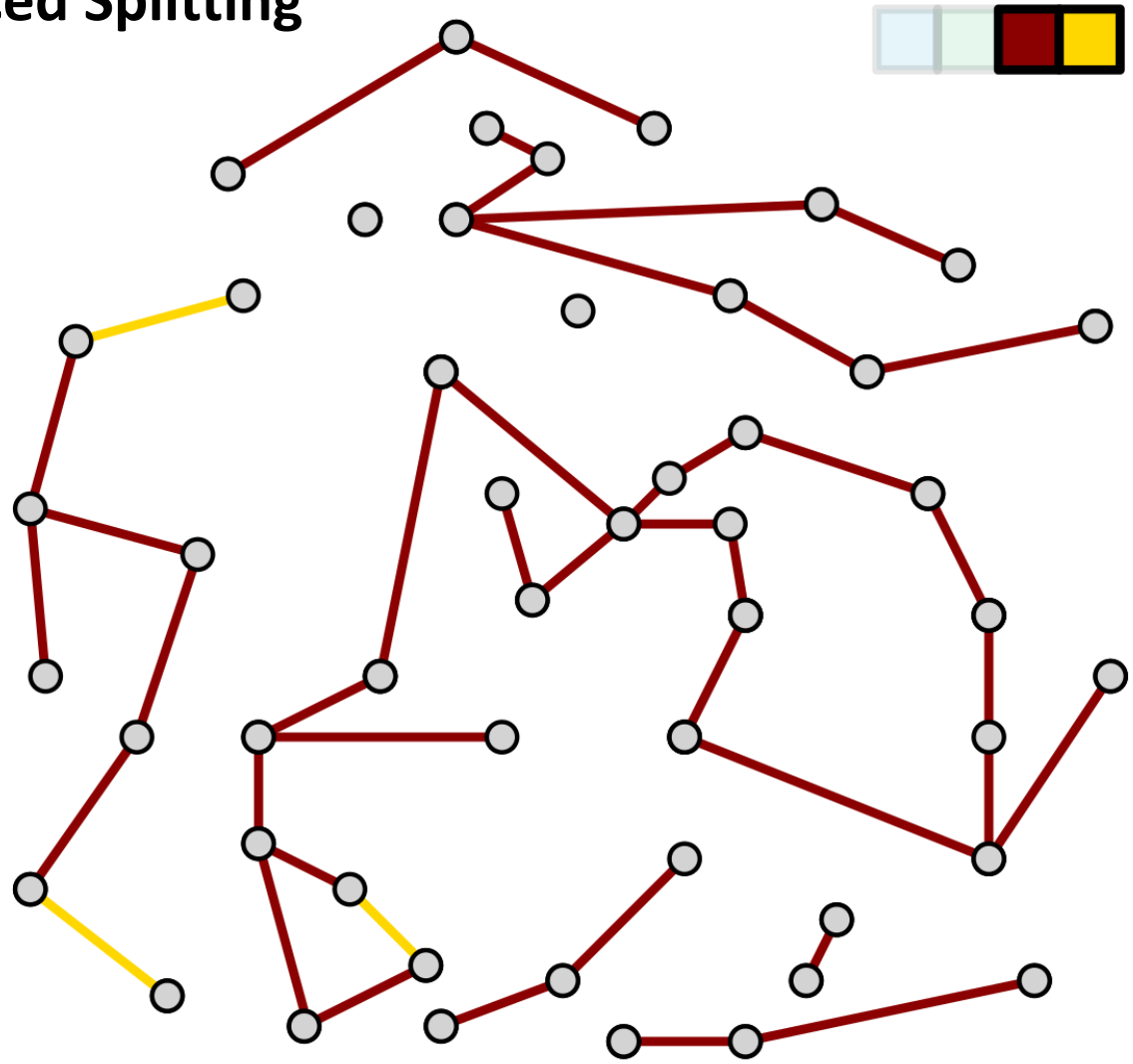

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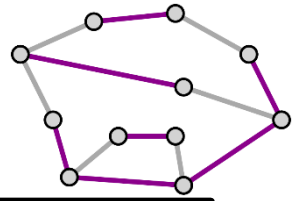

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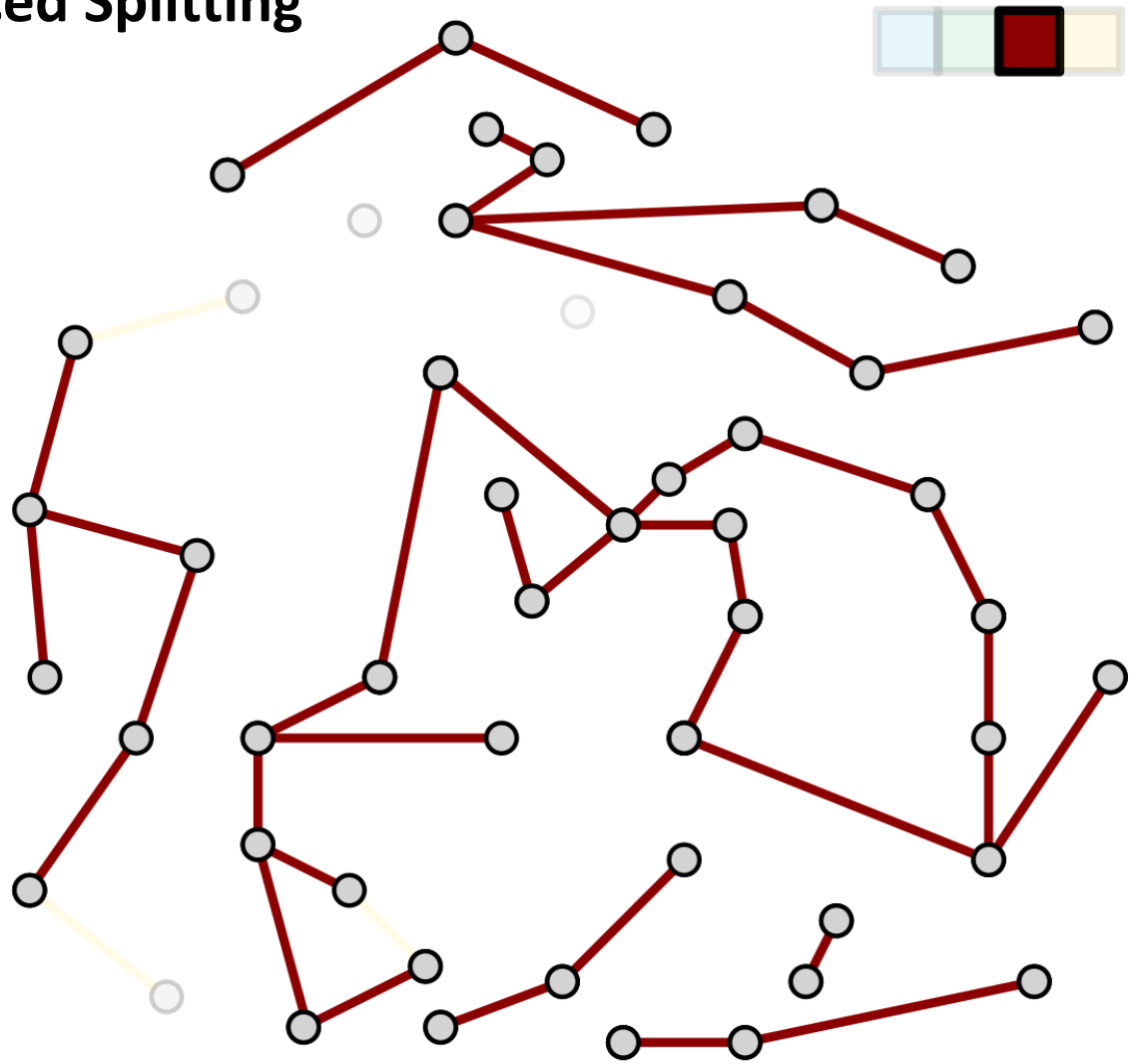

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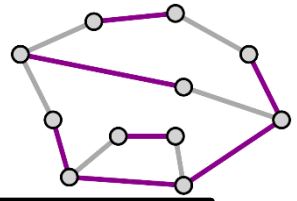

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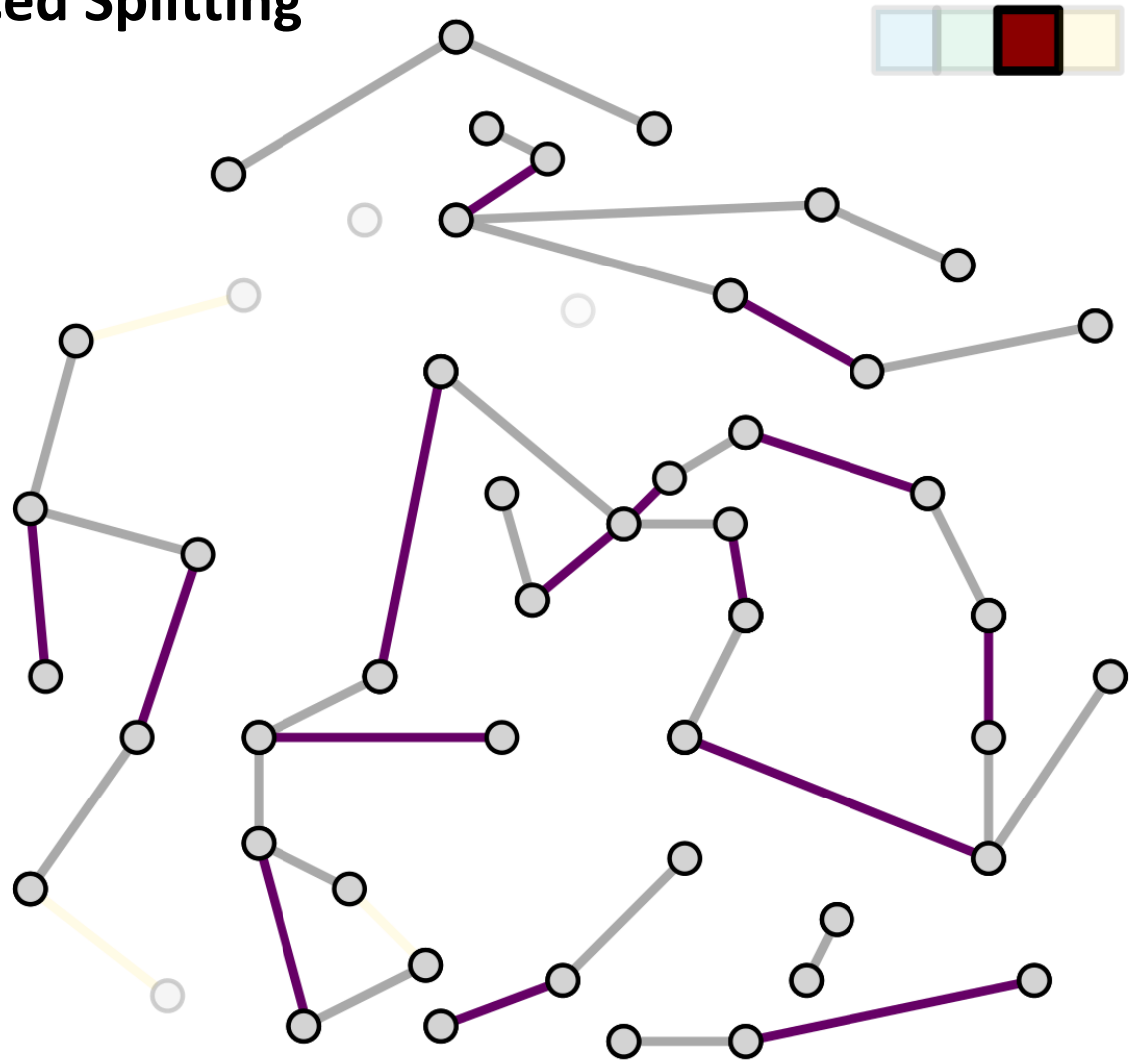

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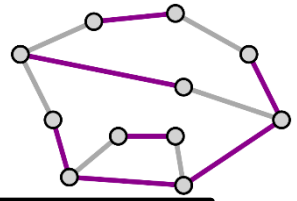

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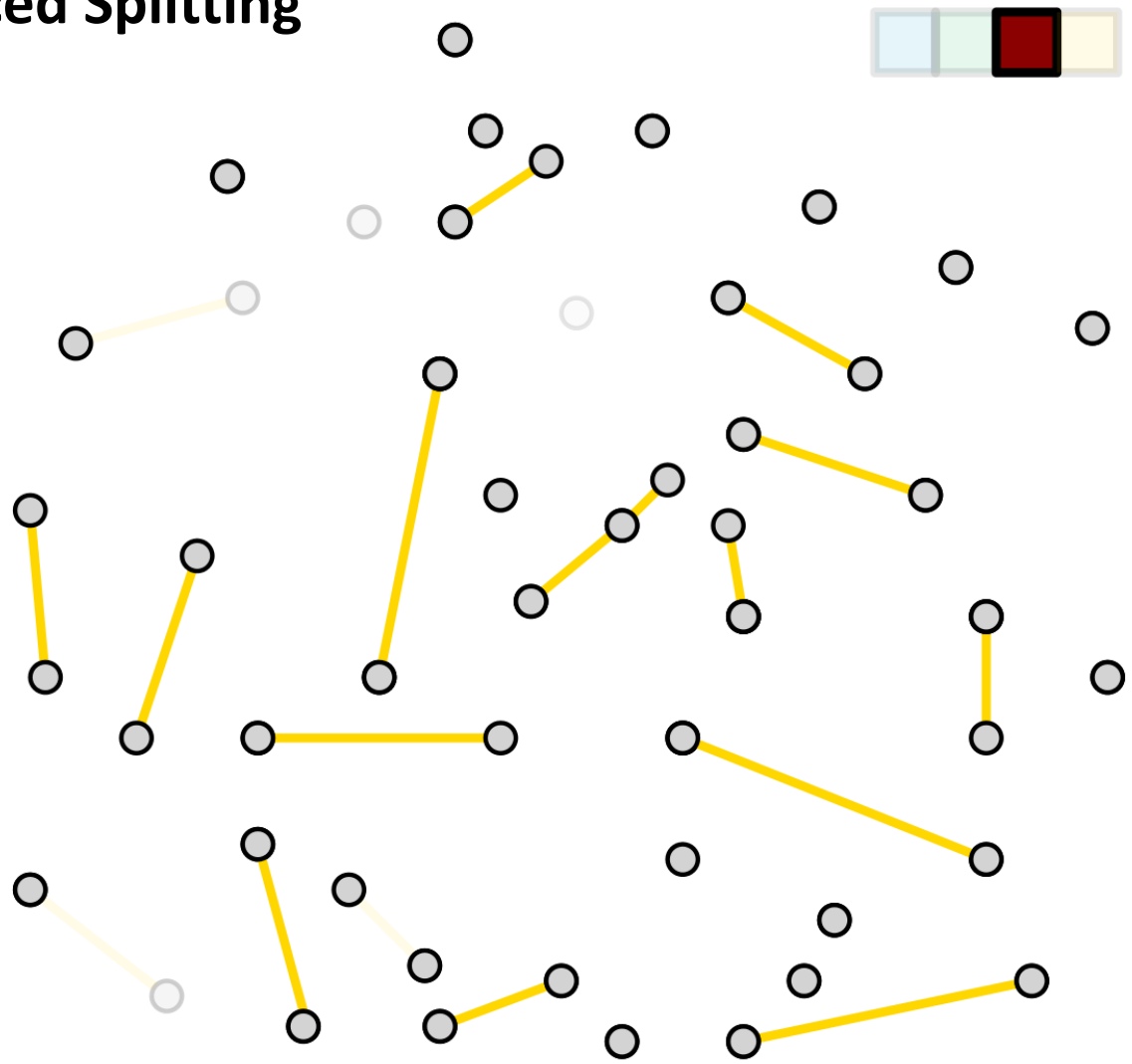

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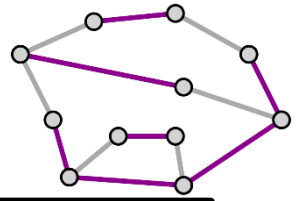

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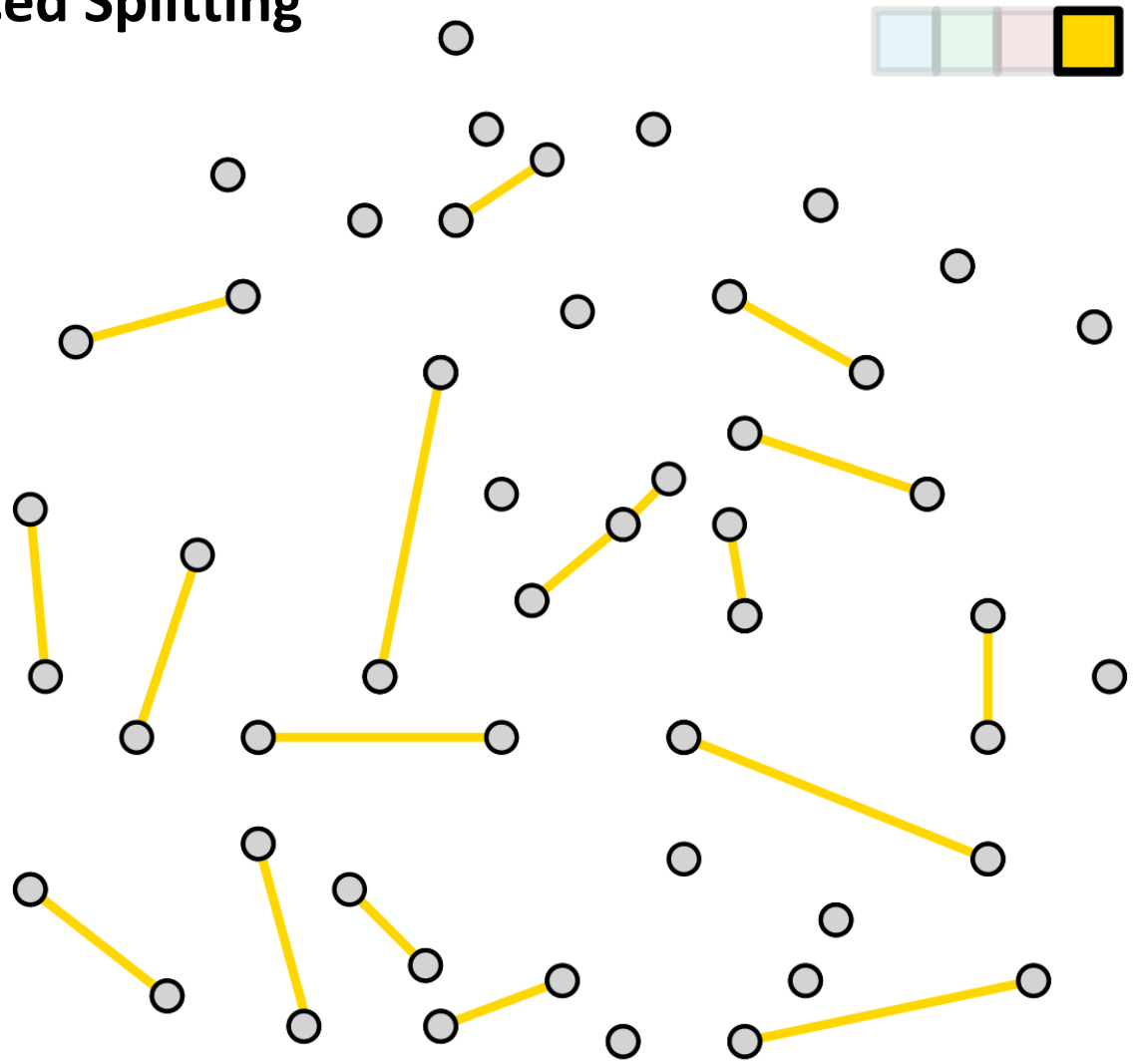

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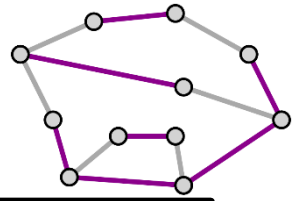

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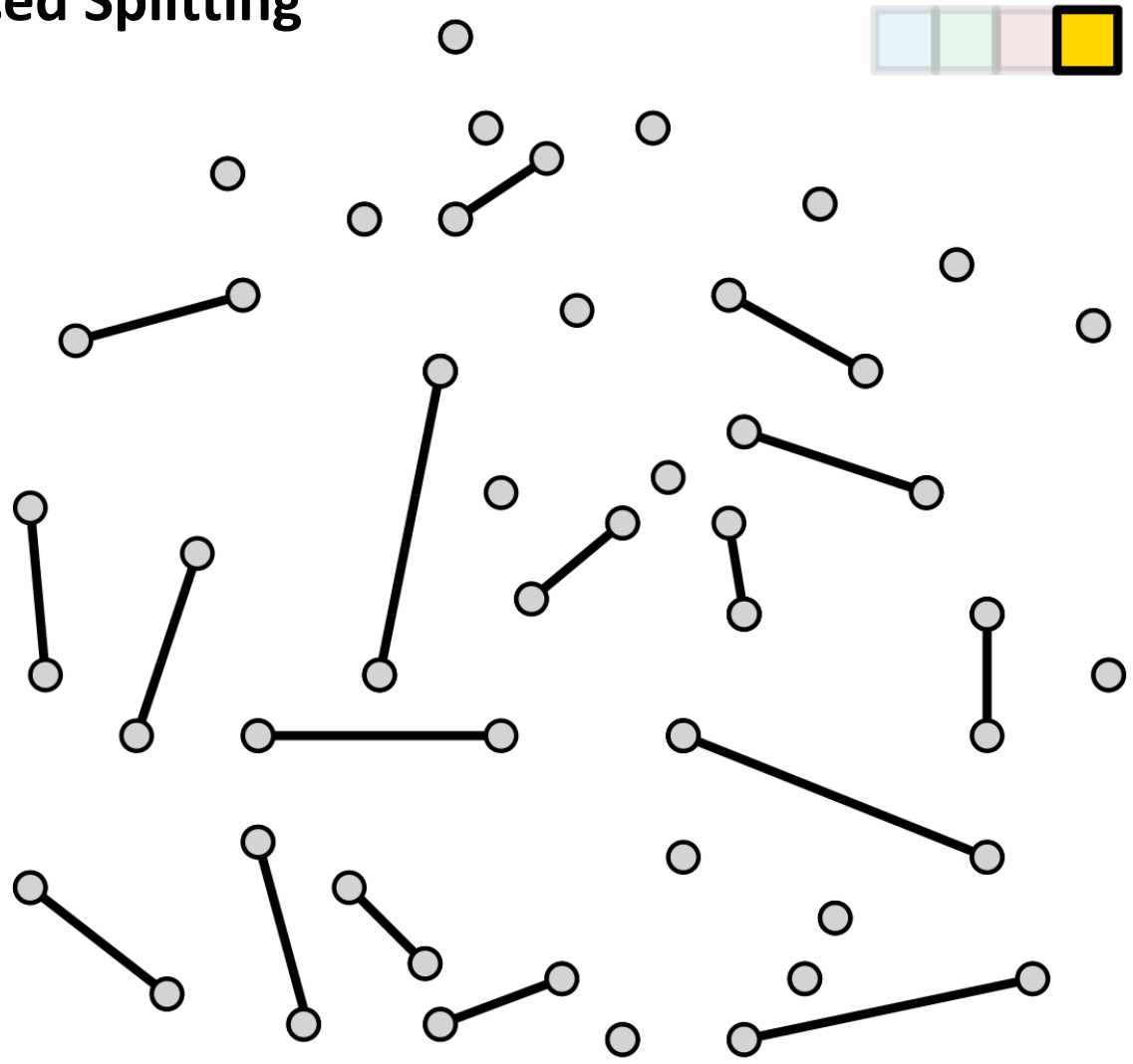

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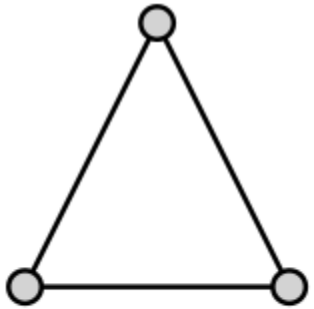

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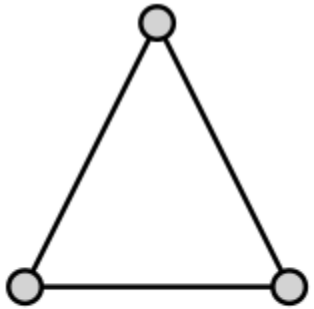

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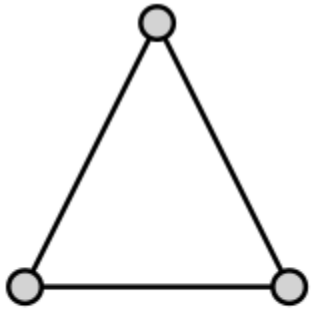

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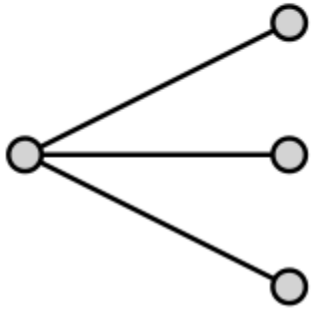

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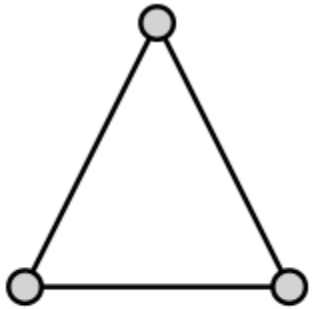
Perfect Splitting not possible in case of…



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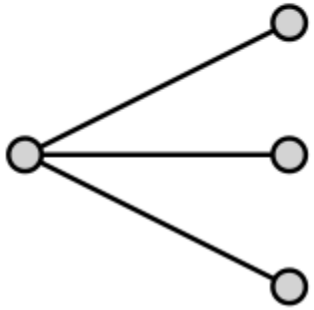

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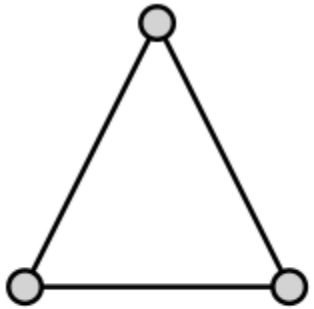

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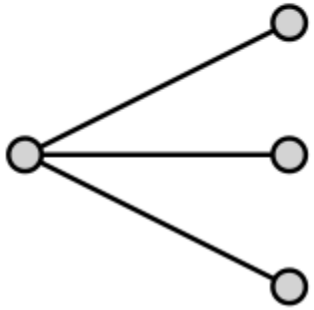

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Suppose that bipartite and even degree!

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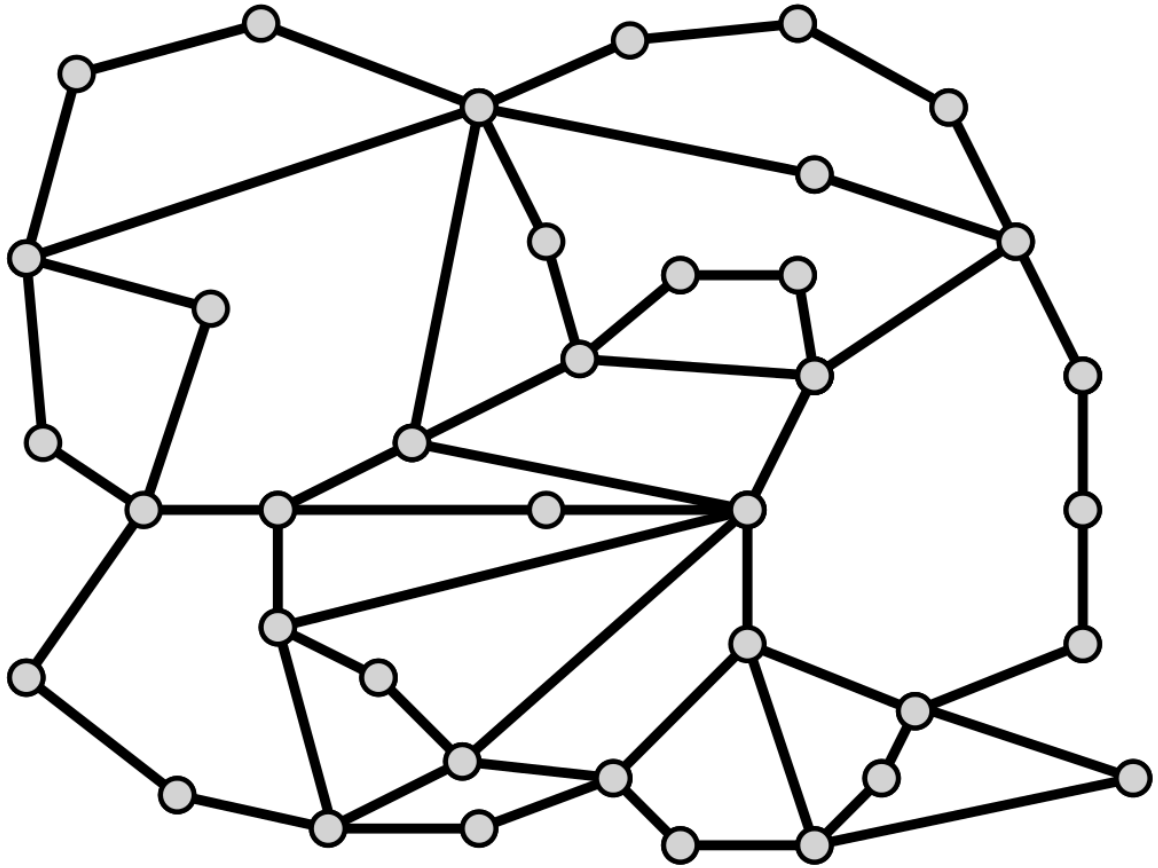

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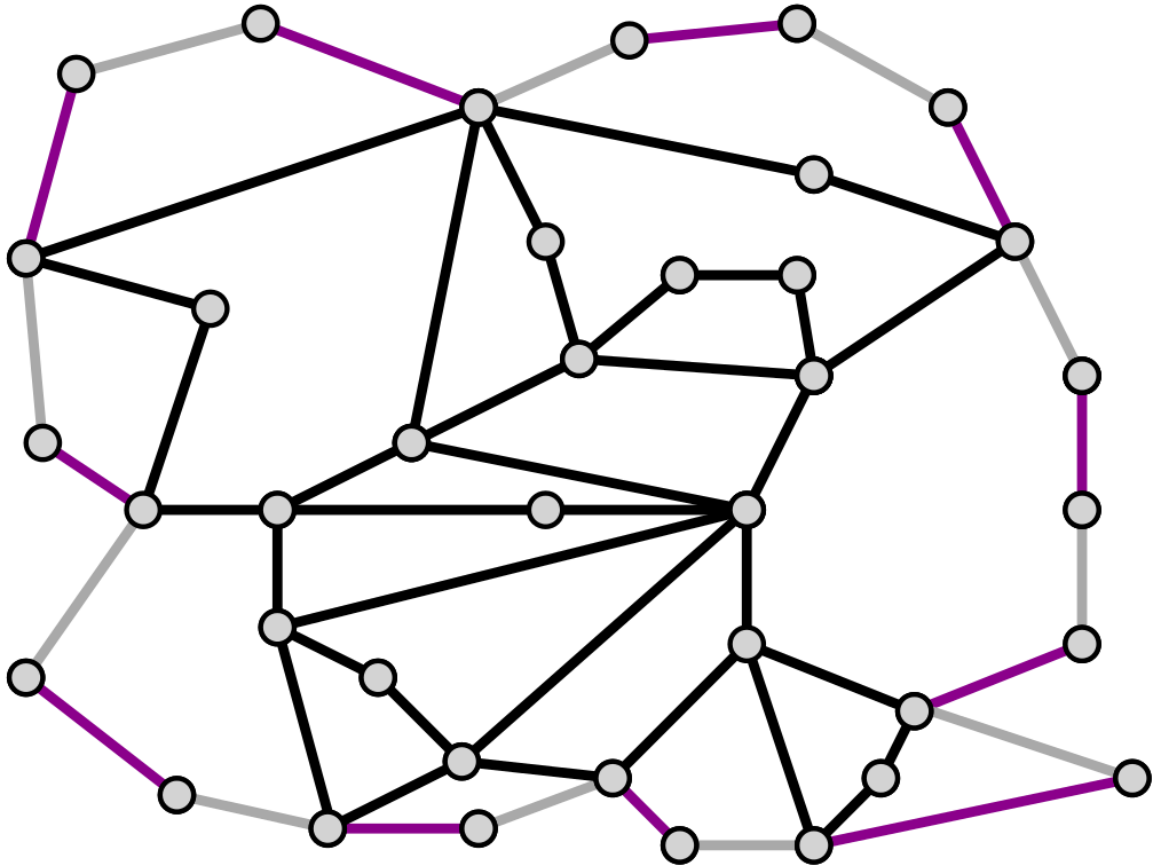

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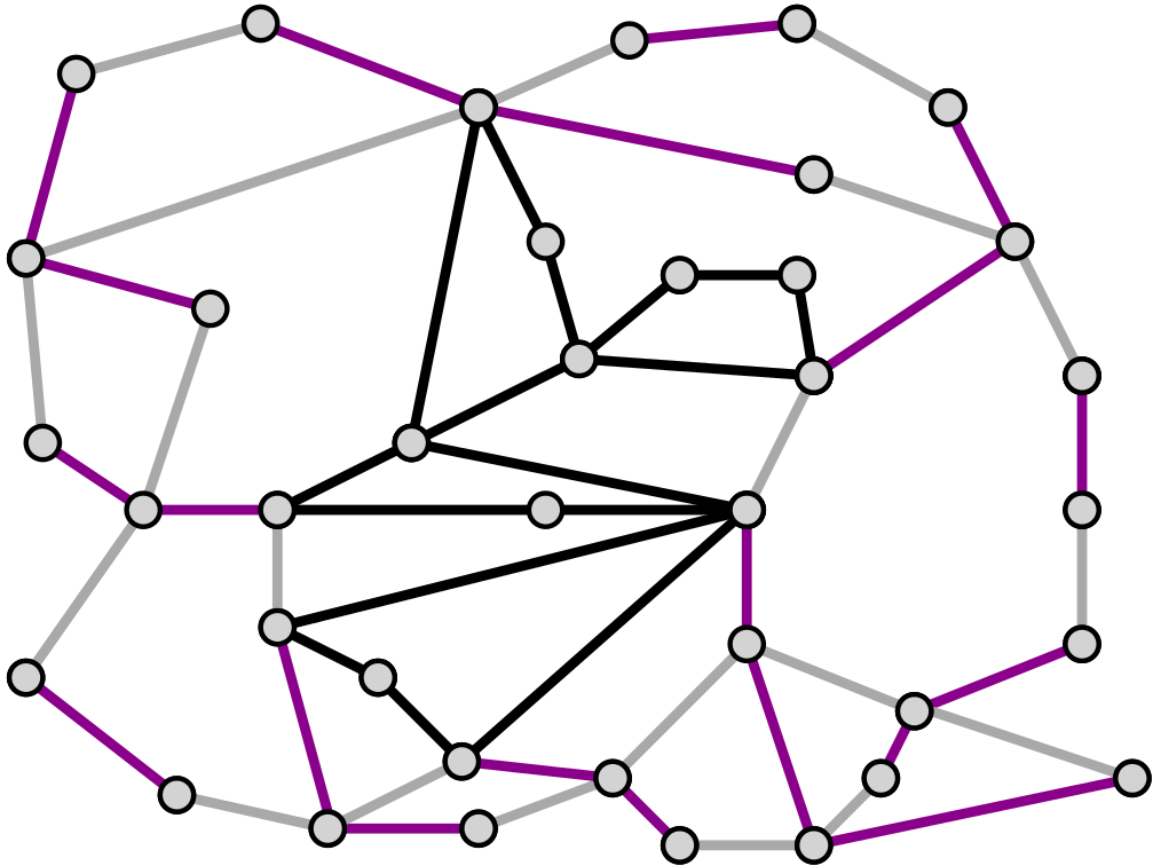

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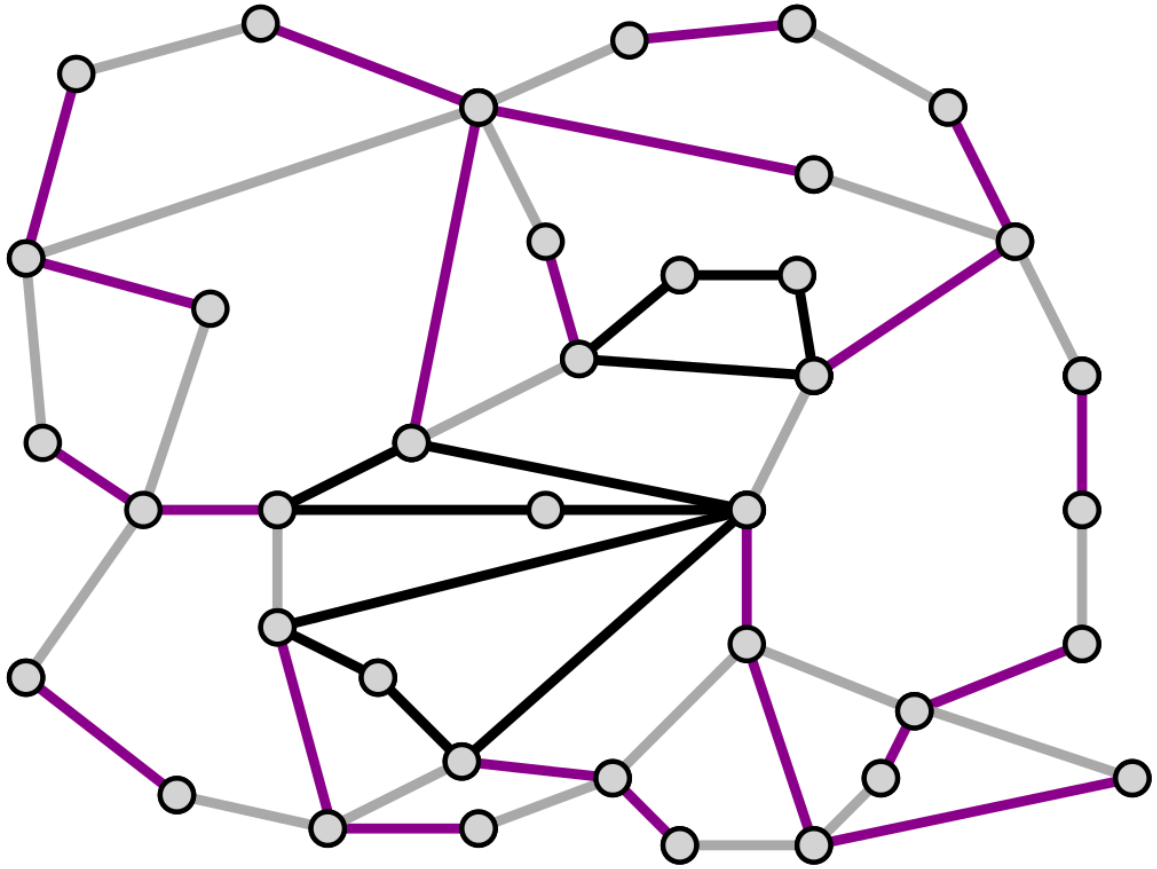

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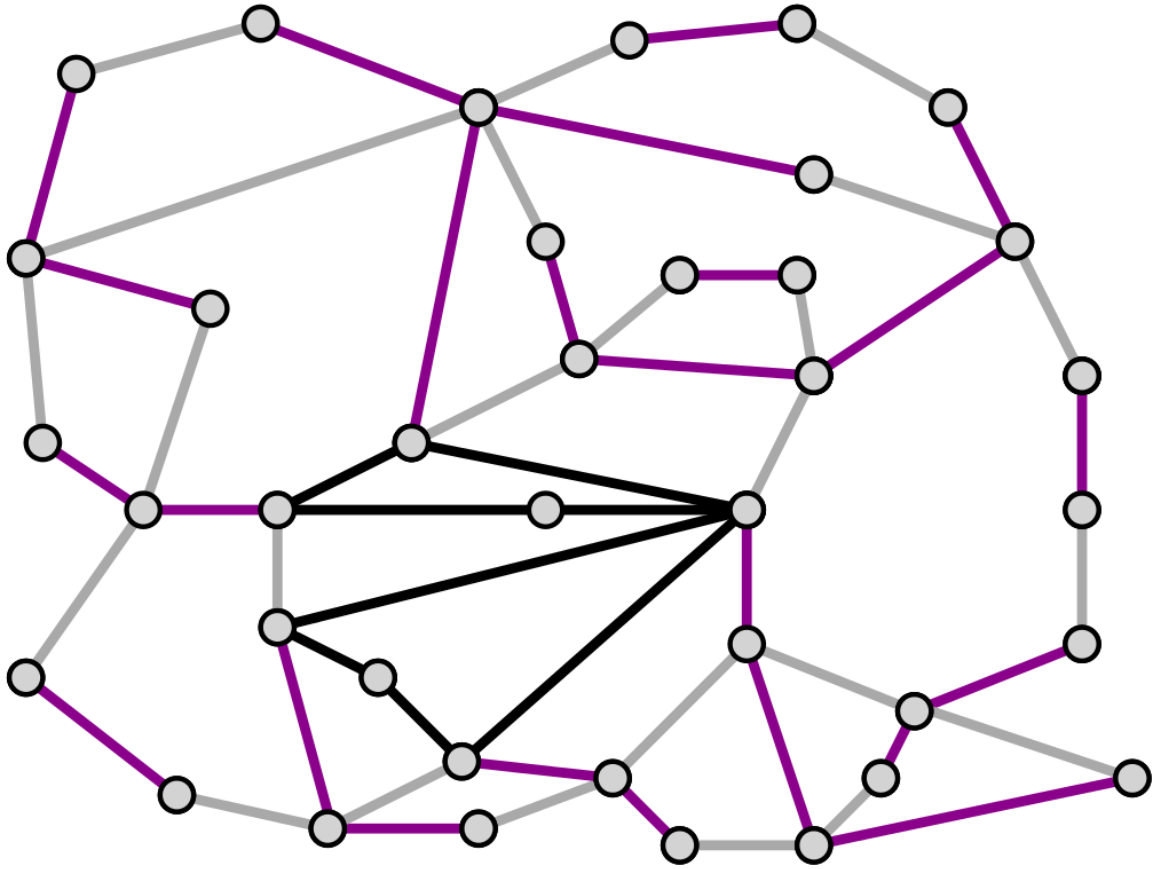

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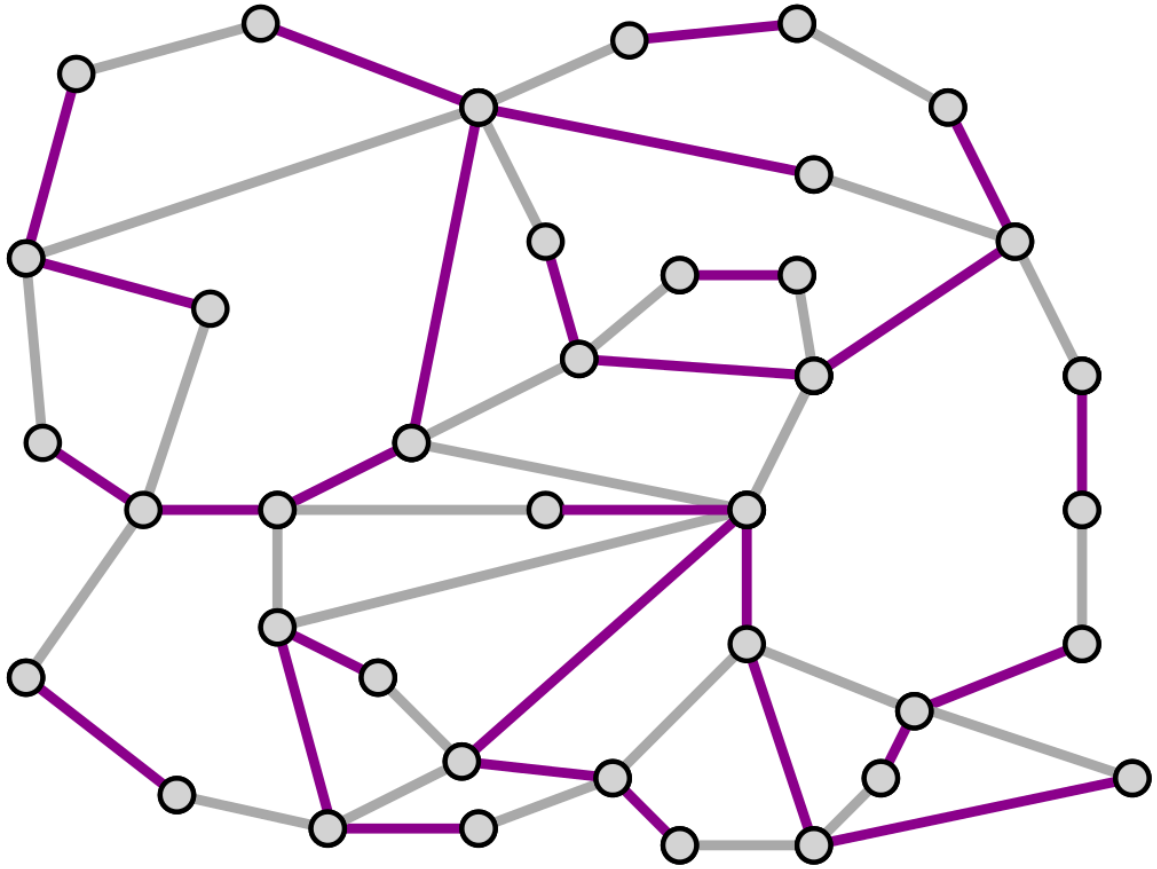

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Decompose into edge-disjoint cycles

In parallel, for all cycles

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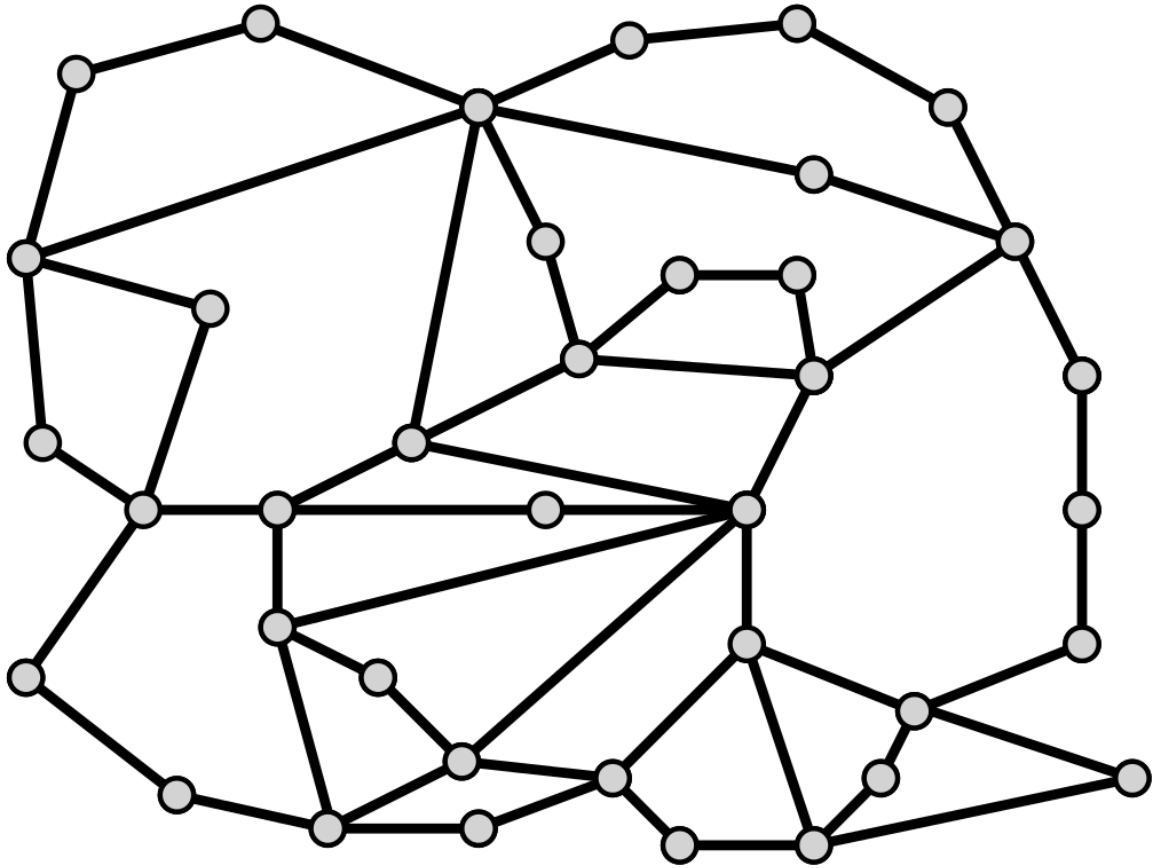

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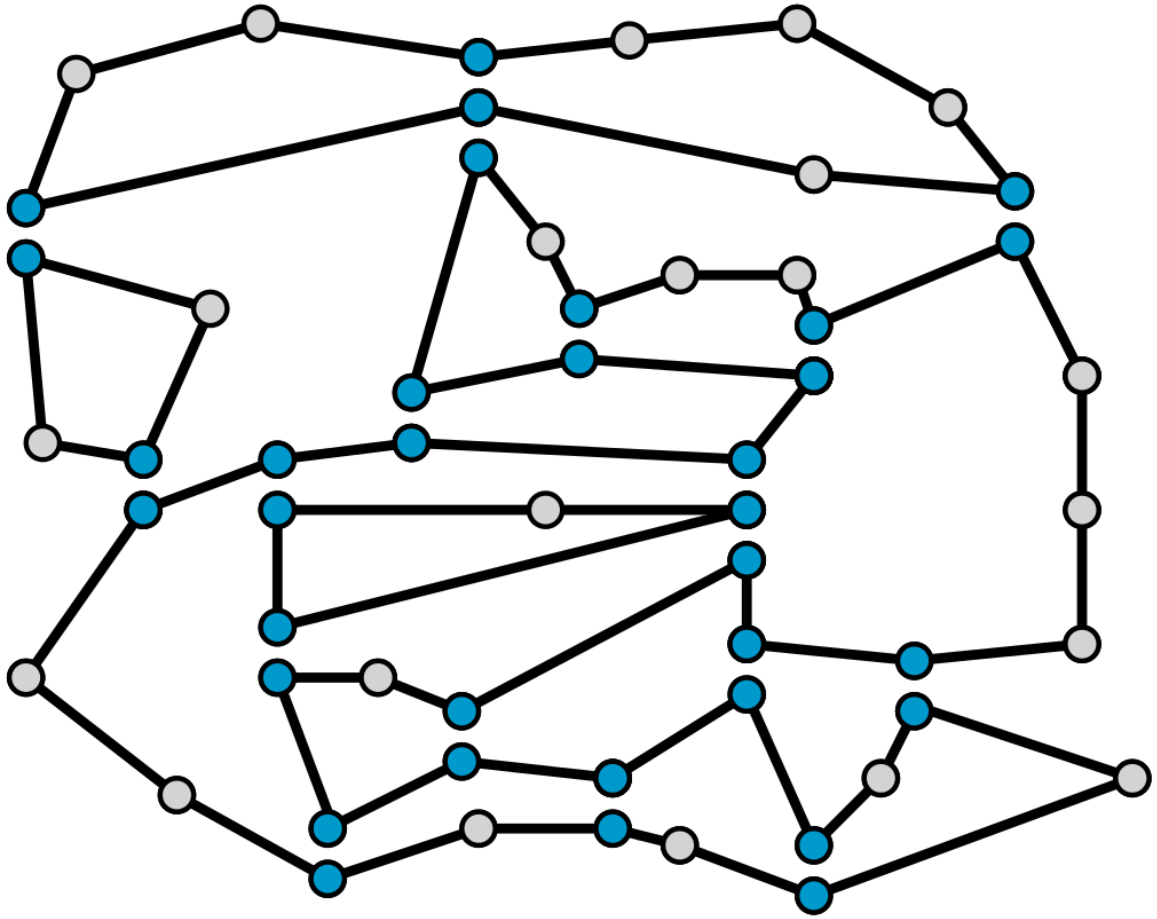

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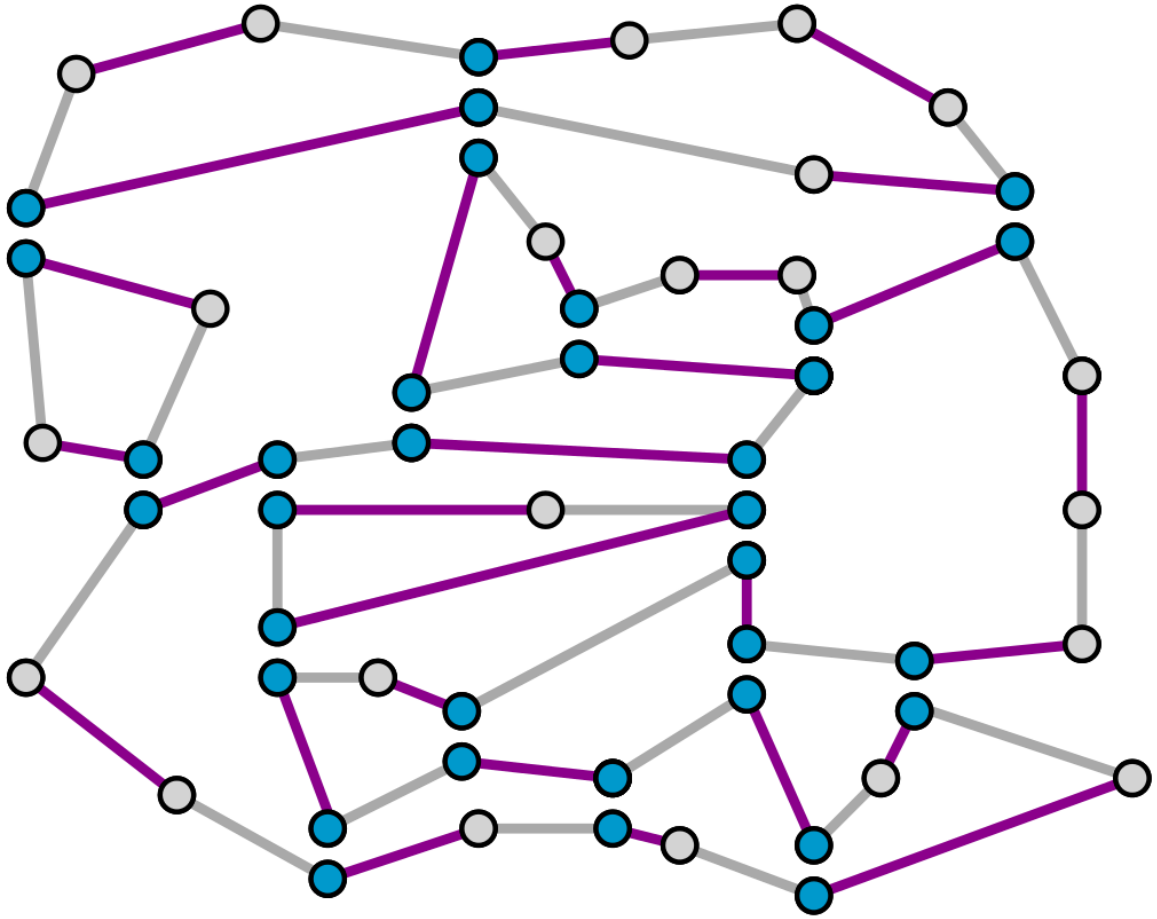

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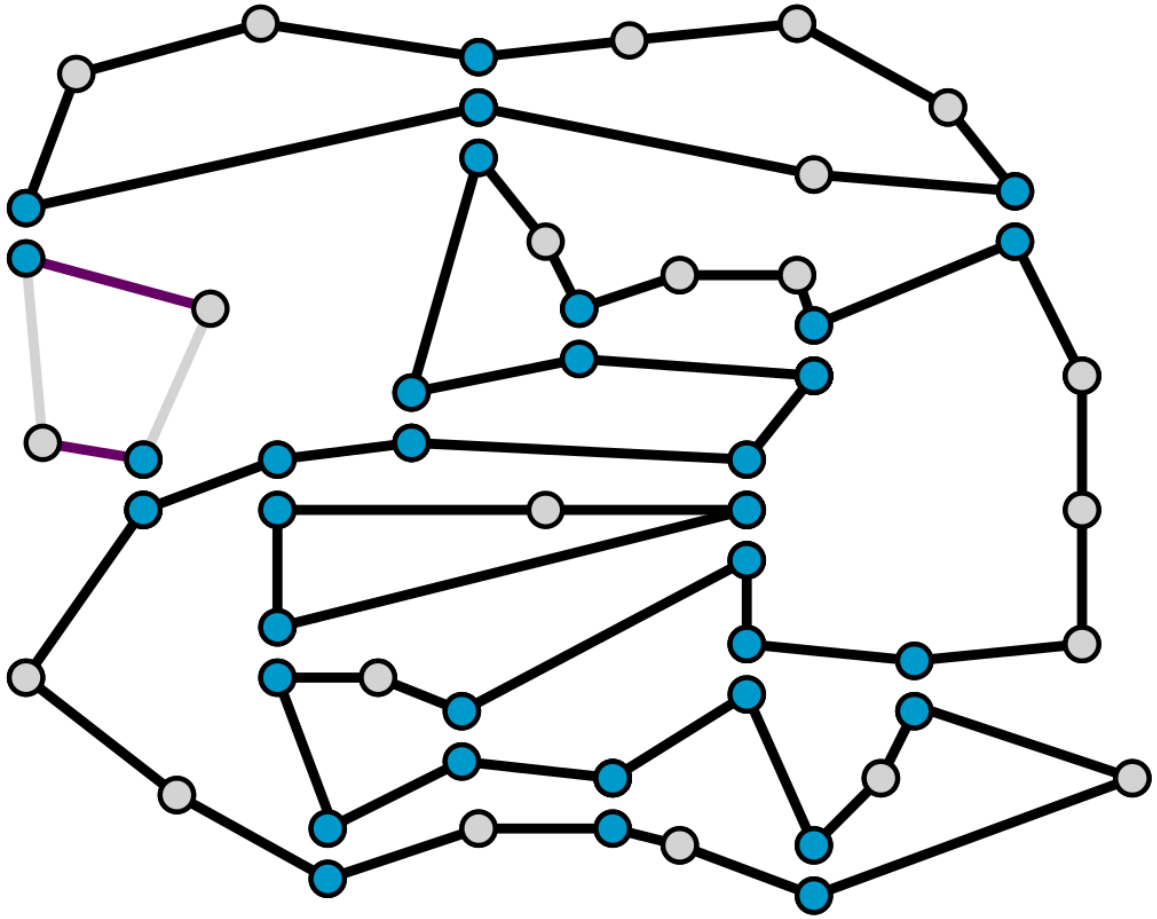

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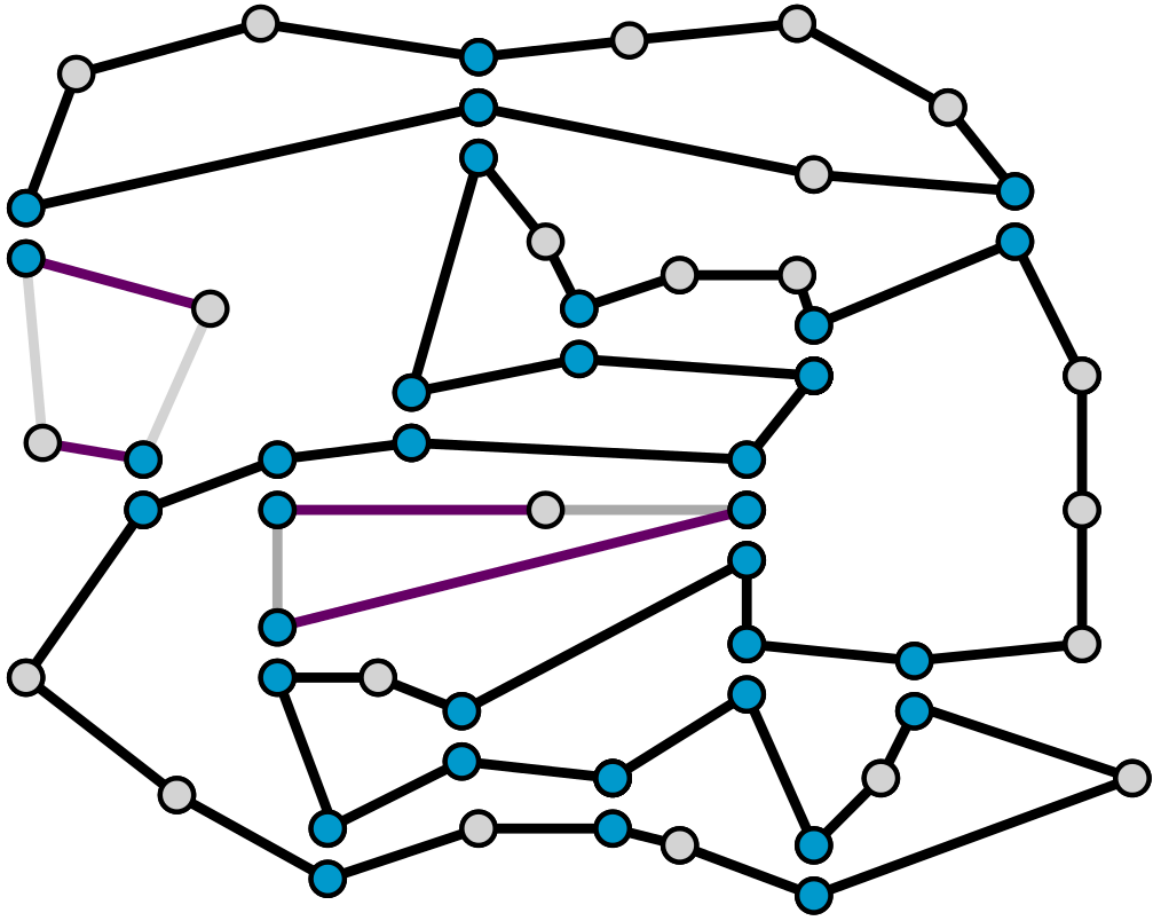

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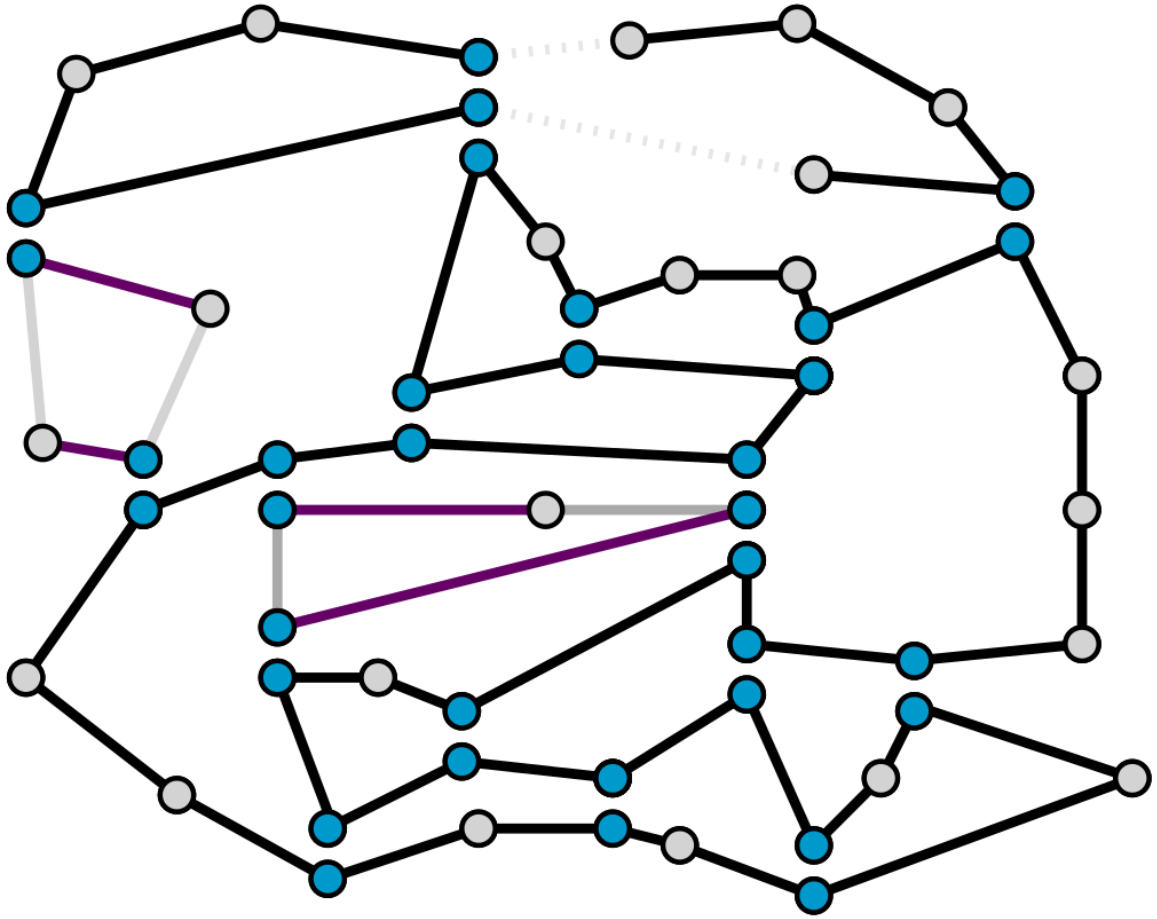

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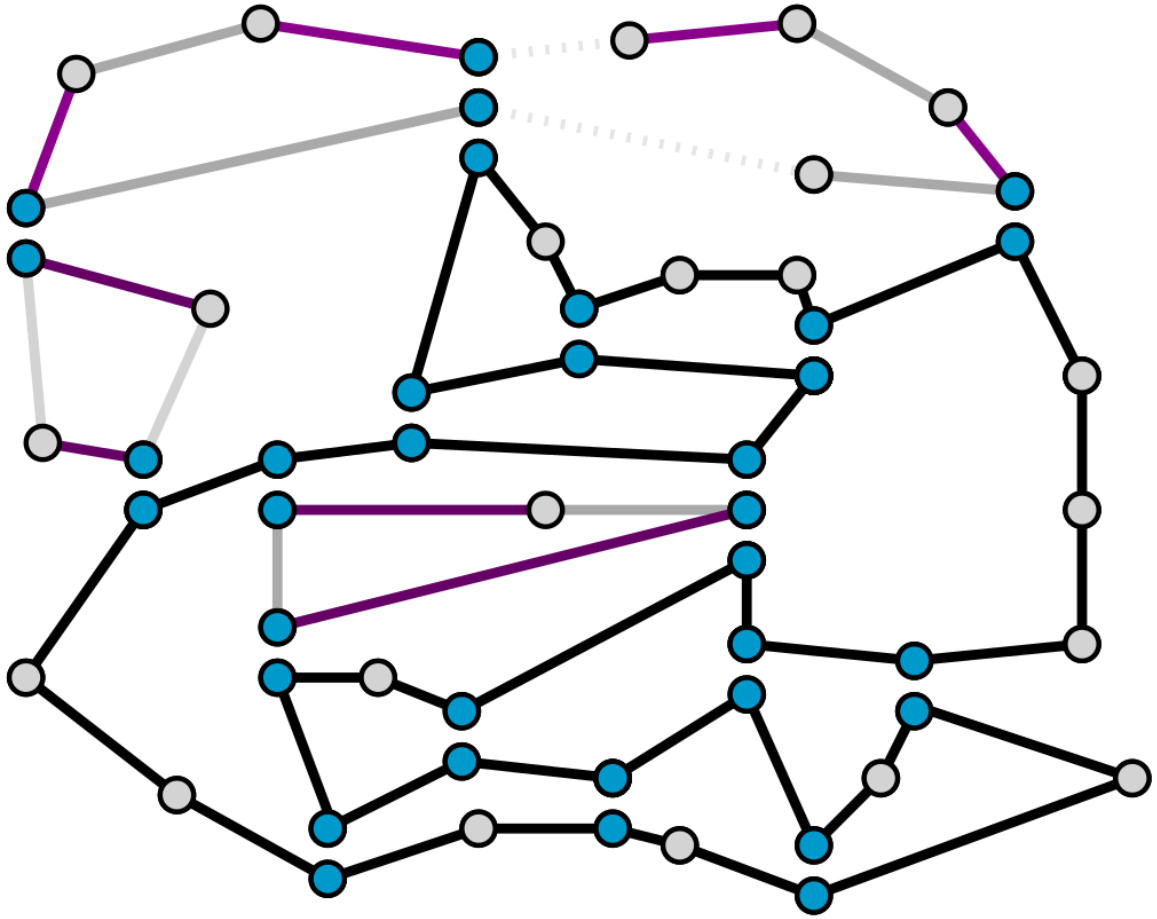

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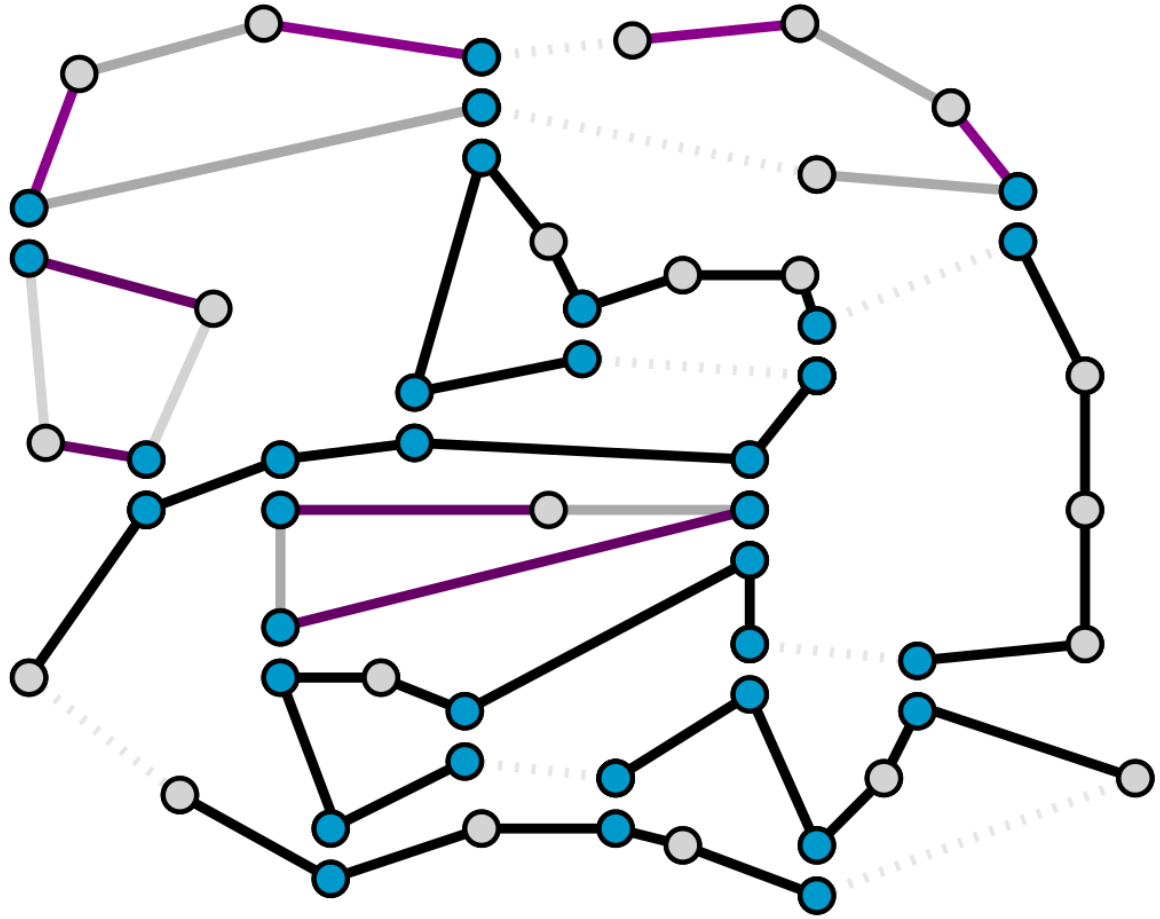

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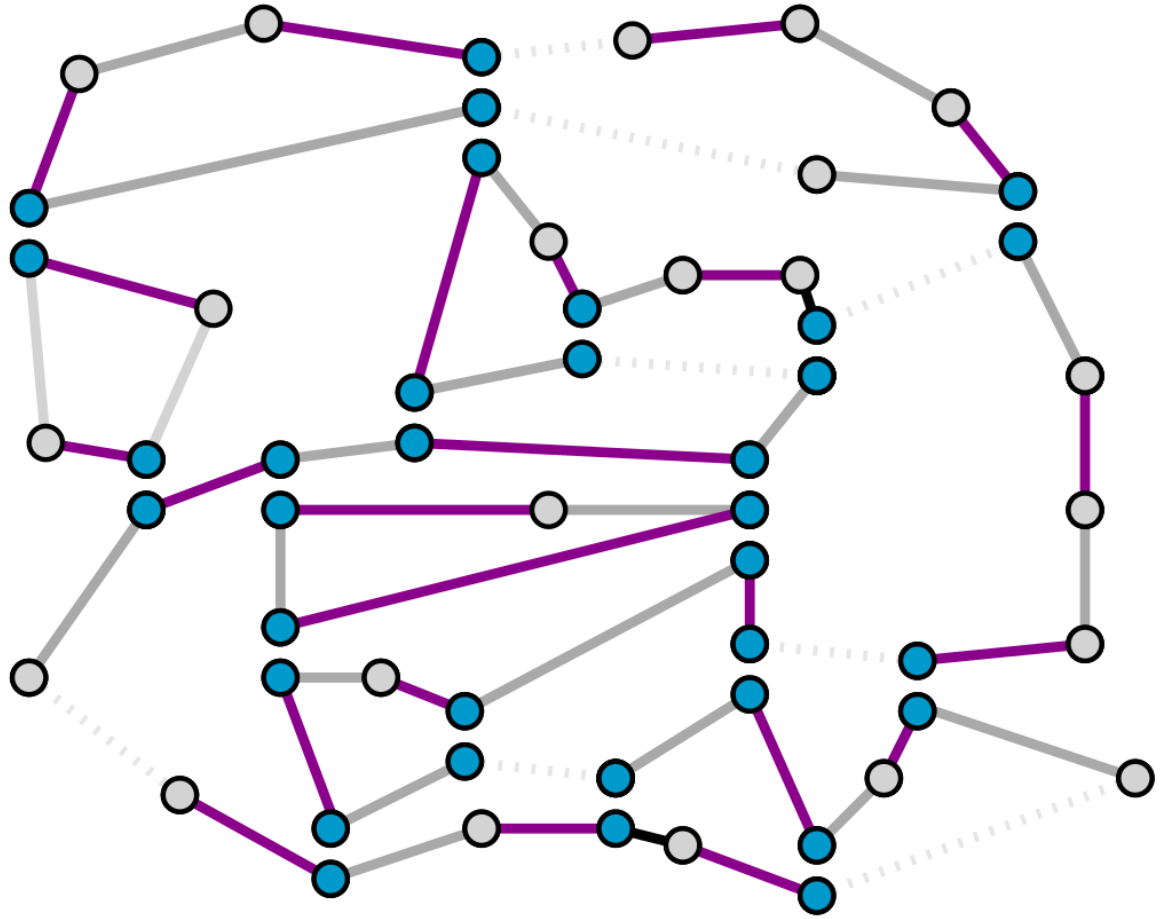

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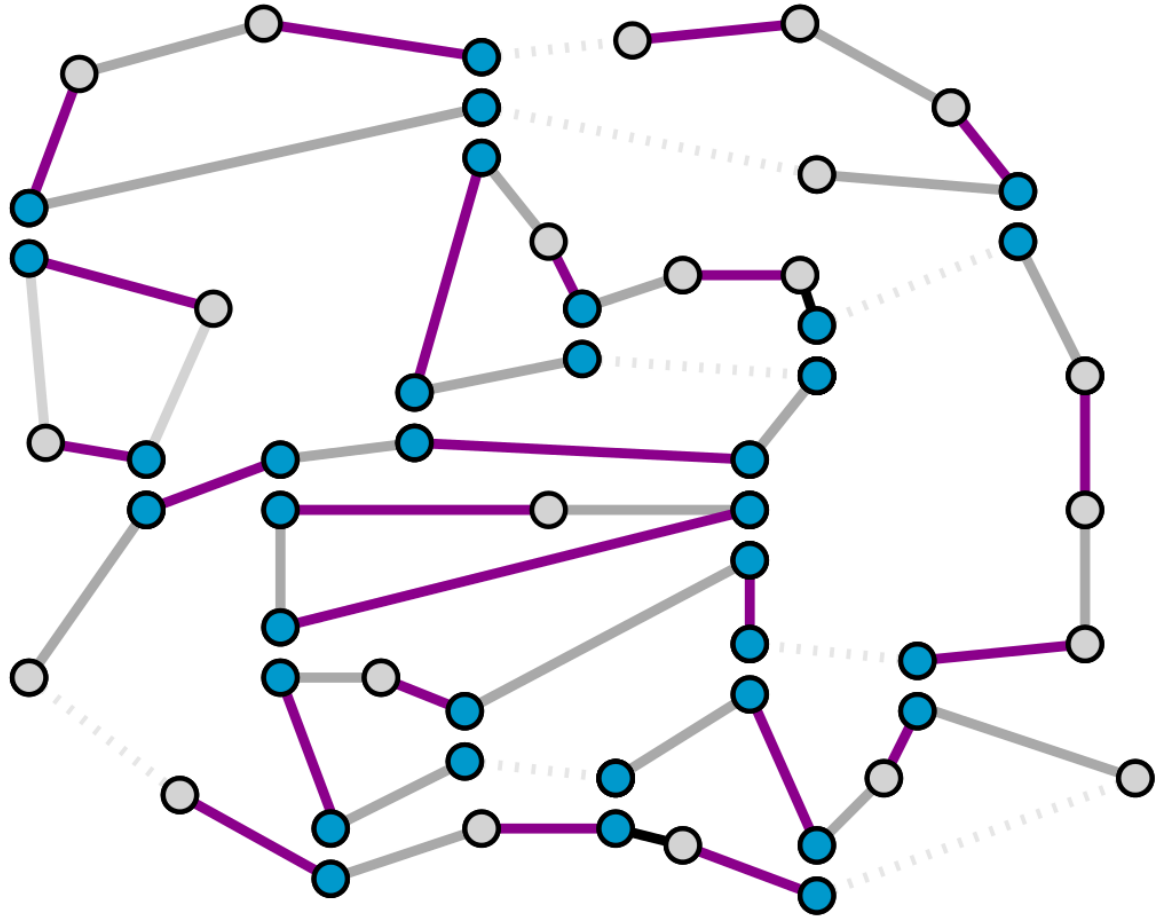

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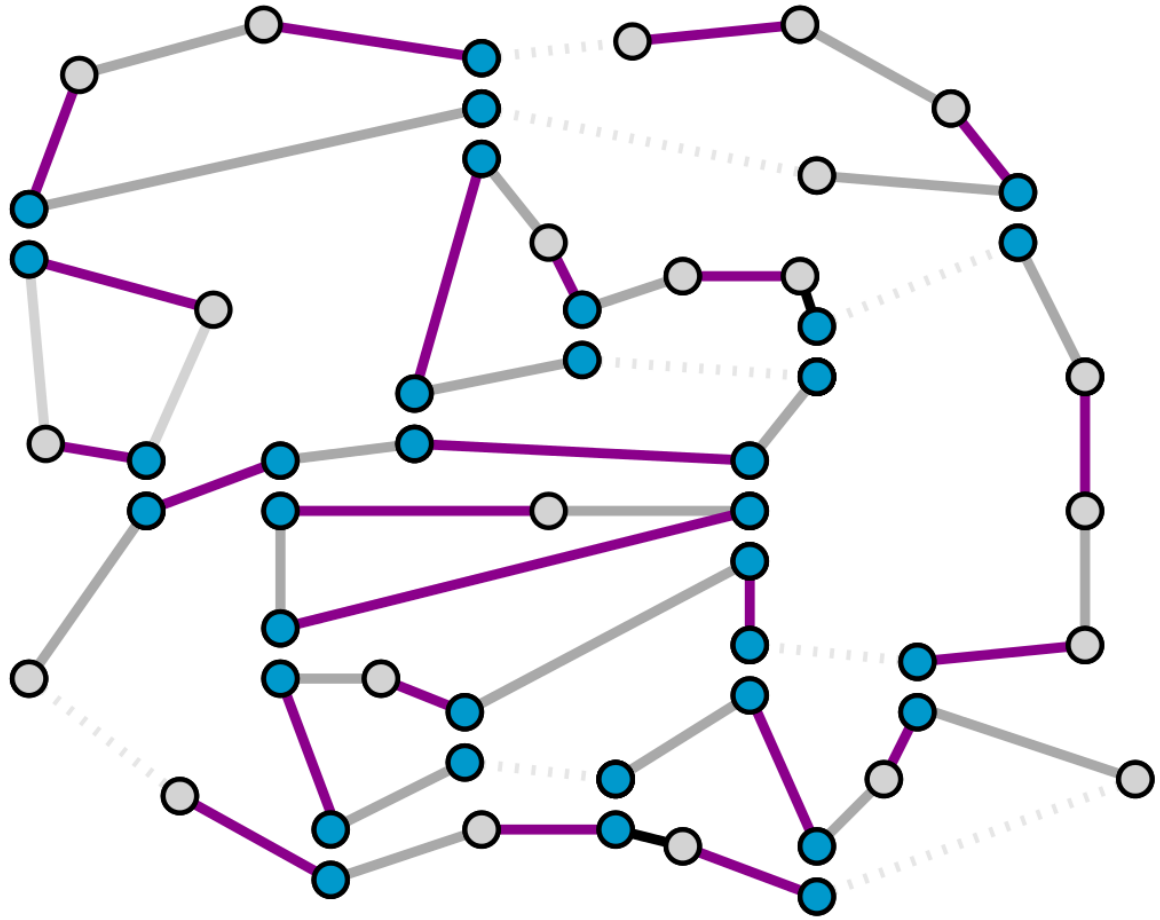

* by *Hańćkowiak, Karoński, Panconesi* [SODA'98,PODC'99] in $O(\log \Delta)$

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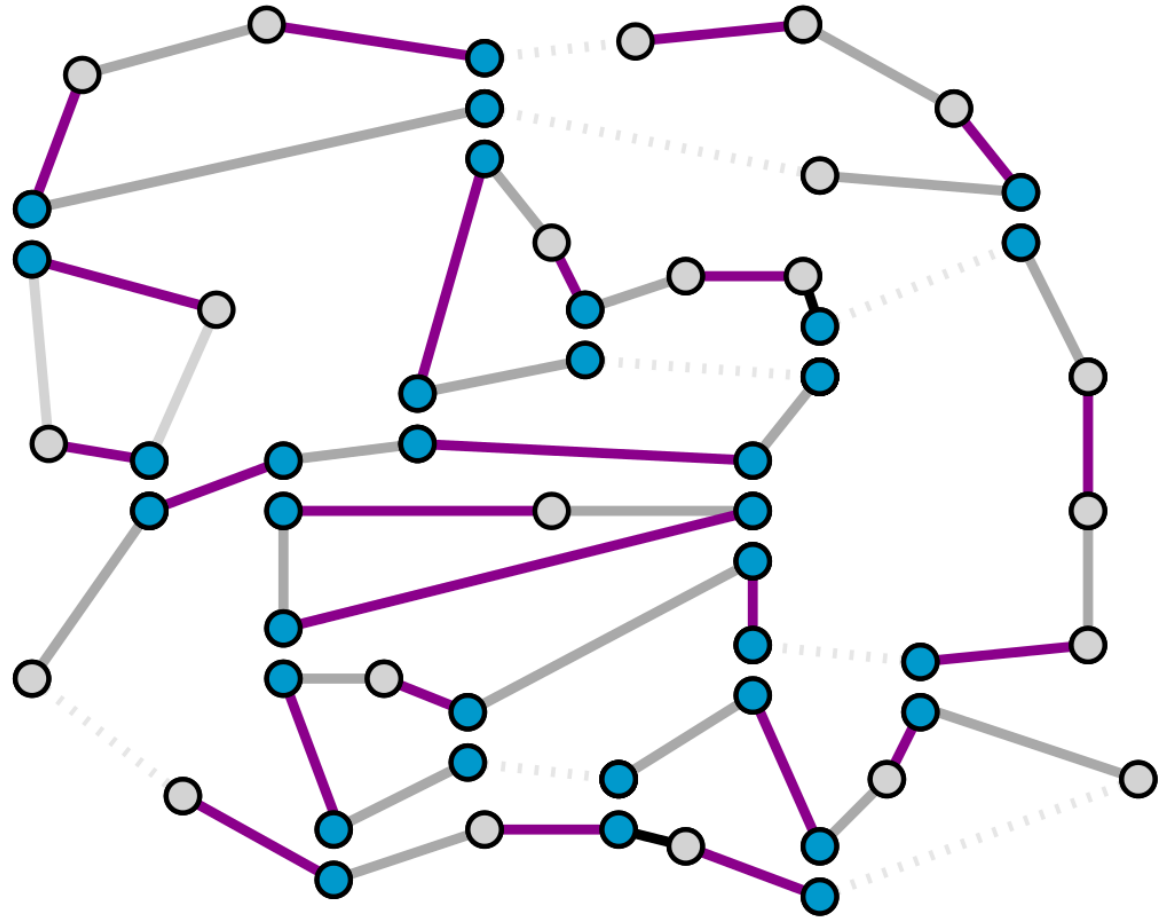

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Over all $O(\log \Delta)$ rounding iterations, total loss still constant!

*** bipartite and even degree!**

Summary

Deterministic LOCAL Complexity of Matching

- $O(\log^2 \Delta \cdot \log n)$ for Maximal Matching
- $O(\log^2 \Delta + \log^* n)$ for constant-approximate Matching
  - exponentially improving over $O(\Delta + \log^* n)$
    Panconesi, Rizzi [DIST'01]
  - close to $\Omega\left(\frac{\log \Delta}{\log \log \Delta} + \log^* n\right)$
    Kuhn, Moscibroda, Wattenhofer [PODC'04]
    Linial [FOCS'87]

Deterministic Rounding

- First efficient deterministic rounding method
- Rounding as **the only obstacle** for efficient deterministic LOCAL graph algorithms.
  Ghaffari, Kuhn, Maus [STOC'17]
- First **efficient deterministic $(2\Delta - 1)$-Edge-Coloring** using rounding for hypergraph matching
  F., Ghaffari, Kuhn [FOCS'17]

Open Problems

- Prove or disprove $\Omega(\log n)$ for Maximal Matching.
- **Conjecture** *Göös, Hirvonen, Suomela* [PODC'14]
  No $o(\Delta) + O(\log^* n)$ algorithm for Maximal Matching.

- Rounding in $O(\log \Delta)$ instead of $O(\log^2 \Delta)$?
- Devise a more general deterministic rounding method.

- **Linial's Question from the 1980s:**
  Is there an efficient deterministic algorithm for MIS?