Informatik II (D-ITET)

Tutorial 2

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Outlook

- Exercise 1 solution discussion
- Exercise 2 tackles (trees, recursion, sorting)
Solution Ex1.Q1

\[
f(a,b) = a \times b = \begin{cases} 
a, & b = 1 \\ 
f(2a, b/2), & b \text{ gerade} \\ 
a + f(2a, (b-1)/2), & \text{sonst}
\end{cases}
\]

- **Is the proof by induction over** \( a \)** possible?**

  It is **not** possible to prove the correctness by induction over \( a \)

  The induction base already fails for \( b > 1 \)!

  The size of \( a \) is ever-growing

  \( \rightarrow \) No conclusion is possible on already proven cases and no induction hypothesis can be formulated

- **Does the algorithm terminate?**

  Yes, if we can make the value of \( b \) reach 1

  Is that the case?

  Yes! Because \( b \) is always halved in each call of the function

  \( \rightarrow \) After \( \lfloor \log_2(b) \rfloor \) step, the value of \( b=1 \) is reached!
Solution Ex1.Q1c

How do we prove the correctness of the algorithm when the smallest case is $b=0$?

$$f(a, b) = a \times b = \begin{cases} 
0 & , b = 0 \\
 f(2a, b/2) & , b \text{ gerade} \\
 a + f(2a, (b-1)/2) & , \text{sonst}
\end{cases}$$

$$\forall a \in \mathbb{N}, \forall b \in \{0, \ldots, n\} : f(a, b) = a \cdot b$$

In 1b) we have shown that the case of $b=1$ is always reached. Since the integer division of 1 by 2 gives 0, then the case of $b = 0$ is also always reached. No change in the proof of this step is required.
Recursion

- **Self-referential definition**
  - The base case returns a value without making any subsequent recursive calls (terminate the recursive process).
  - Recursive case: The reduction step is the central part of a recursive function.

```java
public static int factorial(int N) {
    if (N == 1) return 1;
    return N * factorial(N - 1);
}
```

- factorial(5)
  - factorial(4)
    - factorial(3)
      - factorial(2)
        - factorial(1)
          - return 1
          - return 2*1 = 2
          - return 3*2 = 6
          - return 4*6 = 24
          - return 5*24 = 120
Solution Ex1.Q2a – Recursive method calls

**gerade(int x)**

```java
public static boolean gerade( int x ){
    if( x == 0 ) return true;
    return !gerade( x-1 );
}
```

**verdopple(int x)**

```java
public static int verdopple( int x ){
    if( x == 0 ) return 0;
    return 2 + verdopple( x-1 );
}
```

**halbiere(int x)**

```java
public static int halbiere( int x ){
    if( x == 0 ) return 0;
    if( x == 1 ) return 0;
    return halbiere( x-2 ) + 1;
}
```
Solution Ex1.Q2b

- The total number of calls to the three methods in terms of \(a\) and \(b\) by a single call to \(f\)

```java
private static int f(int a, int b)
{
    if (b == 0) return 0;
    if (gerade(b)) return f(verdopple(a), halbiere(b));
    else return a + f(verdopple(a), halbiere(b));
}
```

- In each case \(\text{gerade}(b)\), \(\text{verdopple}(a)\) and \(\text{halbiere}(b)\) are called. The number of calls (with results from part A2a) is therefore at most

\[
b + 1 + a + 1 + \lfloor b/2 \rfloor + 1 \approx a + 3b/2 + 3
\]
Solution Ex1.Q2c

- Total number of method calls:

It is not \((\# \text{ calls of } f) \times (\# \text{ Total number of calls for a single instance of } f)\)

With the results of 2b), we get:

\[
N(a, b) = \left(a + \frac{3b}{2} + 3\right) + N\left(2a, \frac{b}{2}\right) = \ldots = \sum_{i=0}^{k-1} 2^i a + \sum_{i=1}^{k} \frac{3b}{2^i} + k \cdot 3
\]

The recursion terminates when \(b = 0\). This is reached after \(k = \lfloor \log_2 b \rfloor + 1\) calls, because \(b\) is halved after each step.

In the end, you are going to get \(\approx 2ab - a + 3b\)
/**
 * This function implements the ancient **Egyptian multiplication**.
 *
 * @param a must be a positive integer
 * @param b must be a positive integer
 * @return the product of a and b
 * @throws IllegalArgumentException
 */

public static int mult(int a, int b) throws IllegalArgumentException {
    if (a < 1) throw new IllegalArgumentException("Parameter a must be a positive integer but is " + a);
    if (b < 1) throw new IllegalArgumentException("Parameter b must be a positive integer but is " + b);
    return f(a, b);
}

- A user has entered invalid data.
- A file that needs to be opened cannot be found.
- A network connection has been lost in the middle of communications.
public static int mult(int a, int b) {
    try {
        if (a <= 0)
            throw new IllegalArgumentException("A negativ!");
    } catch (IllegalArgumentException e) // Exception Handler
    {
        . . .
    }
}

Solution Ex1.Q3

try - catch (- finally) outside the function that throws an exception!
Outlook

- Exercise 1 solution discussion
- Exercise 2 tackles (trees, recursion, sorting)
Exercise 2

1. Rooted Trees (theory)
   a. Representation of tree using (i) Klammerdarstellung: brackets and (ii) eingerückte Form: indented
   b. Given brackets representation, (i) draw tree and (ii) give indented format
   c. Can the tree in 1b be reconstructed unambiguously (uniquely reconstructible)? Why/Why not?
   d. For the trees in 1a and 1b: Give (i) height of tree [1 node has height 1], (ii) longest paths [trees are directed!], and (iii) set of leaves

2. Recursive Sorting
   a. Constructor: Create array of given size and fill with random numbers
   b. Build method toString
   c. Create recursiveSort(int until) to sort numbers in descending order

3. Binary Trees
   ▪ Check Trees
Trees

It's a Christmas tree with a heap of presents underneath!

...we're not inviting you home next year.
Exercise 2 – Q 1 & 3

Overview on some different types of trees

- **General Tree**: Every node has X child nodes

- **Binary Tree**: Each node has at most two child nodes

- **Binary/Ternary Search Tree (BST)**: Nodes are saved in an ordered form

- **Trie (from «Retrieval»)**: Not the content but the position of the node that matters, i.e. edge information (e.g. Suffix tree → text autocomplete)

**Task**: Deal with different representations of trees!
Q2: Recursive Sorting

- **Constructor**
  - Produce array of randomly generated numbers
  - Import Random Class (package: `import java.util.Random`)

```java
//RandomGenerator
Random r = new Random();

//Array...

//1 random number generieren:
r.nextInt(1000);
```

- **Method toString()**

  * Example: the string-representation of *int array[] = {1,2,3} is '][1, 2, 3]'*

```java
String s = "[";
for ( int i=0; i < array.length, i++ )
  ...
return s;
```
Q2: Recursive Sorting

- **recursiveSort**(int until)

  - Core idea of recursion is to reduce the problem to smaller instances of the same problem ...

- NOW! Given a list of (N) element

  To order list of i elements in descending order, I need the following...
  ... Sorting the first (i - 1) elements with descending order
  ... Search for the largest element in the list reminder
  ... place it in the first place of the list reminder

- The empty list is a sorted list
recursiveSort(4)

recursiveSort(3)

recursiveSort(2)

recursiveSort(1)

Ist sortiert!

2 <- findLargest(0,3)
swap(0,2)

2 <- findLargest(1,3)
swap(1,2)

3 <- findLargest(2,3)
swap(2,3)

Swap is not necessary anymore...

→ List sorted in descending order!
Q2: Recursive Sorting

c) Implement `recursiveSort(int until)`: 

Sorts numbers in array in descending order

- **Beginning of recursion**: `recursiveSort(length)` sorts the whole array in descending order

- **Assumption of recursion**: `recursiveSort(until-1)` sorts until-1 largest numbers in first until-1 positions

- **Step**: Largest number from the rest of the array is swapped with number at position until-1 → Now the first until numbers are sorted.

- **End of recursion**: An empty array is sorted already
Q3: Binary Tree as an Array

- Binary trees can be saved as arrays given proper interpretation are set

The idea is as follow:
- Set the root of the tree to have an index 0
- The next two array positions from \((i)\) are saved, namely the position \((2i + 1)\) and \((2i + 2)\)
- What is the size of the array that stores the binary tree?

\[2^{\text{height}-1} \leq \text{array.length} < 2^{\text{height}}\]
Q3: Binary Tree as an Array

char[] tree = new char[7];

tree[0] = 'A';
tree[1] = 'B';
tree[2] = 'C';
tree[3] = 'D';
tree[4] = ' ';
tree[5] = 'F';
tree[6] = 'E';

Is this also possible with general (= non-binary) trees?
Q3: Binary Tree as an Array

- Implementation of toString(): Provides trees in indented form
  - toString() call → toString(int node, String indentation)
    - E.g. toString(0," ");

- Method checkTree() hints:
  - Root at index 0
  - Direct successor i to 2i + 1 and 2i + 2
    - $2^{height-1} \leq \text{array.length} < 2^{height}$

- Check if this applies for the passed array
  - Test: Every element has a parent node
    - "The root is its own father."
  - What about the empty nodes?

**Indentation is difficult!**
- Realize toString() recursively and provide number of white spaces as input.
Trees in Computer Science...

Images: http://kitabundsunnah.wordpress.com/, http://www.cs.lmu.edu/courses
Tree traversal...

preOrder(node) {
    print(node)
    if left != null then preOrder(left)
    if right != null then preOrder(right)
}

- **Pre-Order** «root, left, right»
- **In-Order** «left, root, right»
- **Post-Order** «left, right, root»

8, 3, 1, 6, 4, 7, 10, 14, 13
Tree traversal...

```java
inOrder(node) {
    if left != null then preOrder(left)
    print(node)
    if right != null then preOrder(right)
}
```

- **Pre-Order**  «root, left, right»
  - Example: `8, 3, 1, 6, 4, 7, 10, 14, 13`

- **In-Order**  «left, root, right»
  - Example: `1, 3, 4, 6, 7, 8, 10, 13, 14`

- **Post-Order**  «left, right, root»
Tree traversal...

```
postOrder(node) {
    if left != null then preOrder(left)
    if right != null then preOrder(right)
    print(node)
}
```

- **Pre-Order** «root, left, right»
  
  $8, 3, 1, 6, 4, 7, 10, 14, 13$

- **In-Order** «left, root, right»
  
  $1, 3, 4, 6, 7, 8, 10, 13, 14$

- **Post-Order** «left, right, root»
  
  $1, 4, 7, 6, 3, 13, 14, 10, 8$
Eclipse more tricks…

- Display keyboard shortcuts: Control + Shift + L
- Auto-formatting: Control + Shift + F
- Auto-completion: Control + Space
Have Fun!