Chapter -1

How to avoid reading this thesis

This thesis surveys recent developments concerning randomized algorithms for linear programming, related and more general optimization problems. It is meant to be read chapter by chapter. If you would like to avoid this and only want to look up particular algorithms and topics, this chapter gives some advice how and where things can be found.

-1.1 If you just want to know what the title means

Read Chapter 0, the introduction. It explains what randomization means in this thesis and what it does *not* mean. It explains the differences between combinatorial algorithms like simplex and other methods like ellipsoid/interior point, in particular with respect to the model of computation. Finally, it contains a rough account of the kinds of optimization problems studied in this thesis and what the results are concerning these problems.

-1.2 If you want to learn the Simplex Method

Read Chapter 1. You will find a presentation (staying very close to Chvátal's [10]) by example, containing all crucial ideas. No lemmas, no proofs, just small, actual LPs that explain the method in action. Topics covered are the standard and the revised simplex method (including treatment of infeasible, unbounded and degenerate cases) and pivot rules. This is basically the material of Chvátal's Chapters 2, 3 and 7, however with a slightly different (probably more theoretical) focus and much more condensed. Of course, some advanced material is missing; due to similarity of terminology you can easily look it up in Chvátal's book.

-1.3 If you have read my STACS '96 paper with Emo Welzl

In this paper [20], my thesis is referenced several times. Here is where to look for the appropriate part of the text if you want to trace the respective references (ordered by occurence in [20].

- the explicit primitives for the polytope distance and minimum spanning ball problem appear in Chapter 4. They are obtained as specializations of a generic primitive for a certain class of convex optimization problems. The generic primitive appears in Subsection 4.2.4, the specializations together with the actual time bounds can be found in Subsections 4.3.1 and 4.3.2.

- The subexponential bound for LP appears in the more general context of abstract optimization problems in Chapter 5. It is stated in Subsection 5.2.1, Theorem 5.10 and proved in Subsection 5.2.2. The main recurrence that entails the subexponential bound appears in Lemma 5.5, where it is derived for the second time in the thesis (the first time being the proof of Theorem 3.10 (ii), see next item).

- The simple (linear in n, exponential in d) bound for LP-type problems is proved in Chapter 3, Subsection 3.2.2. As well as the subexponential bound from the previous item it is based on the same recurrence relation, which is here solved in an easy but nonoptimal way.

- for the concrete time bounds $(O(d), O(d^3))$ that are mentioned for the primitive operations in the polytope distance and smallest enclosing ball problem see the first item. Please note that in the thesis bounds of $O(d^4)$ for the second primitive is proved. This is due to the fact that the first version of the thesis (which is the one we refer to in [20]) contained an error on which the $O(d^3)$ bound was based. The bound is probably true but the techniques used in the thesis do only prove $O(d^4)$.

- The subexponential bound for the small LP-type problems referred to in [20] appears in Chapter 5, as an analysis for Algorithm 5.13 which is basically identical to the Algorithm `Small_LPtype_E` in the paper. The bound is stated in Subsection 5.3.3 and derived in Subsection 5.3.4.

- The difference between *basis computation* and *basis improvement* is motivated and explained in the beginning of Chapter 3, Section 3.1 where LP-type problems are first introduced as an abstraction of LP. In the thesis, however, they are called LP-type systems and have a slightly different definition (which essentially is equivalent to the one of LP-type problems, as proved in Lemma 3.3).

- The deterministic lower bound for abstract optimization problems is proved in in the lower bound Chapter 6, Section 6.1.

-1.4 If you want to understand subexponential LP in the version of Matoušek, Sharir, Welzl

Do not read this thesis. It contains the results by Matoušek, Sharir and Welzl as a subset, but written up in a quite different way. While [33] develops the subexponential algorithm 'from scratch' and only later remarks that it can be cast as a dual simplex algorithm, I start with the simplex method and obtain the algorithm of [33] as a concrete realization via some particular pivot rule (which turns history upside down). This means, in order to get to the results of [33], you have to dig through the simplex method in chapter 1 at least, and probably through quite some other overhead until you get to the insights you would gain much faster by reading the original paper [33]. However, if you *are* interested in a different view of the results in [33], you might want to spend the effort after all.

-1.5 If you want to understand subexponential LP in the version of Kalai

Kalai was the first to give a subexponential LP algorithm, which incidentally was a simplex algorithm [27]. His paper is not easy to read, and the bounds he gives are not proved in the paper. His main algorithm and the recurrence relation he develops for its runtime are quite similar to my Algorithm 5.13 SMALL-AOP-E and the recurrence developed in Lemma 5.18; in fact, my approach is motivated by Kalai's algorithm. If you manage to follow my analysis, you will be able to apply it to Kalai's recurrence and obtain the bounds he claims. Algorithm 5.13 is a quite abstract realization of Kalai's main idea of 'enlarging the sample space', and this idea is extensively motivated and explained in Subsections 5.3.1 and 5.3.2.