1 Analyze the Ball-Carving with Exponential Clocks

Consider a following process of Ball-Carving:

- every vertex $v$ pick radius $r_v$ according to exponential distribution, that is with probability density $\text{EXP}(x) = \beta \cdot e^{-\beta x}$ for $x \geq 0$.
- every vertex $u$ picks as its ball-center the vertex $v = \arg \max_x (r_x - d(u, x))$ (we say that $u \in B_v$)

(1) Show that each $B_v$ is an induced subgraph of $G$.
(2) Give a reasonable upperbound on the diameter of each $B_v$ (that holds w.h.p.).
(3) Give a reasonable lowerbound on the diameter of each $B_v$ (that holds w.h.p.).

2 Min-Cost Communication Network

2.1 Small average stretch tree

Consider a problem of finding a single tree $T$ such that the average stretch is small. More precisely, given weight function $w : V \times V \rightarrow \mathbb{R}^+$, we say that $T$ has average stretch $\alpha$ if

$\forall u, v \in V \quad d^G(u, v) \leq d^T(u, v)$

$\sum_{u, v \in V} w_{u, v} d^T(u, v) \leq \alpha \sum_{u, v \in V} w_{u, v} d^G(u, v)$.

Adapt the FRT tree embedding algorithm from the lecture to solve this problem (with the same stretch parameter $\alpha = O(\log n)$).

Hint: This is a dual problem to the tree embedding problem.

2.2 Minimal cost communication network

In this problem, we are given $n$ nodes $v_1, \ldots, v_n$ in a metric network, and a set of communication requirement $r_{i,j}, 1 \leq i, j \leq n$. The goal is to find a spanning tree $T$ for nodes that minimizes $\sum_{i,j} d^T(v_i, v_j) \cdot r_{i,j}$.

Hint: sample single tree $T$ and solve the problem in $T$. Project solution back to $G$.

3 Steiner Forest

Given edge weighted graph $G$, and a set of pairs of terminals $(s_1, t_1), \ldots, (s_k, t_k)$, consider problem of finding minimal cost $E'$ such that $s_i, t_i$ are connected in $G[E']$. Use FRT tree embedding algorithm to build $O(\log n)$ approximate algorithm to this problem.

Hint: sample single tree $T$ and solve the problem in $T$. Project solution back to $G$. 