Three Challenges in Distributed Optimization

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Workshop on Local Algorithms WOLA 2019

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Optimization problems
Optimization problems

Minimum vertex cover
Optimization problems

Minimum vertex cover

NP-hard

$(2-\varepsilon)$-approximation is UG-hard

(some algs. with a smaller-than-2 apx)
Distributed optimization

Minimum vertex cover

Complexity ?
Distributed graph algorithms

#Nodes = n
#Bandwidth = B
(typically $O(\log n)$)

Knowledge of neighbors

#Rounds = ?
Distributed graph algorithms

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#Bandwidth = B
(typically $O(\log n)$)

Knowledge of neighbors

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Models:
CONGEST
LOCAL
Distributed graph algorithms

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#Bandwidth = B
(typically O(log n))

Knowledge of neighbors

#Rounds = ?

Models:
CONGEST
LOCAL
Distributed graph algorithms

#Nodes = \( n \)
#Bandwidth = \( B \)
(typically \( O(\log n) \))

Knowledge of neighbors

#Rounds = ?

Models:

CONGEST \( O(m) = O(n^2) \)
LOCAL \( O(D) \)
Challenge 1:
Challenge 1: Distances
Distances

Exact minimum vertex cover: $\Omega(D)$ rounds
Distances

Exact minimum vertex cover: $\Omega(D)$ rounds

Approximations:
**LOCAL**: $(1 + \epsilon)$-approximation in $O(poly(\log n / \epsilon))$ rounds

[Ghaffari, Kuhn, Maus ‘17]
Distances

Exact minimum vertex cover: $\Omega(D)$ rounds

Approximations:

**LOCAL**: $(1 + \epsilon)$-approximation in $O(poly (\log n / \epsilon))$ rounds
[Ghaﬀari, Kuhn, Maus ‘17]

$\Omega(\log \Delta / \log \log \Delta), \Omega(\sqrt{\log n / \log \log n})$ rounds
[Kuhn, Moscibroda, Wattenhofer ‘04]

$\Omega(1/\epsilon)$ [Ben-Basat, Kawarabayashi, Schwartzman ‘18]
Optimal solutions for pieces of the graph

**LOCAL**: \((1 + \epsilon)\)-approximation in \(O(poly(\log n / \epsilon))\) rounds

[Ghaffari, Kuhn, Maus ‘17]

Come to Yannic’s talk tomorrow!
Optimal solutions for pieces of the graph

**LOCAL**: $(1 + \epsilon)$-approximation in $O(poly (\log n /\epsilon))$ rounds

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**LOCAL**: $(1 + \epsilon)$-approximation in $O(poly (\log n / \epsilon))$ rounds

[Ghaffari, Kuhn, Maus ‘17]

Come to Yannic’s talk tomorrow!
Optimal solutions for pieces of the graph

\textbf{LOCAL}: $(1 + \epsilon)$-approximation in $O(poly (\log n / \epsilon))$ rounds
\cite{Ghaffari, Kuhn, Maus '17}

Come to Yannic’s talk tomorrow!
Optimal solutions for pieces of the graph

**LOCAL**: $(1 + \epsilon)$-approximation in $O(poly (\log n / \epsilon))$ rounds

[Ghaffari, Kuhn, Maus ‘17]

Come to Yannic’s talk tomorrow!
Challenge 2:
Challenge 2: Congestion
Congestion

**LOCAL**: $(1 + \epsilon)$-approximation in $O(poly(\log n / \epsilon))$ rounds

[Ghaffari, Kuhn, Maus ‘17]

Cannot collect dense neighborhoods quickly
(2+\varepsilon)-approximation in \(O(\log \Delta / \log \log \Delta)\) rounds
[Bar-Yehuda, C., Schwartzman ‘16]

\(\Omega(\log \Delta / \log \log \Delta)\), \(\Omega(\sqrt{\log n / \log \log n})\) rounds for approximation
[Kuhn, Moscibroda, Wattenhofer ‘04]
CONGEST

(2+\(\varepsilon\))-approximation in \(O(\log \Delta / \log \log \Delta)\) rounds
[Bar-Yehuda, C., Schwartzman ‘16]

2-approximation [Ben-Basat, Even, Kawarabayashi, Schwartzman ‘18]

\(o(n^2)\), (2-\(\varepsilon\))-approximation [Ben-Basat, Kawarabayashi, Schwartzman ‘18]

\(\Omega(\log \Delta / \log \log \Delta)\), \(\Omega(\sqrt{\log n / \log \log n})\) rounds for approximation
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(2+\(\varepsilon\))-approximation in \(O(\log \Delta / \log \log \Delta)\) rounds
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\(\Omega(\log \Delta / \log \log \Delta)\), \(\Omega(\sqrt{\log n / \log \log n})\) rounds for approximation
[Kuhn, Moscibroda, Wattenhofer ‘04]

\(\Omega(n^2 / \text{poly} \log n)\) rounds for exact [C., Khoury, Paz ‘17]
Minimum vertex cover in CONGEST

#rounds

\(\tilde{\Omega}(n^2)\)
[C., Khoury, Paz ‘17]

[Ben-Basat, Even, Kawarabayashi, Schwartzman ‘18]

[Kuhn, Moscibroda, Bar-Yehuda, C., Wattenhofer ‘04] Schwartzman ‘16]
Minimum vertex cover in \textsc{CONGEST}

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$\tilde{\Omega}(n^2)$

[C., Khoury, Paz ‘17]

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[C., Khoury, Paz ‘17]

[Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]

[Ben-Basat, Even, Kawarabayashi, Schwartzman ‘18]

[Kuhn, Moscibroda, Wattenhofer ‘04] [Bar-Yehuda, C., Schwartzman ‘16]
Minimum vertex cover in CONGEST

#rounds

\[ \tilde{\Omega}(n^2) \]
[C., Khoury, Paz ‘17]

[Ben-Basat, Even, Kawarabayashi, Schwartzman ‘18]

[Kuhn, Moscibroda, Bar-Yehuda, C., Schwartzman ‘04]
[Wattenhofer ‘04, Schwartzman ‘16]

[Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]

1 (exact) 1.5 2 2+\varepsilon

approximation
Minimum vertex cover in CONGEST

#rounds

\(\tilde{\Omega}(n^2)\)

[C., Khoury, Paz ‘17]

[Ben-Basat, Even, Kawarabayashi, Schwartzman ‘18]

[Kuhn, Moscibroda, Wattenhofer ‘04]  [Bar-Yehuda, C., Schwartzman ‘16]

[Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]

1 (exact)  1.5  2  2+\(\varepsilon\)
2-Party communication

[Yao ‘79]
History
History

[Peleg, Rubinovich ‘97] [Lotker, Patt-Shamir, Peleg ’01] [Elkin ‘04] [Das-Sarma, Holzer, Kor, Korman, Nanongkai, Pandurangan, Peleg, Wattenhofer ‘11] [Frischknecht, Holzer, Wattenhofer ‘12] [Ghaffari, Kuhn ’13] [Drucker, Kuhn, Oshman ‘14] [Nanongkai, Das-Sarma, Pandurangan ‘14] [Das-Sarma, Molla, Pandurangan ‘15] [Holzer, Pinsker, ‘15] [Pandurangan, Peleg, Scquizzato ‘16] [Pandurangan, Robinson, Scquizzato ‘16] [C., Kavitha, Paz, Yehudayoff ‘16] [Fischer, Gonen, Kuhn, Oshman ‘18] [C., Dory ‘17] [C., Dory ‘18]
2-Party communication

Alice: $x = x_1, \ldots, x_k$  
Bob: $y = y_1, \ldots, y_k$

Goal: $f(x, y)$
2-Party communication

Alice: \( x = x_1, \ldots, x_k \)
Bob: \( y = y_1, \ldots, y_k \)

Goal: \( f(x, y) \)

Set-Disjointess:

\[ \exists i: x_i = y_i = 1 \, ? \]

\[ \text{cost} = \Omega(k) \]

[Kalyanasundaram and Schnitger ‘87]
[Razborov ‘90]
[Bar-Yossef et al. ‘04]
2-Party communication ➔ CONGEST

Alice: \( x=x_1, \ldots, x_k \)  
Bob: \( y=y_1, \ldots, y_k \)

Graph property \( P \) of \( G_{x,y} \) determines \( f(x,y) \)
2-Party communication ➔ CONGEST

Alice: $x=x_1,\ldots,x_k$  Bob: $y=y_1,\ldots,y_k$

Graph property $P$ of $G_{x,y}$ determines $f(x,y)$

$\text{Rounds} \cdot \text{cut} \cdot B = \Omega(\text{cost}(f(k)))$
2-Party communication ➔ CONGEST

Alice: $x=x_1,\ldots,x_k$  Bob: $y=y_1,\ldots,y_k$

Graph property $P$ of $G_{x,y}$ determines $f(x,y)$

$$\text{Rounds} \cdot \text{cut} \cdot B = \Omega(\text{cost}(f(k)))$$

$$\text{Rounds} = \Omega(\frac{\text{cost}(f(k))}{\text{cut} \cdot B})$$
Warm-up: MVC, CONGEST

$$\tilde{n}$$

Cut = $$\tilde{n} = n/6$$
Warm-up: MVC, CONGEST

Add edge iff input is 0

cut = \tilde{n} = n/6
Warm-up: MVC, CONGEST

Add edge iff input is 0

cut = \tilde{n} = n/6

k = \tilde{n}^2
Warm-up: MVC, CONGEST

\[ \text{MVC} = 4\tilde{n} - 2 \]

inputs not disjoint
Warm-up: MVC, CONGEST

\[ \text{MVC} = 4\tilde{n} - 2 \quad \text{inputs not disjoint} \]

\[ \text{MVC} \geq 4\tilde{n} - 1 \quad \text{inputs disjoint} \]
Warm-up: MVC, CONGEST

\[ MVC = 4\tilde{n} - 2 \]
inputs not disjoint

\[ MVC \geq 4\tilde{n} - 1 \]
inputs disjoint

Lower bound:
\[ \Omega\left(\frac{k}{n \log n}\right) = \tilde{\Omega}(n) \]
rounds
Minimum vertex cover in CONGEST

- \(\tilde{\Omega}(n^2)\) [C., Khoury, Paz ‘17]
- [Ben-Basat, Even, Kawarabayashi, Schwartzman ‘18]
- [Kuhn, Moscibroda, Wattenhofer ‘04] [Bar-Yehuda, C., Schwartzman ‘16]

\#rounds

[Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]

approximation

1 (exact) 1.5 2 2+\(\varepsilon\)
Small cuts for MVC

[C., Khoury, Paz ‘17]
Bit-gadget used for, e.g.,
diameter in [Abboud, C.,
Khoury ‘16]
Small cuts for MVC

[C., Khoury, Paz ‘17]

MVC = 4(\tilde{n} - 1 + \log \tilde{n})

inputs not disjoint
Small cuts for MVC

[C., Khoury, Paz ‘17]

\( k = \Theta(n^2), \text{cut} = \Theta(\log n) \)

Rounds

\[ = \Omega\left(\frac{\text{cost}(f(k))}{\text{cut} \cdot \log n}\right) \]

\[ = \Omega\left(\frac{n^2}{\log^2 n}\right) \]
Minimum vertex cover in CONGEST

#rounds

$\tilde{\Omega}(n^2)$

[C., Khoury, Paz ‘17]

[Ben-Basat, Even, Kawarabayashi, Schwartzman ‘18]

[Kuhn, Moscibroda, Bar-Yehuda, C., Wattenhofer ‘04] [Schwartzman ‘16]

[Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]

approximation
Why not for 1.5-approximation?

- Alice/Bob compute $\text{OPT}_A$ and $\text{OPT}_B$ for their parts
- The smaller is at most $\text{OPT}/2$
Why not for 1.5-approximation?

• Alice/Bob compute $\text{OPT}_A$ and $\text{OPT}_B$ for their parts
• The smaller is at most $\text{OPT}/2$
Why not for 1.5-approximation?

- Alice/Bob compute $\text{OPT}_A$ and $\text{OPT}_B$ for their parts
- The smaller is at most $\text{OPT}/2$
- The other side fills in the rest, at most $\text{OPT}$
Optimization problems
Maximum independent set in CONGEST

#rounds

\( \tilde{\Omega}(n^2) \)
[C., Khoury, Paz ‘17]

[Kawarabayashi, Khoury, Schild, Schwartzman ‘19]
[Boppana, Halldorsson, Rawitz ‘18]
Maximum independent set in CONGEST

#rounds

\[ \tilde{\Omega}(n^2) \]
[C., Khoury, Paz ‘17]

[Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]

[Kawarabayashi, Khoury, Schild, Schwartzman ‘19]

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\[O(1/\Delta)\]
(approximation)

1 7/8 (exact)
Maximum independent set in **CONGEST**

- #rounds
  - $\tilde{\Omega}(n^2)$
    - [C., Khoury, Paz ‘17]

- Approximation
  - [Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]

- Exact
  - 1
  - $7/8$
  - $1/2$
  - $O(1/\Delta)$

- [Kawarabayashi, Khoury, Schild, Schwartzman ‘19]
- [Boppana, Halldorsson, Rawitz ‘18]
7/8-approximation for MaxIS

[Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]

Bit-gadget
7/8-approximation for MaxIS

[Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]

Code-gadget: non-binary codewords with a polylog distance

$\tilde{\Omega}(n^2)$ rounds
Optimization problems

- Minimum vertex cover
- Maximum independent set
Optimization problems

• Minimum vertex cover
• Maximum independent set
• Max cut

[Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]
  – Requires $\tilde{\Omega}(n^2)$ rounds
  – $(1 - \epsilon)$-approximation in $\tilde{O}(n)$ rounds [Zelke ‘09]
Optimization problems

• Minimum vertex cover
• Maximum independent set
• Max cut
  [Bachrach, C., Dory, Efron, Leitersdorf, Paz ‘19]
  – Requires $\tilde{\Omega}(n^2)$ rounds
  – $(1 - \varepsilon)$-approximation in $\tilde{O}(n)$ rounds [Zelke ‘09]
• FT-BFS
  [Ghaffari, Parter ‘16]
  – 2-approximation in $\tilde{O}(D)$ rounds
  – sequential approximation implies set cover
Challenge 3:
Challenge 3: ???
Alice-Bob limitations

• For triangle detection [Drucker, Kuhn, Oshman ‘14]
  – [Izumi, Le Gall ‘17]: $\tilde{O}(n^{2/3})$
  – [Chang, Pettie, Zhang, ’18]: $\tilde{O}(n^{1/2})$
  – [Chang, Saranurak ‘19]: $\tilde{O}(n^{1/3})$

• For weighted APSP above $\Omega(n)$ [C., Khoury, Paz ’17]
  – [Elkin ’17]: $\tilde{O}(n^{5/3})$
  – [Huang, Nanongkai, Saranurak ’17]: $\tilde{O}(n^{5/4})$
  – [Agarwal, Ramachandran, King, Pontecorvi ’18]: $\tilde{O}(n^{3/2})$ det.
  – [Bernstein, Nanongkai ‘19]: $\tilde{O}(n)$

• For 4-clique detection above $\Omega(n^{1/2})$ [Czumaj, Konrad ‘18]
  – [Eden, Fiat, Fischer, Kuhn, Oshman ‘19]: first sublinear algorithm
Distributed optimization

1. Distances
2. Congestion
3. ???

Can we have better approximation factors in the CONGEST model?
Distributed optimization

1. Distances
2. Congestion
3. ???

Can we have better approximation factors in the CONGEST model?

CONGESTED CLIQUE? MPC? Streaming?
Distributed optimization

1. Distances
2. Congestion
3. ???

Can we have better approximation factors in the CONGEST model?
CONGESTED CLIQUE? MPC? Streaming?
Distributed optimization

1. Distances
2. Congestion
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Can we have better approximation factors in the CONGEST model?

CONGESTED CLIQUE? MPC? Streaming?

Thank you!

A challenge for WOLA 2020