Advanced Algorithms 2021 12/10, 2021

Exercise 11

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1 Steiner Forest [Leftover]

Given a weighted graph G with positive edge weights, and a set of pairs of terminals $(s_1, t_1), \dots, (s_k, t_k)$, consider problem of finding minimal cost E' such that s_i, t_i are connected in $G[E']$. Use the tree embedding algorithm from the lecture to build $\mathcal{O}(\log n)$ approximate algorithm in expectation to this problem.

Hint:

Sample single tree T and solve the problem in T. Project solution back to G.

2 L1 embedding of cycle [Recommended]

You are given an unweighted cycle C on n vertices.

1. Find a randomized algorithm that embeds the cycle C to $\mathbb{R}$ such that the expected stretch of every edge is constant. That is, your randomized algorithm maps each vertex $u \in C$ to some number $f(u)$. For every pair $u, v \in C$ it has to be the case that $d_C(u,v)/K \leq \mathbb{E}[|f(u) - f(v)|] \leq d_C(u,v)$ for some constant K.
2. Find a deterministic algorithm that embeds C to $\mathbb{R}^2$ with L_1 norm such that the stretch of every edge is constant. That is, you should map each vertex $u \in C$ to some point $f(u) \in \mathbb{R}^2$. For every pair $u, v \in C$ it has to be the case that $d_C(u,v)/K \leq ||f(u) - f(v)||_1 \leq d_C(u,v)$ for some constant K.

3 Minimum bisection cut

A *bisection cut* is a cut (S, S') such that $|S| = |S'| = n/2$. An *r-balanced cut* is a cut where $r \cdot n \leq |S| \leq (1-r) \cdot n$. A size of a cut is the number of edges that go across the cut.

Give a polynomial-time algorithm that, given a graph G as input, outputs a 1/3-balanced cut whose size is $\mathcal{O}(\log n)$ factor from the size of the smallest-size bisection cut of G.

Hint:

Find a black box reduction to the result you saw in the lecture via a greedy algorithm.