Time hierarchies of distributed complexities

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**Topic: distributed graph problems**

- **Family** of graph problems
- **Focus on locality**
  - How much does an entity need to know about the graph in order to solve a graph problem?
  - How local are these problems?
  - Does randomness help?
LOCAL model

- Entities = nodes
- Communication links = edges
- Input graph = communication graph
LOCAL model

- Each node has a unique identifier from 1 to $\text{poly}(n)$
- No bounds on the computational power of the entities
- No bounds on the bandwidth
LOCAL model

• At each **synchronized** round:
  • **Send** messages to neighbours
  • **Receive** messages from neighbours
  • Perform **local computation**
LOCAL model

- After $t$ rounds:
  - knowledge of the graph up to distance $t$
- Focus on locality:
  - time = number of rounds = distance
LOCAL model

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• Randomized LOCAL:
  • each node can generate an unbounded number of random bits
LOCAL model

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  • time = number of rounds = distance
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Everything can be solved in Diameter time!
LOCAL model: initial knowledge

• **Initial knowledge** of a node:
  • \( n \) = the total number of nodes in the graph
  • \( \Delta \) = the maximum degree of the graph
  • Its unique ID
  • A **port numbering** of its incident edges
Locally Checkable Labelings (LCLs)

A family of graph problems that includes many important problems
Maximal Independent Set, Maximal Matching, vertex coloring, edge coloring...
Locally Checkable Labelings (LCLs)

• **Input**
  • Graph of constant maximum degree $\Delta$
  • Node labels from a constant-size set $X$

• **Output**
  • Node labels from a constant-size set $Y$, such that each node satisfies some local constraints

• **Correctness**
  • A solution is globally correct if it is correct in all constant-radius neighborhoods

[Naor and Stockmeyer, 1995]
Example: weak 2-coloring

- **Output**: color nodes from a palette of 2 colors
- **Constraint**: each node must have a different color from at least 1 neighbor
Landscape of LCLs

• Which time complexities are possible for LCLs?
• How local are LCLs?
• Does randomness help in solving an LCL faster?
Randomised

Deterministic
Randomised
Deterministic

\[ \log \log n \]

\[ \log n \]

\[ \log^* n \]

\[ n \]
$\Theta(\log n)$ deterministic

$\Theta(\log^* n)$ randomized
Paths/Cycles

Randomised

Deterministic

1

log* n

log n

n

n

\log n

\log \log n
Paths/Cycles

- Trivial
- Trivial
- Cole & Vishkin 1986
- Linial 1992

Graph: Points at 1, log* n, log n, and n. Lines indicate deterministic and randomised algorithms.
Paths/Cycles

- Cole & Vishkin 1986
- Linial 1992
- Naor & Stocmayer 1995
- Chang et al. 2016
Gaps

- $\omega(1) - o(\log^* n)$ gap:
  - Every algorithm $A$ that solves an LCL $P$ in $o(\log^* n)$ rounds can be automatically sped up into an algorithm $A'$ that solves $P$ in $O(1)$ rounds.

- $\omega(\log^* n) - o(n)$ gap:
  - Every algorithm $A$ that solves an LCL $P$ in $o(n)$ rounds can be automatically sped up into an algorithm $A'$ that solves $P$ in $O(\log^* n)$ rounds.
Trees and general graphs?
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Trees and general graphs?
Lots of progress since 2016

- Brandt, Fischer, Hirvonen, Keller, Lempiäinen, Rybicki, Suomela, Uitto [STOC 2016]
- Chang, Kopelowitz, Pettie [FOCS 2016]
- Ghaffari, Su [SODA 2017]
- Brandt, Hirvonen, Korhonen, Lempiäinen, Östergård, Purcell, Rybicki, Suomela, Uznański [PODC 2017]
- Fischer, Ghaffari [DISC 2017]
- Chang, Pettie [FOCS 2017]
- Chang, He, Li, Pettie, Uitto [SODA 2018]
- B., Hirvonen, Korhonen, Lempiäinen, Olivetti, Suomela [STOC 2018]
- Ghaffari, Hirvonen, Kuhn, Maus [PODC 2018]
- B., Brandt, Olivetti, Suomela [DISC 2018]
- B., Brandt, Olivetti, Suomela [Unpublished 2019]
Trees

Randomised

Deterministic

$\log \log n$

$n^{1/3}$

$n^{1/k}$

$\log n$

$\log \log n$

$\log^* n$

$log \log^* n$

$1$

Randomised

Deterministic
Trees

Randomised

Deterministic

$1$, $\log^* n$, $\log n$, $\log \log n$, $\log \log^* n$

$\log^* n$, $\log n$, $\log \log n$, $\log \log^* n$

$n^{1/k}$, $n^{1/3}$, ...

$n^{1/2}$, $n^{1/3}$, ...

$n$, $n^{1/2}$, $n^{1/k}$, $n^{1/3}$, ...

$1/2$, $1/3$, $1/k$, ...

$1$, $\log^* n$, $\log n$, $\log \log n$, $\log \log^* n$
Trees

Randomised

Deterministic

1

log log* n

log* n

log n

log log n

log log* n

1

1

n

n

n^{1/2}

n^{1/3}

n^{1/k}

n

Chang & Pettie 2017

Balliu et al. 2018

Naor & Stocmayer 1995

Chang et al. 2016

Chang et al. 2016

Ghaffari & Su 2017

Chang et al. 2018

Chang & Pettie 2017

Chang & Pettie 2017

Chang & Pettie 2017

Chang & Pettie 2017

Chang et al. 2016

Chang et al. 2016

Unpublished

gap: Chang & Pettie 2017

gap: Chang et al. 2016

gap: Chang et al. 2018

gap: Chang et al. 2016

gap: Chang et al. 2016

gap: Chang et al. 2017

gap: Chang & Pettie 2017

gap: Chang & Pettie 2017

gap: Chang (unpublished)
Trees

Open problem
Trees

Homogeneous LCLs

[B., Hirvonen, Olivetti, Suomela 2019]
General graphs
General graphs
General graphs

Randomness may help, but not by much
Randomised
Deterministic

Thank you!