Finding Monotone Patterns in Sublinear Time

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Testing Monotonicity of an Array: *Sortedness*

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Given query access to an unknown $f : [n] \to \mathbb{R}$ and a parameter $\varepsilon > 0$:

- If f monotone, accept w.p. $> 2/3$;
- If f is ε-far from monotone, reject w.p. $> 2/3$;

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  - $i, j \in [n]$ where $i < j$ and $f(i) > f(j)$. *one-sided* error.

ε-Far-From-Monotone Sequences

- f is ε-far from monotone: for any monotone $g\colon [n] \to \mathbb{R}$,

$$\frac{1}{n}\sum_{i=1}^{n} \mathbf{1}\{f(i) \neq g(i)\} \geq \varepsilon.$$

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Lemma

Let $f : [n] \to \mathbb{R}$ be ε-far from monotone. There exists set of disjoint pairs,

$$T = \left\{(i,j) \in [n]^2 : i < j \text{ and } f(i) > f(j)\right\}$$

of size $|T| \geq \varepsilon n/2$.

Monotonicity Testing

Theorem (Ergün, Kannan, Kumar, Rubinfeld, Viswanathan 99)

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- $\Omega((\log n)/\varepsilon)$ queries needed for adaptive algorithms too! [Fischer 04].

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Let $k \in \mathbb{N}$ and $\pi = (\pi_1, \ldots, \pi_k)$ be a permutation of $[k]$. Given $f : [n] \to \mathbb{R}$, the k-tuple $(i_1, \ldots, i_k)$ has *order pattern* π if:

$$f(i_\ell) < f(i_m) \text{ whenever } \pi_\ell < \pi_m.$$

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For fixed k and π, query complexity of testing π-freeness?

- Some sublinear in n upper bounds for general π.
- $\pi = (132)$ requires $\Omega(\sqrt{n})$ queries for non-adaptive, one-sided algorithms.
- [Ben-Eliezer, Canonne 18] Many π have complexity a $n^{1-1/(k-\Theta(1))}$.

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Theorem (NRRS17)

There is a non-adaptive algorithm with query complexity $((\log n)/\varepsilon)^{O(k^2)}$.

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Theorem (Upper bound)

For $k \in \mathbb{N}$, there exists a non-adaptive algorithm with query complexity

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Theorem (Lower bound)

Any non-adaptive algorithm needs to make $\Omega\left((\log n)^{\lfloor \log_2 k \rfloor}\right)$ queries.

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Let $f : [n] \to \mathbb{R}$ and disjoint subset of pairs T of size $\varepsilon n/2$ with

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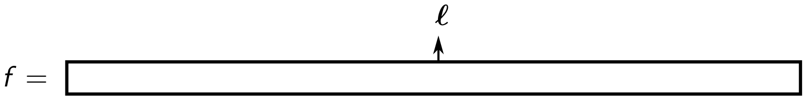

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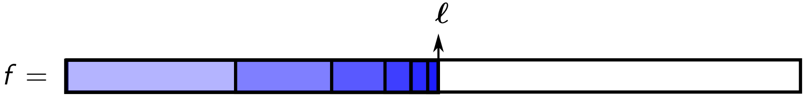

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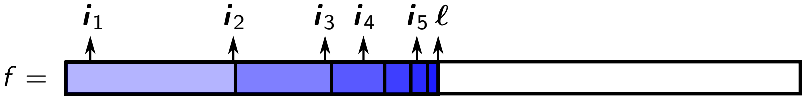

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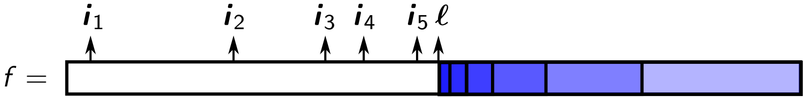

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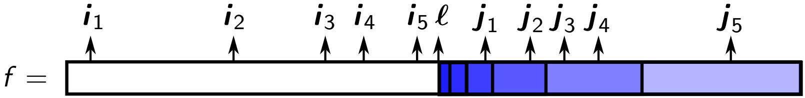

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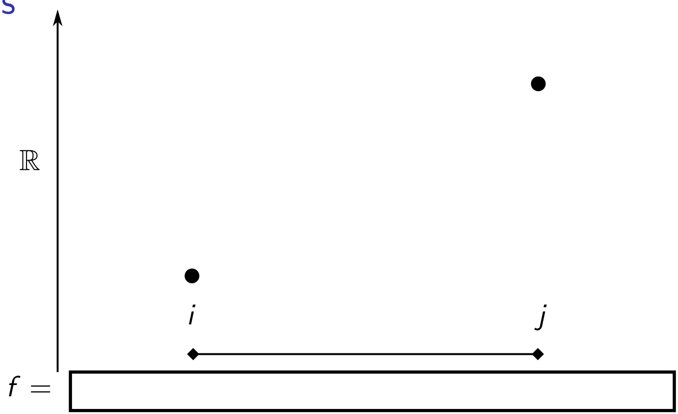

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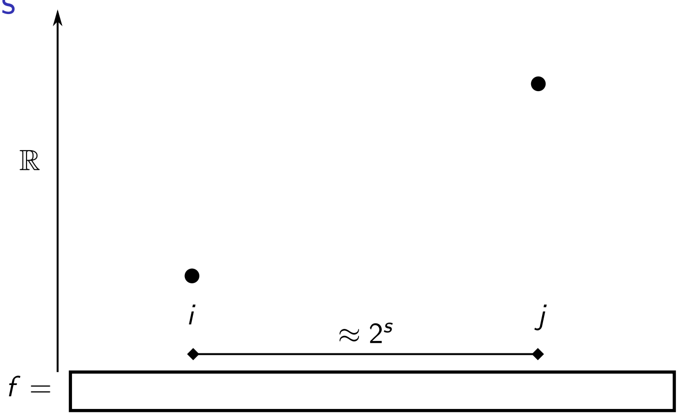

- Let T be set of monotone pairs.
- $(i, j) \in T$. (i, j) *scale s* when $j - i \approx 2^s$.

$$\Pr_{\ell \sim [n]} [i \leq \ell \leq j] \approx \frac{2^s}{n}.$$

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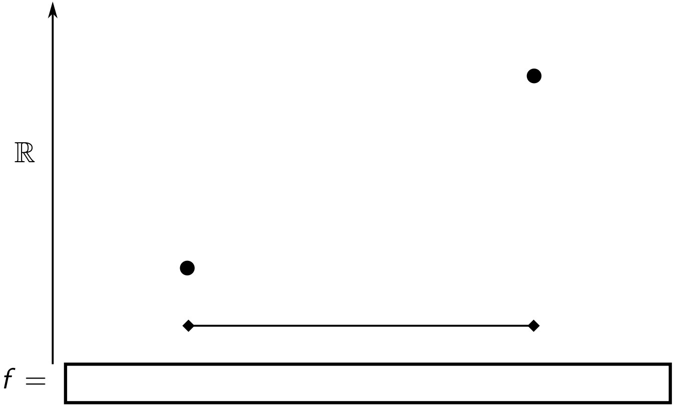

$$\mathop{\mathbb{E}}_{\ell \sim [n]} \left[\sum_{s=1}^{\log n} \frac{\#\ (i,j) \text{ scale } s \text{ cut}}{2^{s+1}} \right] \gtrsim \varepsilon.$$

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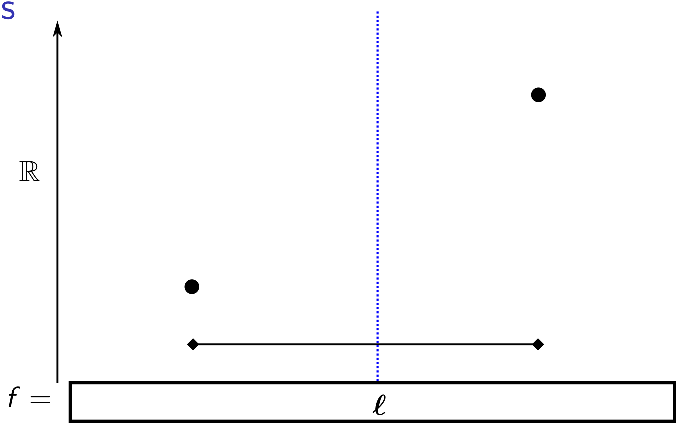

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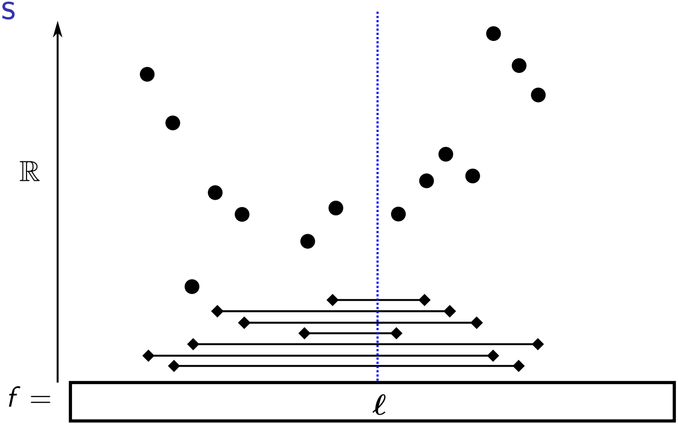

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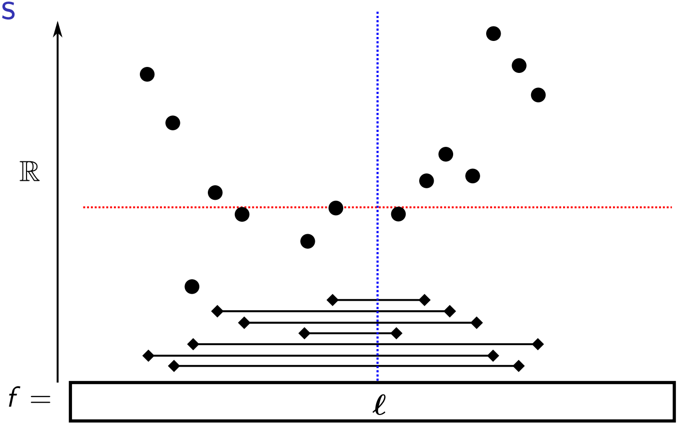

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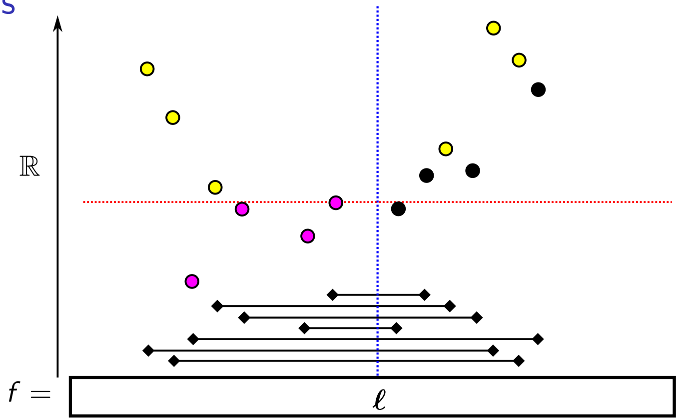

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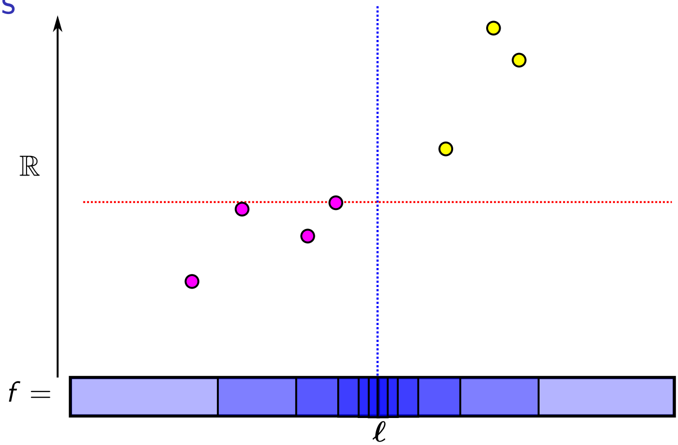

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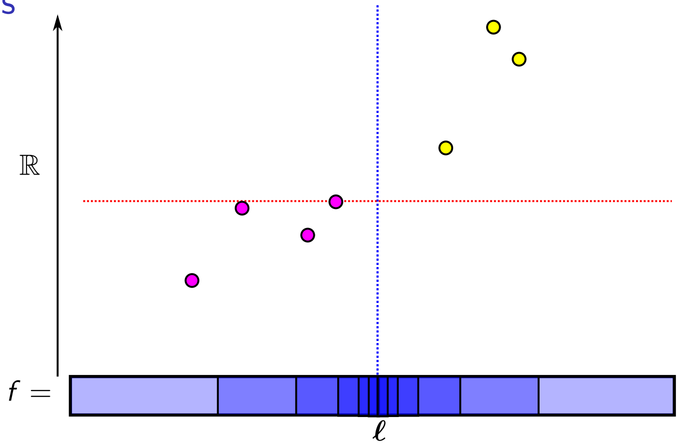

- Density δ_s^- and δ_s^+: $\sum_{s=1}^{\log n} \delta_s^- \geq \varepsilon$ and $\sum_{s=1}^{\log n} \delta_s^+ \geq \varepsilon$.

$$\Pr_{\substack{\boldsymbol{i}_1,\ldots,\boldsymbol{i}_t \\ \boldsymbol{j}_1,\ldots,\boldsymbol{j}_t}} [\text{avoid pair}] \leq \left(\frac{1}{\log n} \sum_{s=1}^{\log n} (1 - \delta_s^-) \right)^t + \left(\frac{1}{\log n} \sum_{s=1}^{\log n} (1 - \delta_s^+) \right)^t \leq \frac{1}{3}.$$

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- Query $f(i)$ for all $i \in \boldsymbol{A}$.

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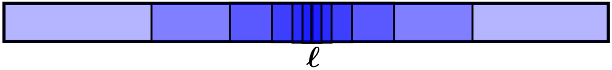
- Round r, $2 \leq r \leq \lfloor \log_2 k \rfloor + 1$: For each $i \in \boldsymbol{A}$ and $s \in \{1, \dots, \log n\}$, sample $O(1/\varepsilon)$ indices from $[i - 2^s, i + 2^s]$ and include them in $\boldsymbol{A}$.
- Query $f(i)$ for all $i \in \boldsymbol{A}$.

$f =$ 

Monotone patterns for $k > 2$?

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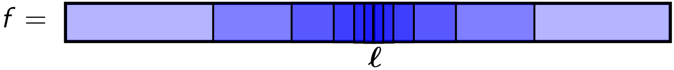

Query Complexity: $\quad O(1/\varepsilon) \cdot (O(\log n/\varepsilon))^{\lfloor \log_2 k \rfloor}$.

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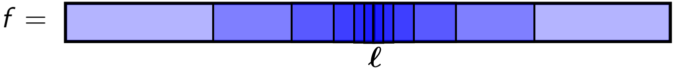

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Theorem

Suppose $f : [n] \to \mathbb{R}$ contains $\varepsilon n/k$ disjoint $(12\dots k)$-patterns, algorithm finds one w.p $\ge 2/3$.

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Thanks!