

SC22

Dallas, TX | hpc
accelerates.

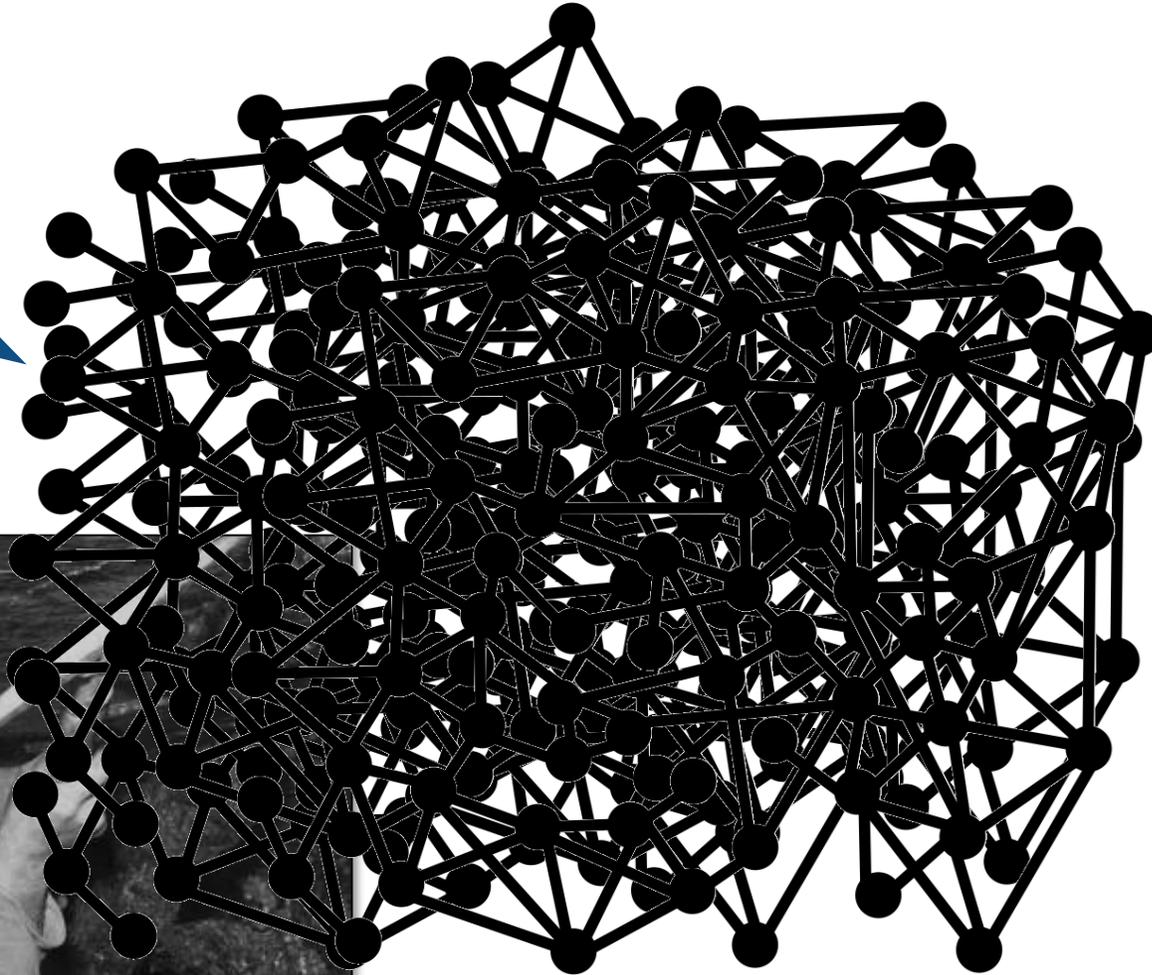
ProbGraph: High-Performance and High-Accuracy Graph Mining with Probabilistic Set Representations

M. BESTA, C. MIGLIOLI, P. S. LABINI, J. TĚTEK, P. IFF,
R. KANAKAGIRI, S. ASHKBOS, K. JANDA, M. PODSTAWSKI,
G. KWASNIEWSKI, N. GLEINIG, F. VELLA, O. MUTLU, T. HOEFLER.

Graph Mining

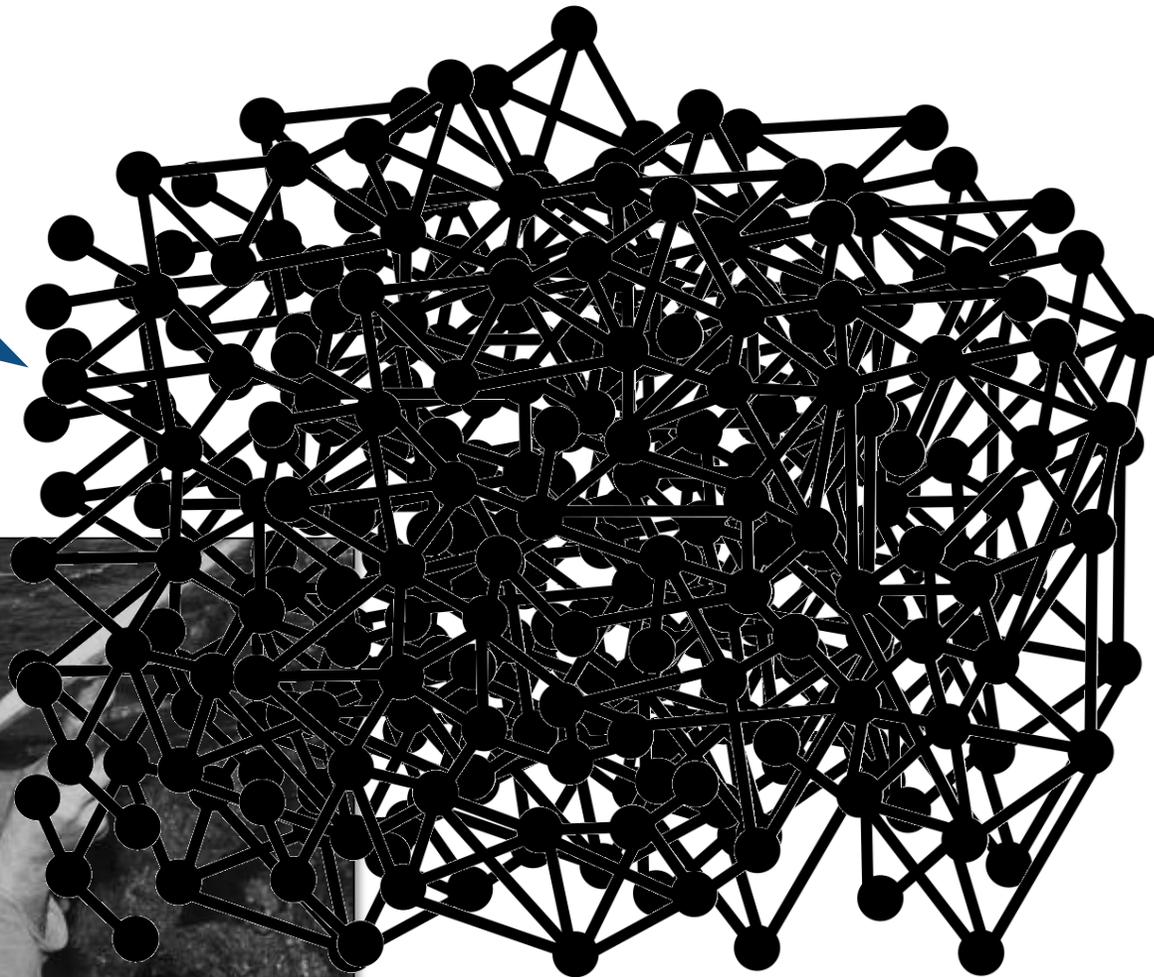
Graph Mining

A huge & complex graph dataset

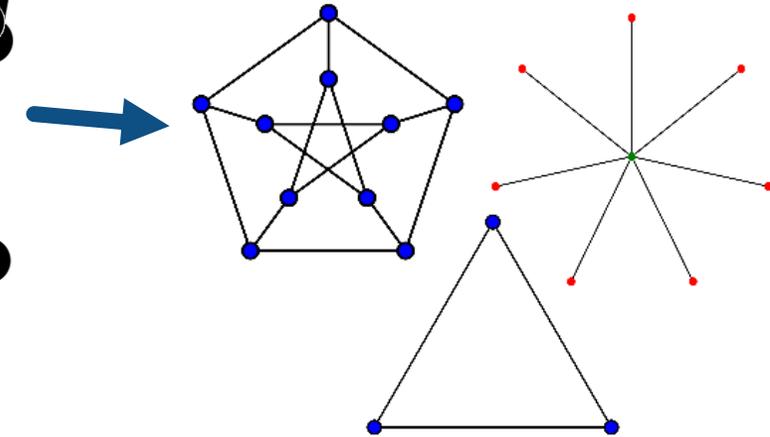


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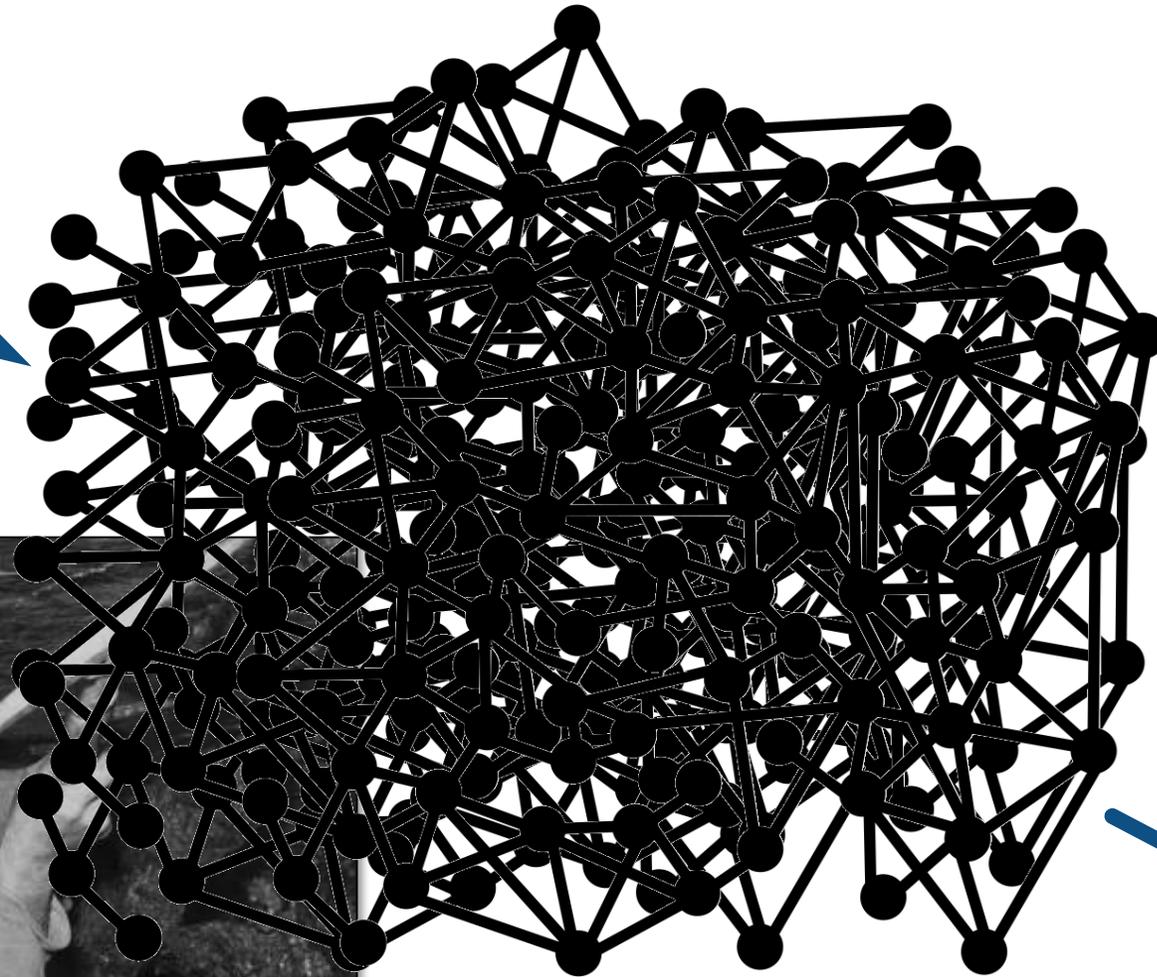


Pattern counting
 (triangles, higher-order cliques, dense subgraphs, ...)

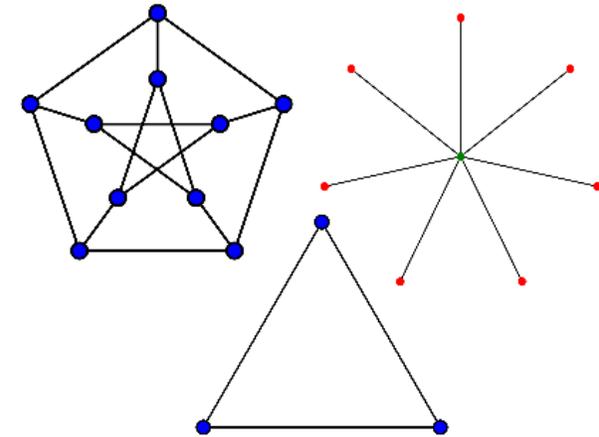


Graph Mining

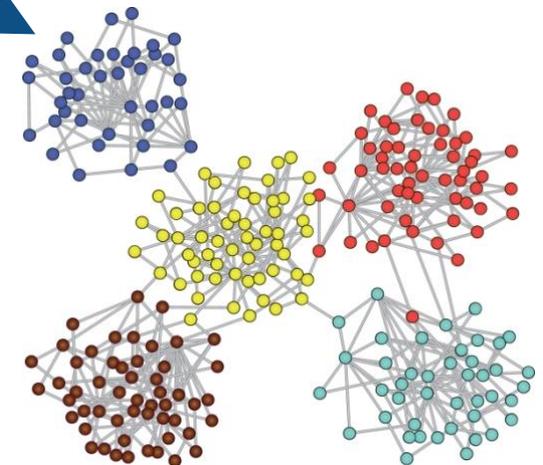
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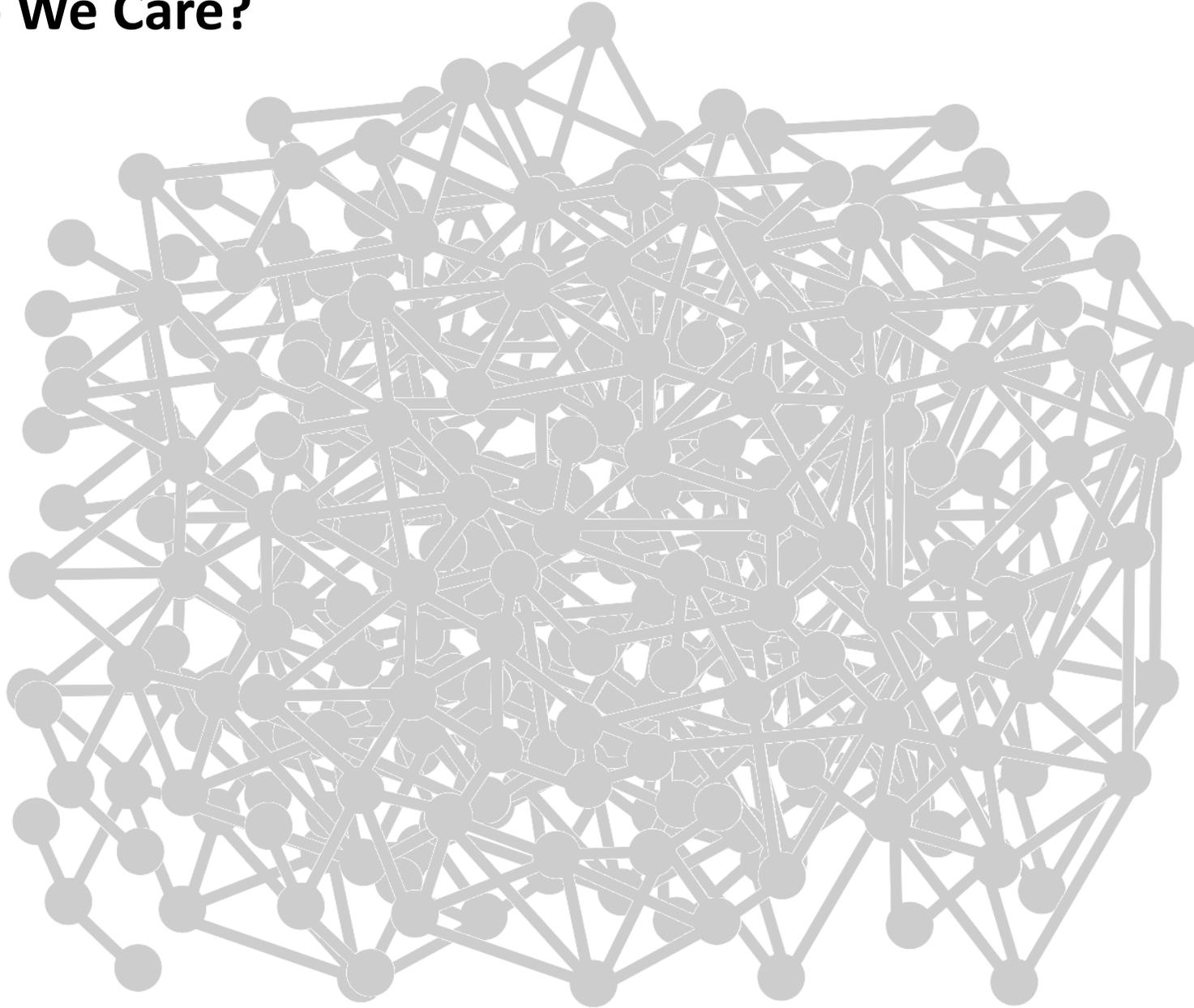
Pattern counting
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Clustering, Link Prediction, Vertex Similarity, ...

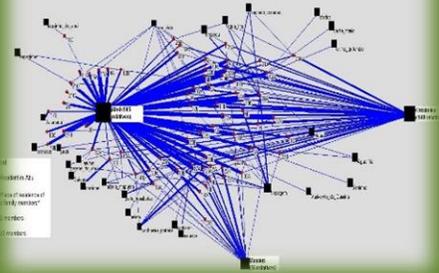


Graph Mining: Do We Care?



Graph Mining: Do We Care?

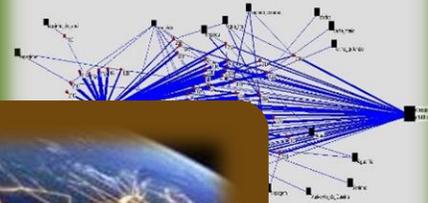
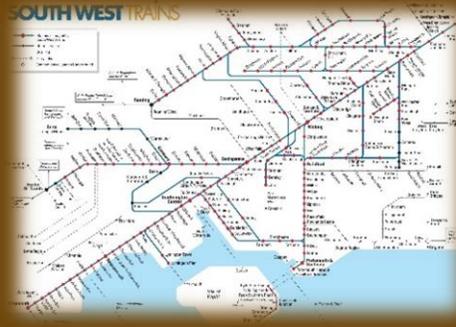
Social sciences



Graph Mining: Do We Care?

Social sciences

Engineering



Graph Mining: Do We Care?

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Biology

Chemistry

Engineering

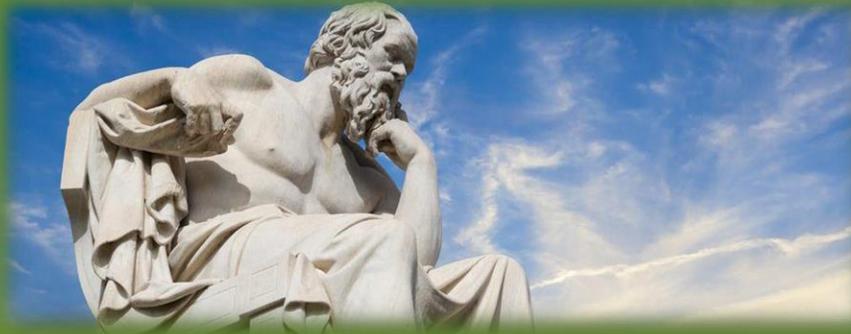
Communication

Web graph analysis

Medicine

Cybersecurity

...even philosophy 😊



Modeling a Philosophical Inquiry: from MySQL to a graph database

The short story of a long refactoring process

- 📍 Track: Graph Processing devroom
- 📍 Room: AW1.126
- 📅 Day: Saturday
- 🕒 Start: 12:45
- 🕒 End: 13:35

Bruno Latour wrote a book about philosophy (an inquiry into modes of existence). He decided that the paper book was no place for the numerous footnotes, documentation or glossary, instead giving access to all this information surrounding the book through a web application which would present itself as a reading companion. He also offered to the community of readers to submit their contributions to his inquiry by writing new documents to be added to the platform. The first version...

Graph Mining: Do We Care?

Social sciences

Biology

Chemistry

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Communication

Web graph analysis



Challenges

...even philosophy



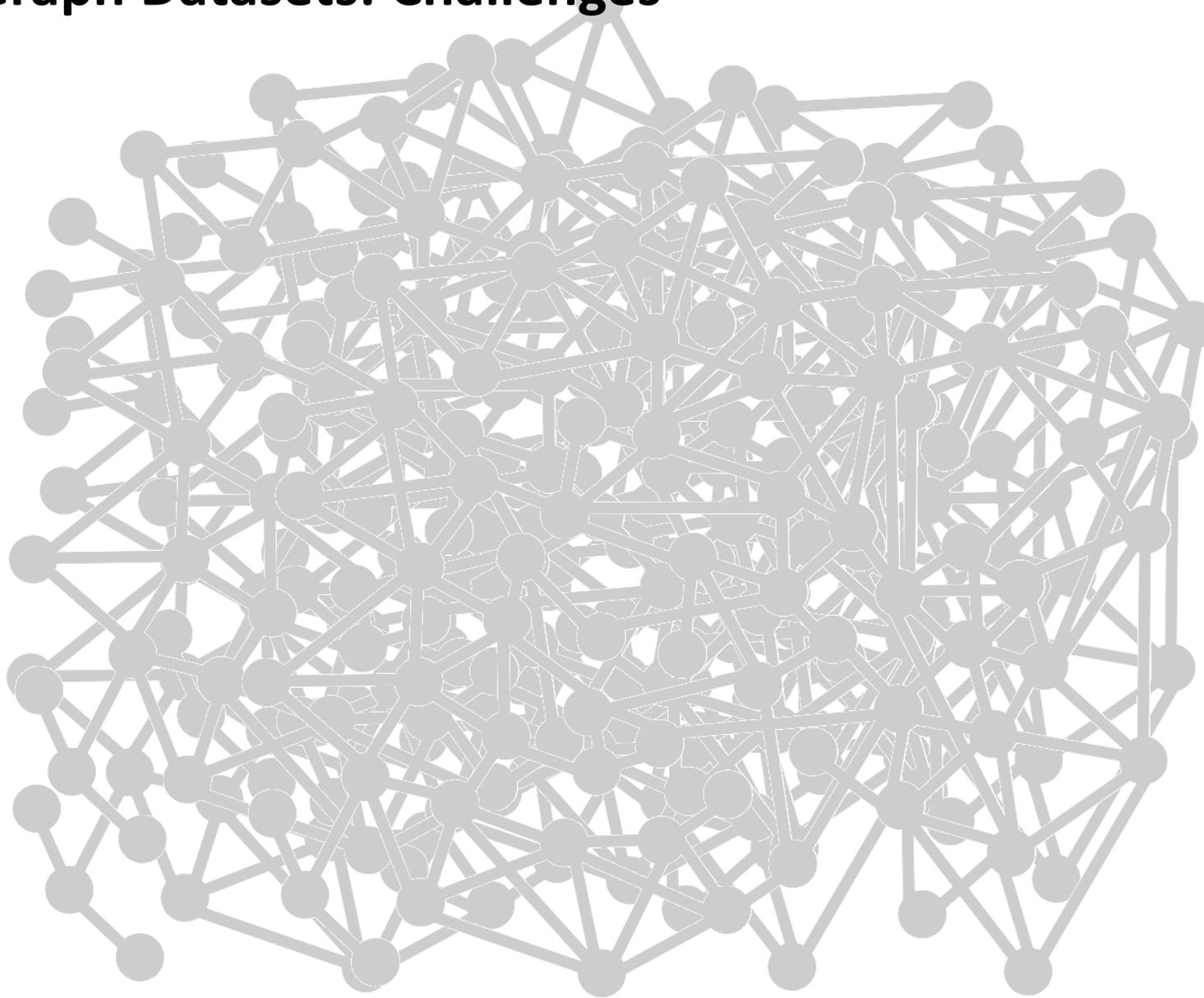
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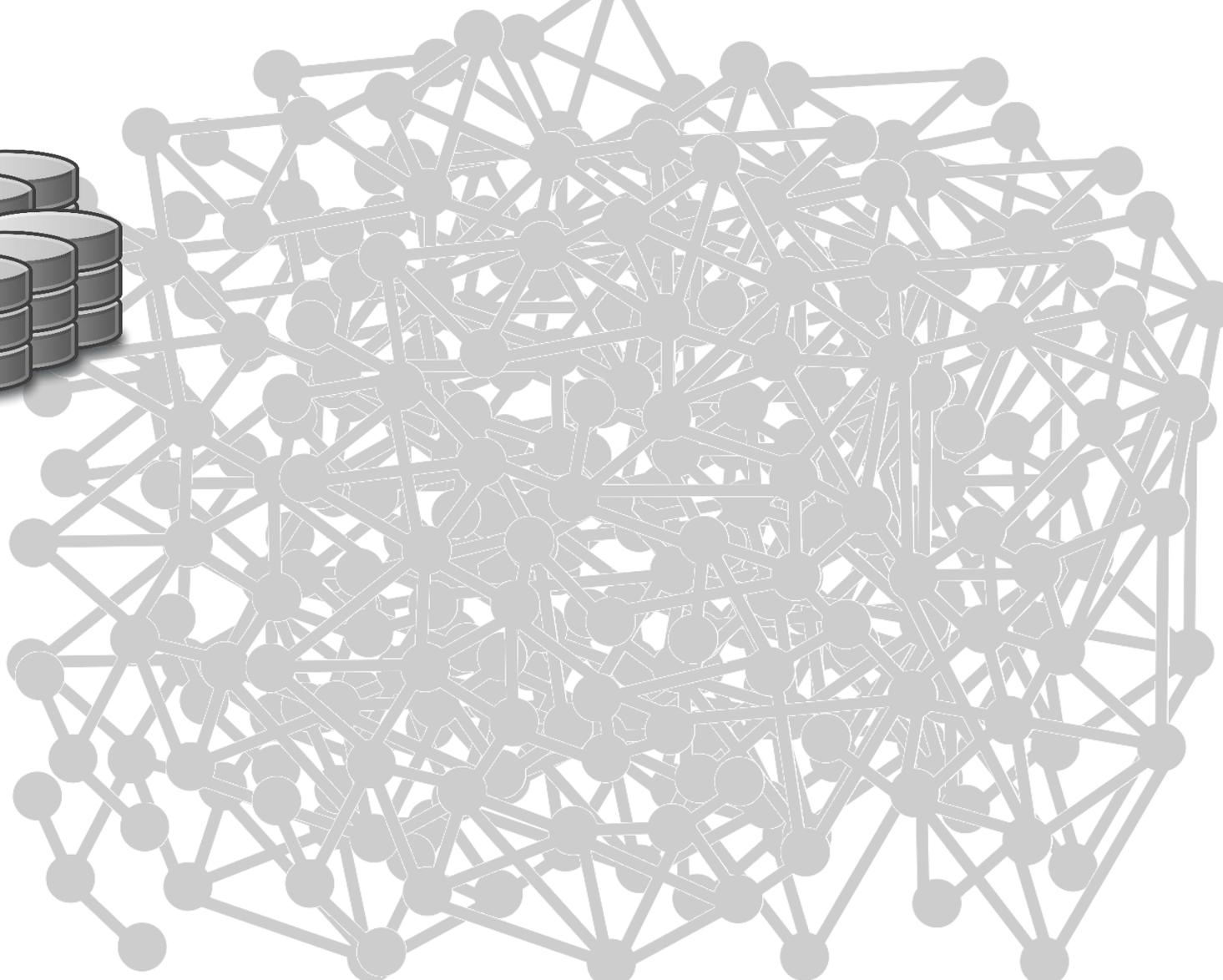
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Graph Mining & Graph Datasets: Challenges



Graph Mining & Graph Datasets: Challenges

Huge

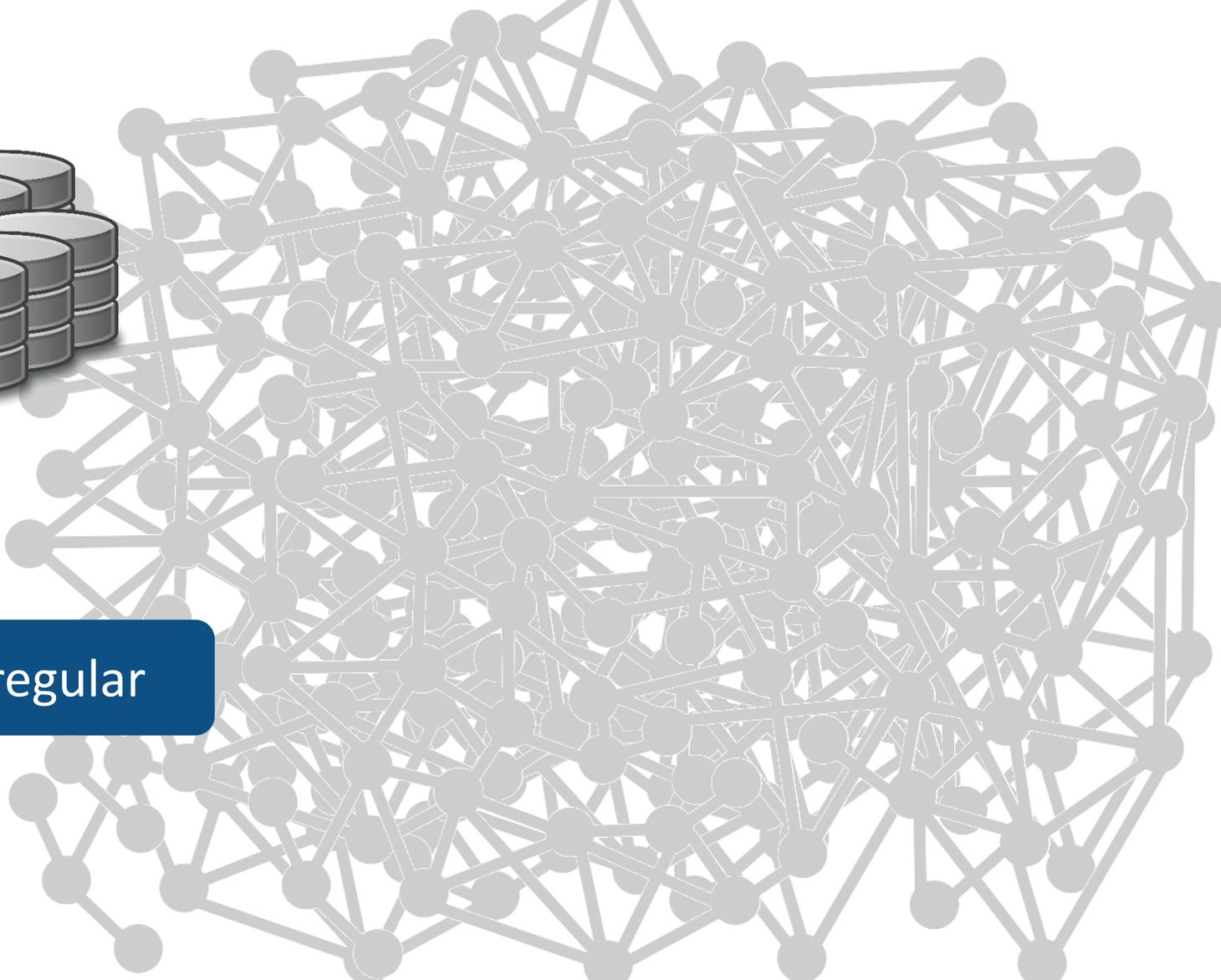
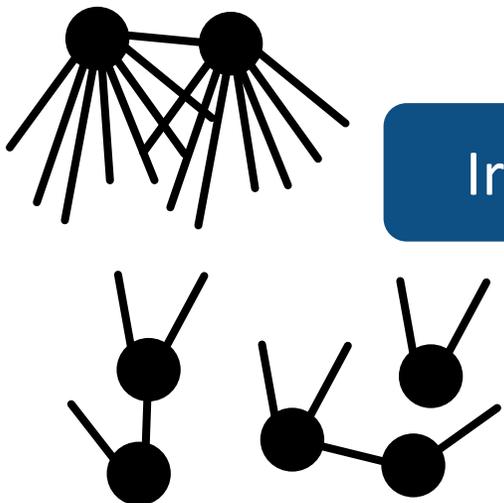


Graph Mining & Graph Datasets: Challenges

Huge



Irregular



Graph Mining & Graph Datasets: Challenges

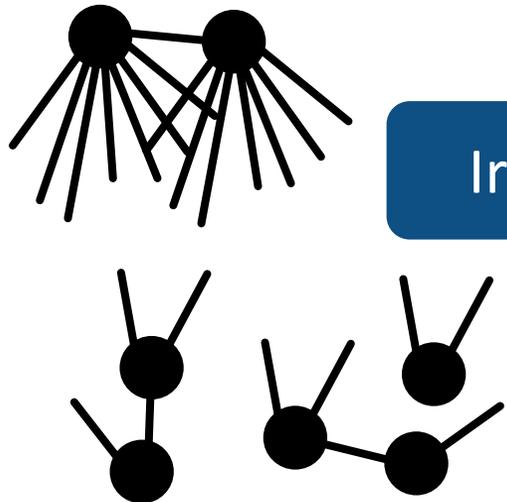
Huge



Communication-heavy

Synchronization-heavy

Irregular



Graph Mining & Graph Datasets: Challenges

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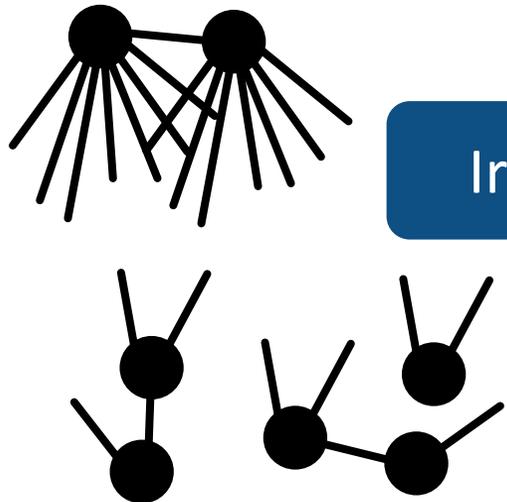
Power-hungry



Synchronization-heavy



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Graph Mining & Graph Datasets: Challenges

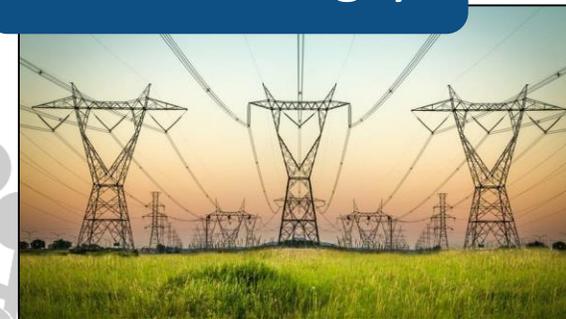
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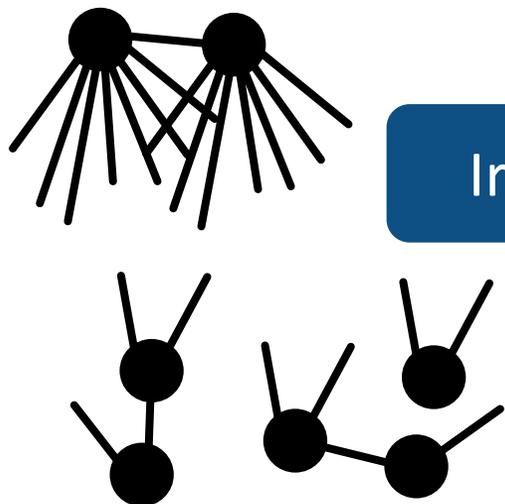
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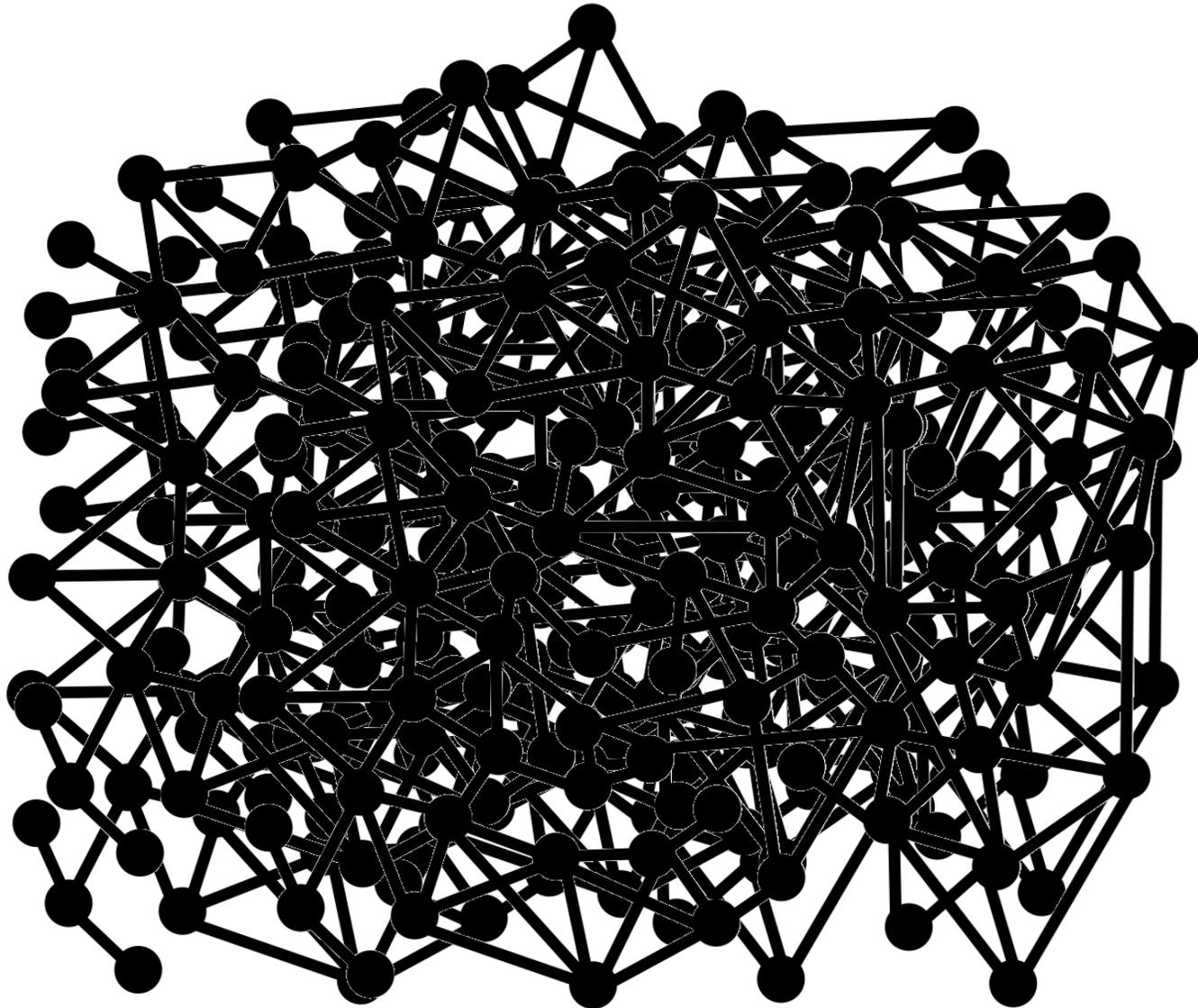
Time complexities often $O(n^k)$ for $k \geq 2$



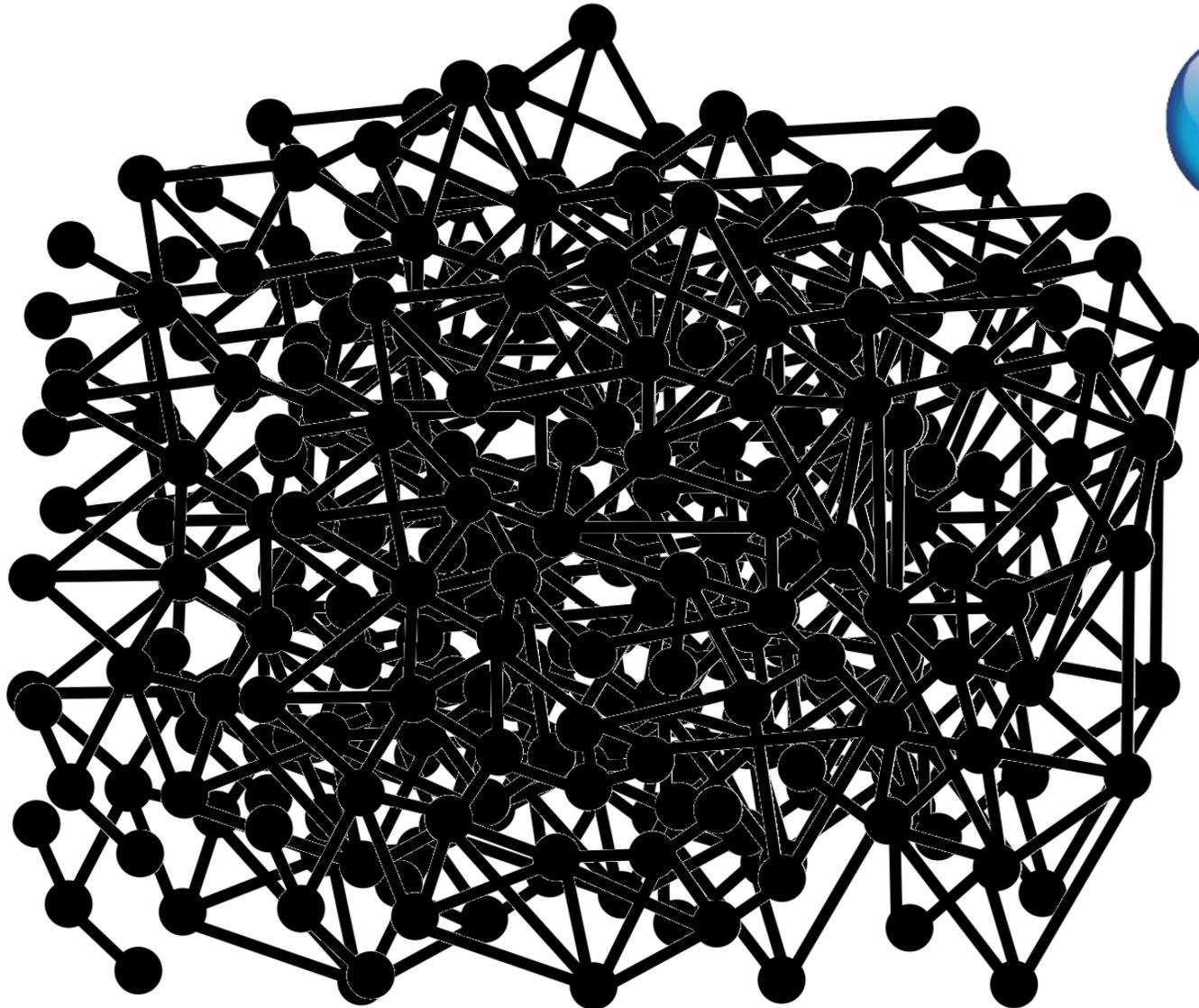
Irregular



Goal: Making Graph Mining Radically Faster

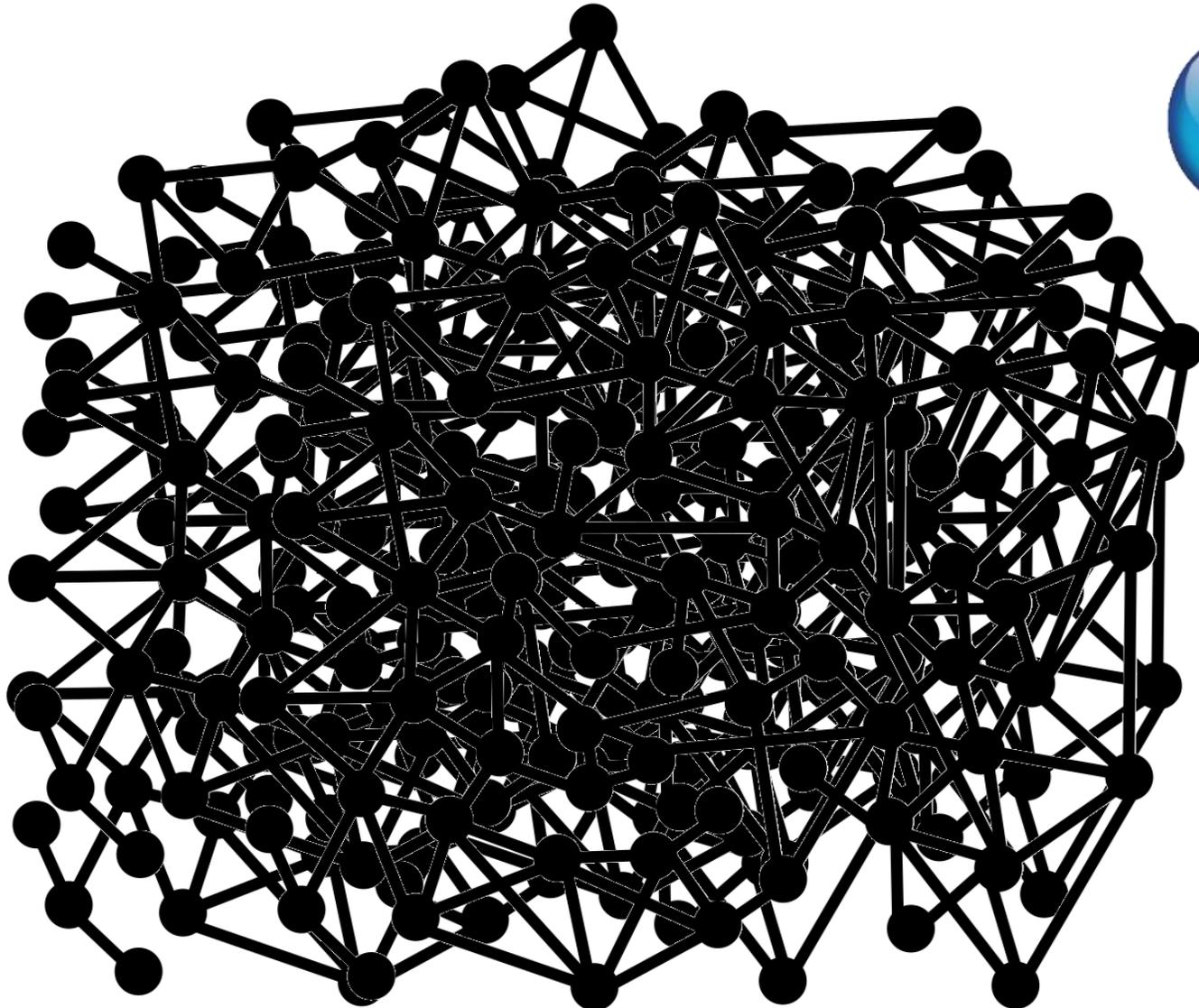


Goal: Making Graph Mining Radically Faster



Do we need 100% accurate results in all cases?

Goal: Making Graph Mining Radically Faster



Do we need 100% accurate results in all cases?

Let's say we can choose between...

Find all the patterns (e.g., cliques) in 1 day

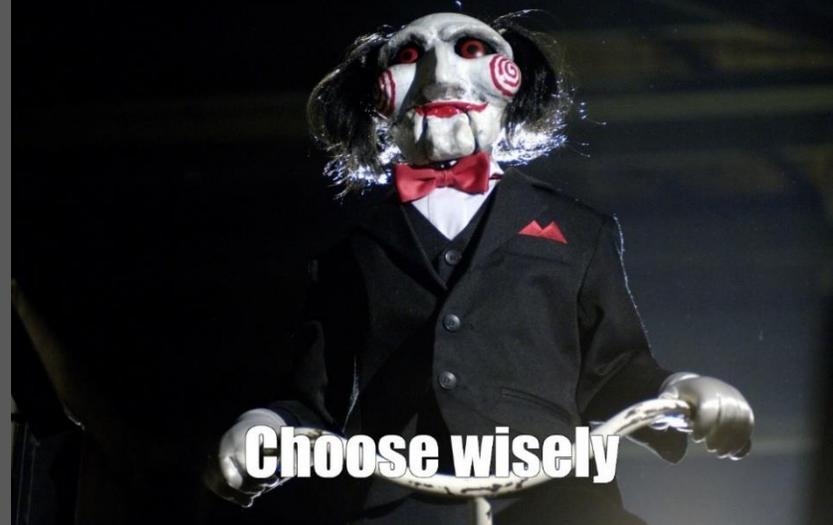
Find $\geq 90\%$ of all the patterns in 30 minutes

Goal: Making Graph Mining Radically Faster



Do we need 100% accurate results in all cases?

The choice is yours



Choose wisely

Let's say we can choose between...

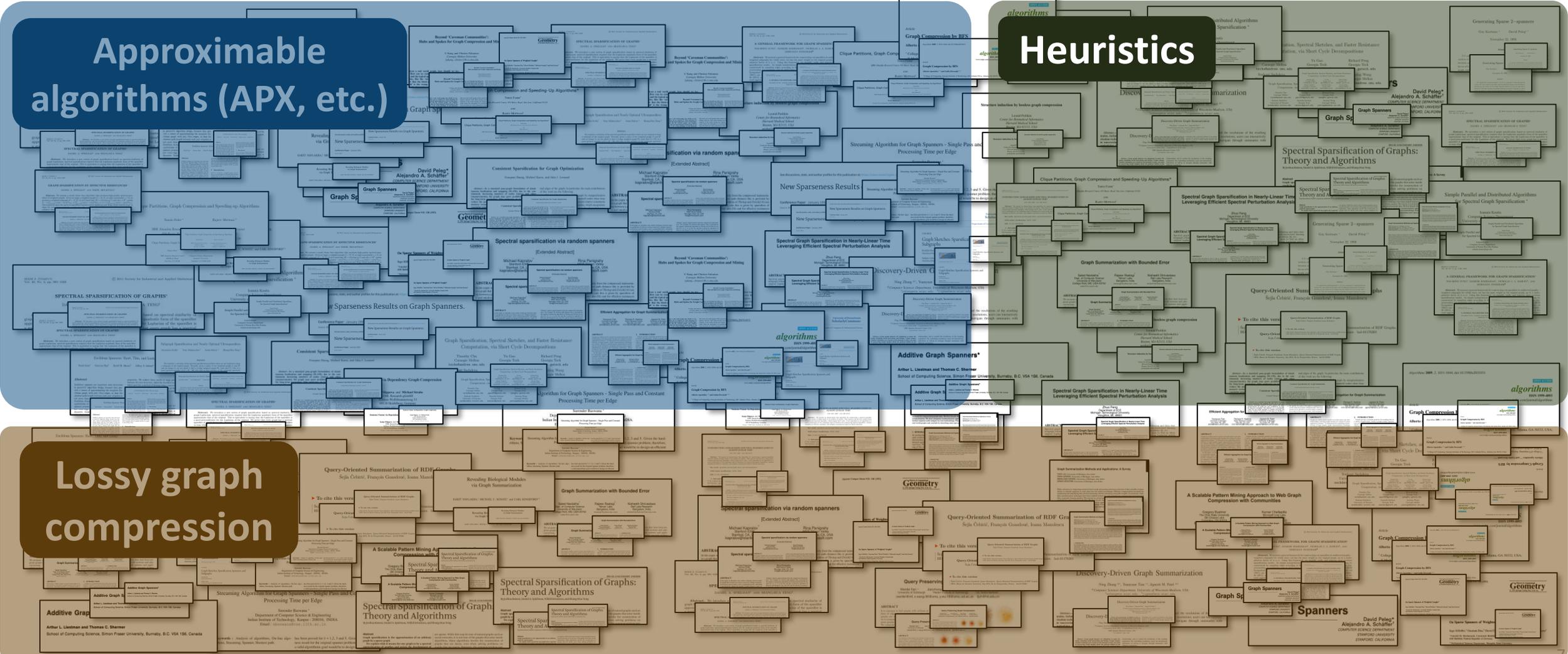
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Approximate Graph Processing: State & Challenges

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We analyzed > 500 works and identified three classes of schemes...



Approximate Graph Processing: State & Challenges

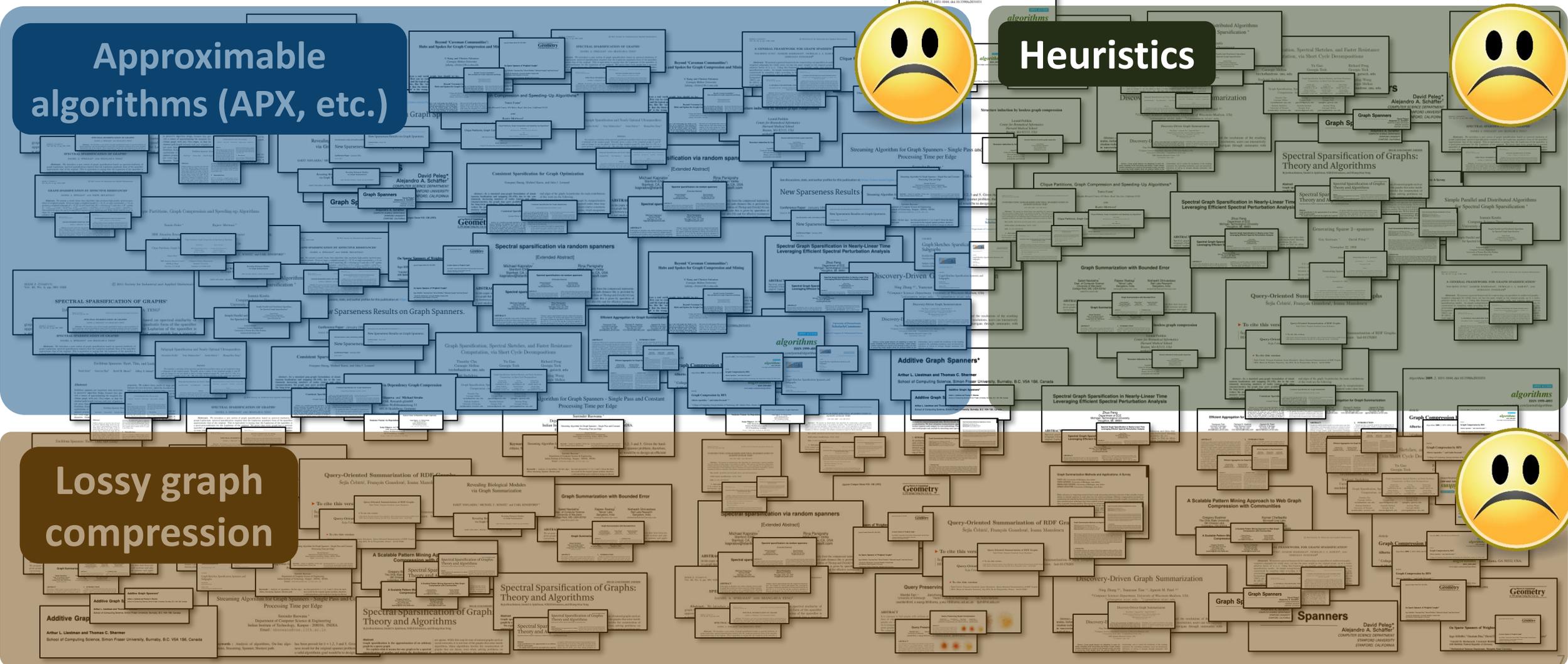
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...they all have problems

Approximable algorithms (APX, etc.)



Heuristics



Lossy graph compression



Approximate Graph Processing: State & Challenges

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Little parallelism

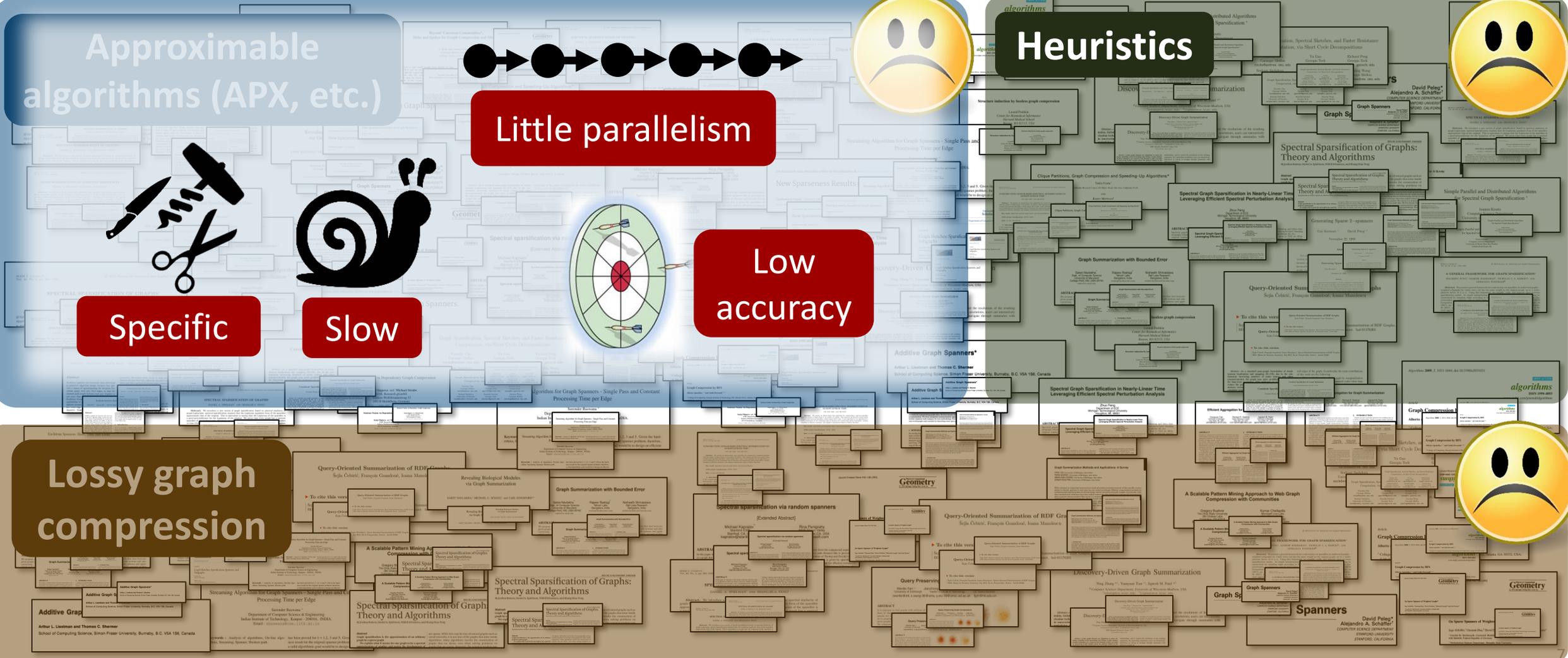
Heuristics

Specific

Slow

Low accuracy

Lossy graph compression



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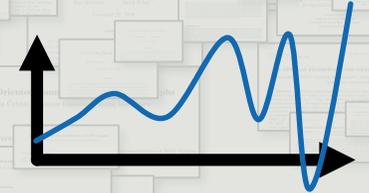
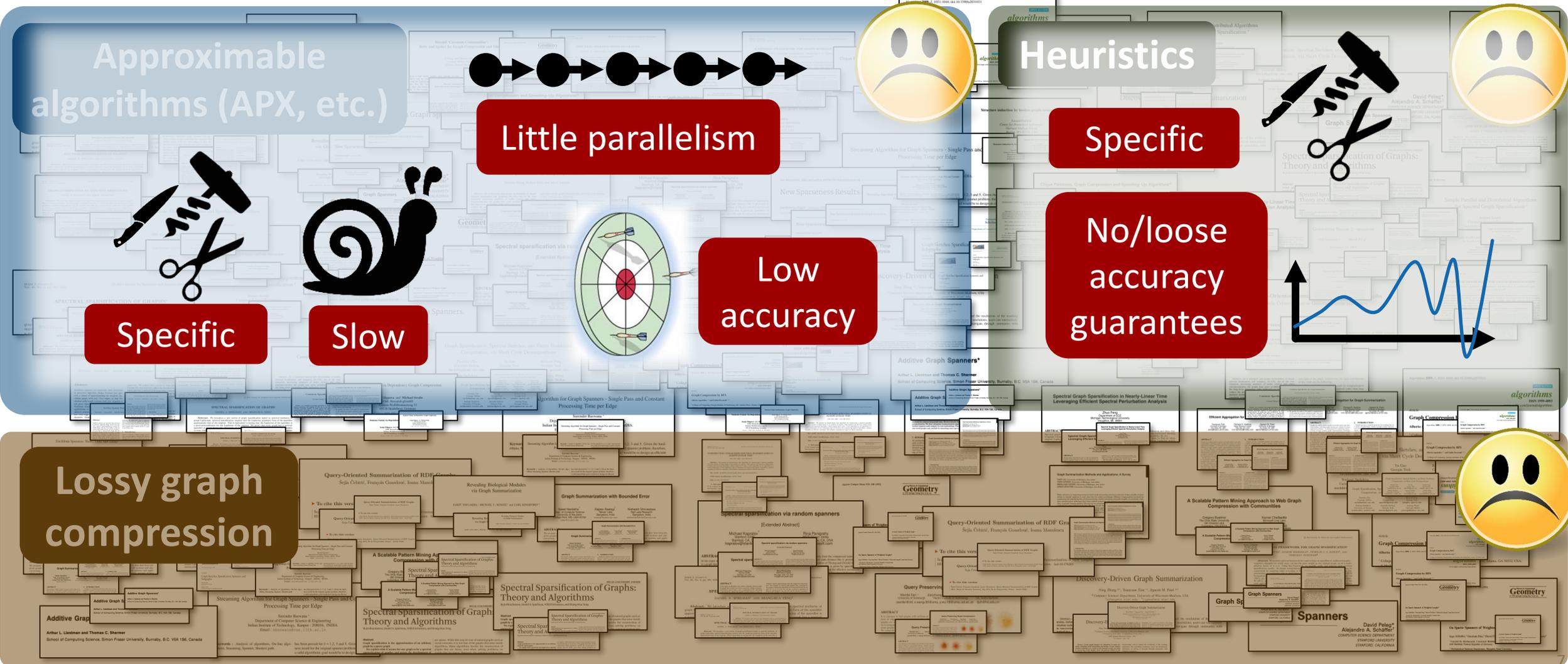
No/loose accuracy guarantees

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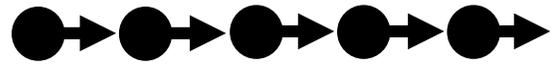
Large memory overheads

No/loose accuracy guarantees

Slow

Approximate Graph Processing: Current Issues & Our Objectives

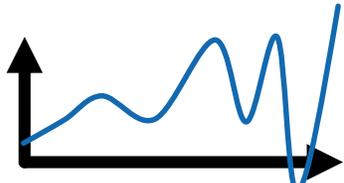
Approximate Graph Processing: Current Issues & Our Objectives



Little parallelism



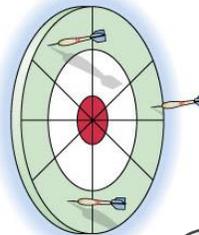
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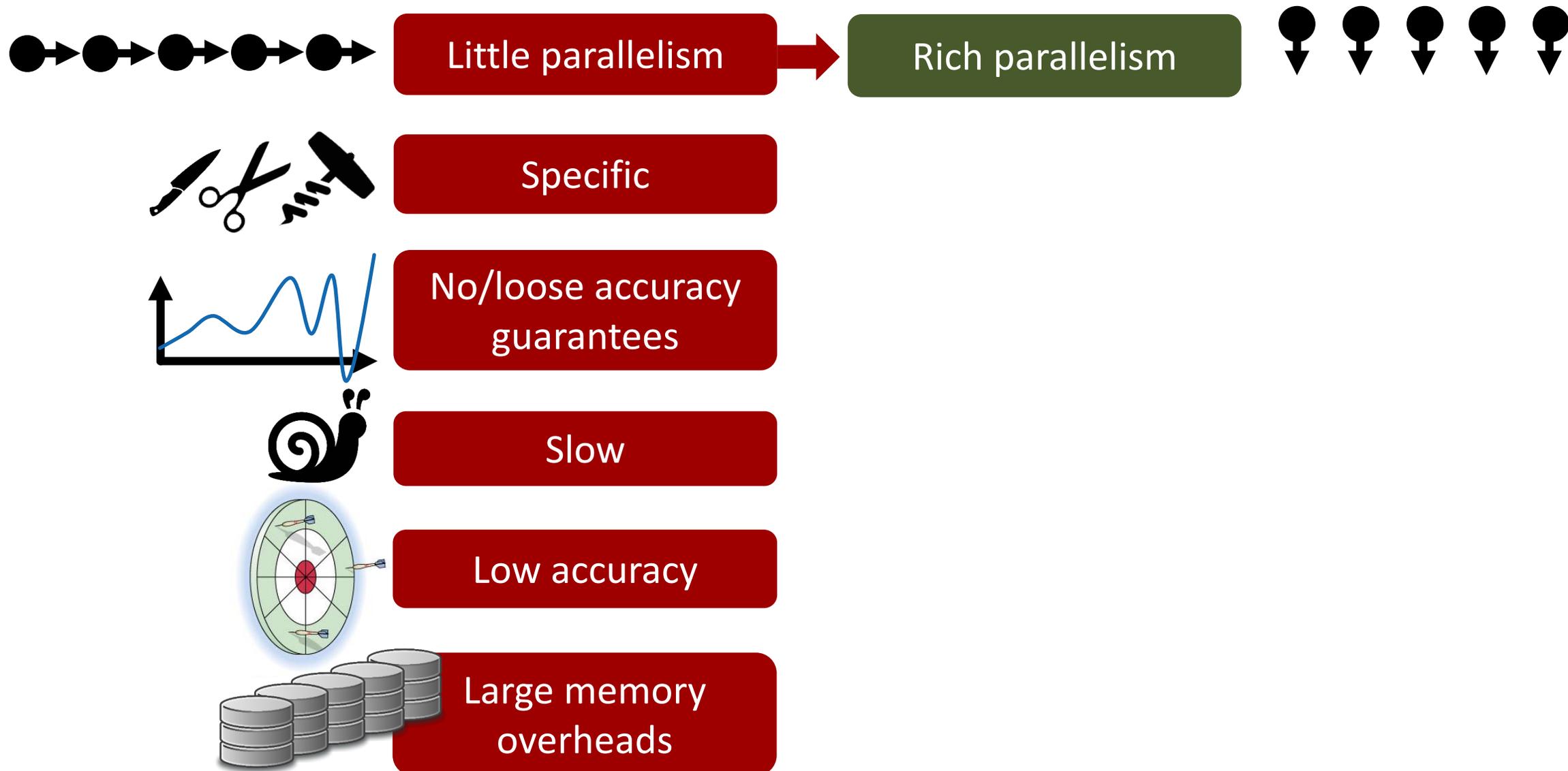


Low accuracy

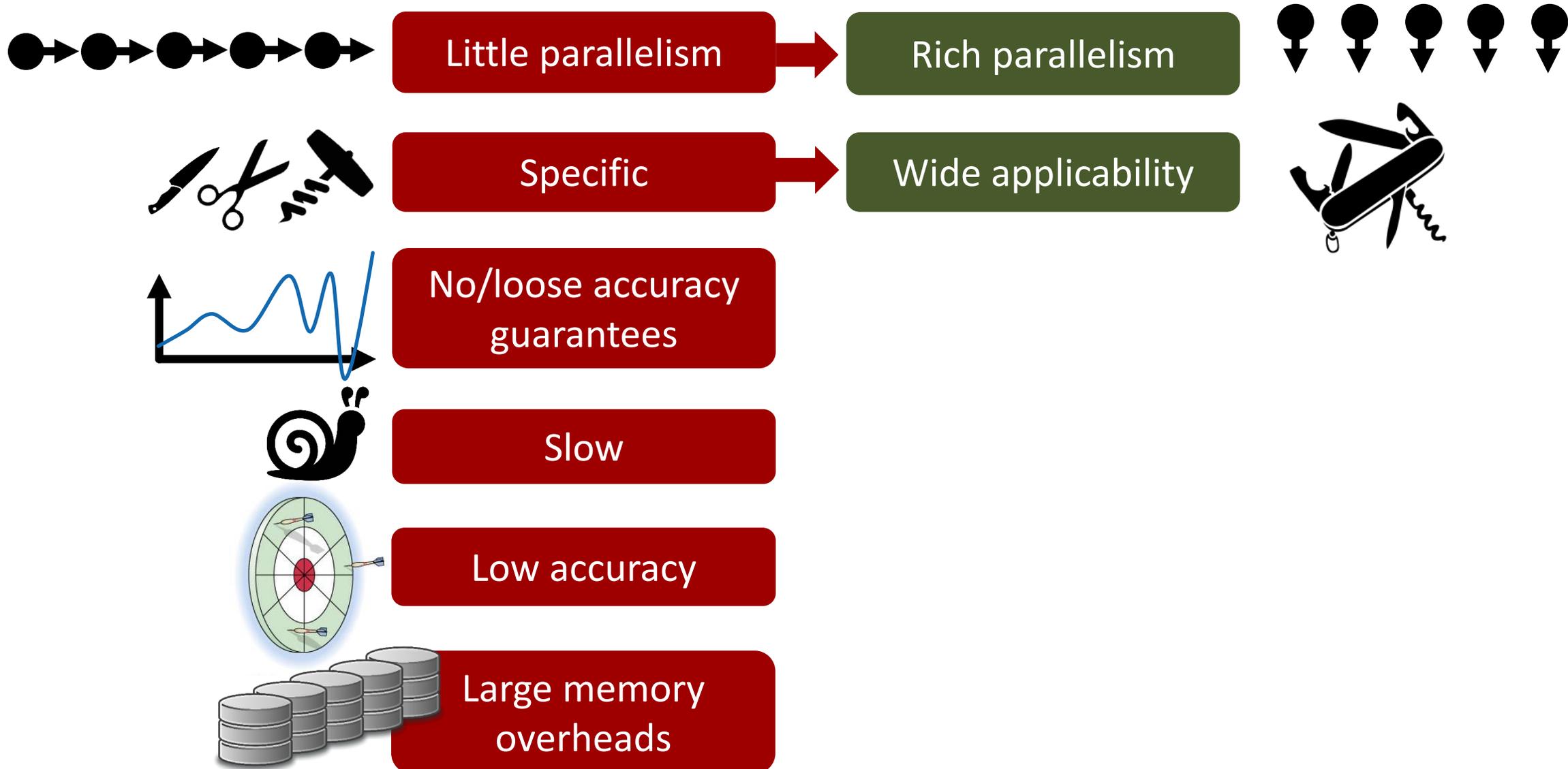


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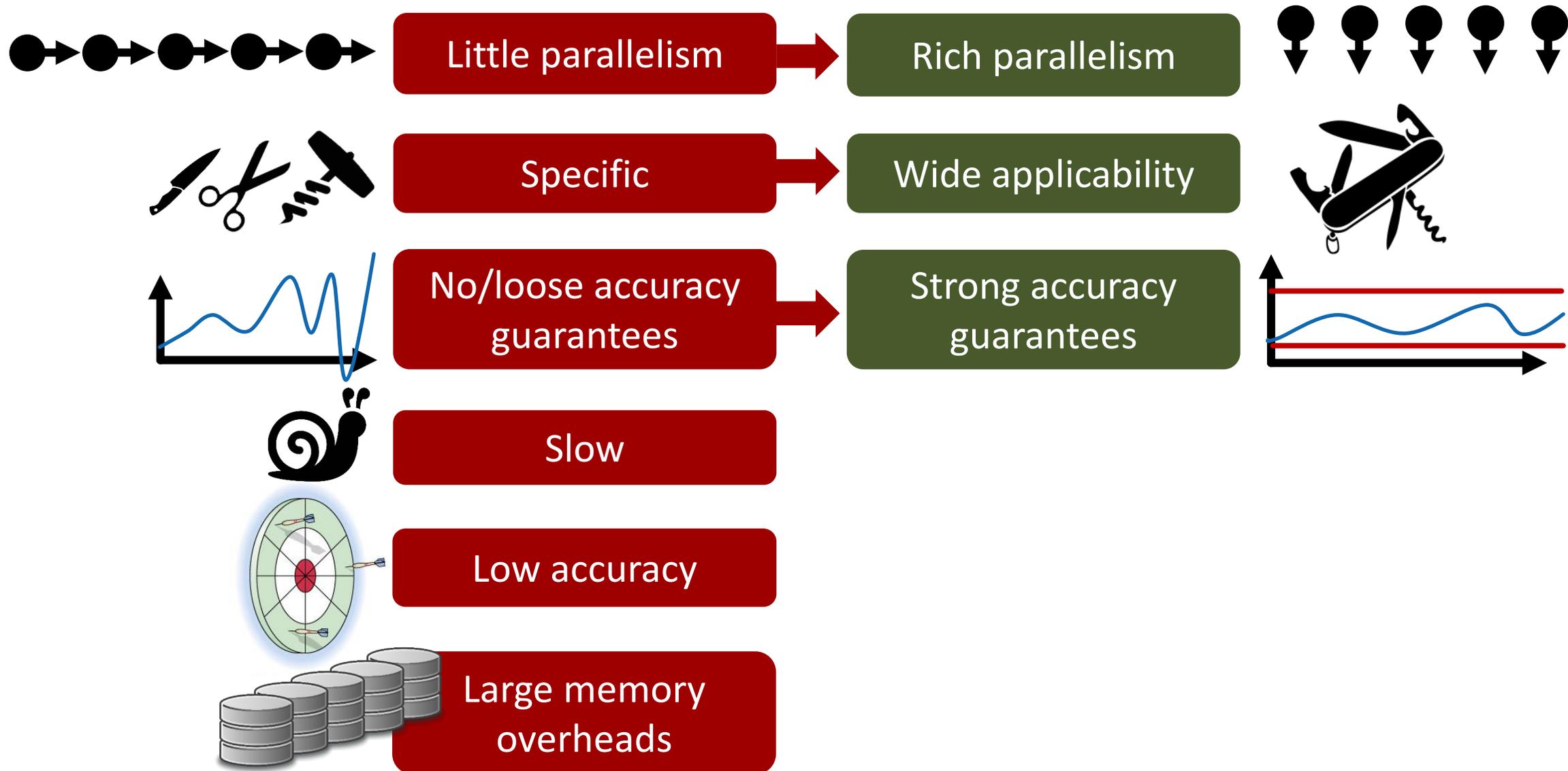
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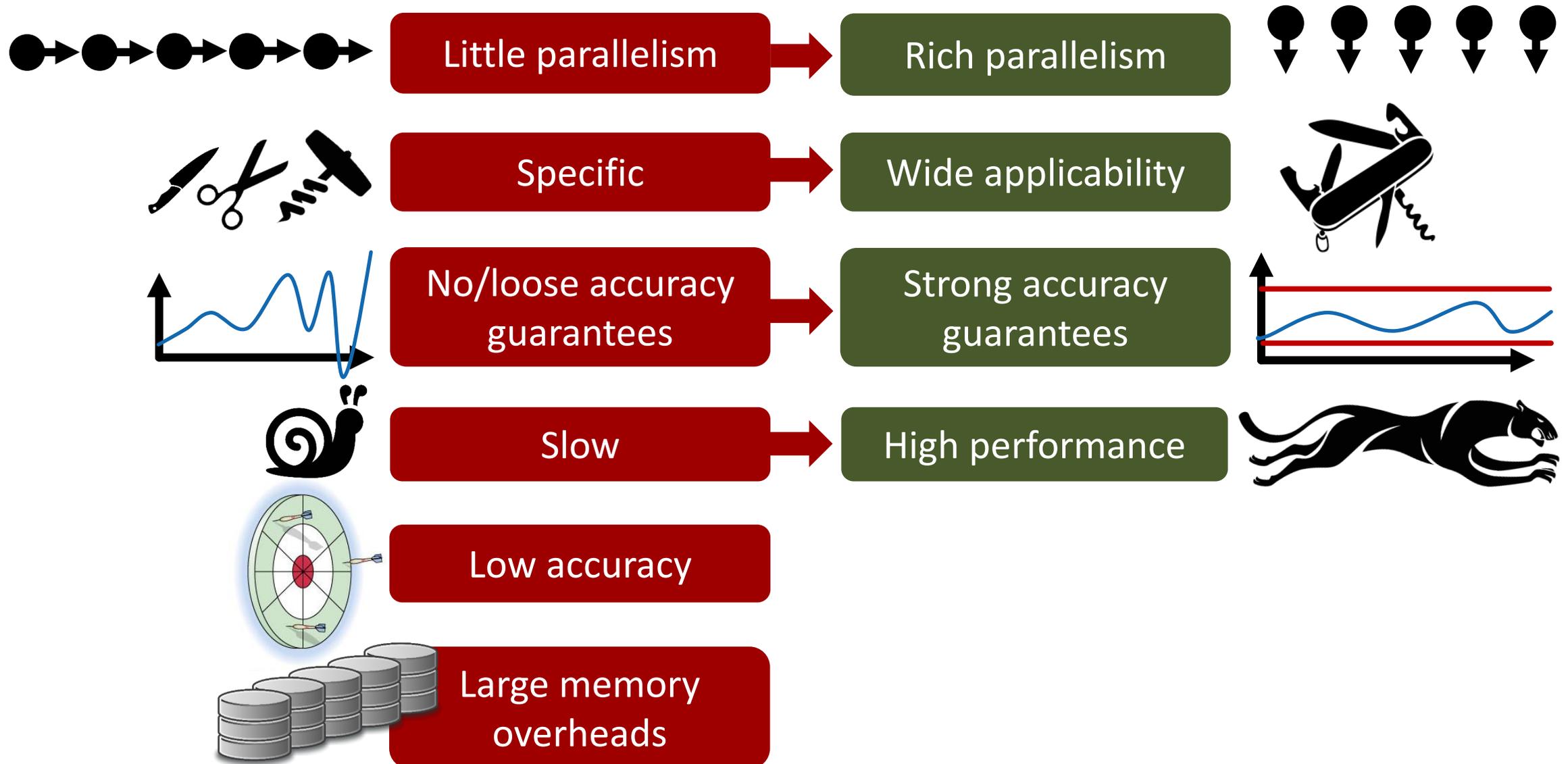
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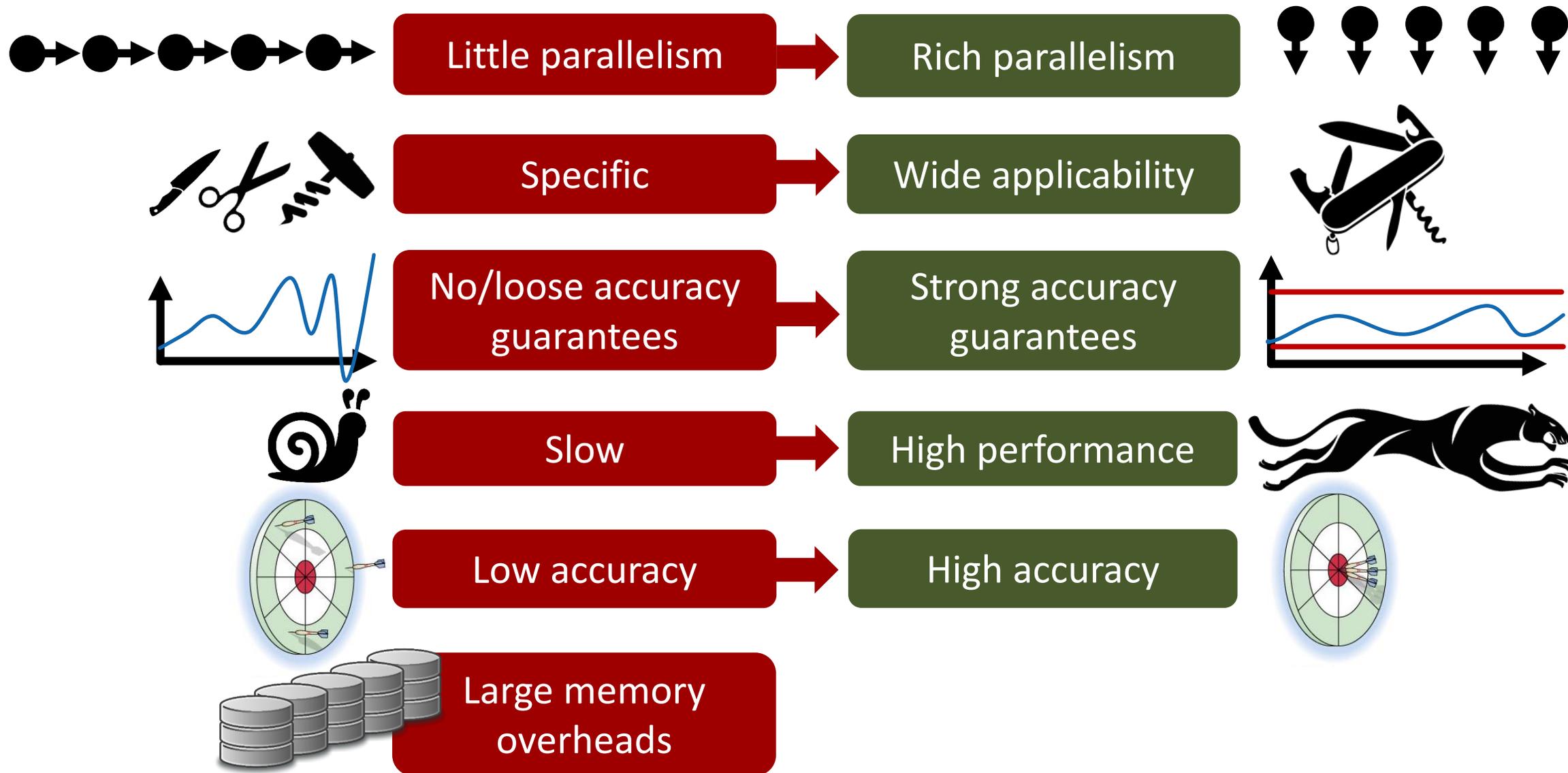
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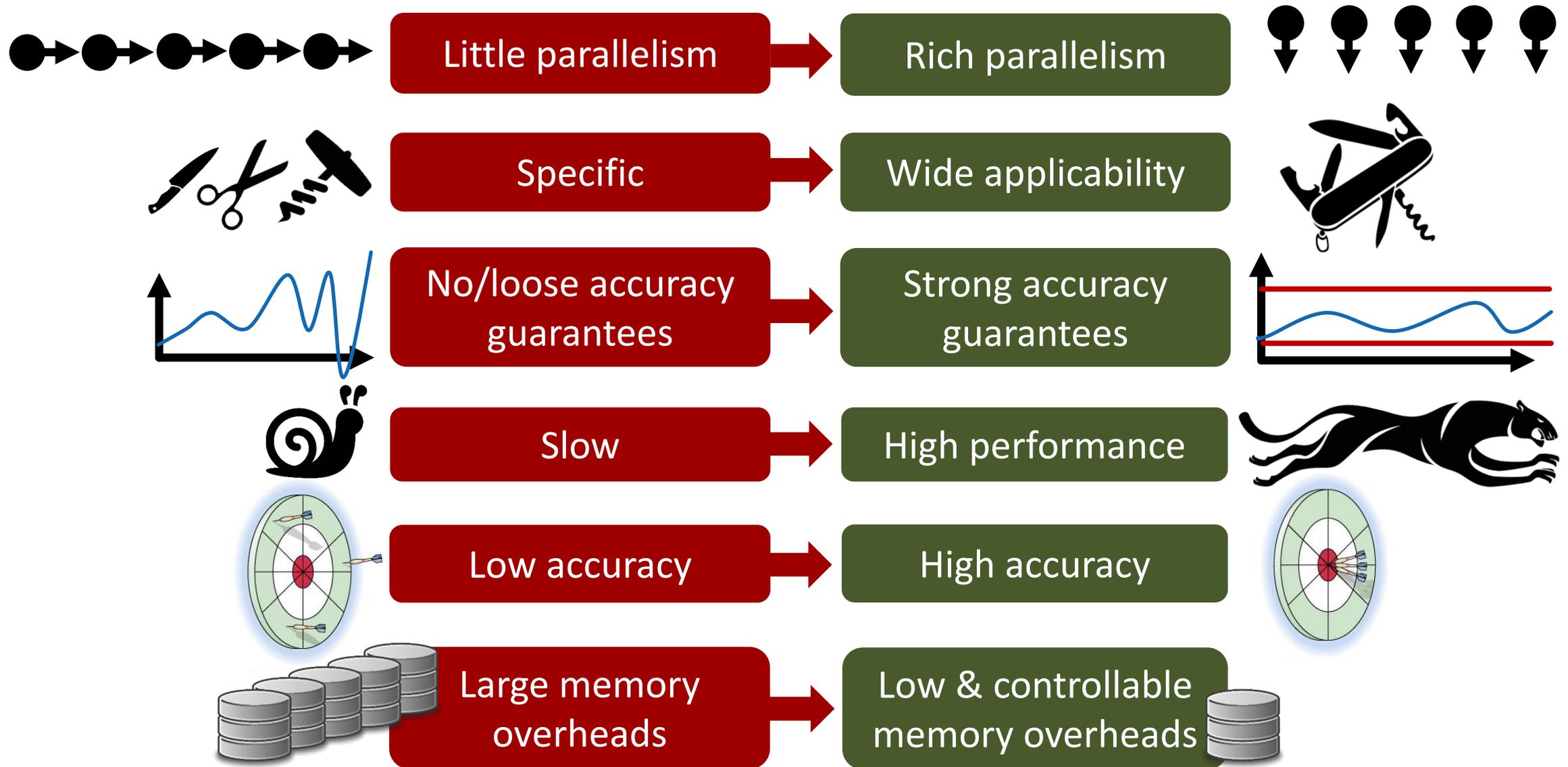
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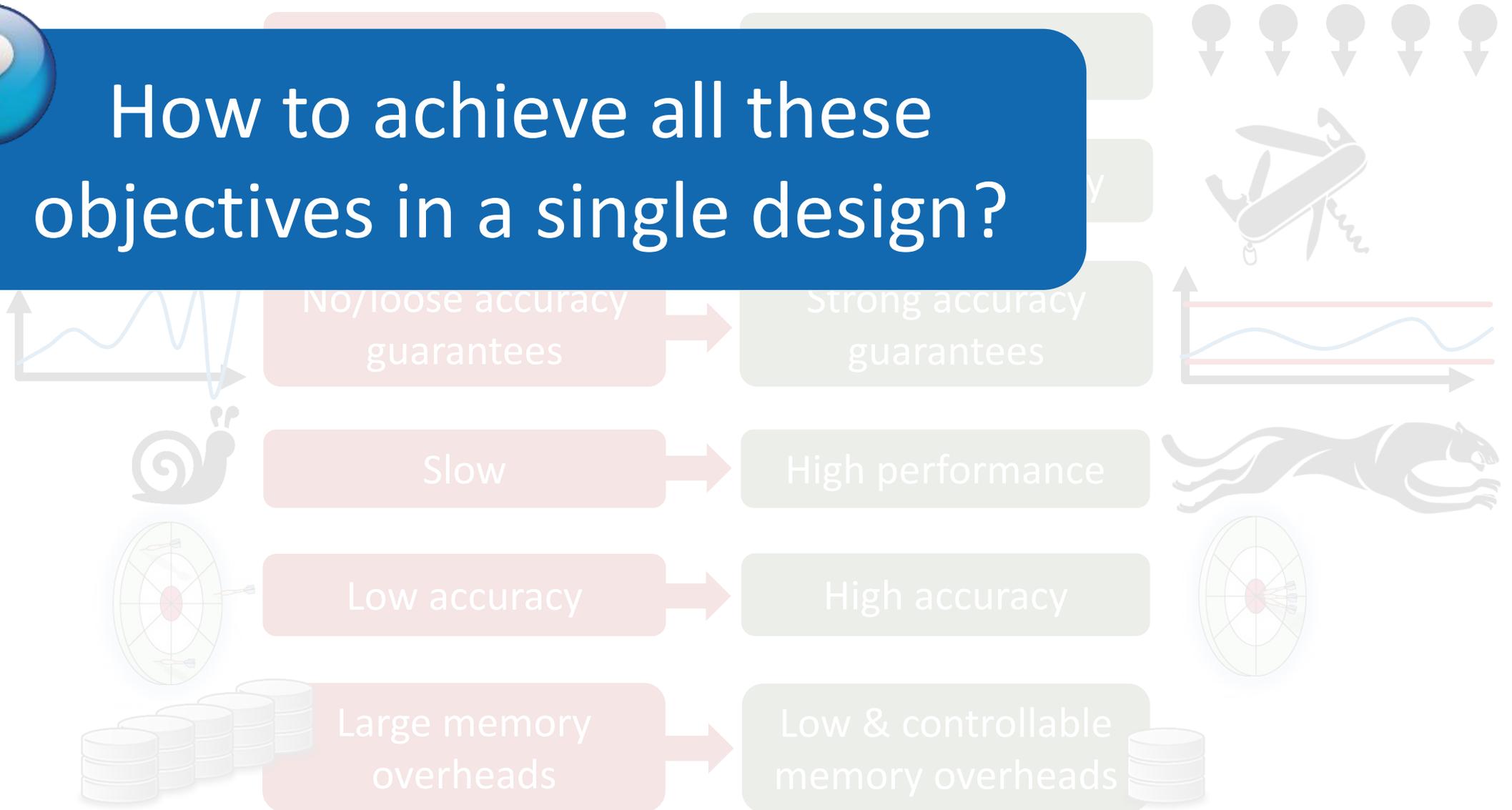
Approximate Graph Processing: Current Issues & Our Objectives



Approximate Graph Processing: Current Issues & Our Objectives



How to achieve all these objectives in a single design?



Approximate Graph Processing: Current Issues & Our Objectives



How to achieve all these objectives in a single design?

We develop **ProbGraph**: a graph representation that uses probabilistic set representations (aka sketches)



No/loose accuracy guarantees



Strong accuracy guarantees

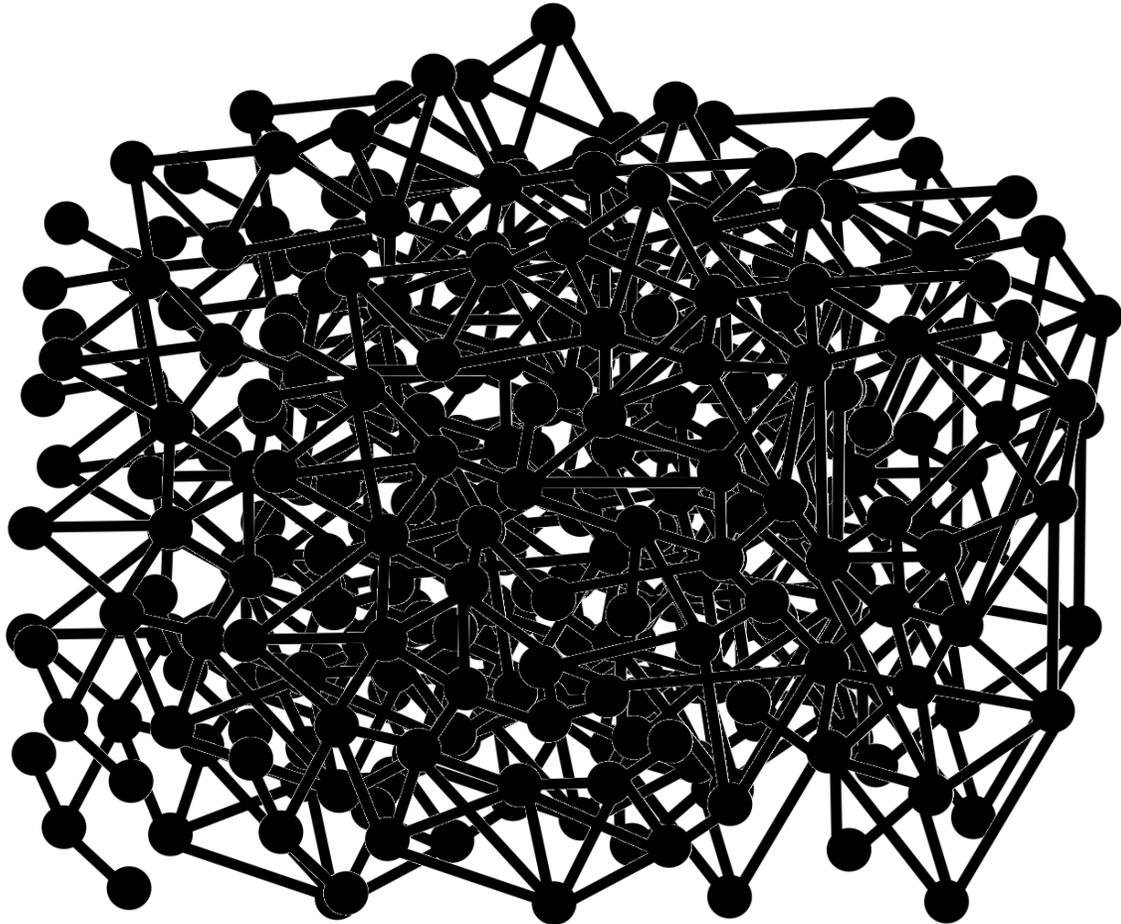


overheads

memory overheads

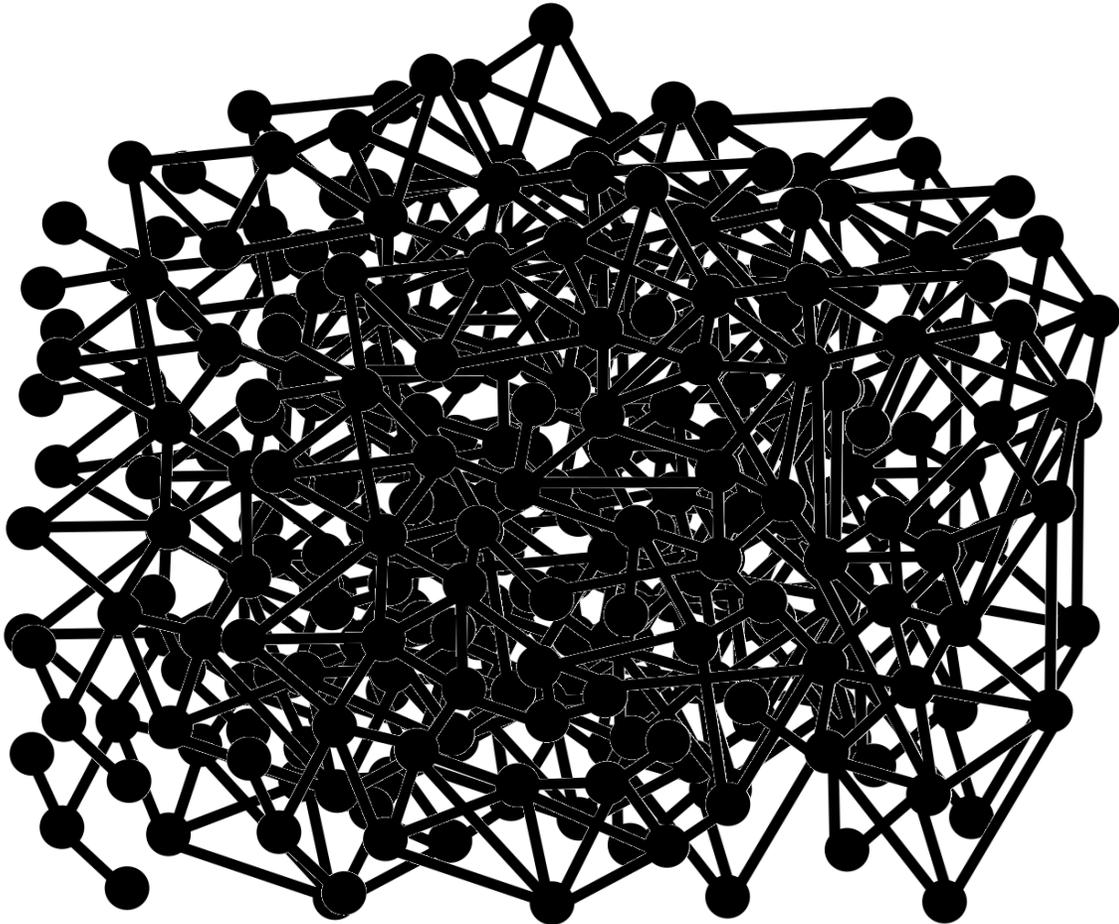


High-Level Approach Taken in ProbGraph



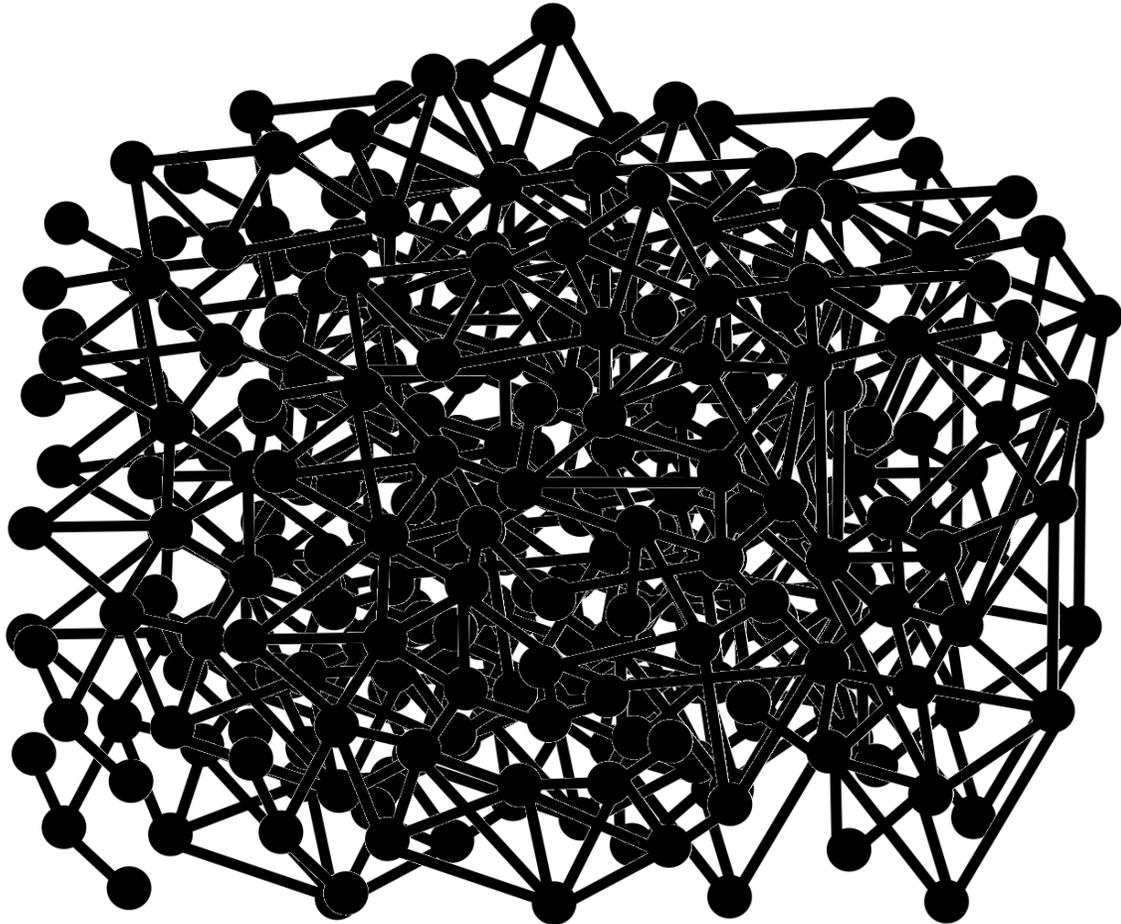
High-Level Approach Taken in ProbGraph

Keep the original graph

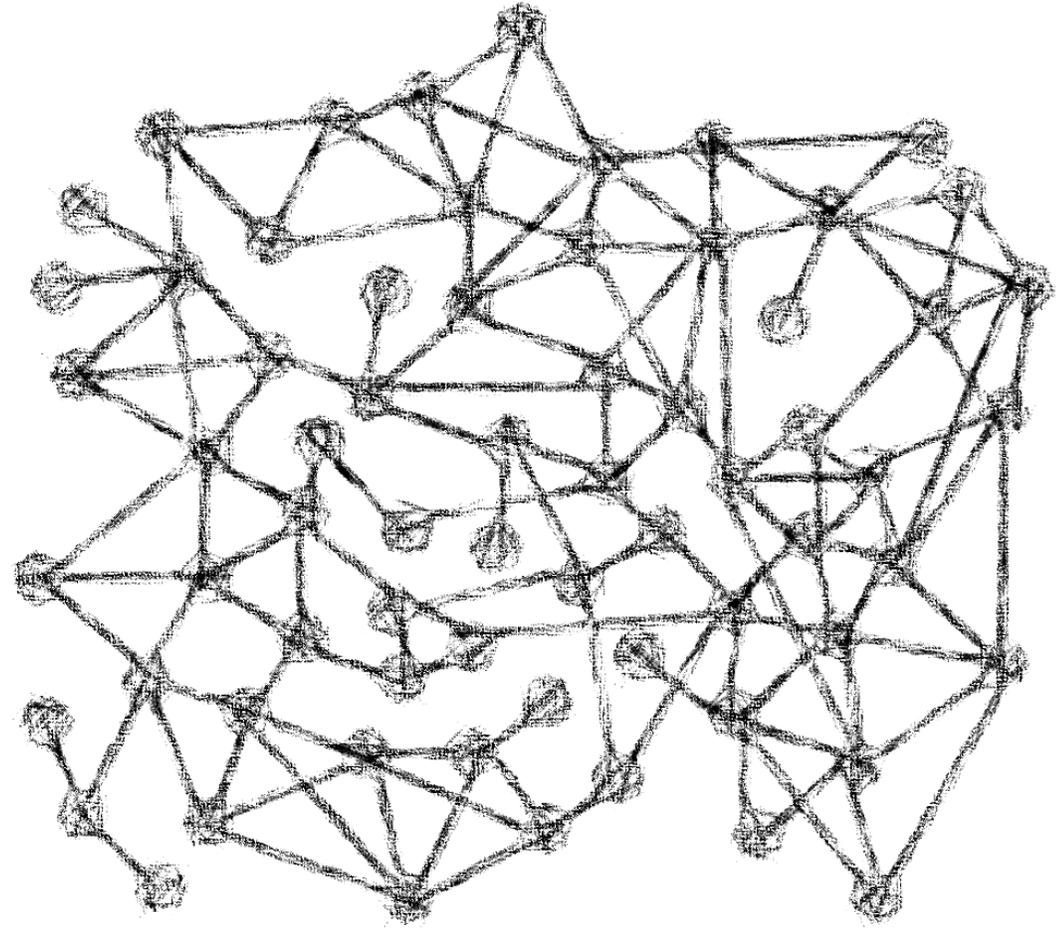


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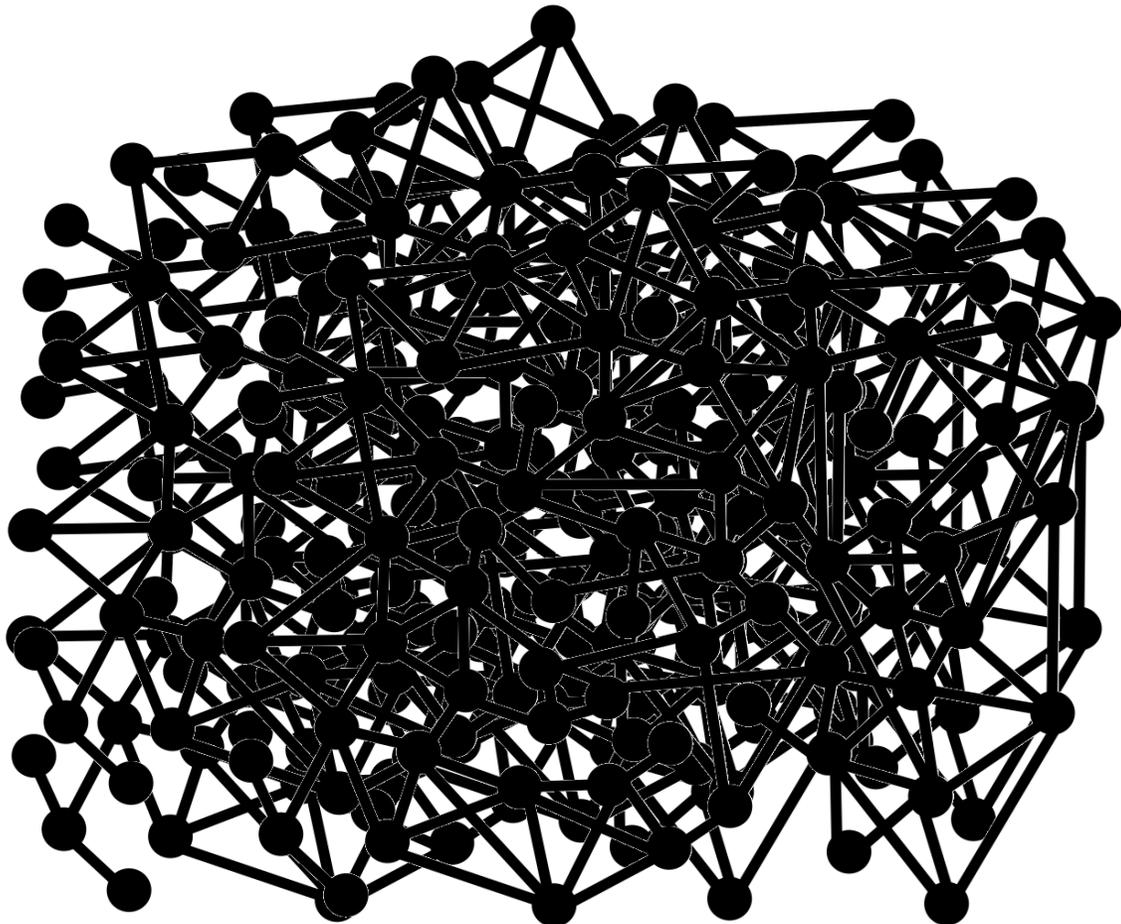
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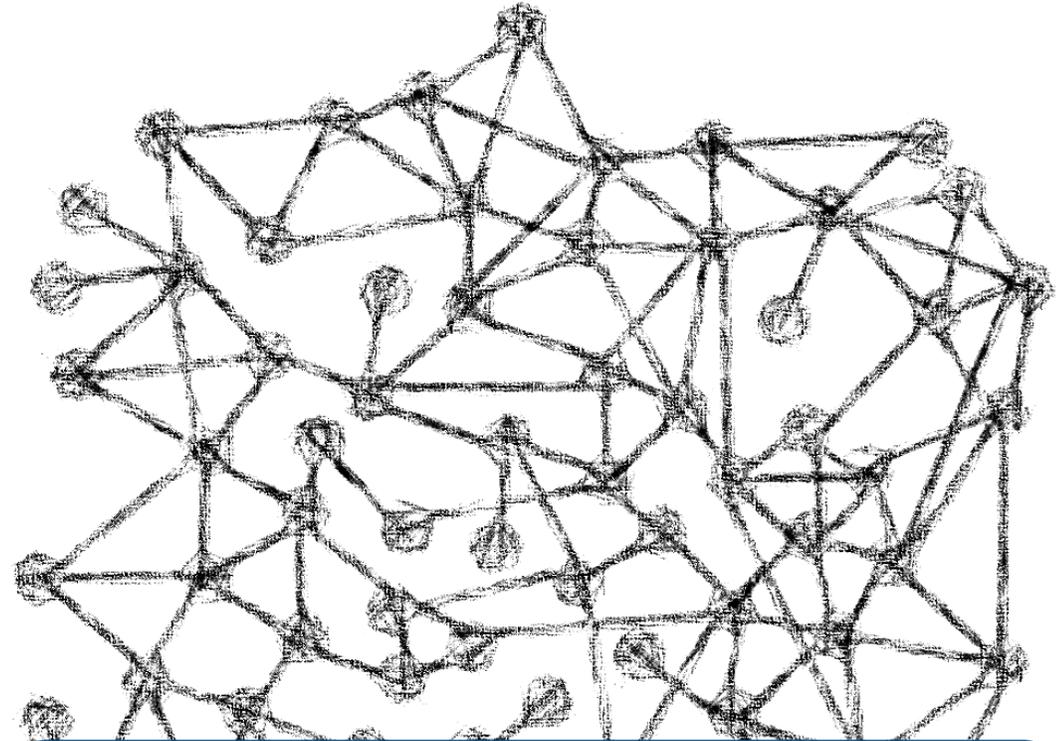
Maintain a very small
“sketch” of a graph

High-Level Approach Taken in ProbGraph

Keep the original graph



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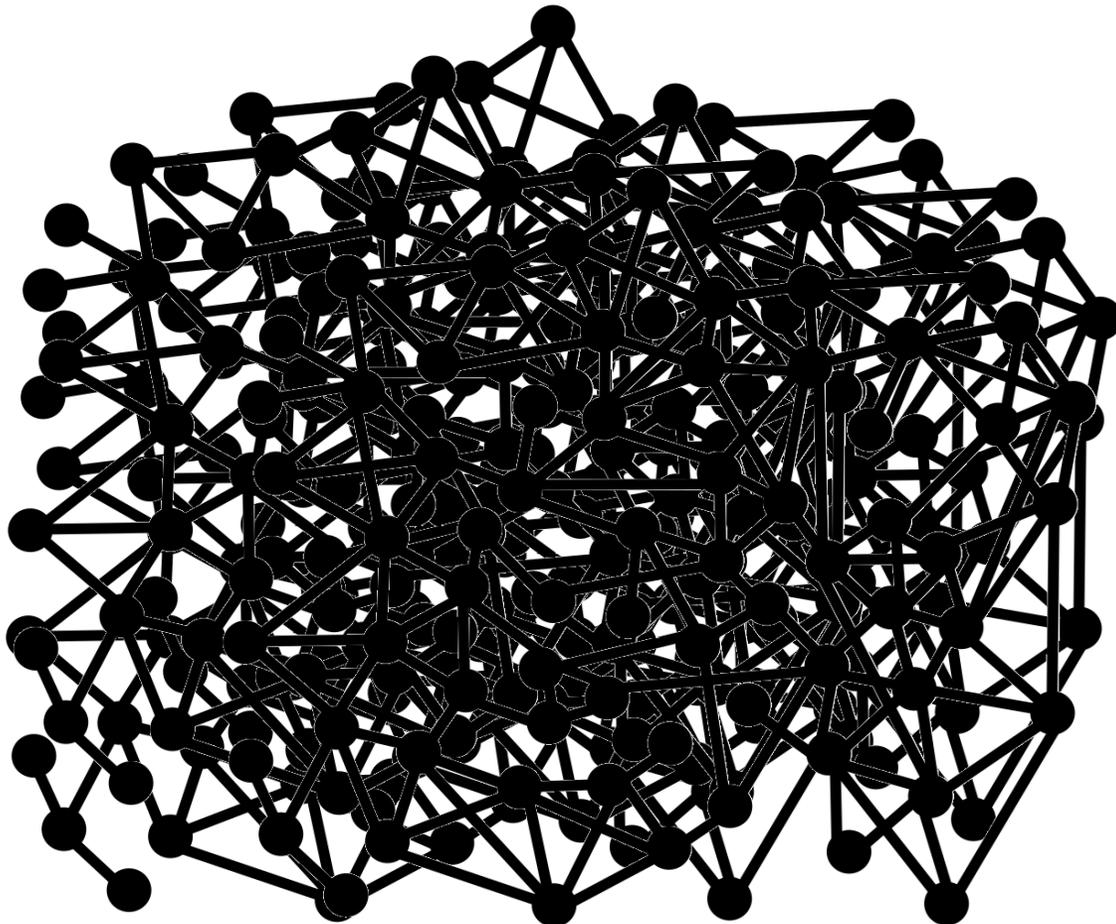


Maintain a very small
“sketch” of a graph

Use the sketch to answer
performance critical queries

High-Level Approach Taken in ProbGraph

Keep the original graph



+

Maintain a very small
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What design to use
for the sketch, to
satisfy all the goals?

Use the sketch to answer
performance critical queries

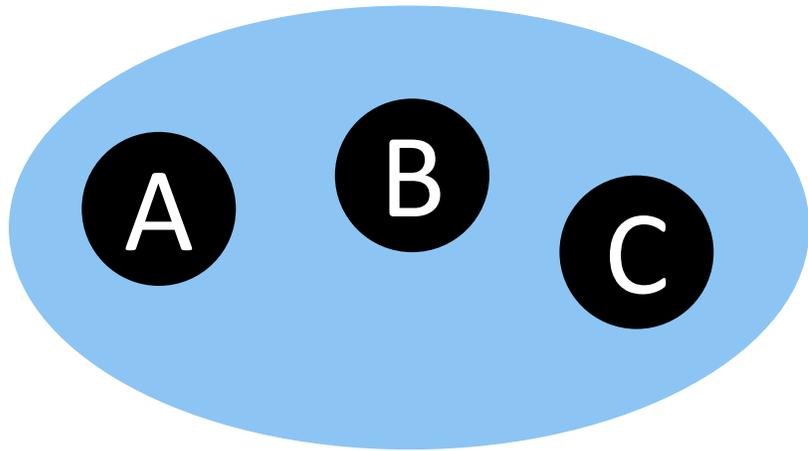
ProbGraph key idea: Use probabilistic set representations (set sketches)



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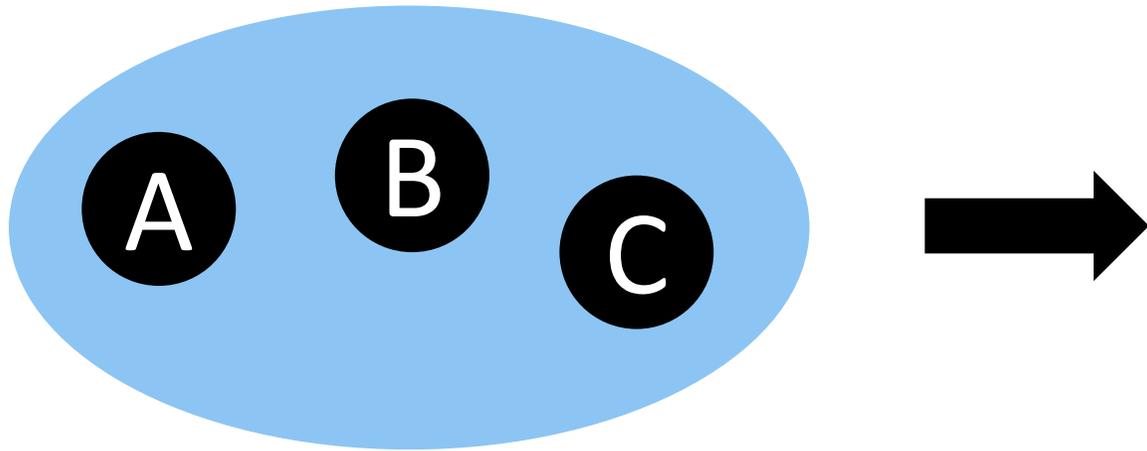
A set = {A, B, C}



ProbGraph key idea: Use probabilistic set representations (set sketches)



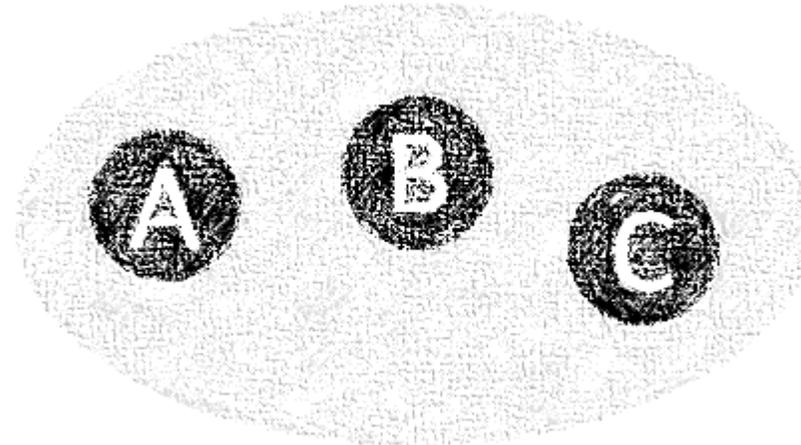
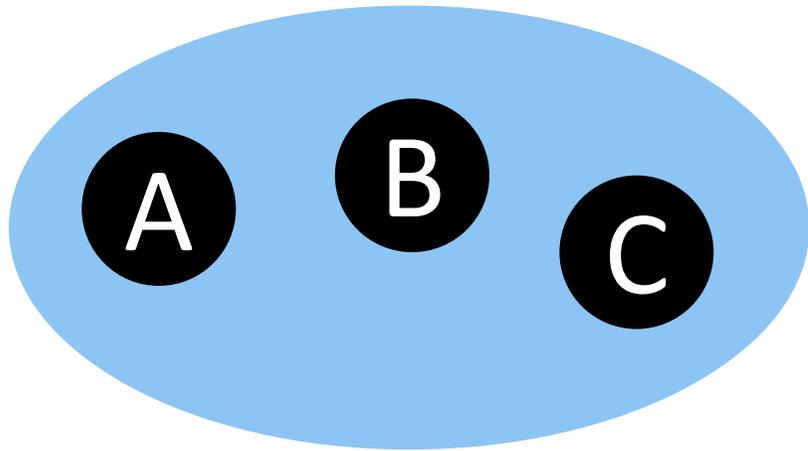
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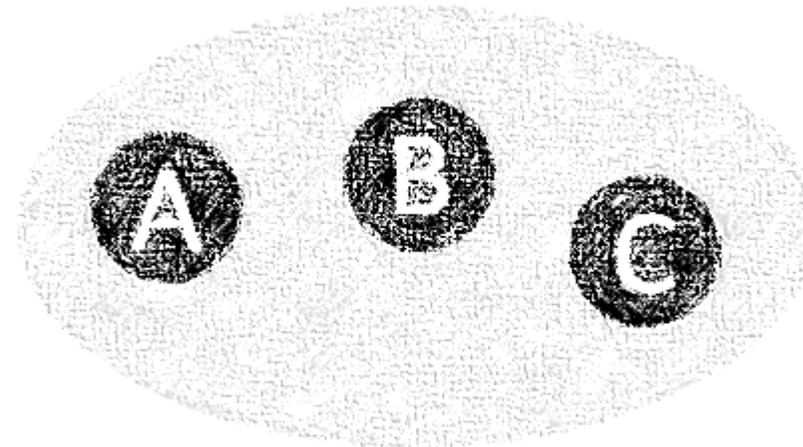
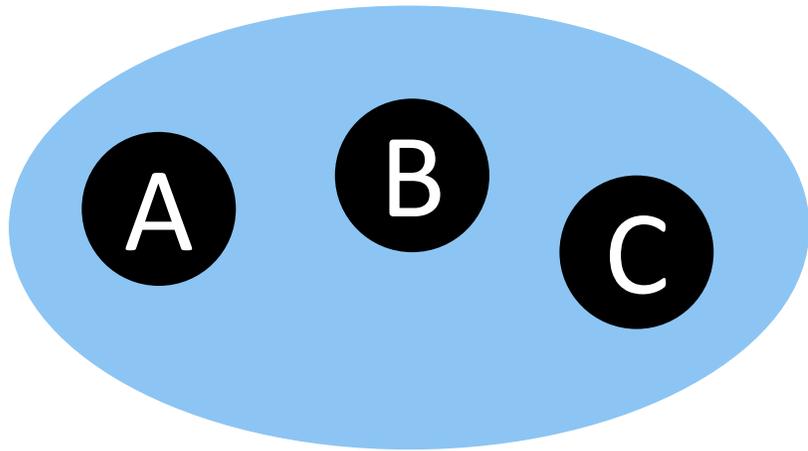
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Bloom filters
(BF) [1]

MinHash [2]

K Minimum
Values [3]

[1] B. H. Bloom, "Space/time trade-offs in hash coding with allowable errors", CACM, 1970.

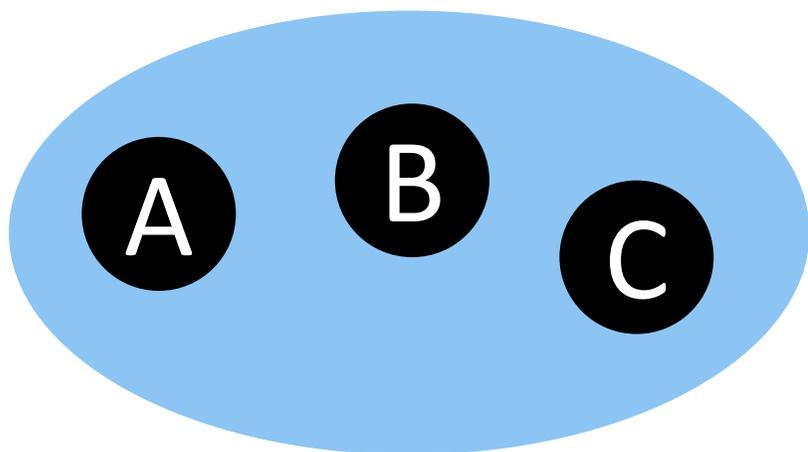
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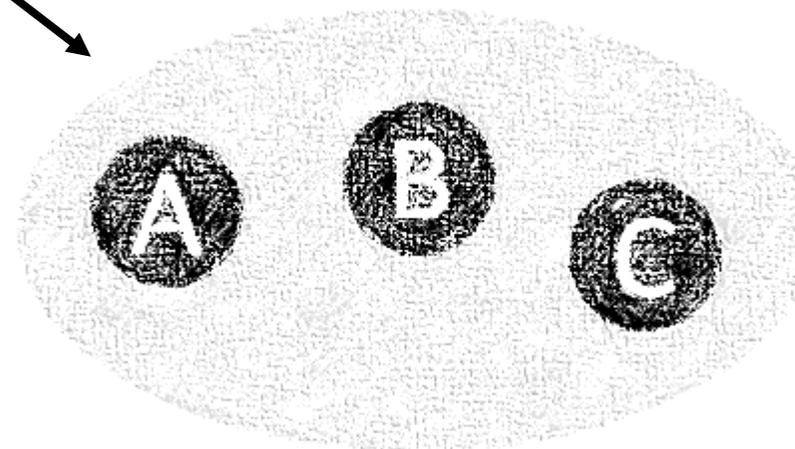
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Less space

Faster operations

Accuracy loss



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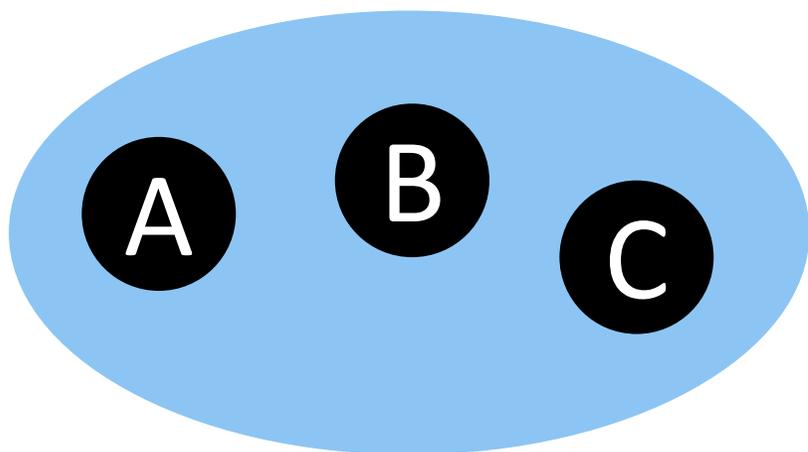
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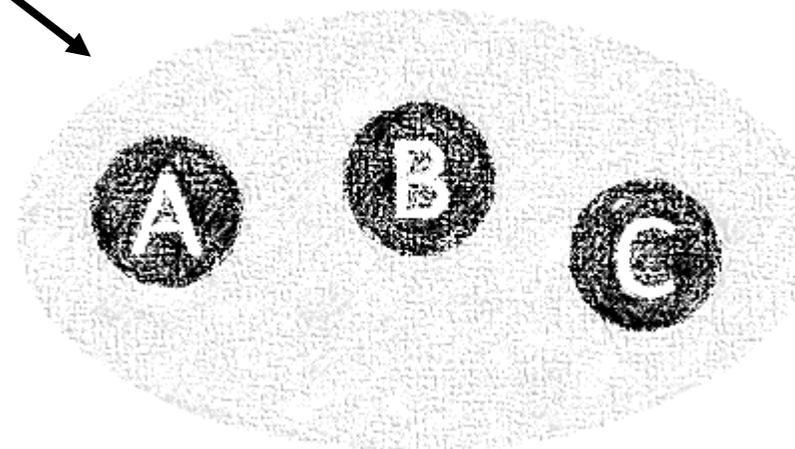
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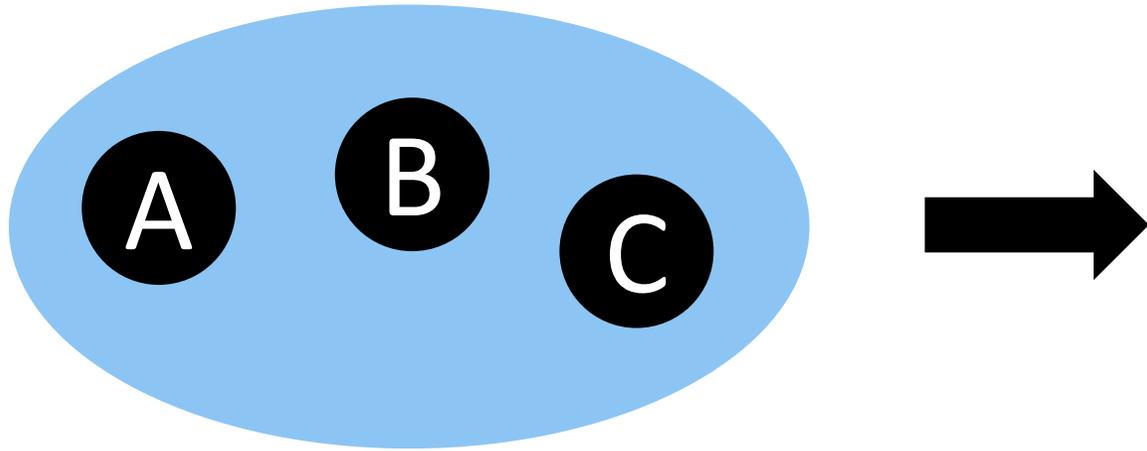
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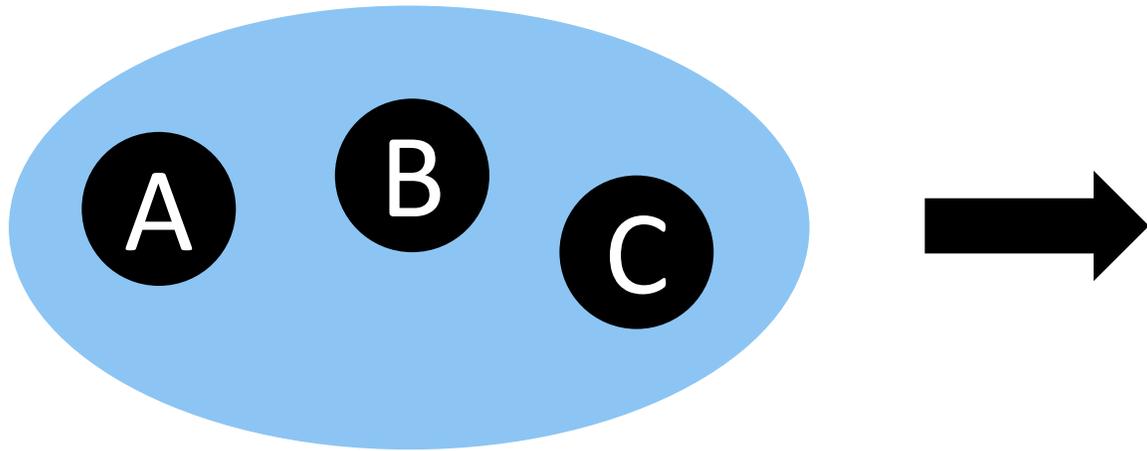
Bloom Filters for Graph Mining

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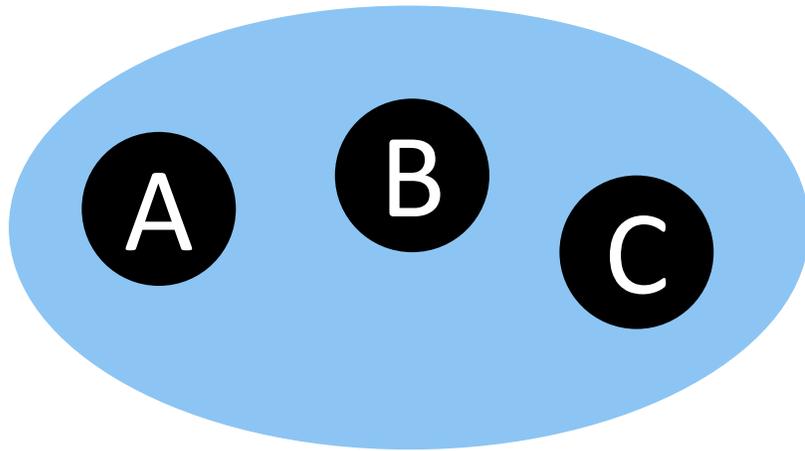


Bloom filter \mathcal{B}_X of X

Bitvector of size B_X [bits]

Bloom Filters for Graph Mining

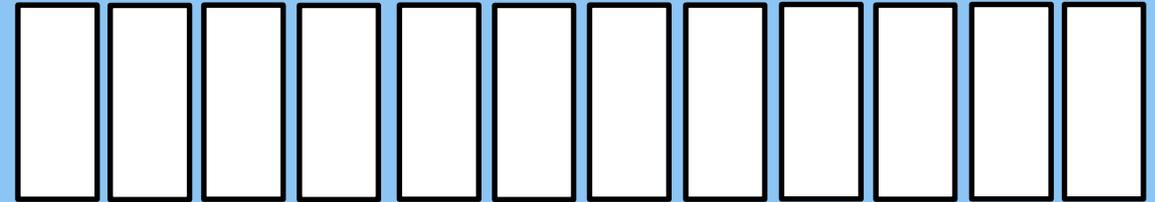
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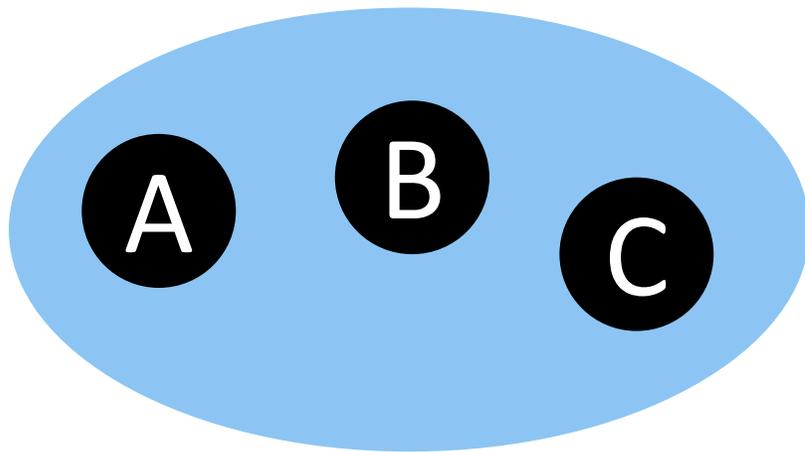
Bitvector of size B_X [bits]

$B_X = 12$



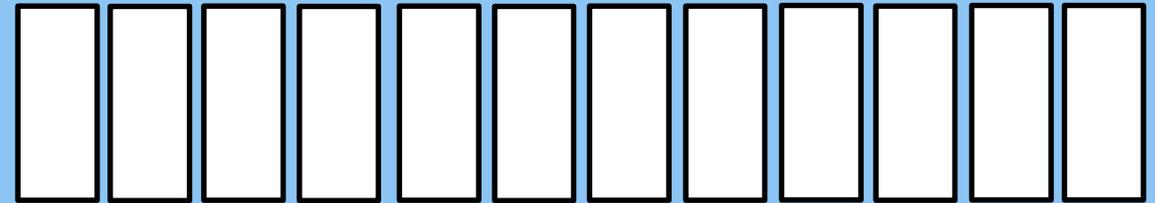
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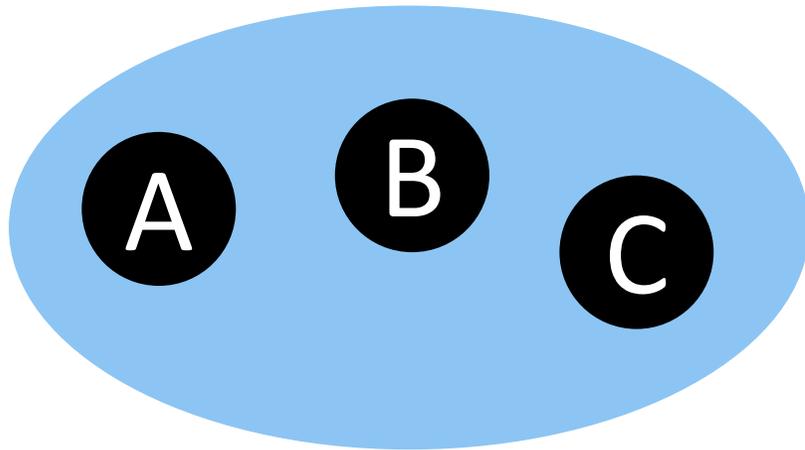
Bitvector of size B_X [bits] $B_X = 12$



Hash functions h_1, \dots, h_b
 $h_i : X \rightarrow \{1, \dots, B_X\}$

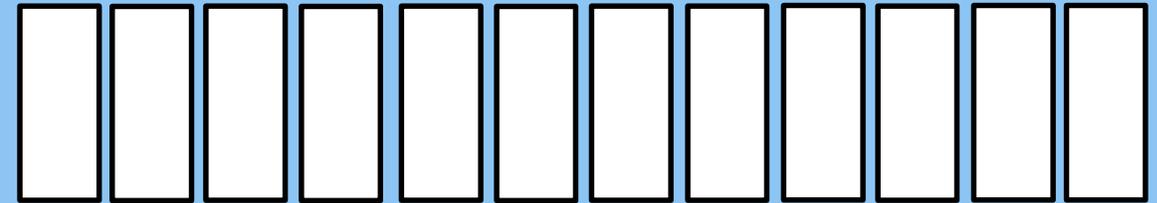
Bloom Filters for Graph Mining

A set = {A, B, C}



Bloom filter \mathcal{B}_X of X

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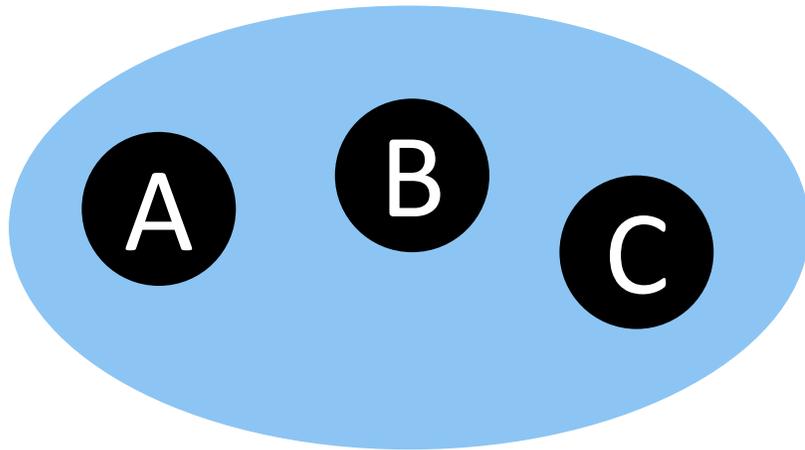
$h_i : X \rightarrow \{1, \dots, B_X\}$

$b = 2$

$h_2, h_1 : X \rightarrow \{1, \dots, 12\}$

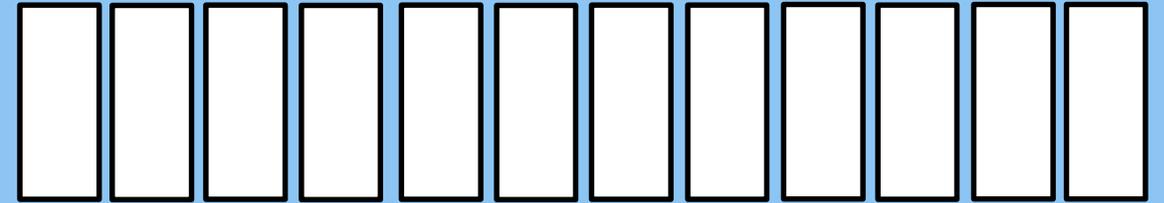
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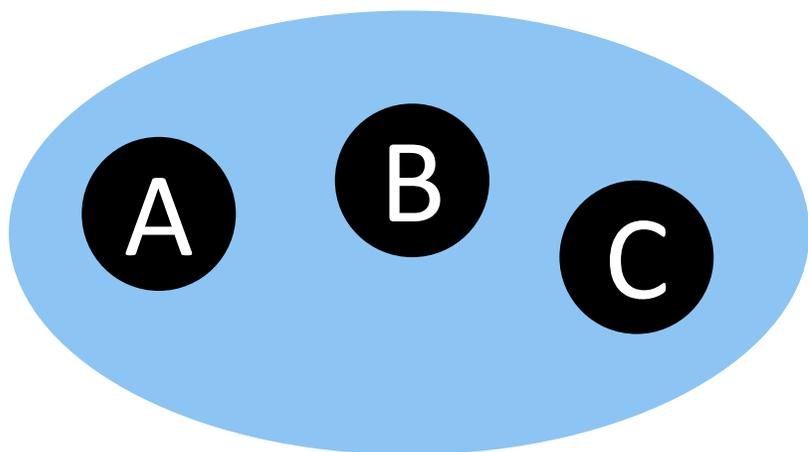
$h_2, h_1 : X \rightarrow \{1, \dots, 12\}$

$h_1(\text{A}) = 3$

$h_2(\text{A}) = 5$

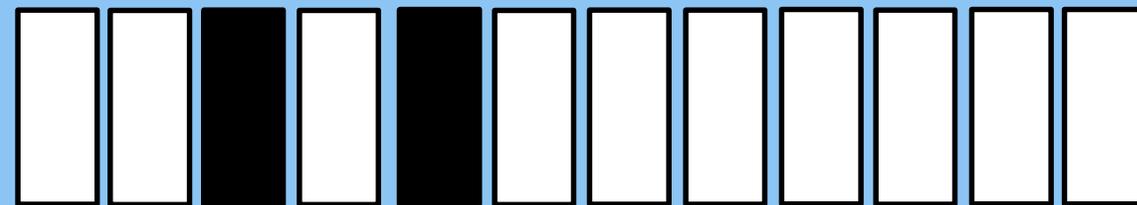
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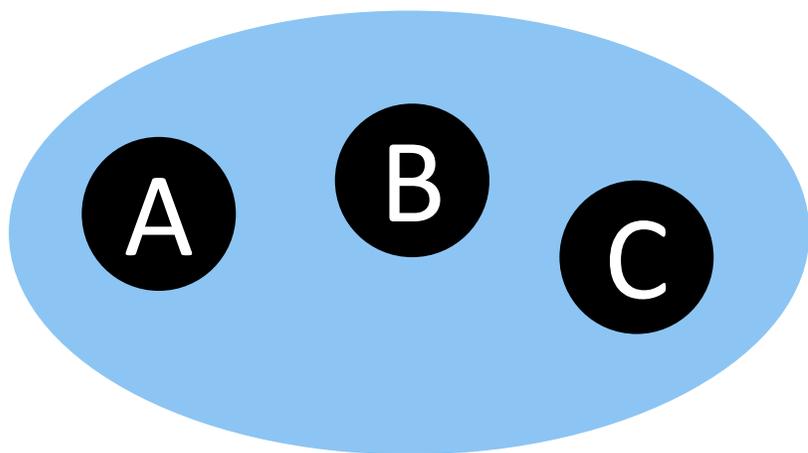
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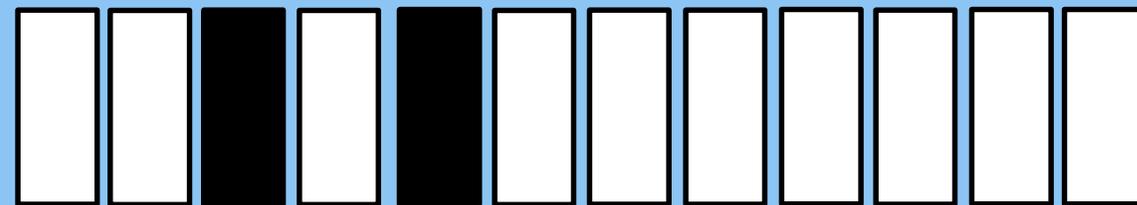
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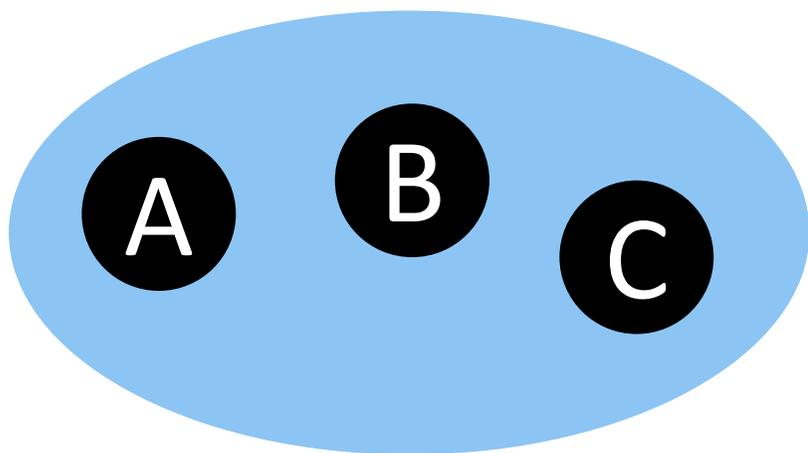
$h_2, h_1 : X \rightarrow \{1, \dots, 12\}$

$h_1(\text{A}) = 3$ $h_1(\text{B}) = 1$

$h_2(\text{A}) = 5$ $h_2(\text{B}) = 8$

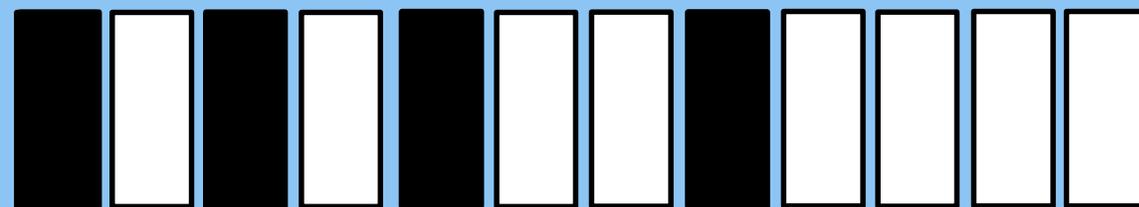
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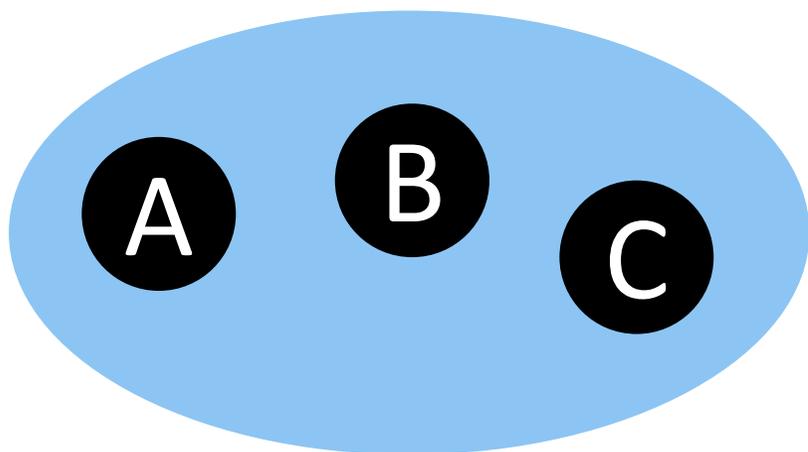
$h_2, h_1 : X \rightarrow \{1, \dots, 12\}$

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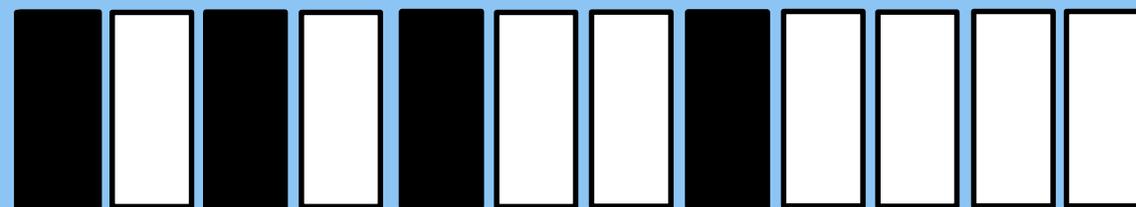
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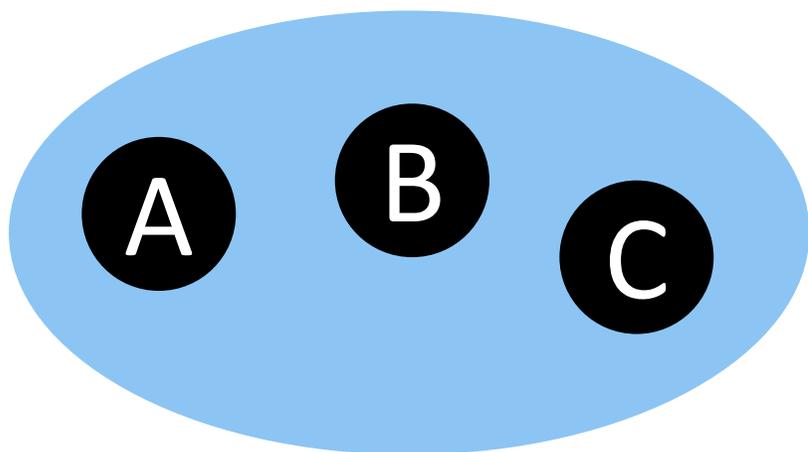
$h_2, h_1 : X \rightarrow \{1, \dots, 12\}$

$h_1(\text{A}) = 3$ $h_1(\text{B}) = 1$ $h_1(\text{C}) = 4$

$h_2(\text{A}) = 5$ $h_2(\text{B}) = 8$ $h_2(\text{C}) = 11$

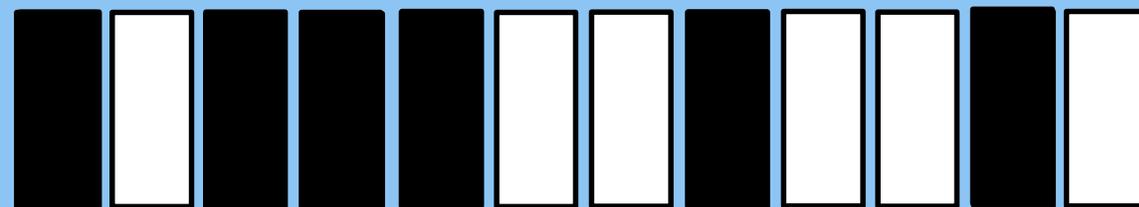
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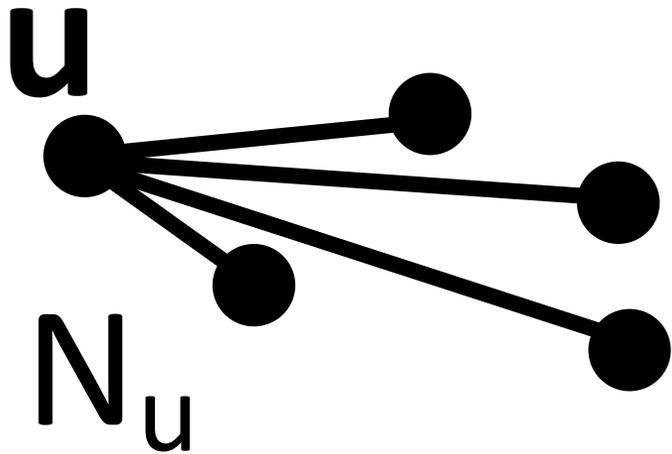
$b = 2$

$h_2, h_1 : X \rightarrow \{1, \dots, 12\}$

$h_1(\mathbf{A}) = 3$ $h_1(\mathbf{B}) = 1$ $h_1(\mathbf{C}) = 4$

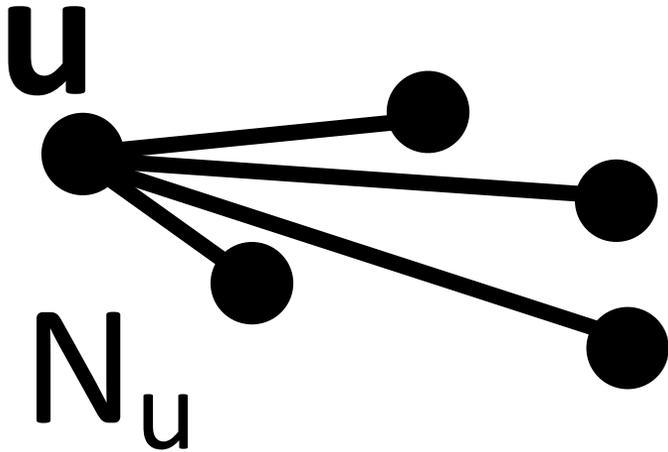
$h_2(\mathbf{A}) = 5$ $h_2(\mathbf{B}) = 8$ $h_2(\mathbf{C}) = 11$

Bloom Filters for Graph Mining



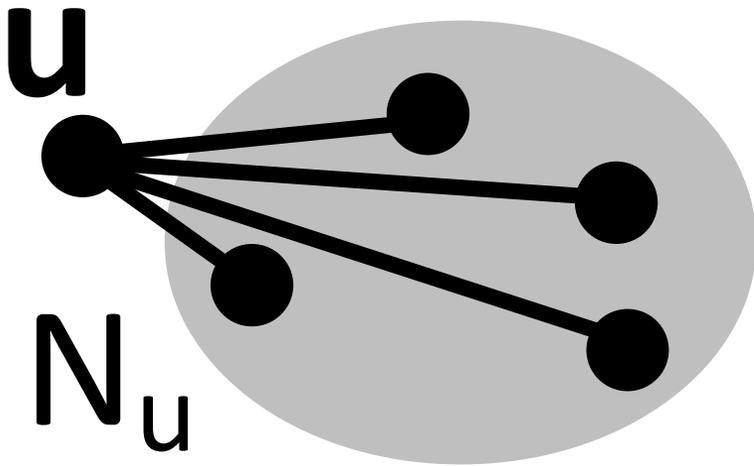
Bloom Filters for Graph Mining

Each neighborhood
 N_u is a set of
vertices



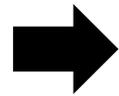
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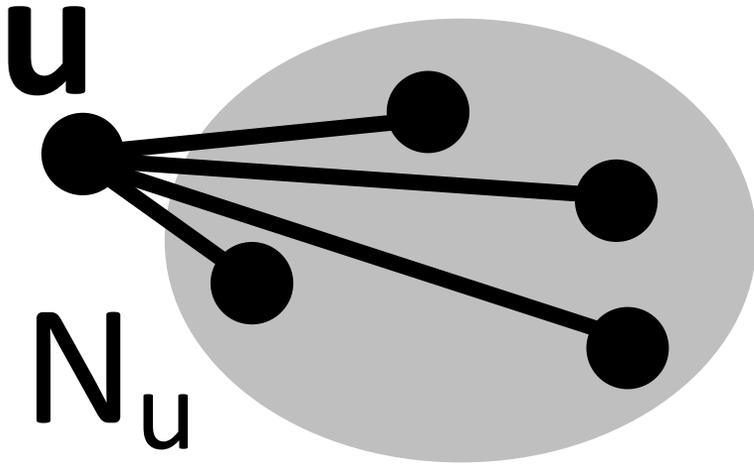


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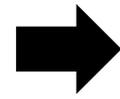


„Sketch” each N_u with a Bloom filter

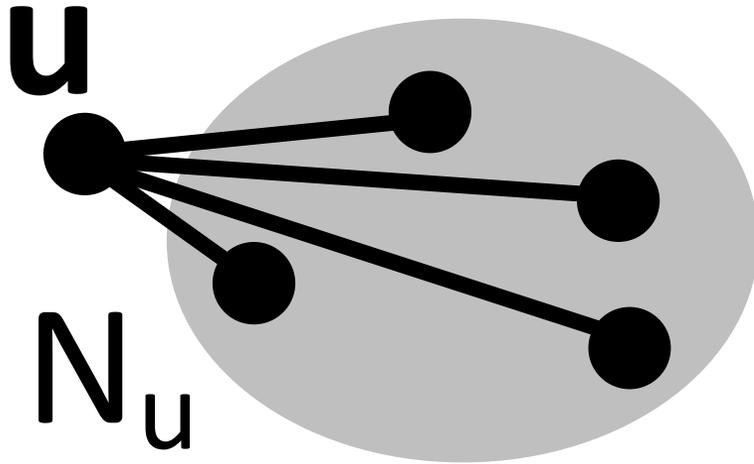


Bloom Filters for Graph Mining

Each neighborhood N_u is a set of vertices



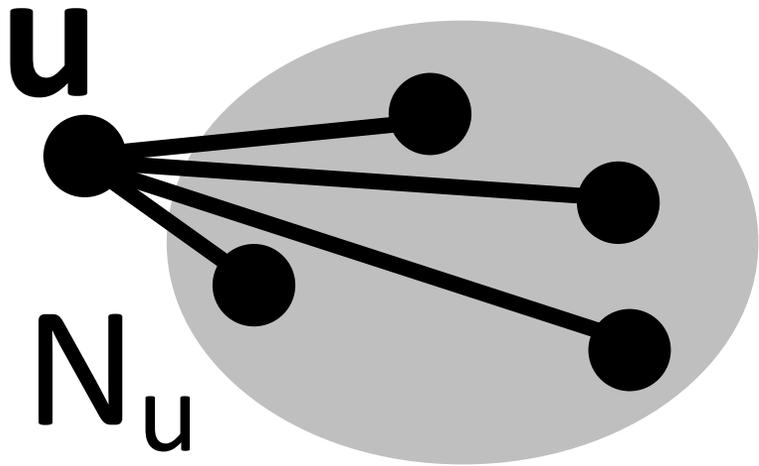
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Bloom Filters for Graph Mining

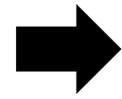
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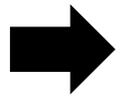
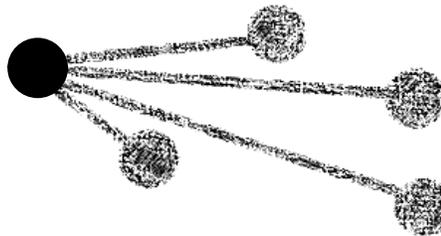
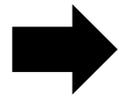
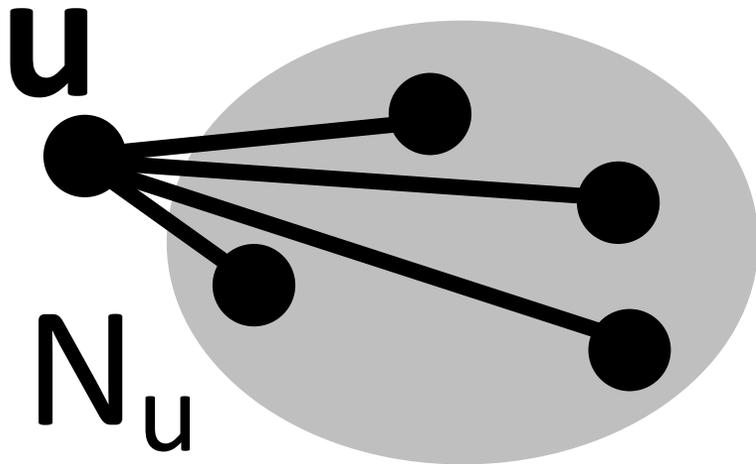


Bloom Filters for Graph Mining

Each neighborhood N_u is a set of vertices

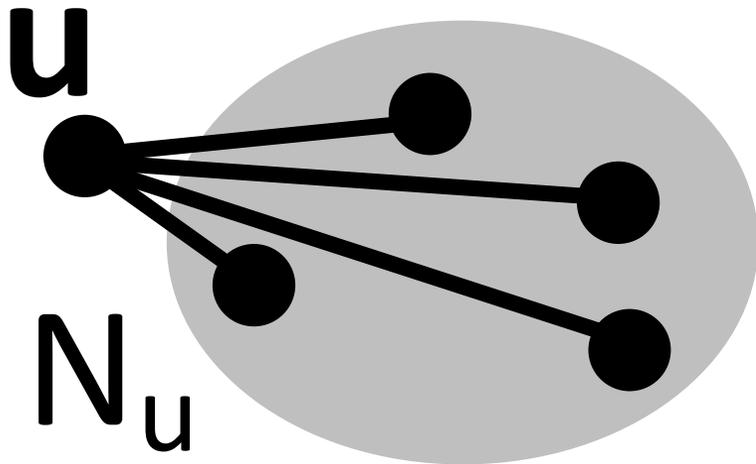


„Sketch” each N_u with a Bloom filter

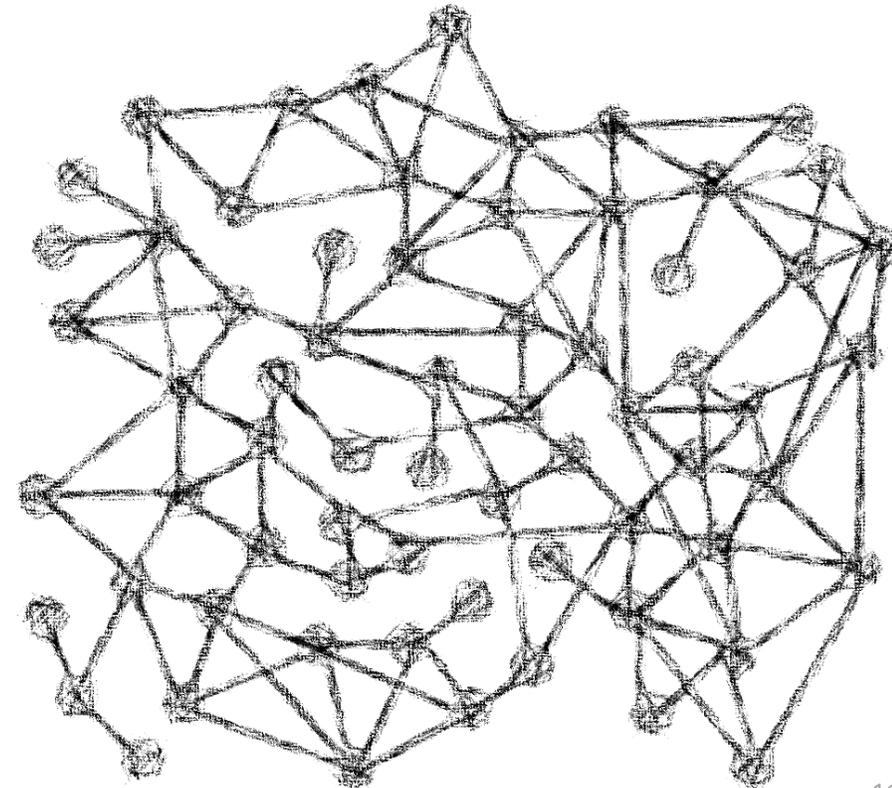
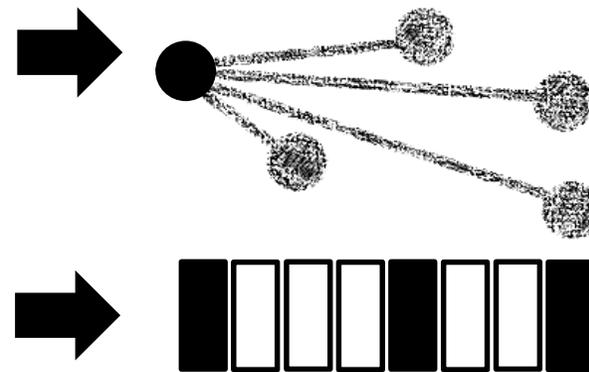


Bloom Filters for Graph Mining

Each neighborhood N_u is a set of vertices



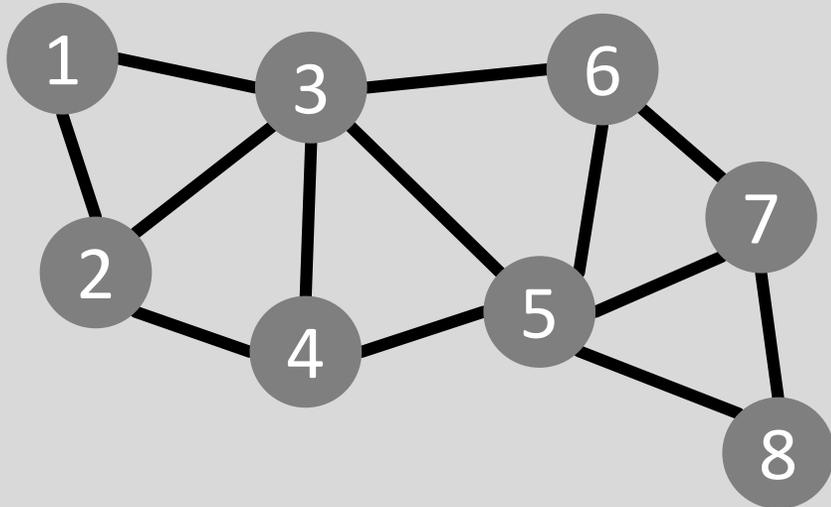
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ProbGraph: Summary of Design

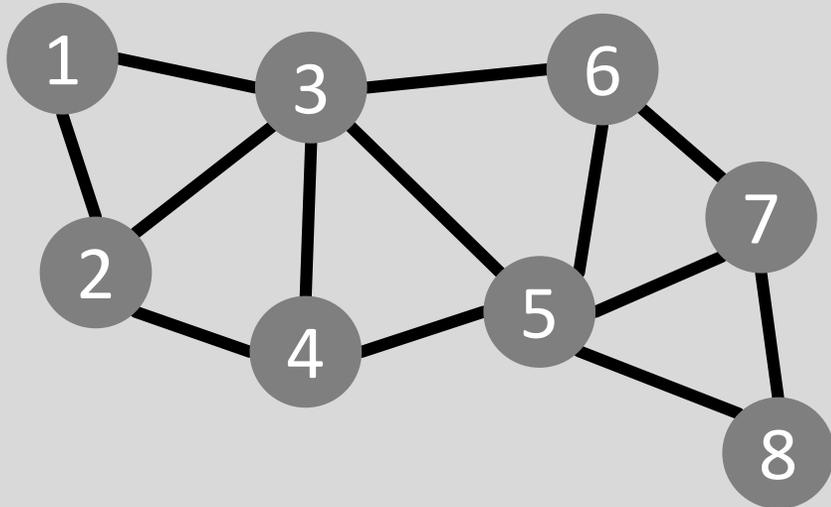
ProbGraph: Summary of Design

Input graph G



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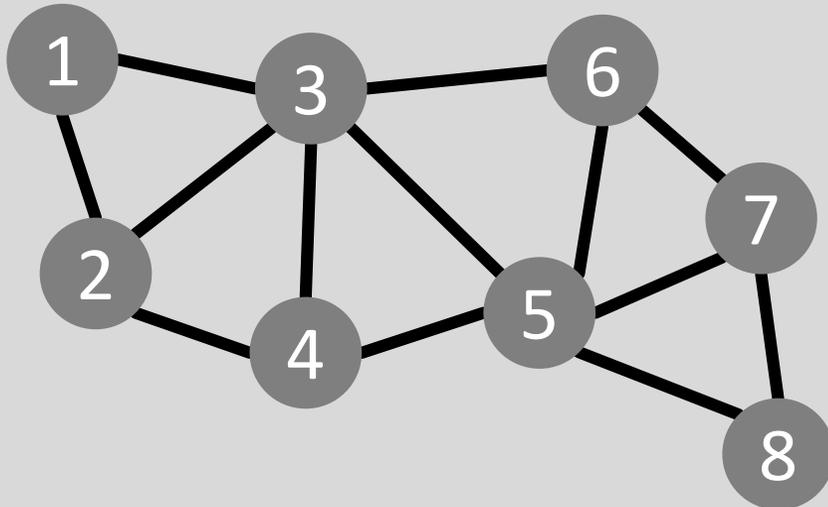
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Standard graph representation (e.g., CSR)

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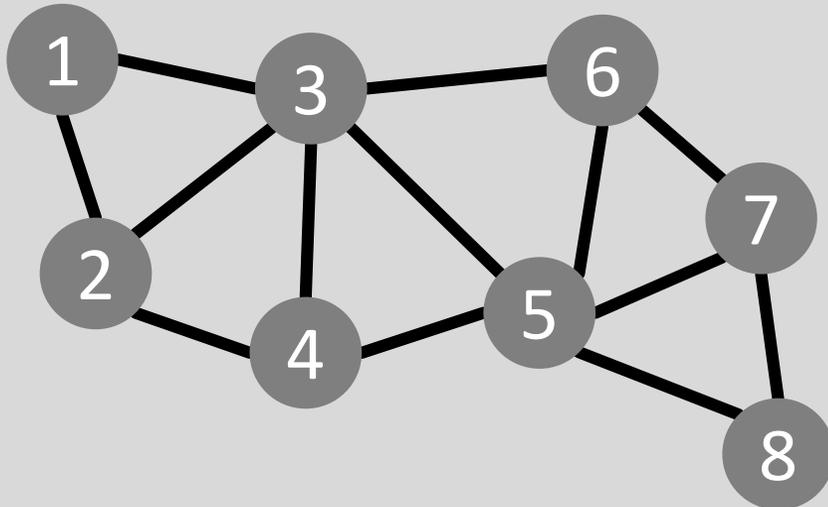


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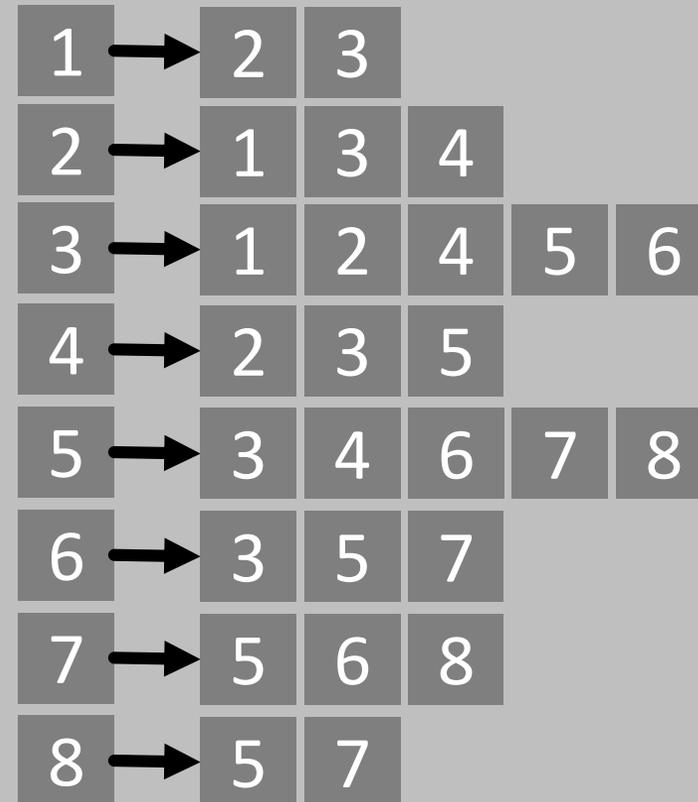
1	→	2	3						
2	→	1	3	4					
3	→	1	2	4	5	6			
4	→	2	3	5					
5	→	3	4	6	7	8			
6	→	3	5	7					
7	→	5	6	8					
8	→	5	7						

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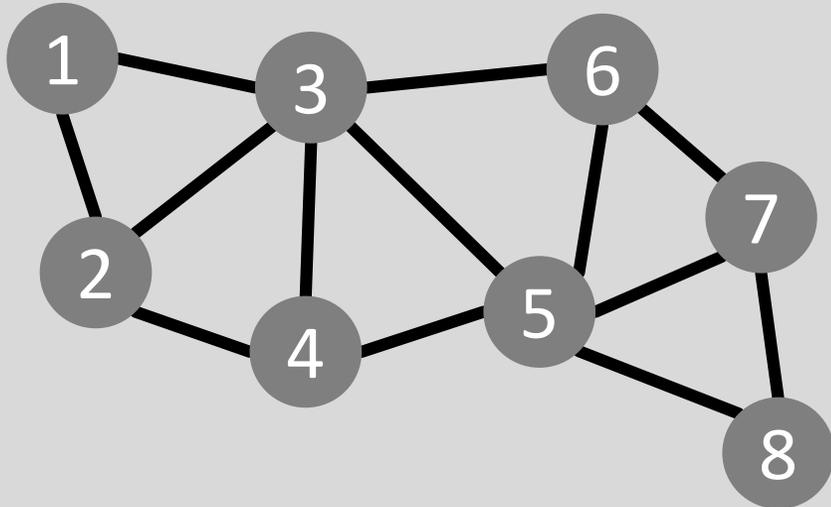


ProbGraph representation

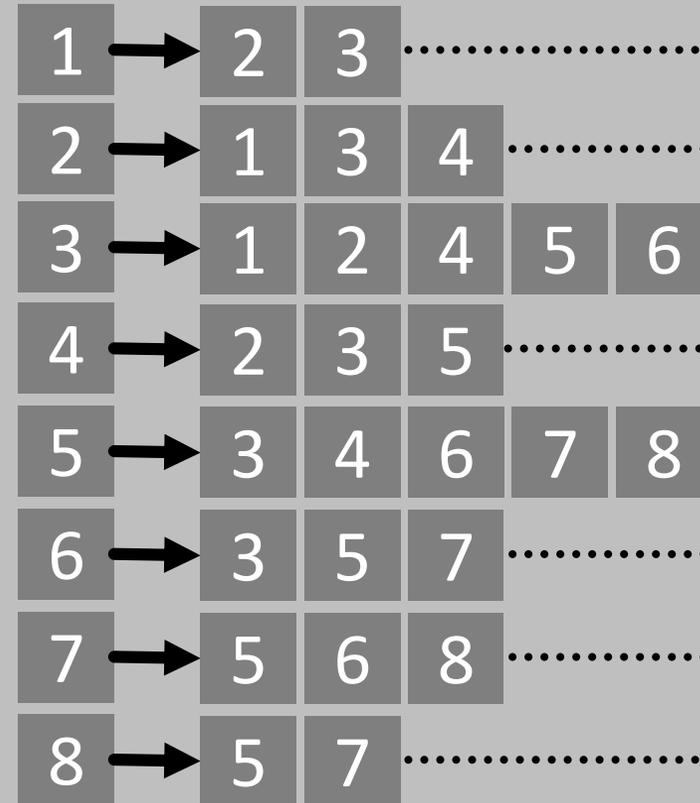


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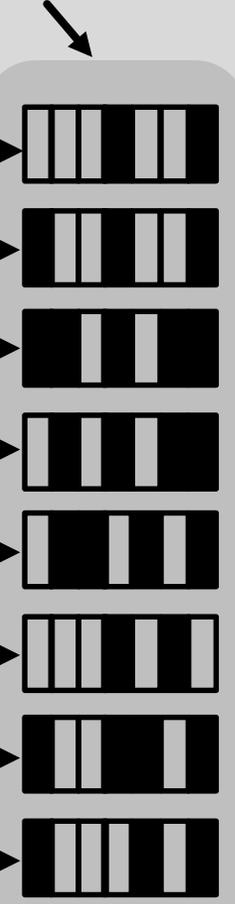
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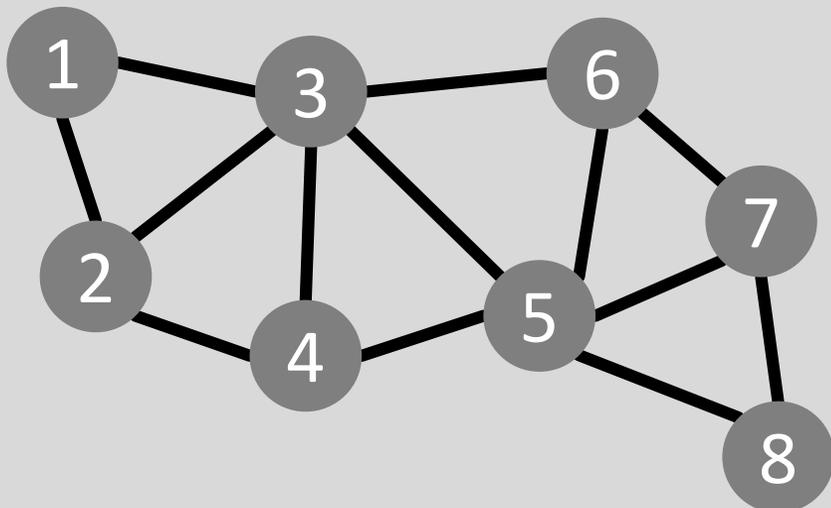


Bloom filters

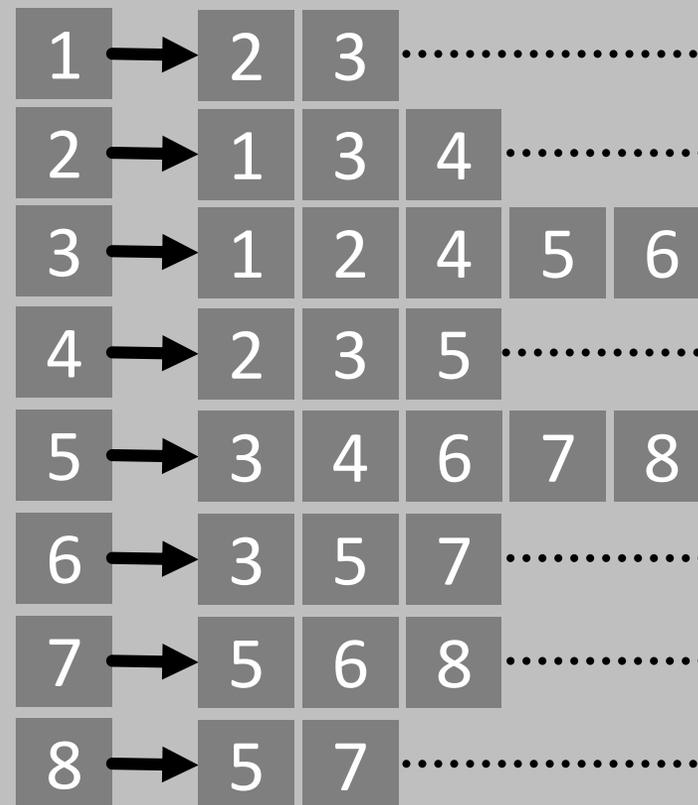


ProbGraph: Summary of Design

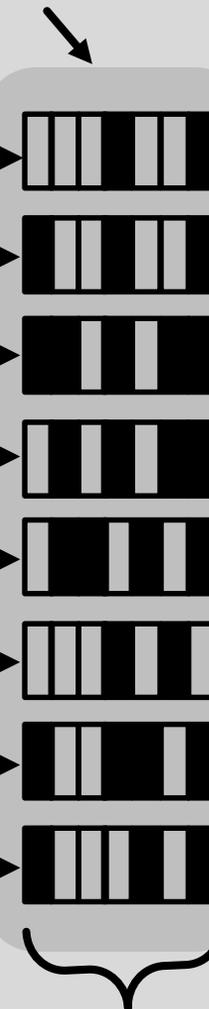
Input graph G



ProbGraph representation



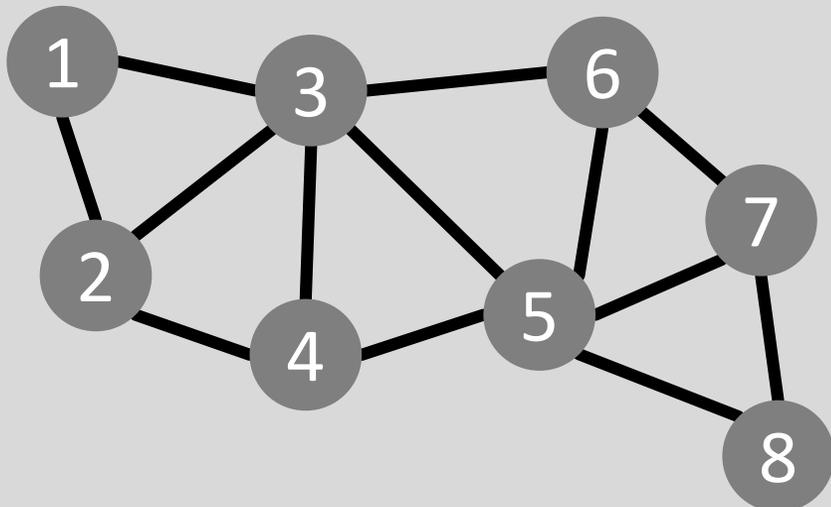
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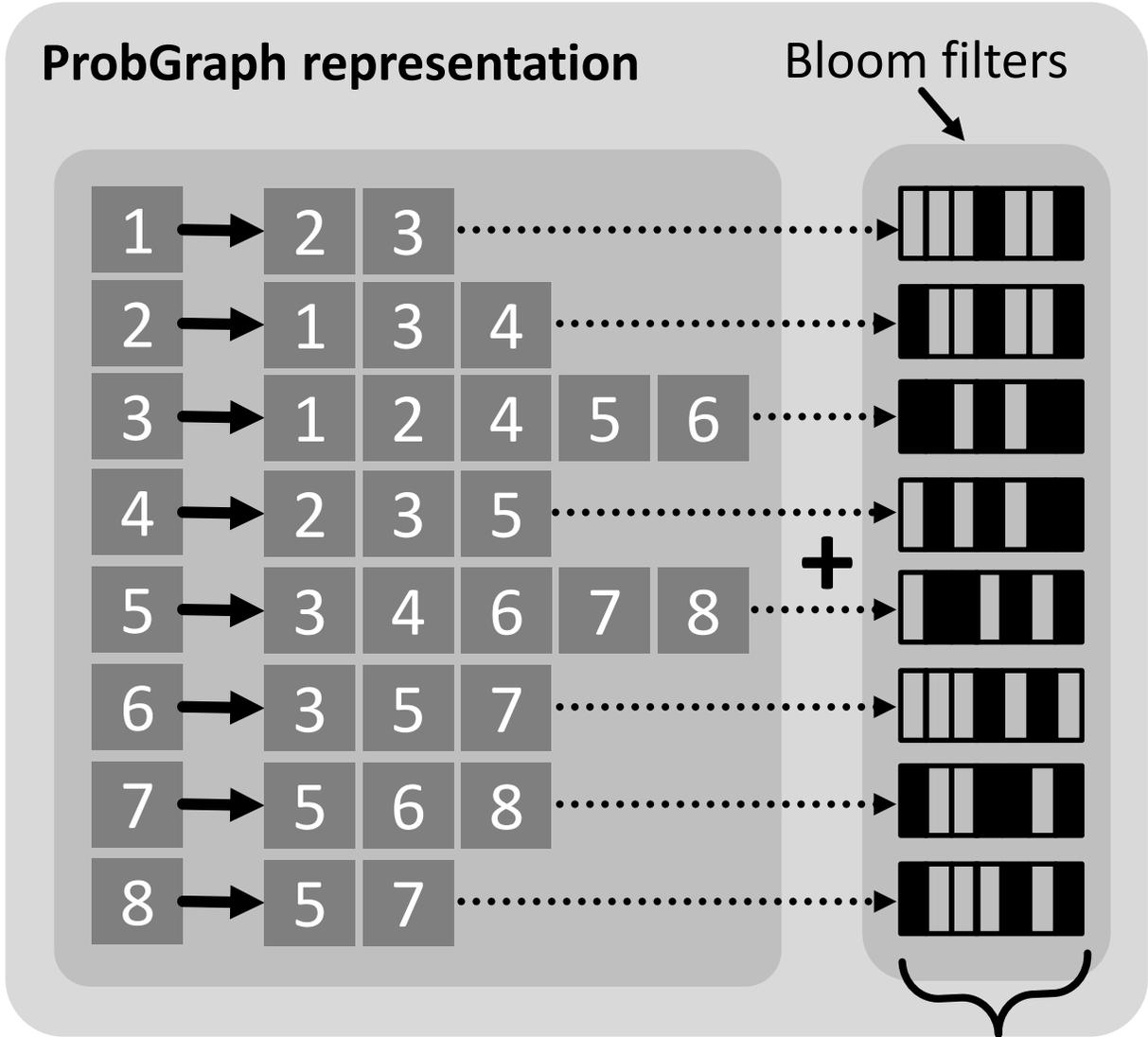
Larger B_x : more accuracy & more storage required. Lower B_x : vice versa. $\longrightarrow B_x$ [bits]

ProbGraph: Summary of Design

Input graph G

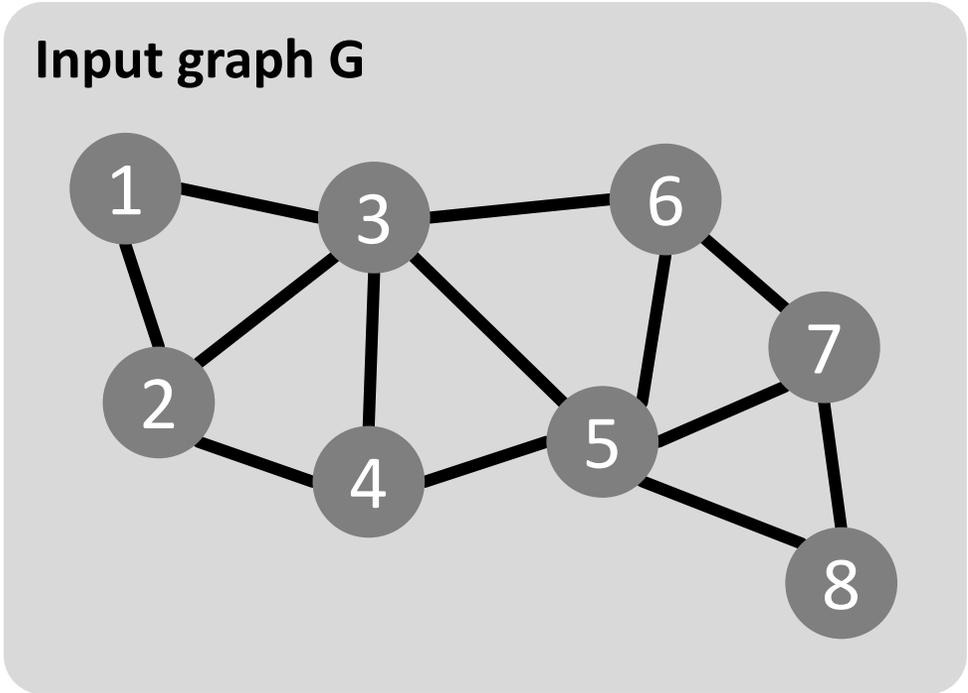


B_x is often small \rightarrow little storage



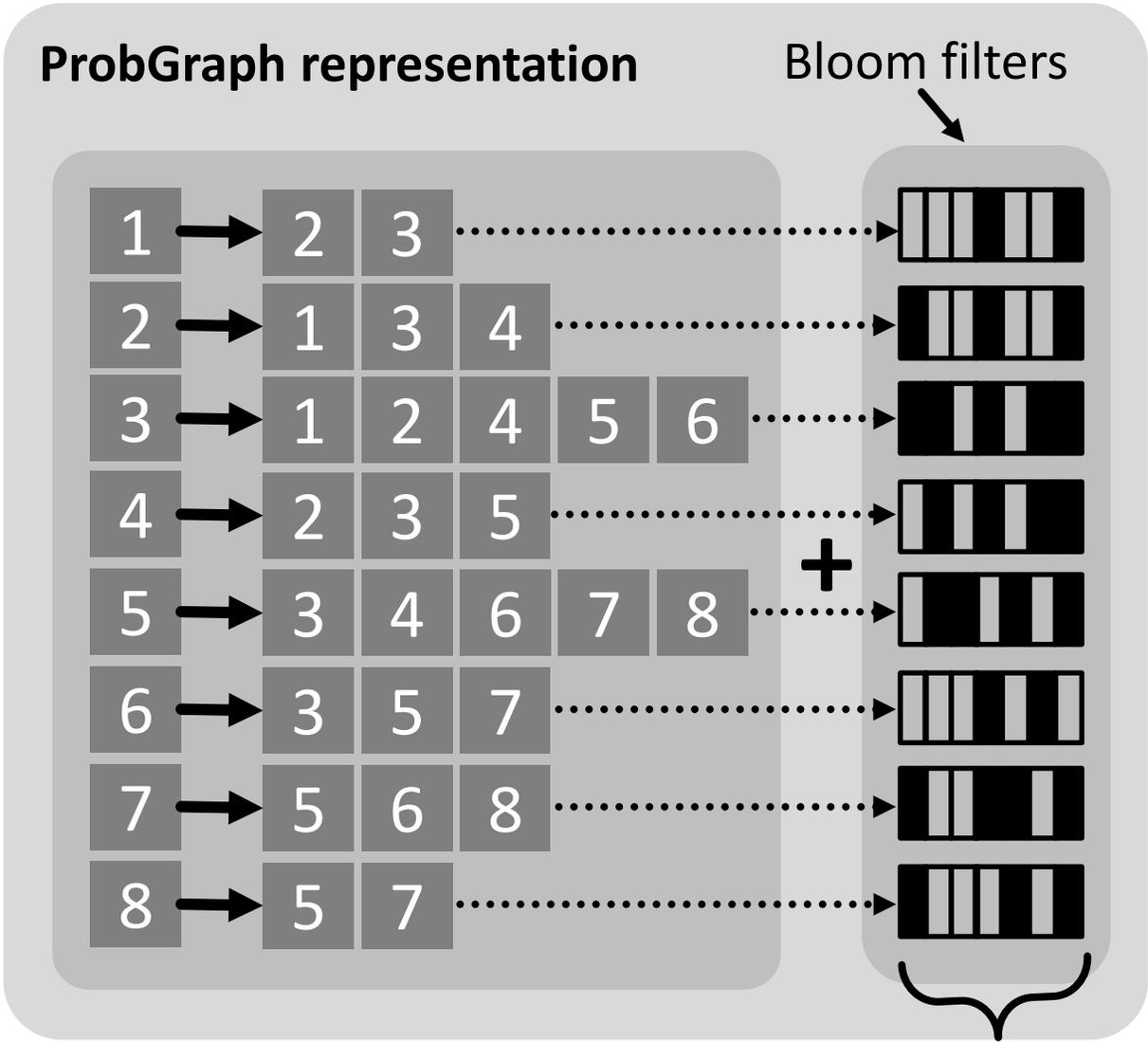
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ProbGraph: Summary of Design



B_x is often small \rightarrow little storage

BFs have the same size \rightarrow great load balancing



Larger B_x : more accuracy & more storage required. Lower B_x : vice versa. $\rightarrow B_x$ [bits]



How does our idea compare to other Bloom filter use cases?



How does our idea compare to other Bloom filter use cases?

Traditional BF use case: presence tracking



How does our idea compare to other Bloom filter use cases?

Traditional BF use case: presence tracking

Data stored somewhere

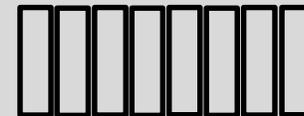




How does our idea compare to other Bloom filter use cases?

Traditional BF use case: presence tracking

A BF cache tracking the presence of data



Data stored somewhere



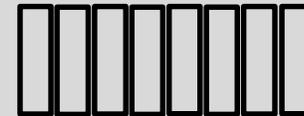


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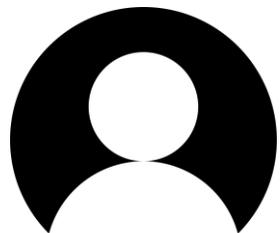
Data stored somewhere





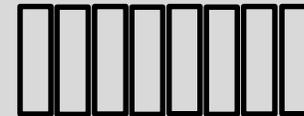
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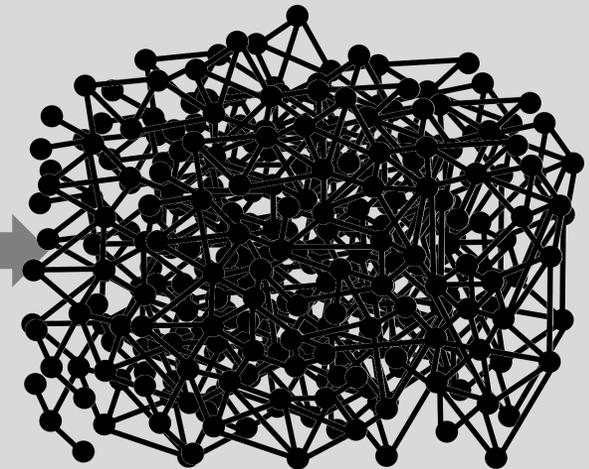


Insert an element

A BF cache tracking the presence of data



Data stored somewhere





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Traditional BF use case: presence tracking



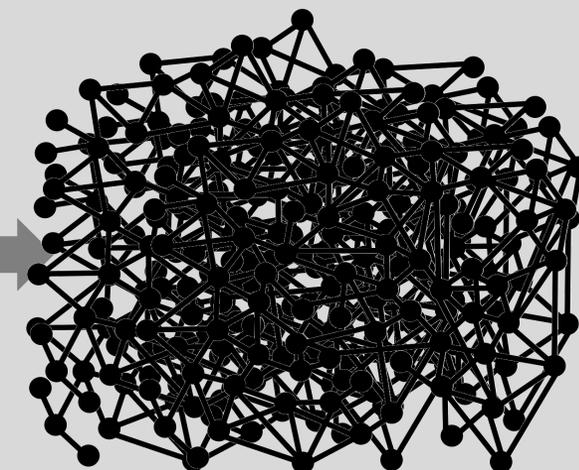
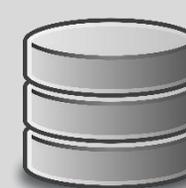
Set the appropriate BF bits

A BF cache tracking the presence of data



Insert an element

Data stored somewhere





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Traditional BF use case: presence tracking



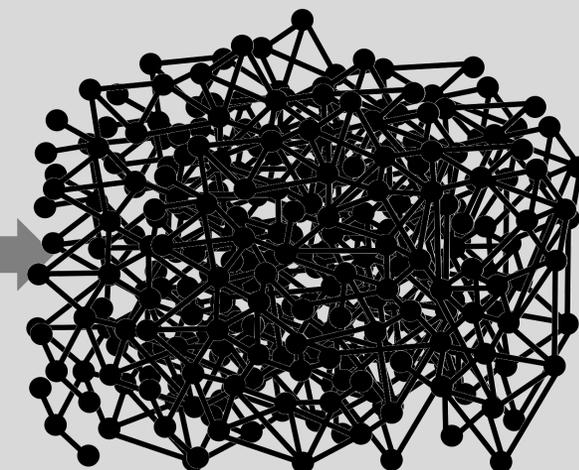
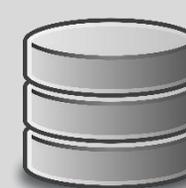
Set the appropriate BF bits

A BF cache tracking the presence of data



Insert an element

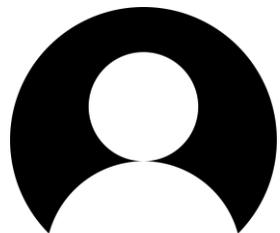
Data stored somewhere





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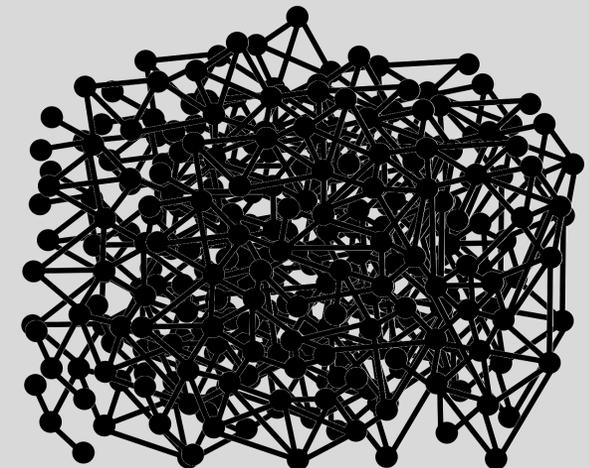
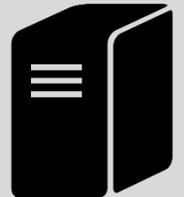
Traditional BF use case: presence tracking



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Data stored somewhere





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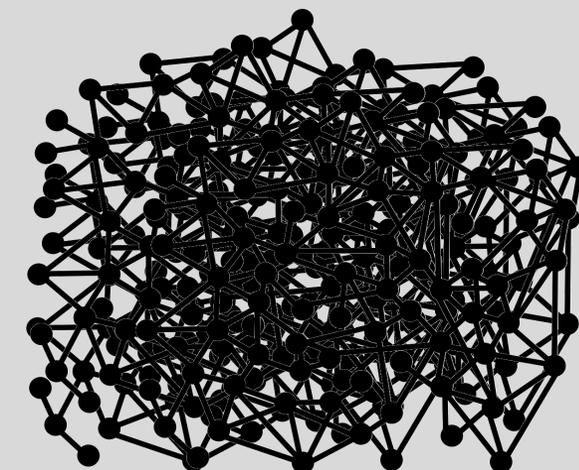
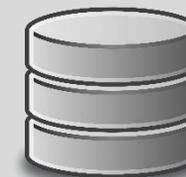
Is the data in question over there?

A BF cache tracking the presence of data



Yes

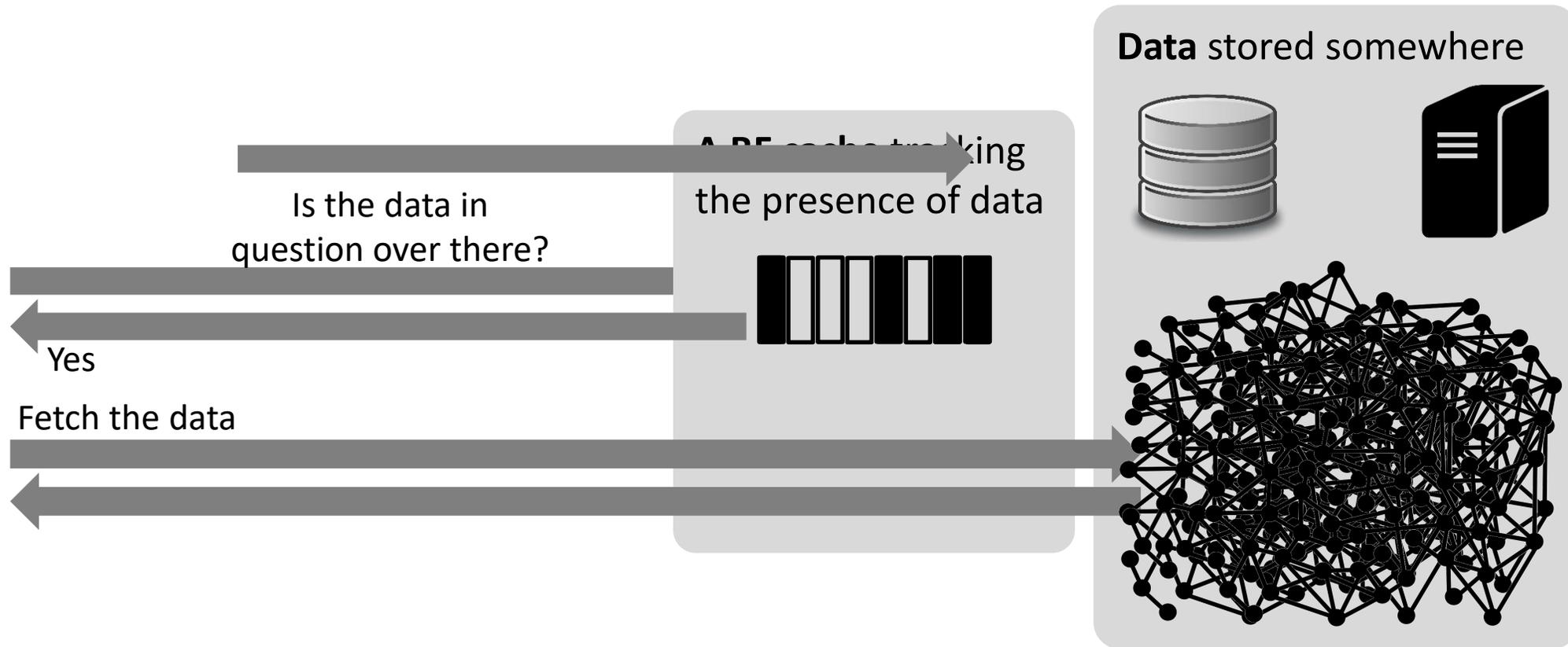
Data stored somewhere





How does our idea compare to other Bloom filter use cases?

Traditional BF use case: presence tracking





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Traditional BF use case: presence tracking



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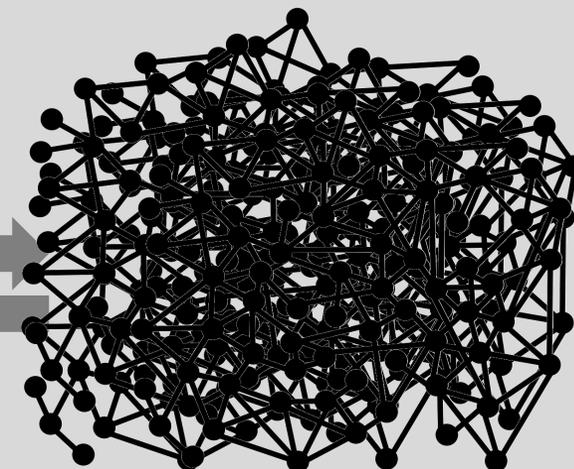
A BF cache tracking the presence of data



Yes

Fetch the data

Data stored somewhere

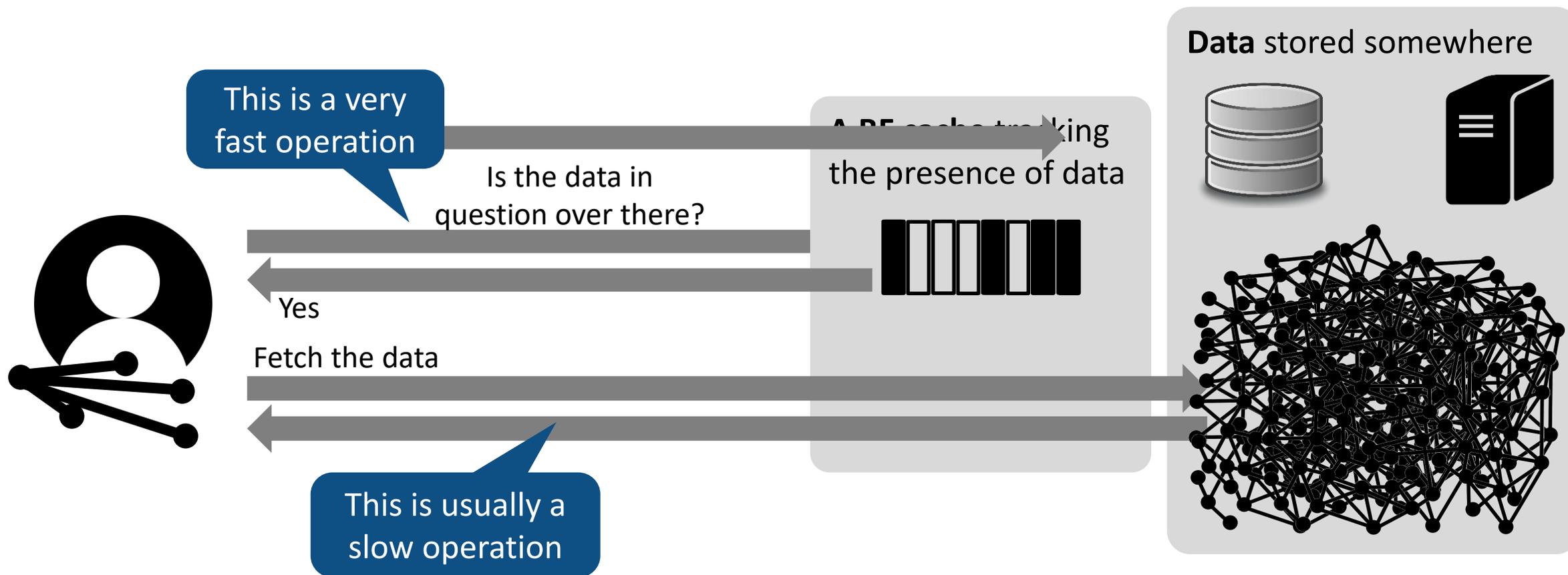


This is usually a slow operation



How does our idea compare to other Bloom filter use cases?

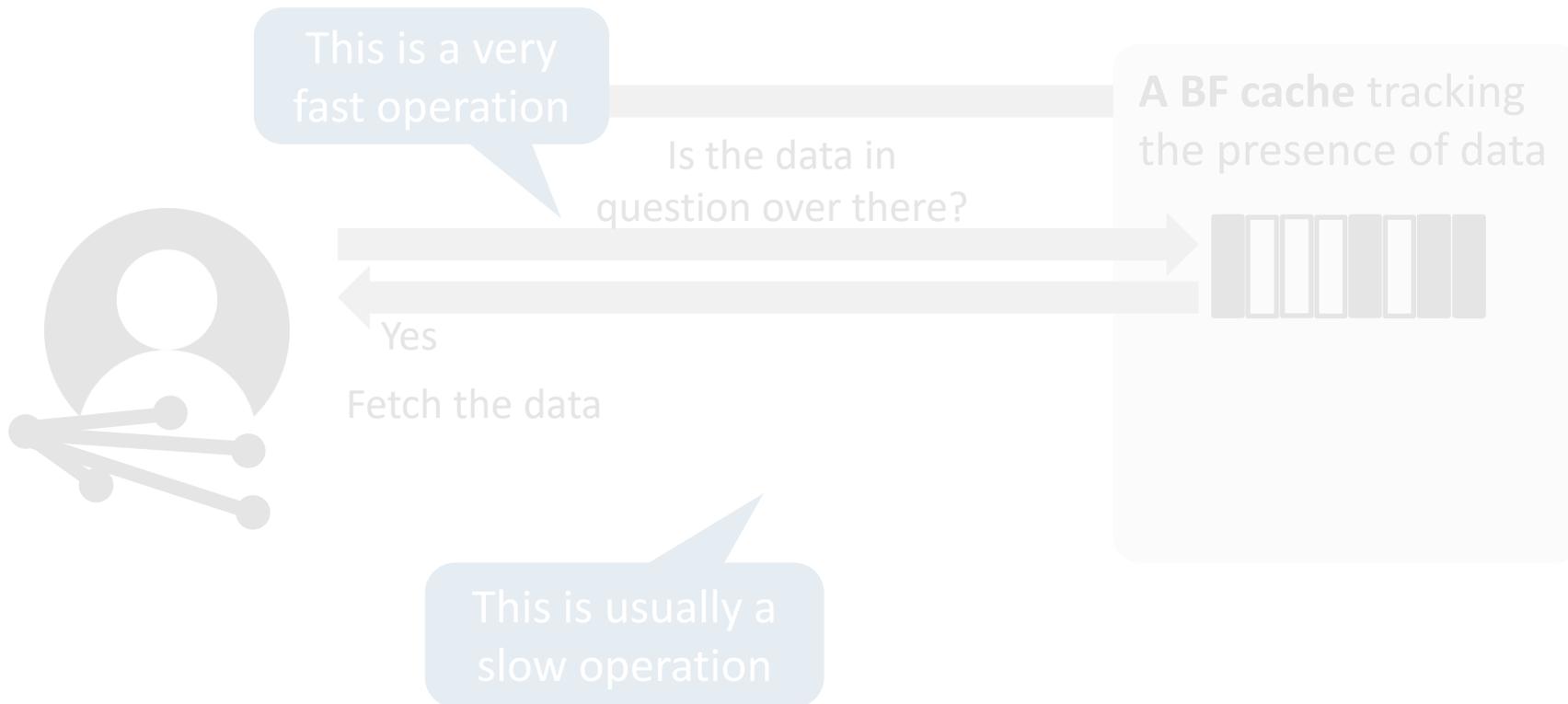
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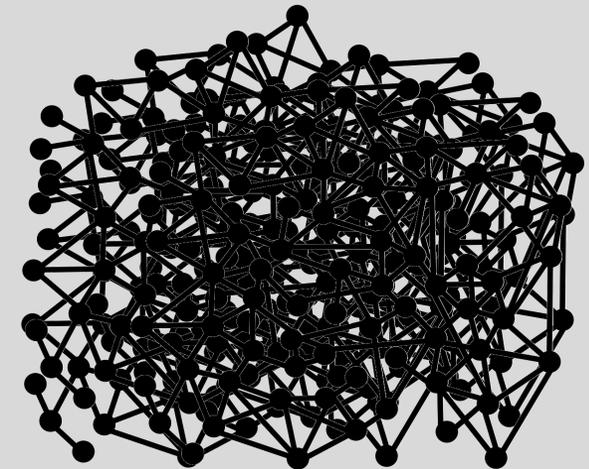


The novelty of ProbGraph

Other Bloom filter use cases?



Data stored somewhere





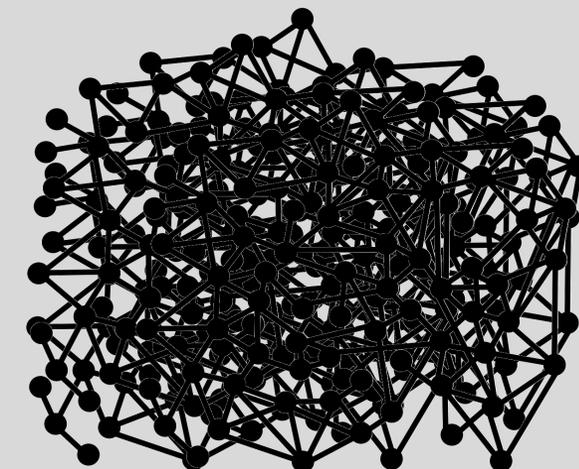
The novelty of ProbGraph

Other Bloom filter use cases?

We use BFs as a sketch of the actual dataset

Sketching the presence of data

Data stored somewhere



Yes
Fetch the data

This is usually a slow operation



The novelty of ProbGraph

Other Bloom filter use cases?

We use BFs as a sketch of the actual dataset

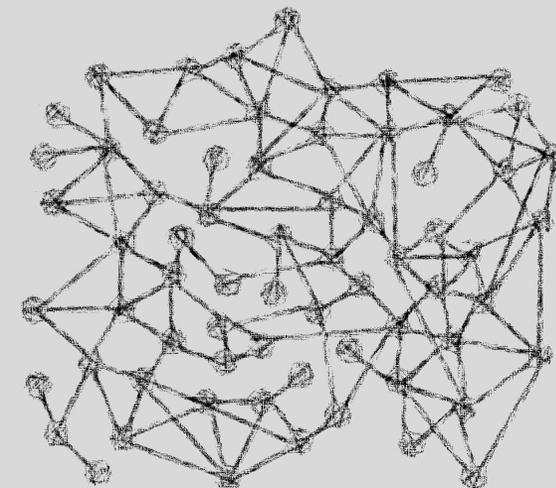
Bloom filter cache tracking
the presence of data



Fetch the data

This is usually a slow operation

Data stored somewhere





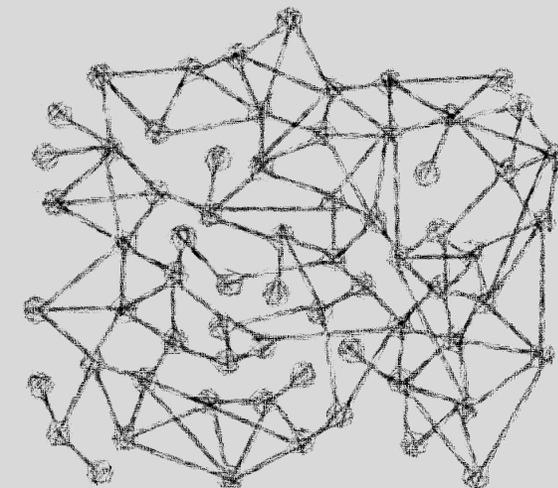
The novelty of ProbGraph

Other Bloom filter use cases?

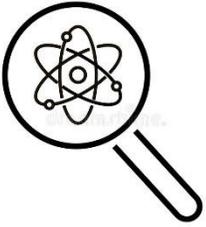
We use BFs as a sketch of the actual dataset

How do we exactly use these sketches to benefit graph mining?

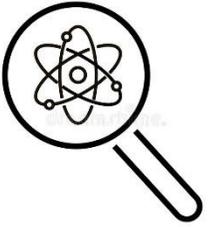
Data stored somewhere



Observation: Set Intersection Cardinality is Prevalent in Graph Mining

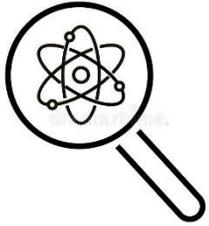


Observation: Set Intersection Cardinality is Prevalent in Graph Mining



$$|X \cap Y|$$

Observation: Set Intersection Cardinality is Prevalent in Graph Mining



spcl.inf.ethz.ch
@spcl_eth

Graph Mining

A huge & complex graph dataset

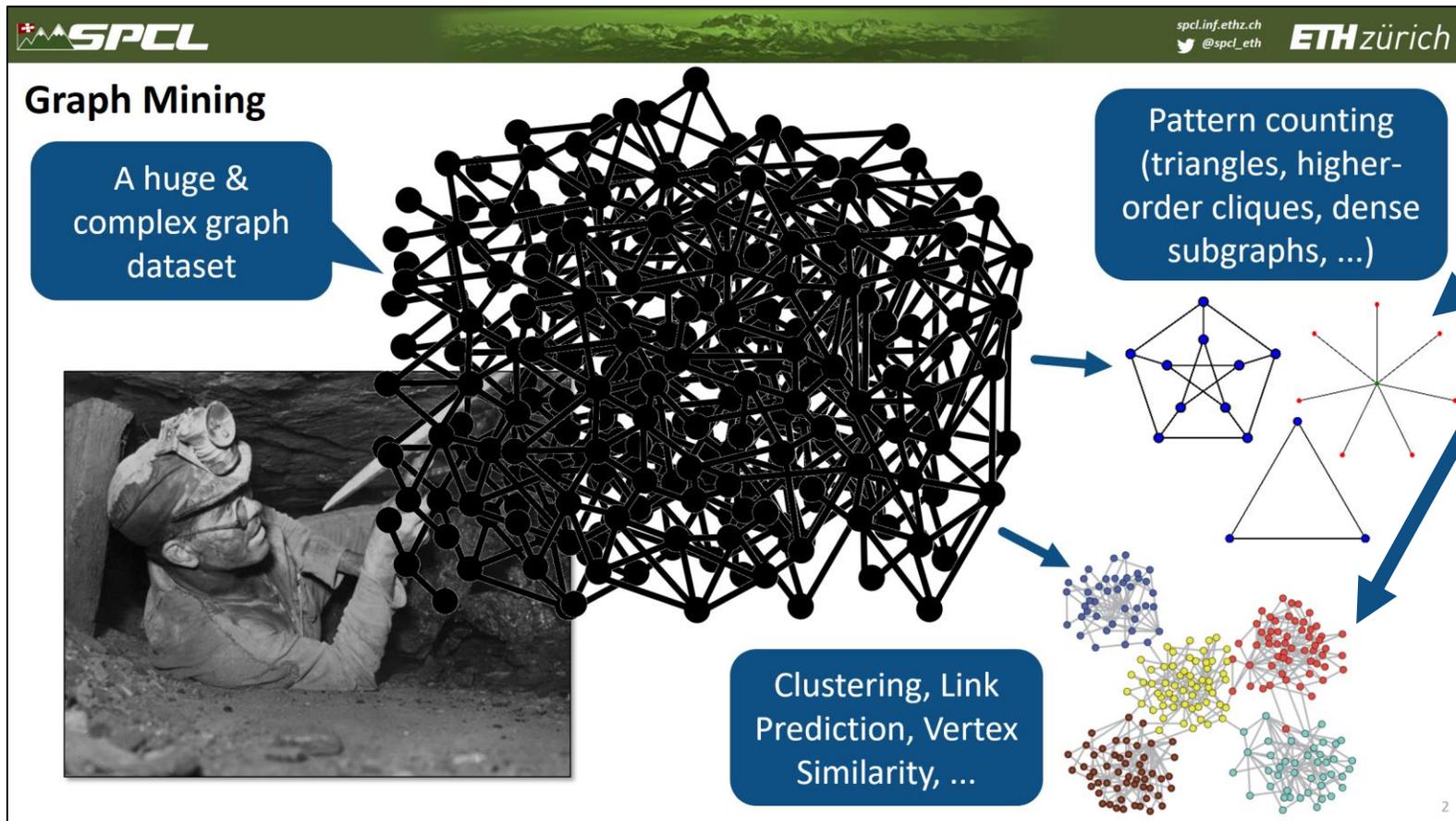
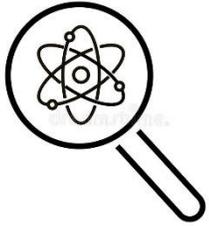
Pattern counting
(triangles, higher-order cliques, dense subgraphs, ...)

Clustering, Link Prediction, Vertex Similarity, ...

2

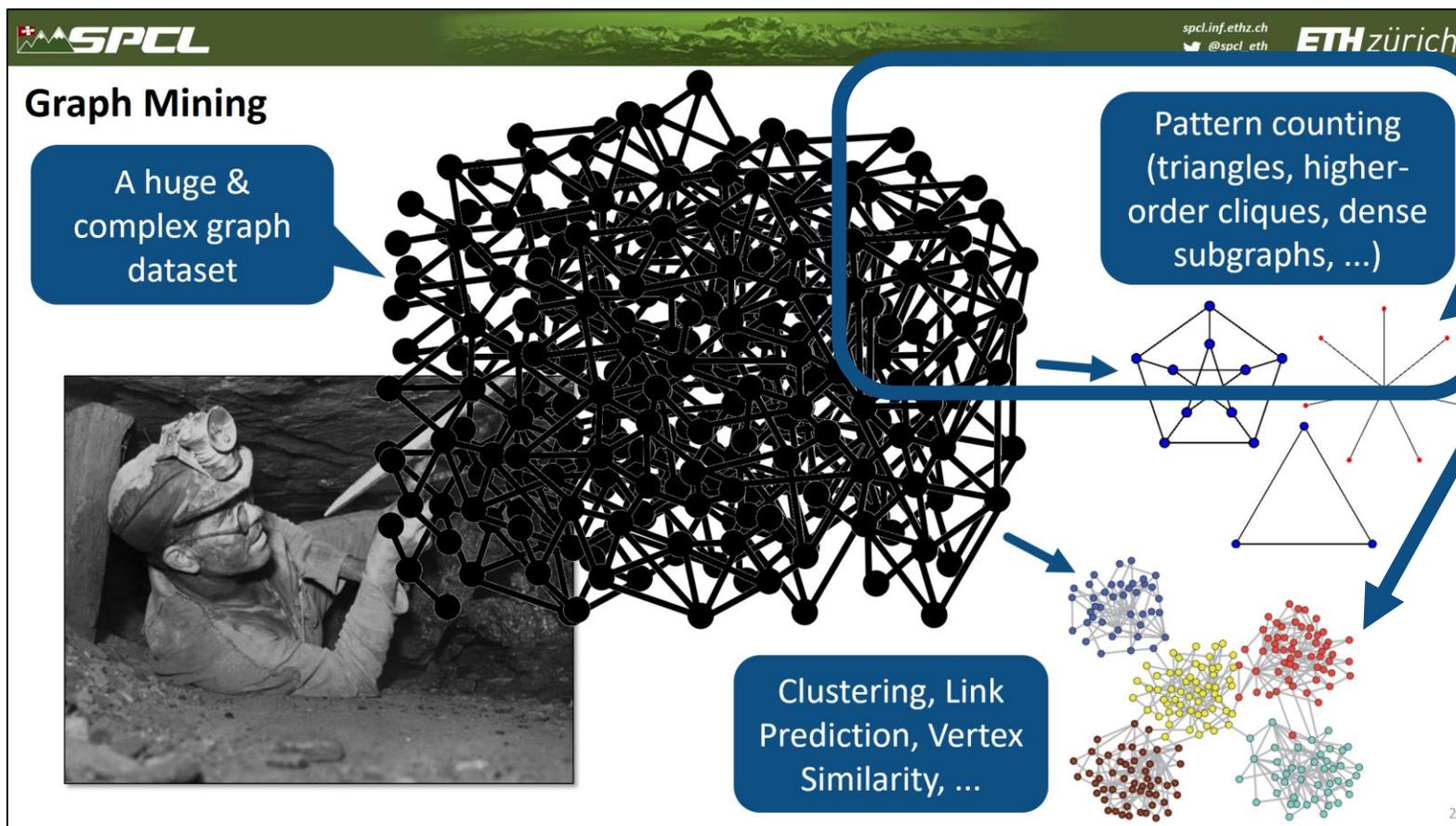
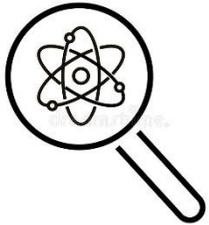
$$|X \cap Y|$$

Observation: Set Intersection Cardinality is Prevalent in Graph Mining



$$|X \cap Y|$$

Observation: Set Intersection Cardinality is Prevalent in Graph Mining



$$|X \cap Y|$$

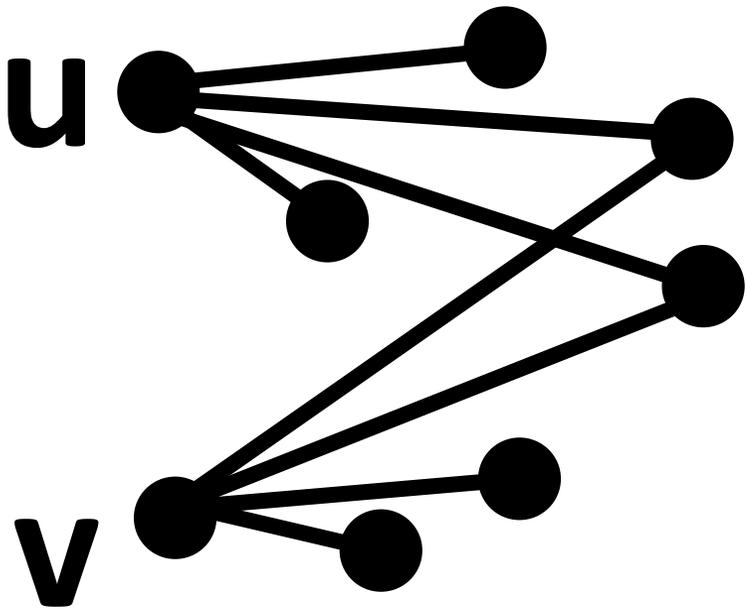
We greatly accelerate $|X \cap Y|$ with BFs



ProbGraph key idea, continued

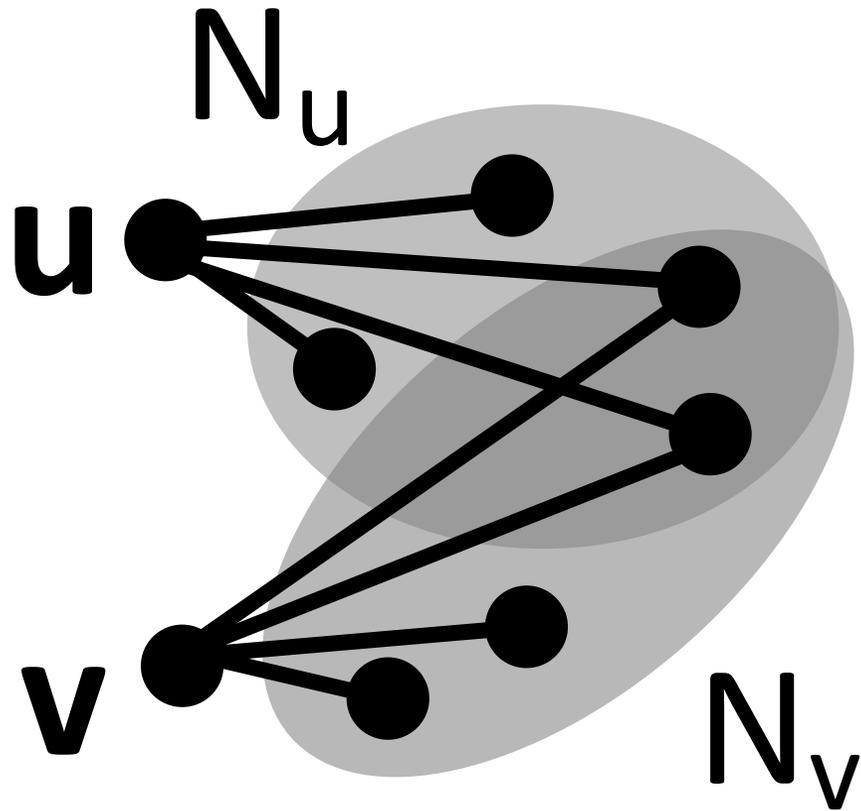


ProbGraph key idea, continued



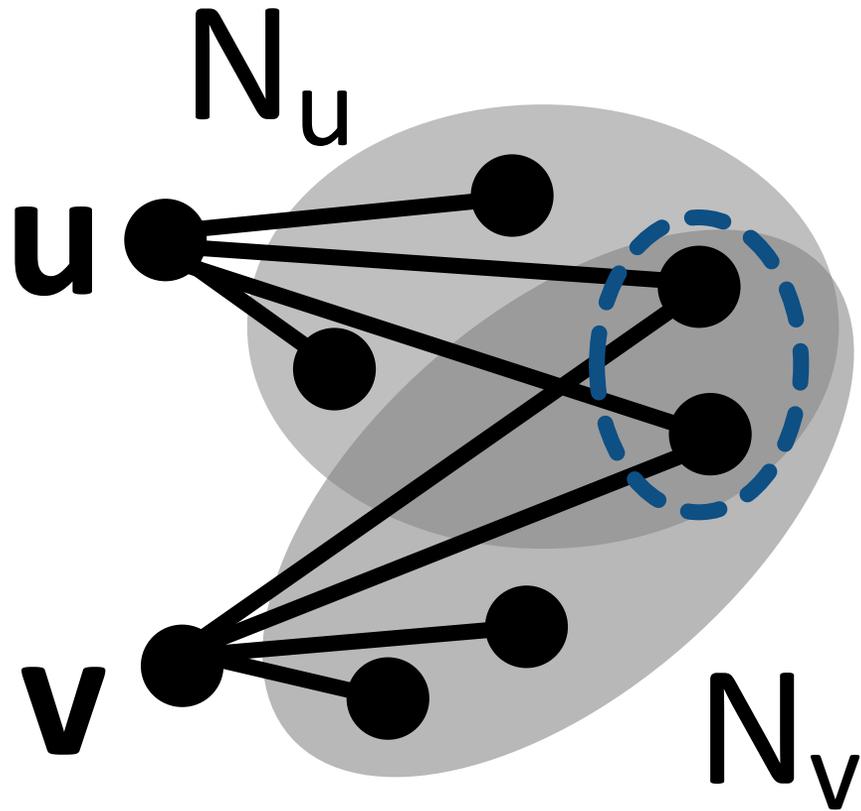


ProbGraph key idea, continued



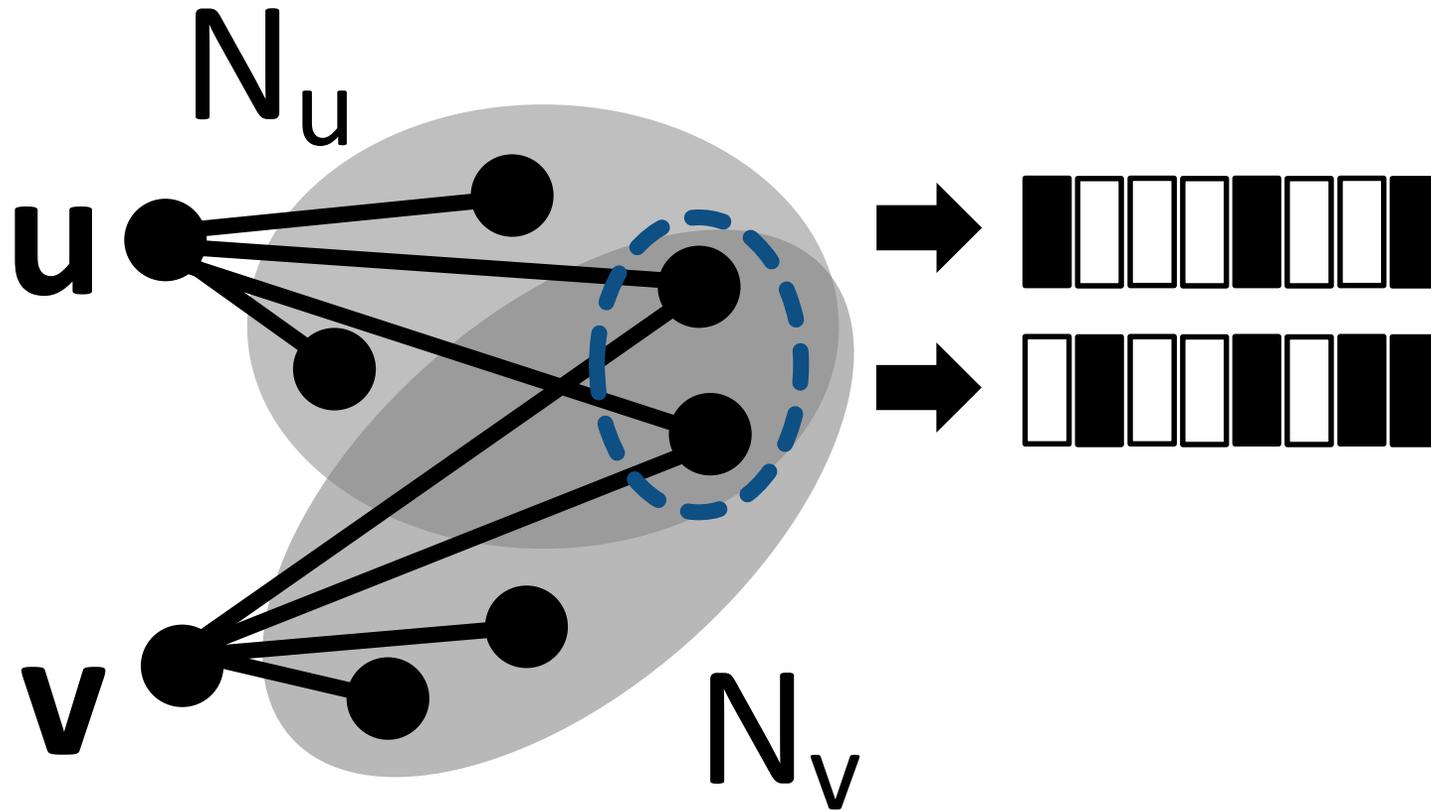


ProbGraph key idea, continued



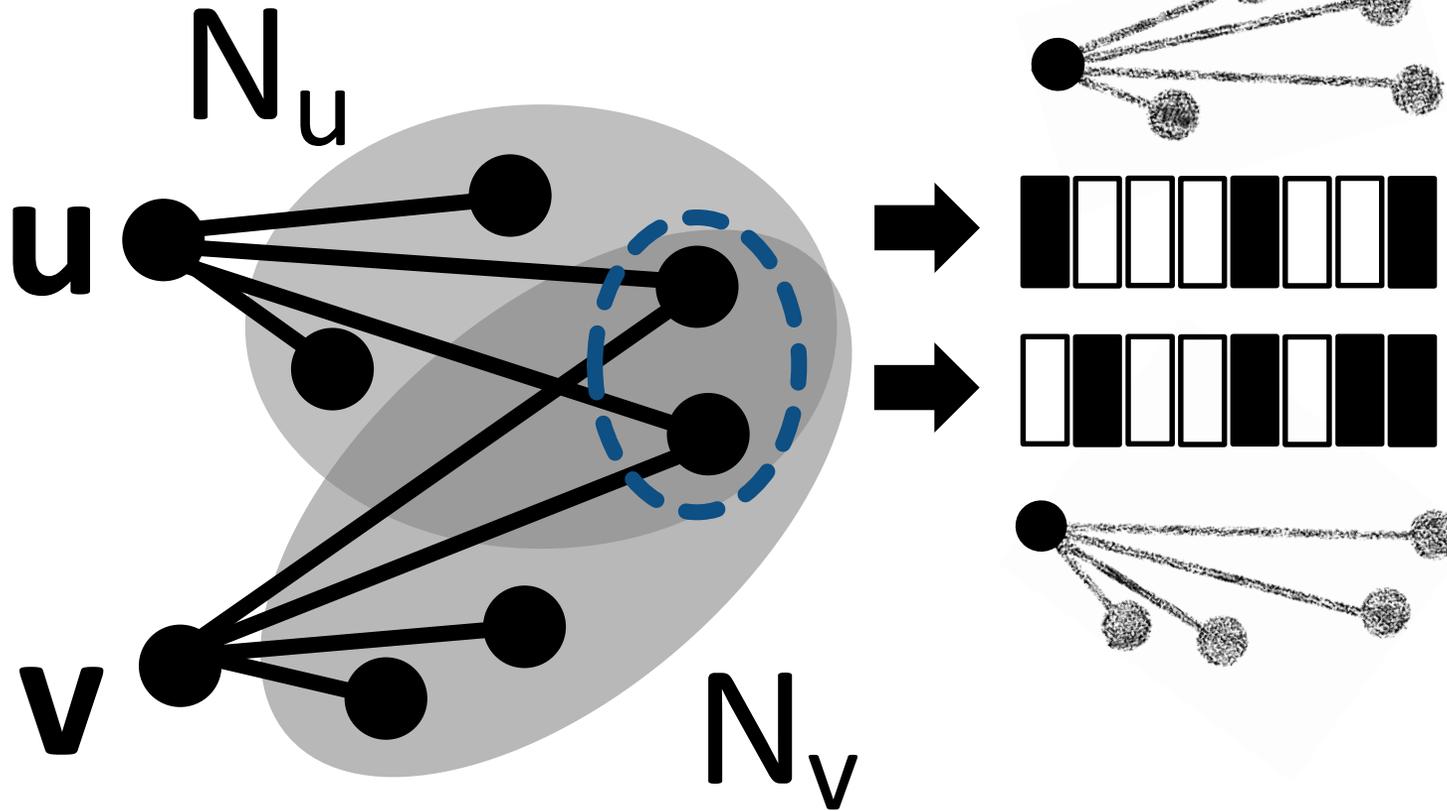


ProbGraph key idea, continued



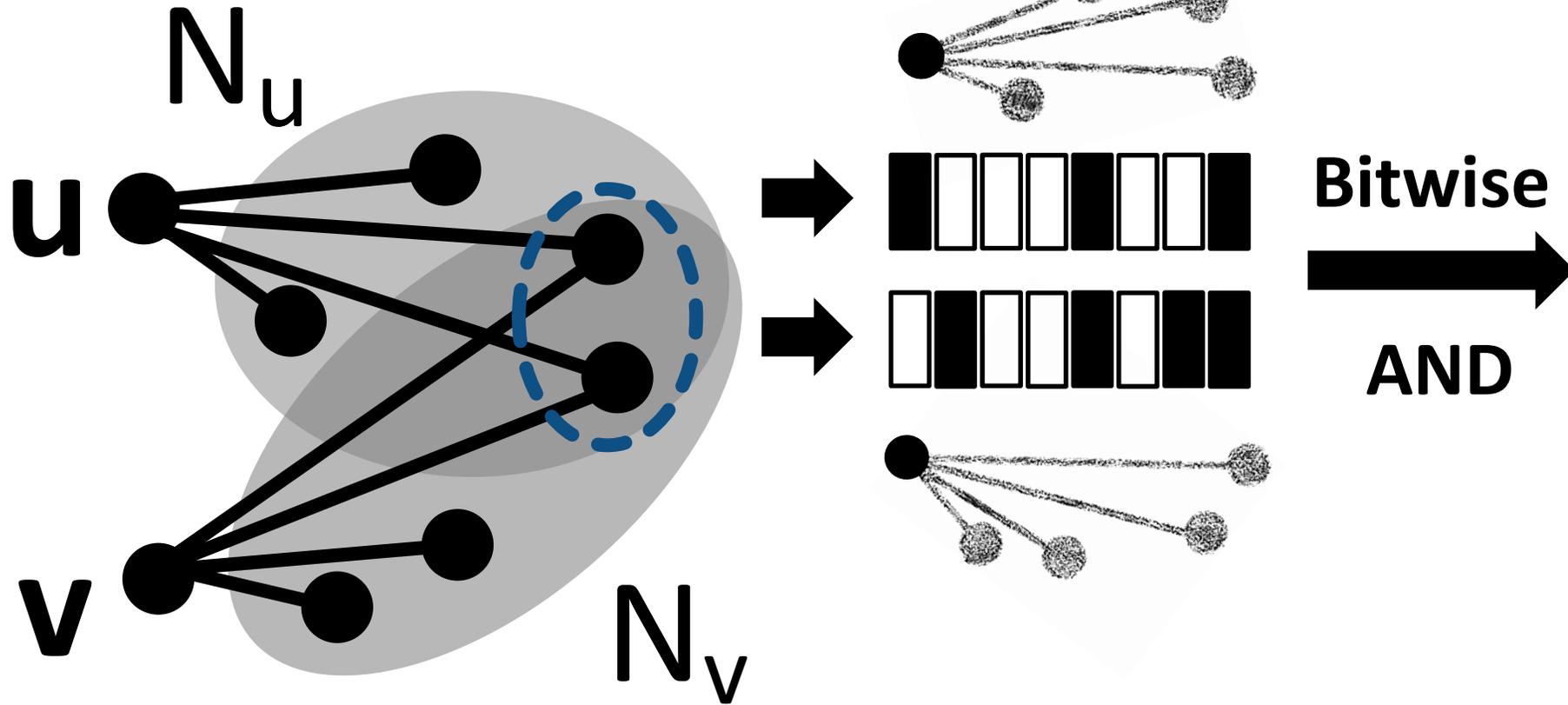


ProbGraph key idea, continued



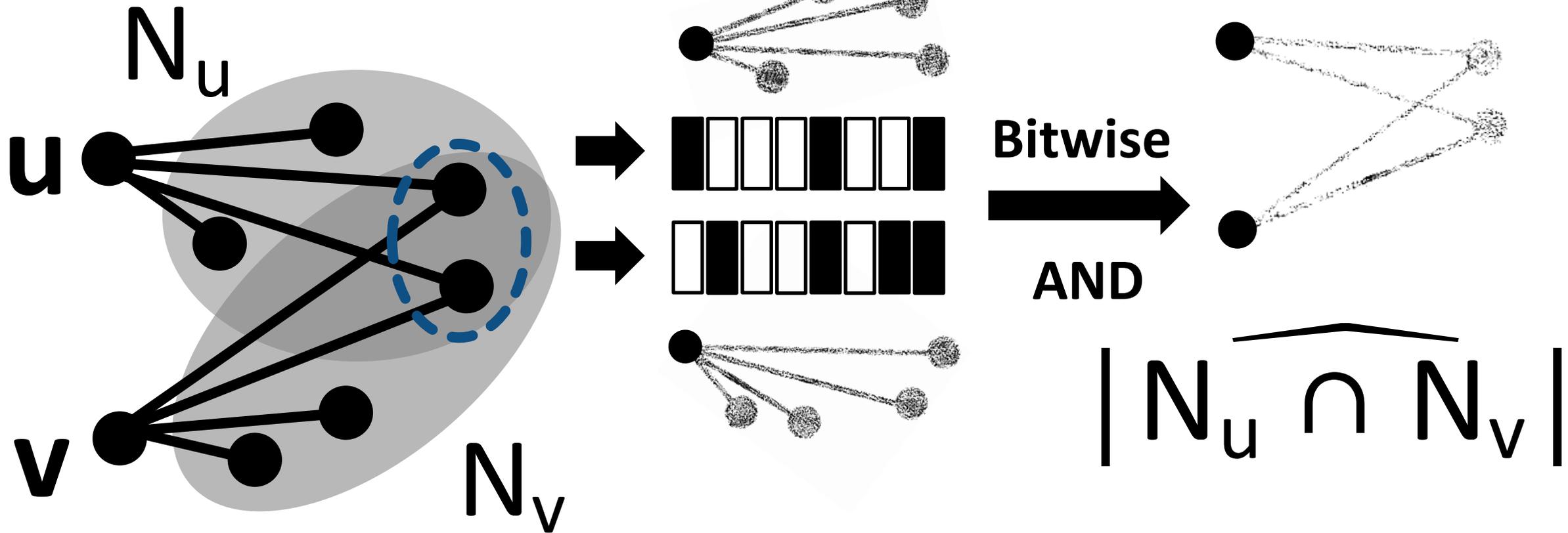


ProbGraph key idea, continued

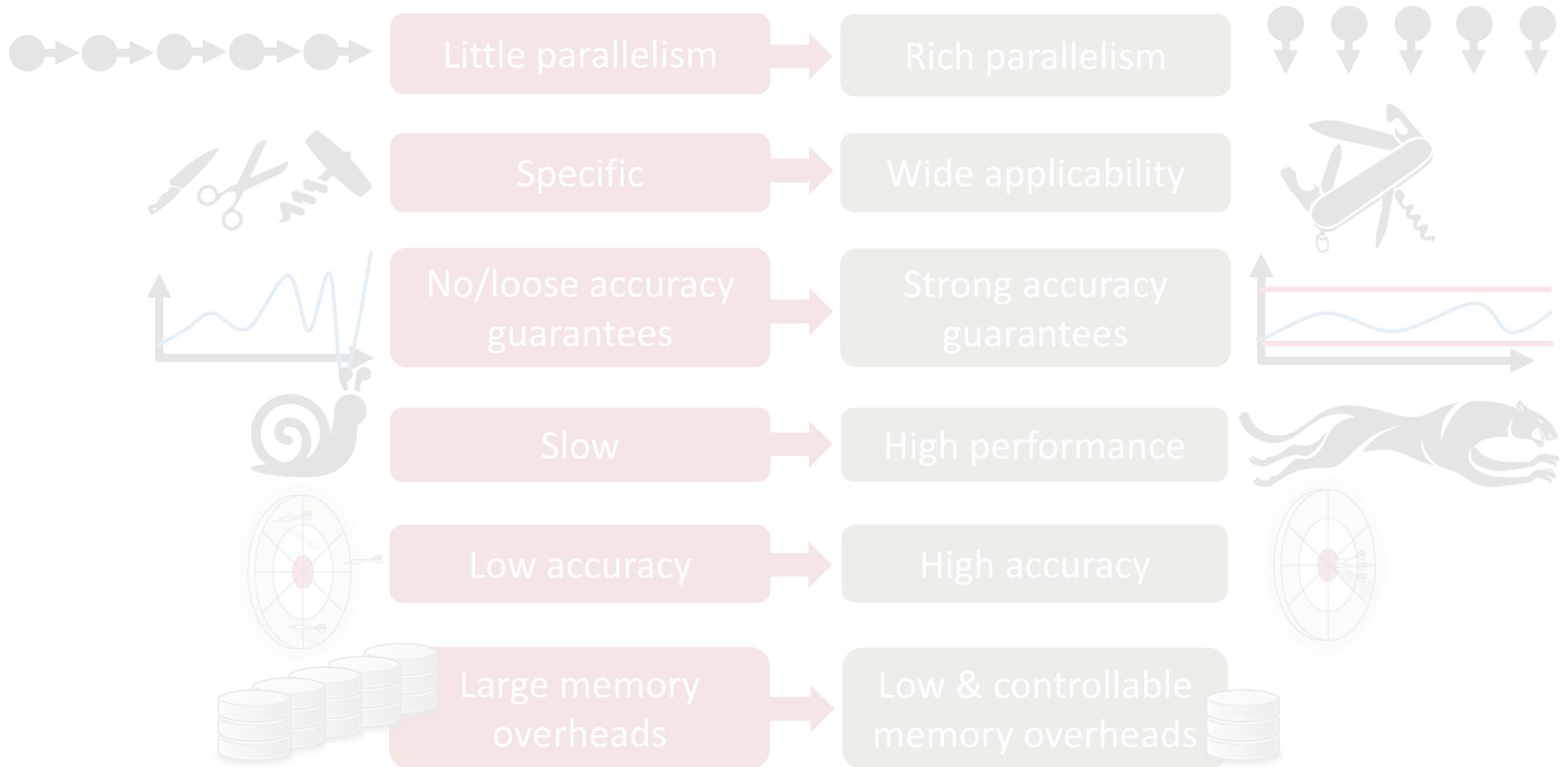




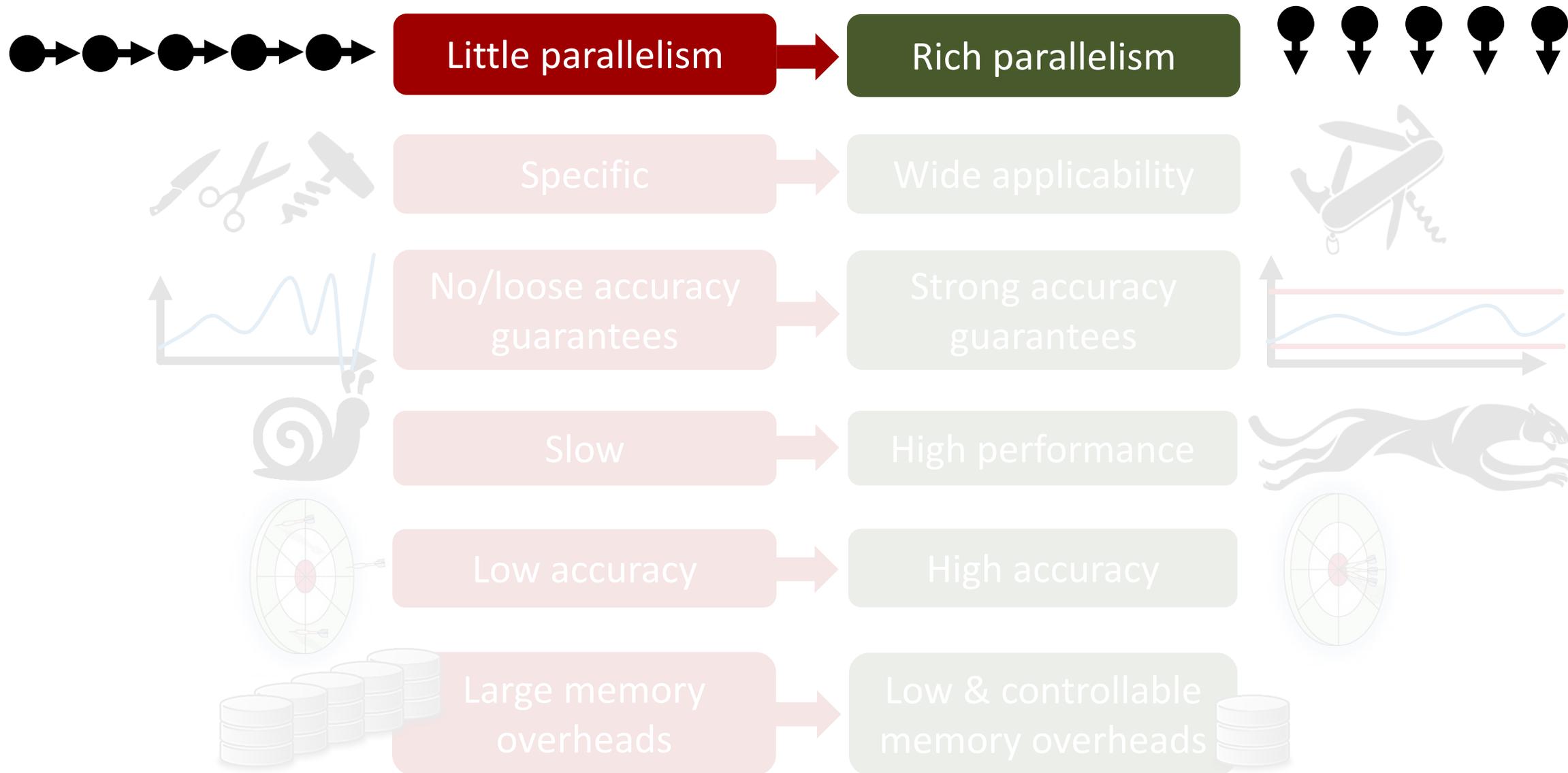
ProbGraph key idea, continued



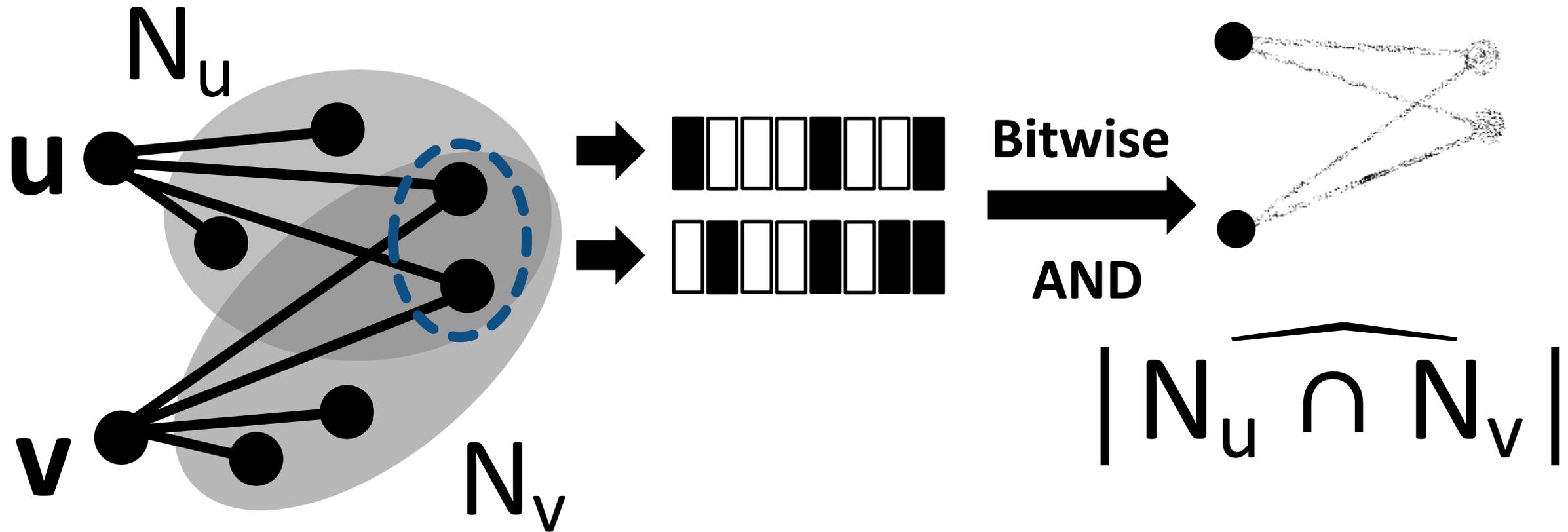
Approximate Graph Processing: Our Objectives



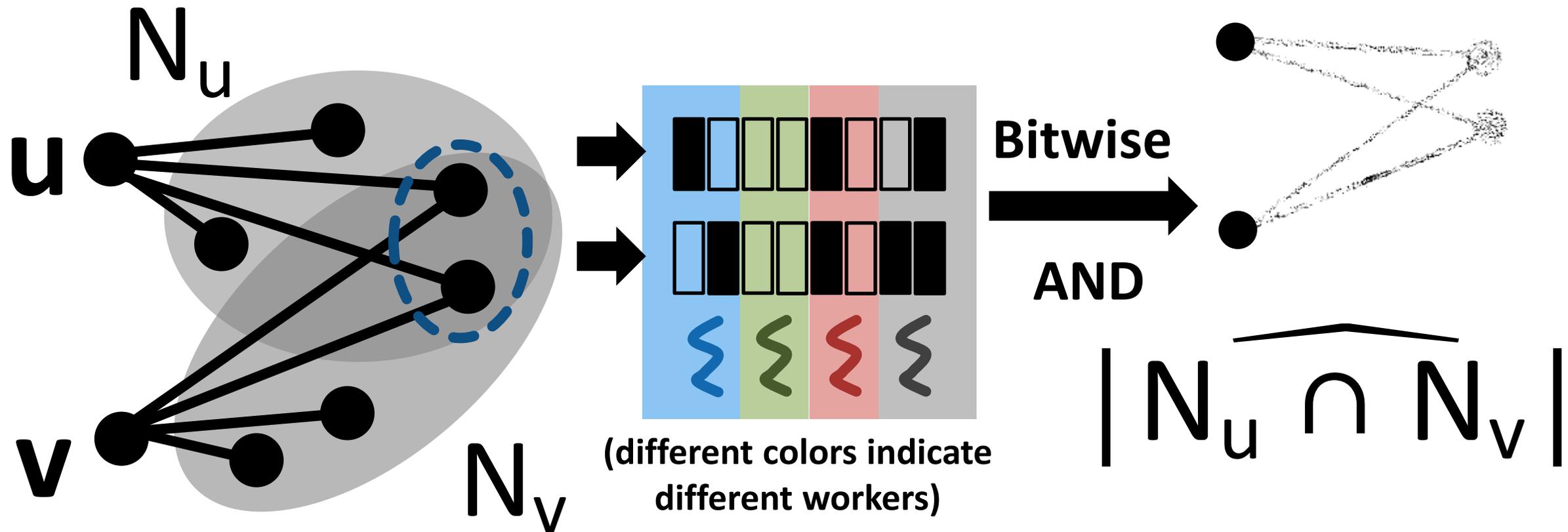
Approximate Graph Processing: Our Objectives



ProbGraph: Fast & Parallel Execution

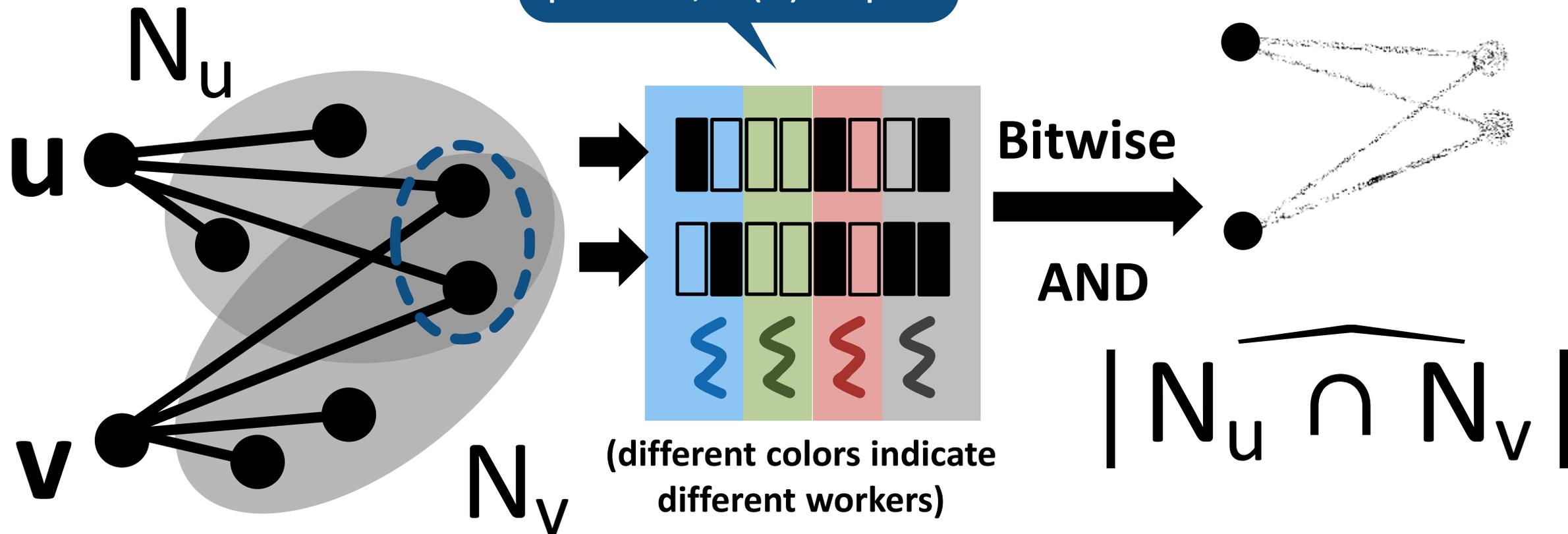


ProbGraph: Fast & Parallel Execution

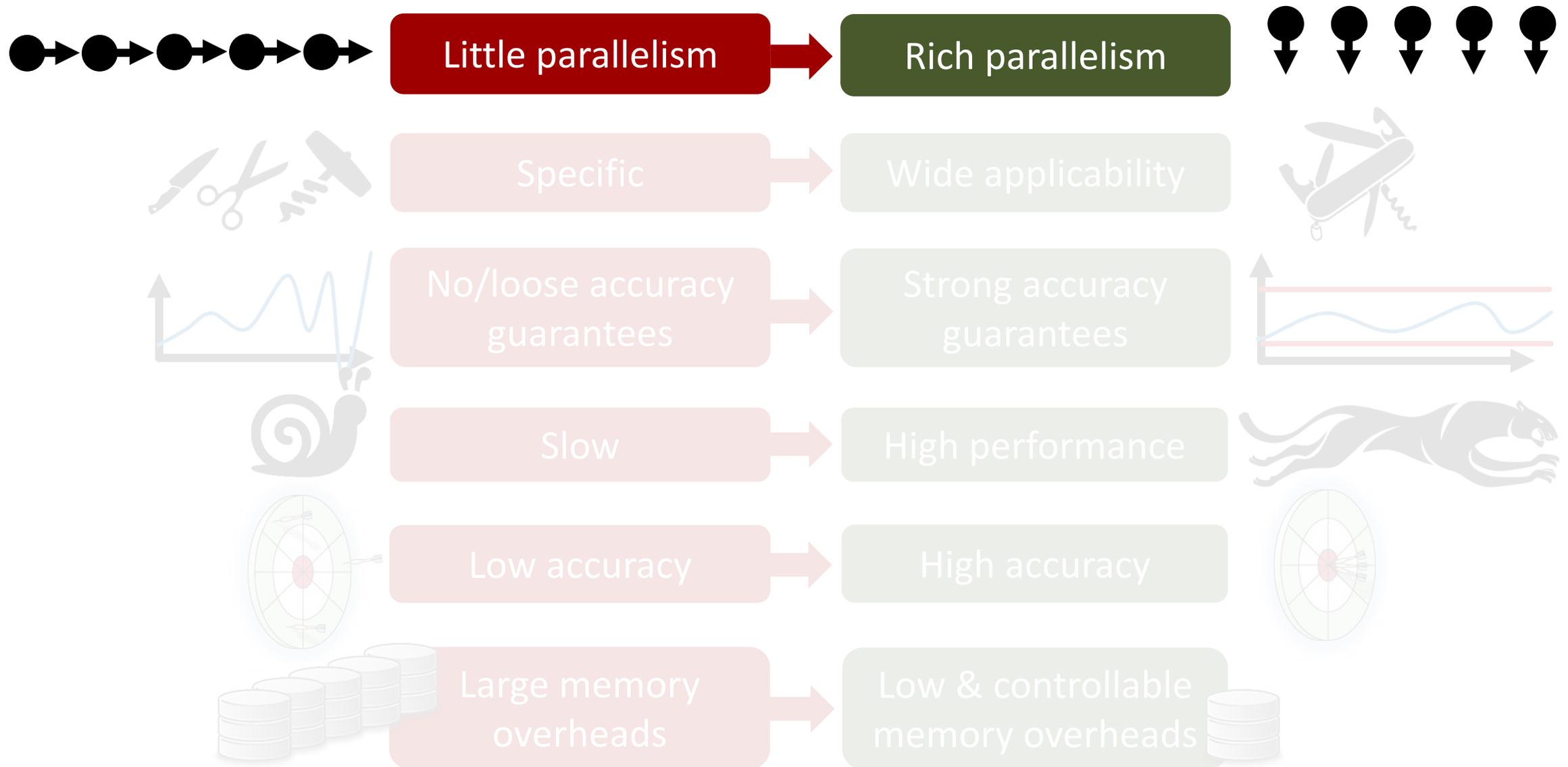


ProbGraph: Fast & Parallel Execution

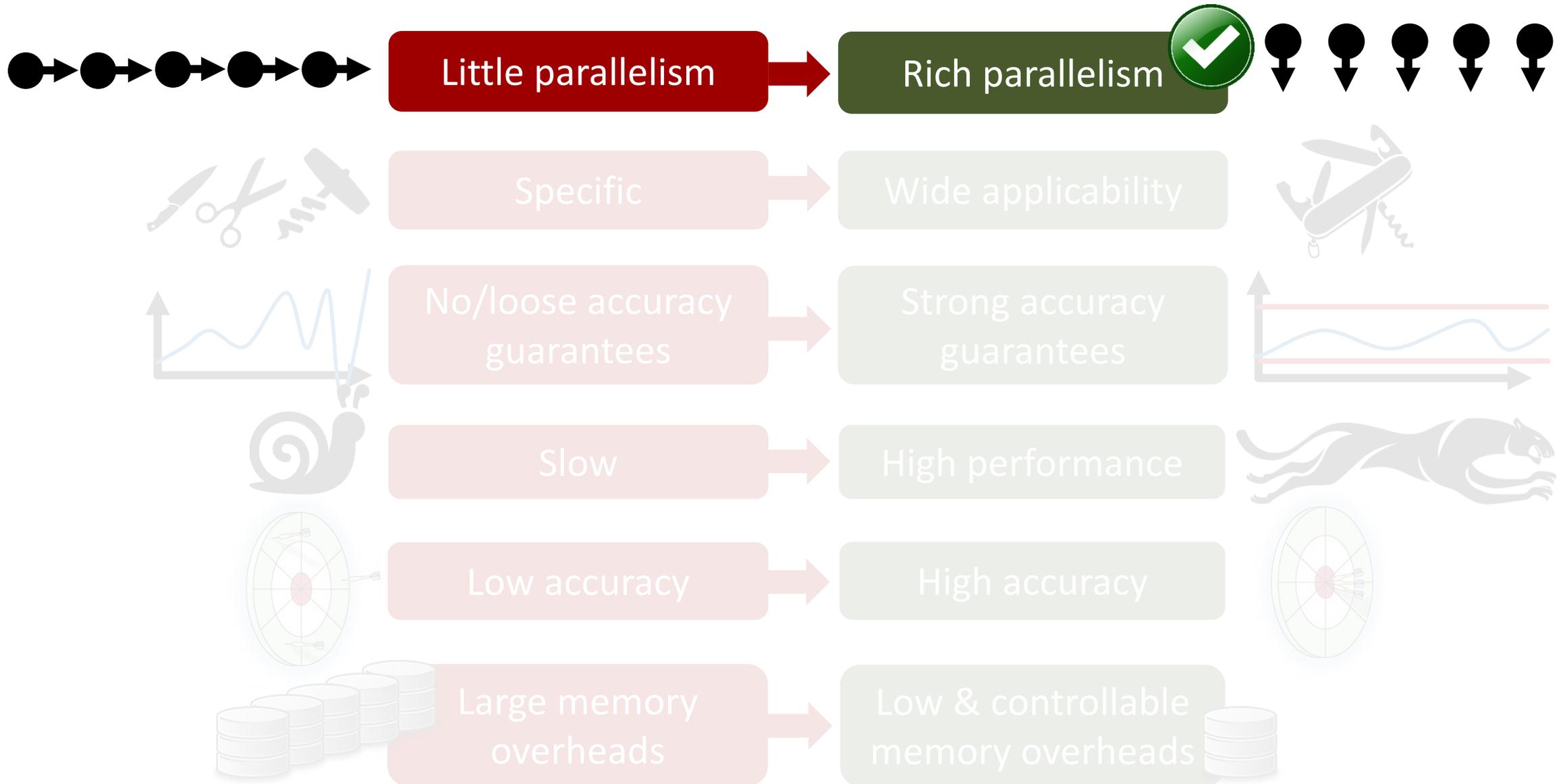
Embarrassingly parallel, $O(1)$ depth



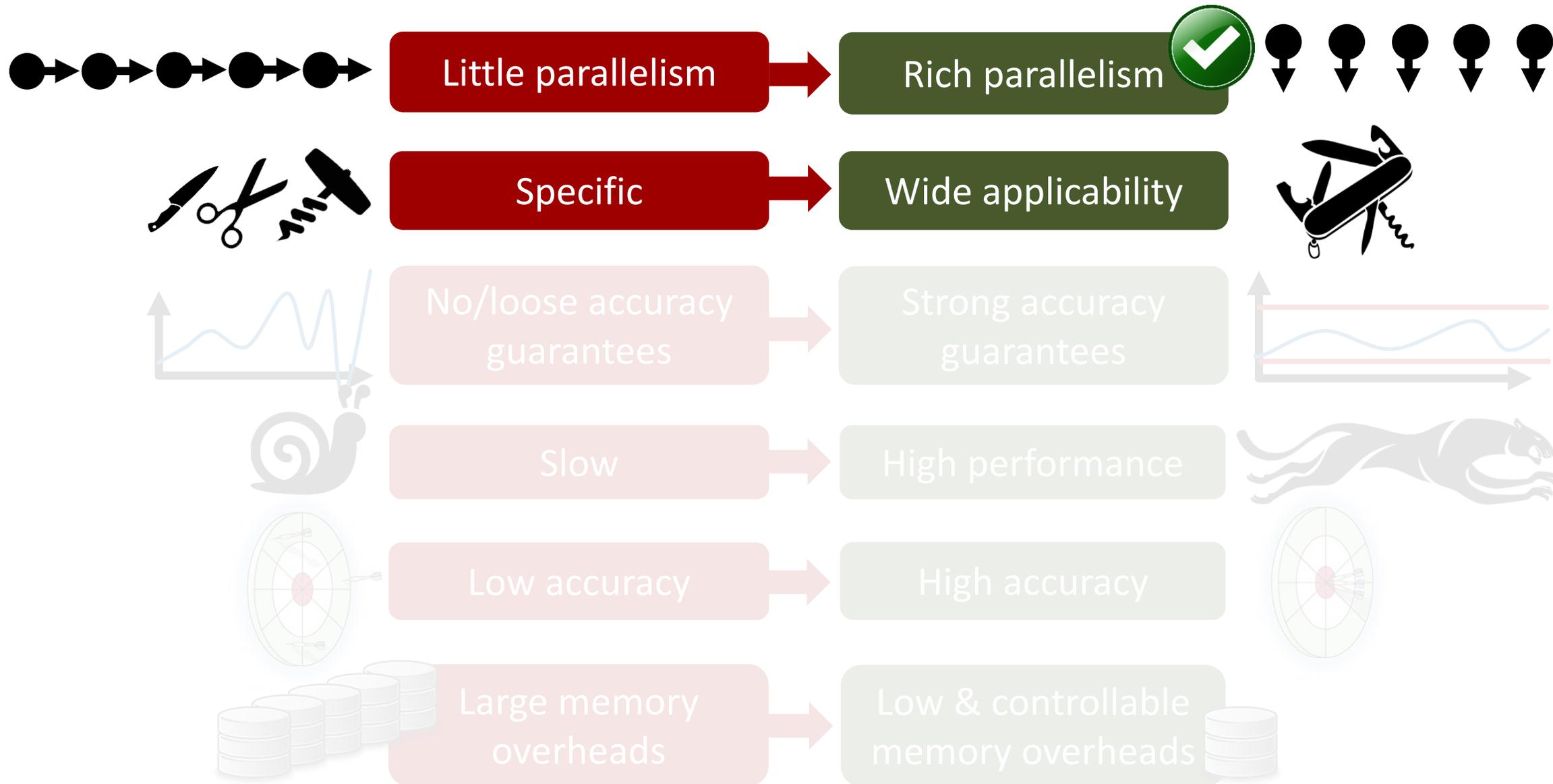
Approximate Graph Processing: Our Objectives



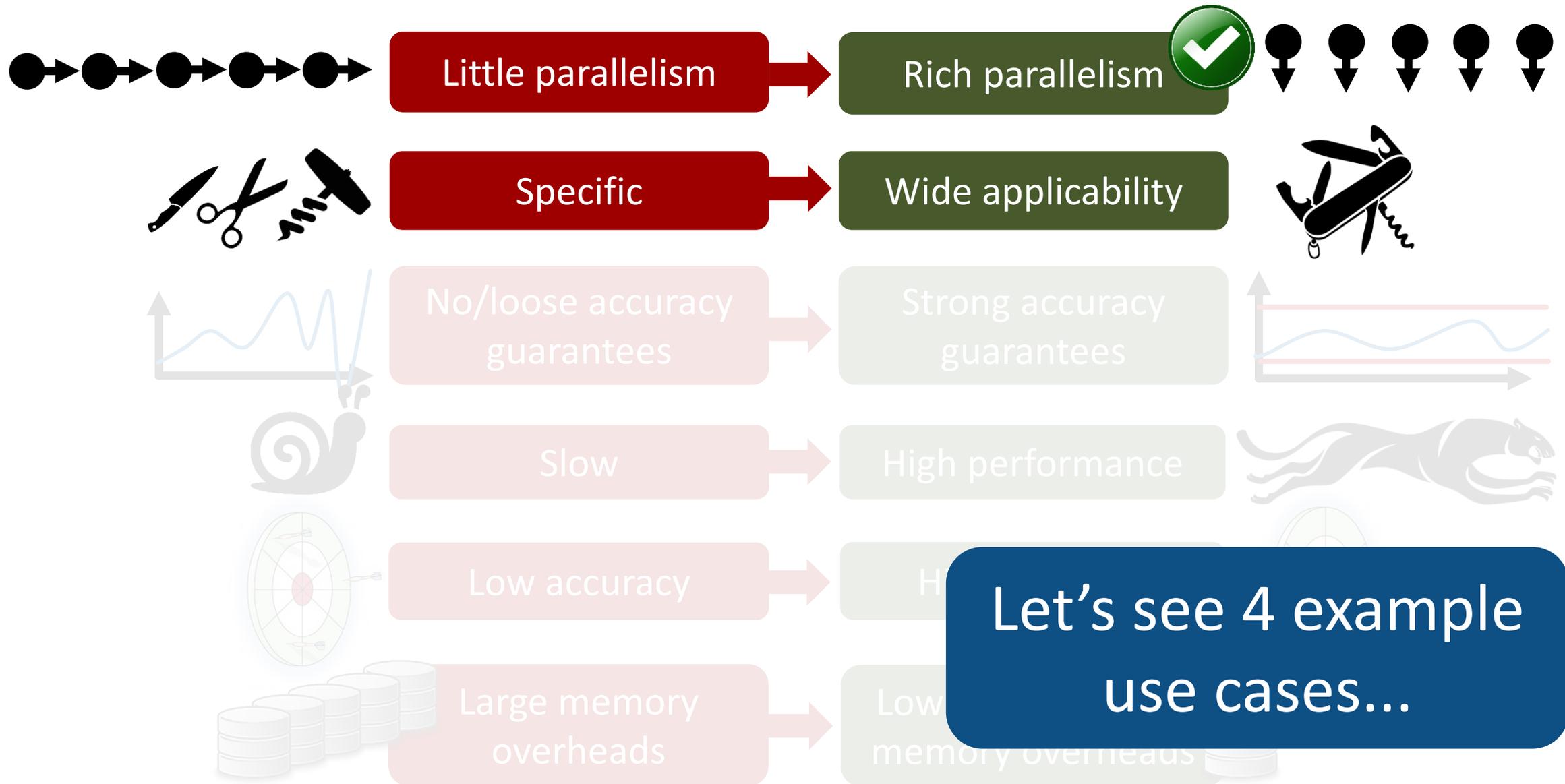
Approximate Graph Processing: Our Objectives



Approximate Graph Processing: Our Objectives



Approximate Graph Processing: Our Objectives

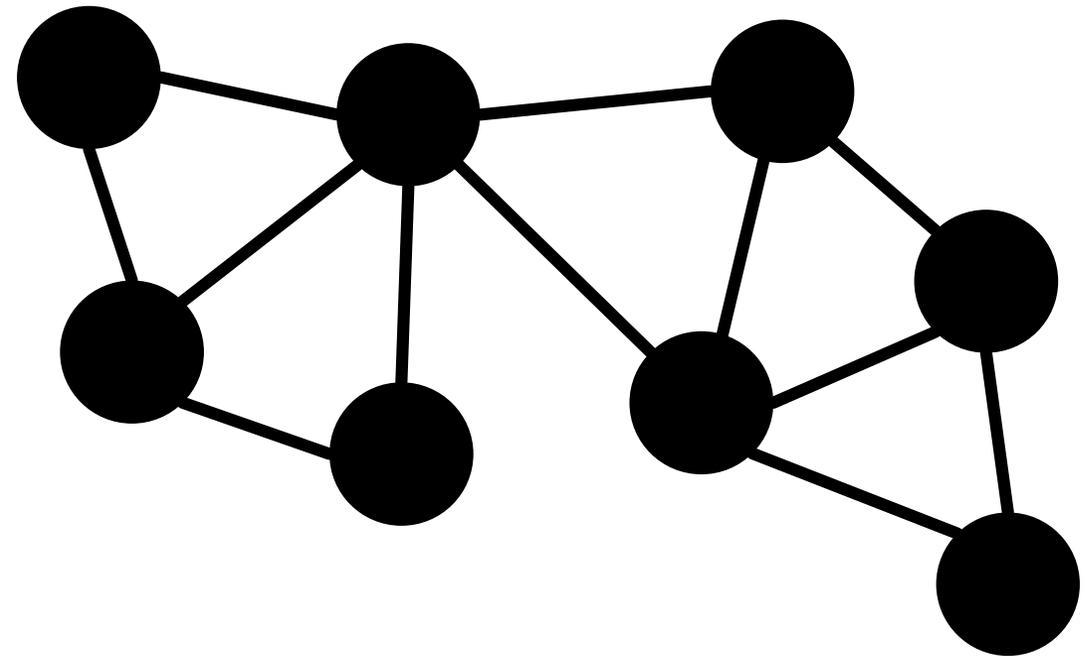


Let's see 4 example use cases...

Use Case 1: Link Prediction

Which links
will appear?

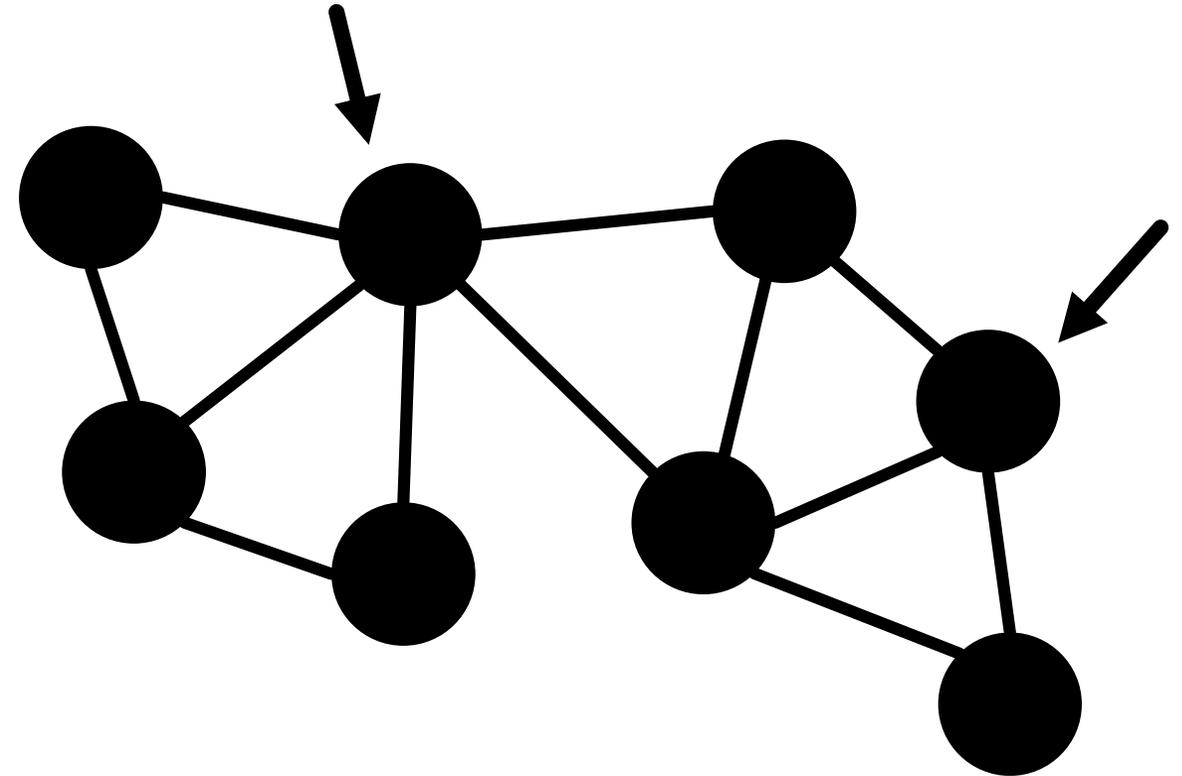
Which links
are missing?



Use Case 1: Link Prediction

Which links will appear?

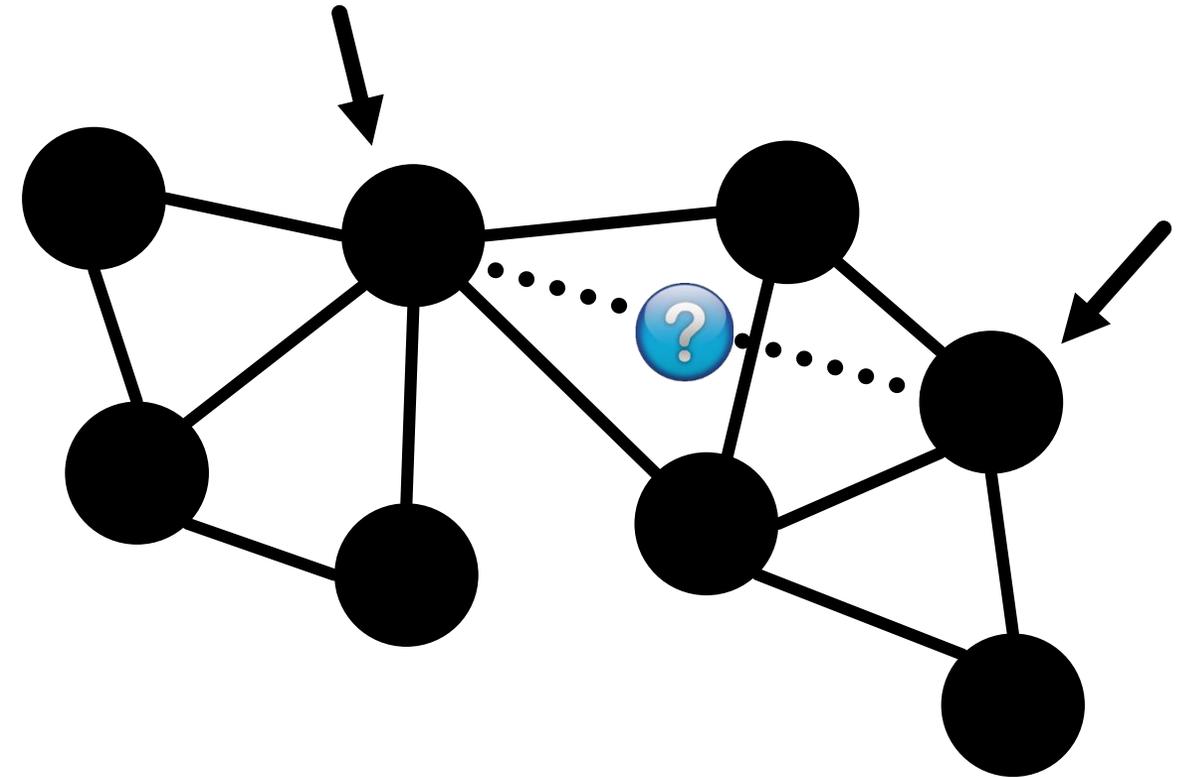
Which links are missing?



Use Case 1: Link Prediction

Which links
will appear?

Which links
are missing?



Use Case 1: Link Prediction

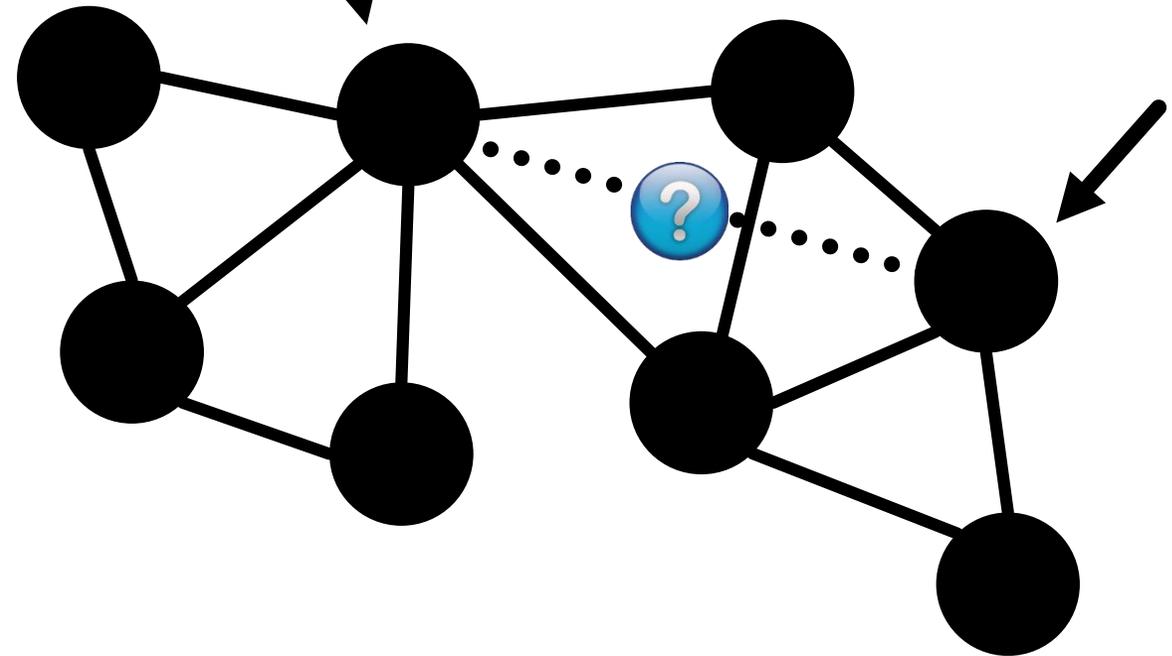
Which links will appear?

Which links are missing?

Predict future data



Fixing missing data



Use Case 1: Link Prediction

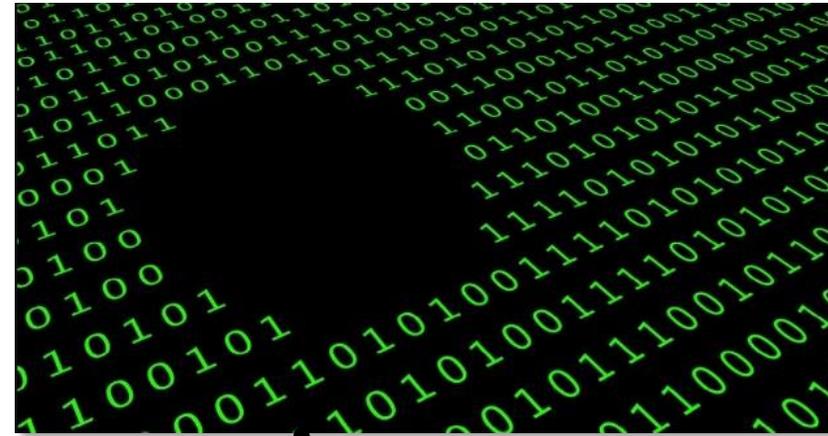
Which links will appear?



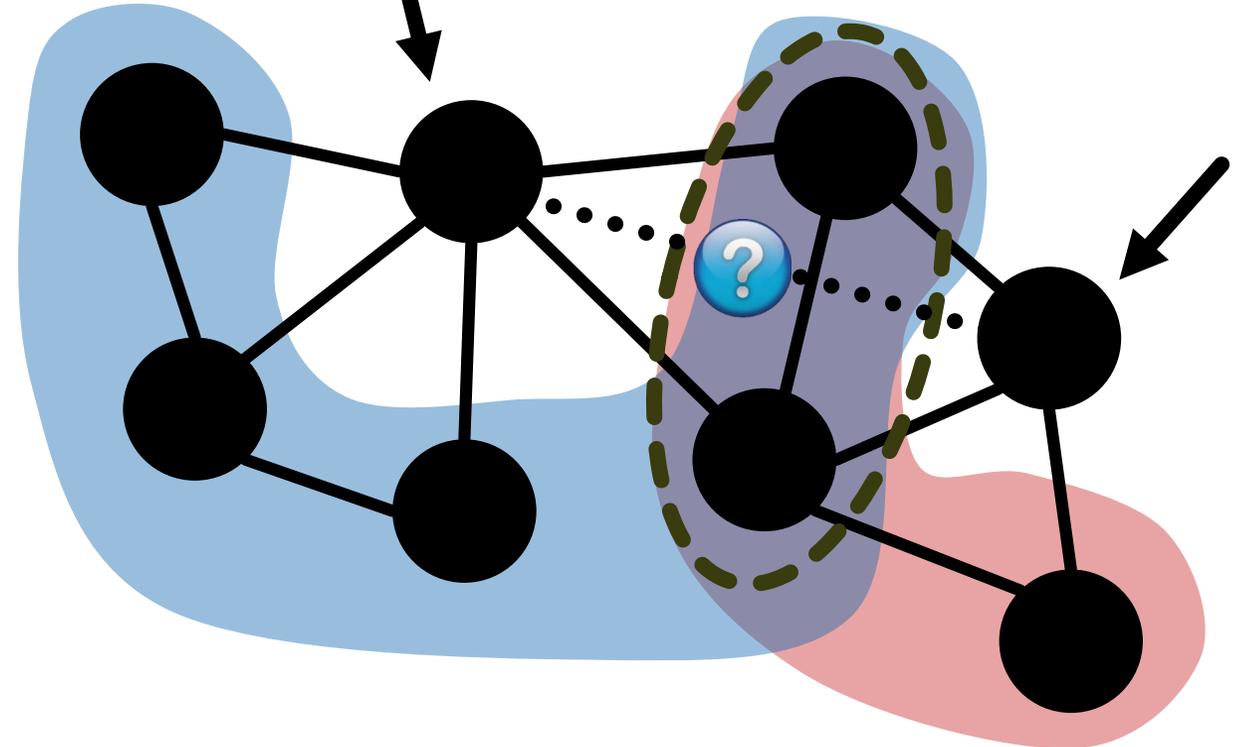
Which links are missing?



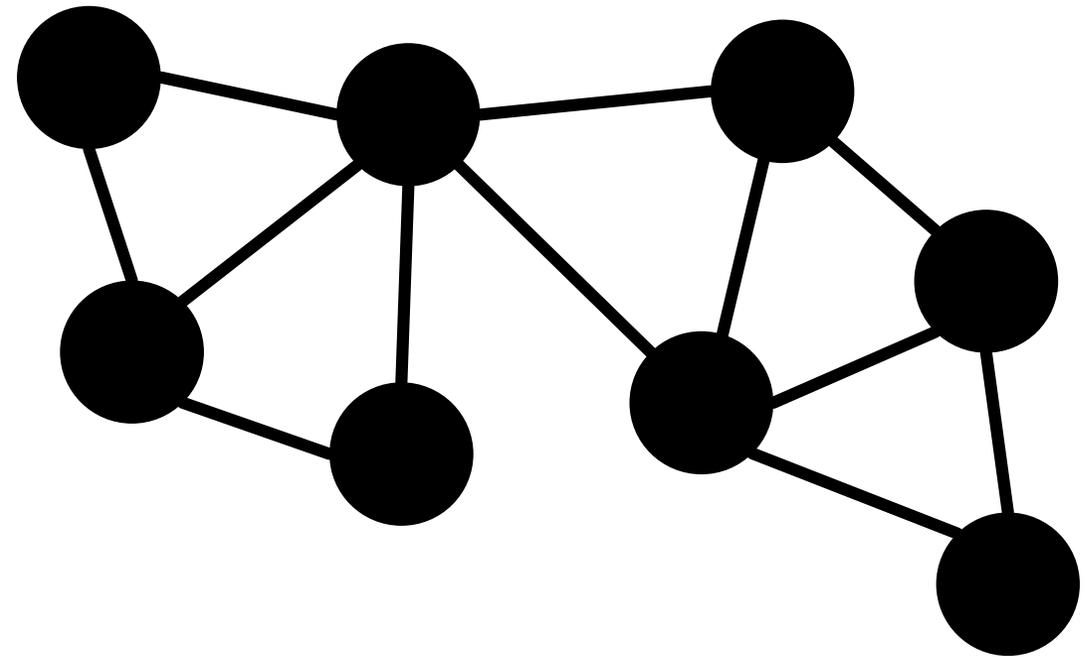
Predict future data



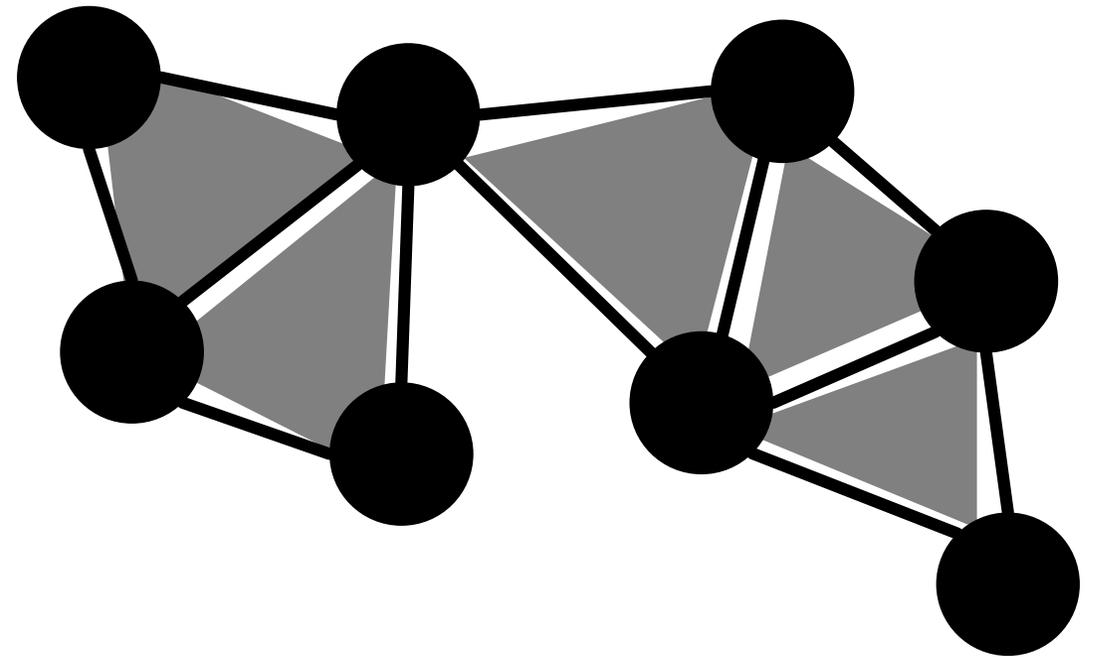
Fixing missing data



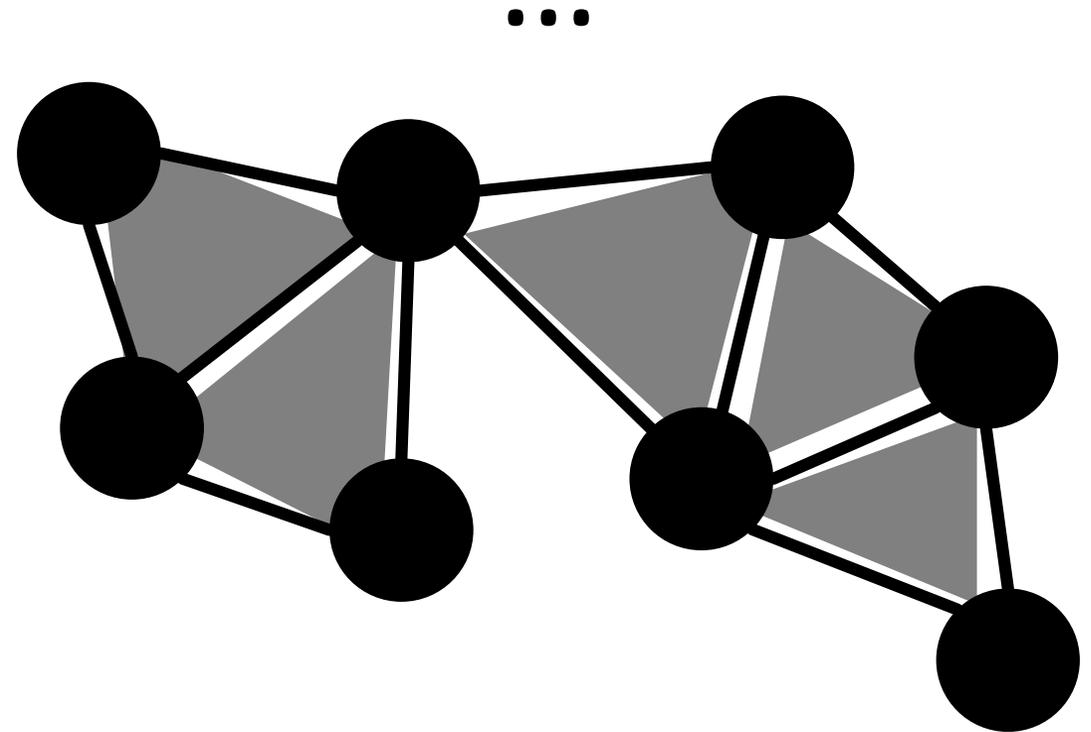
Use Case 2: Clique Counting



Use Case 2: Clique Counting

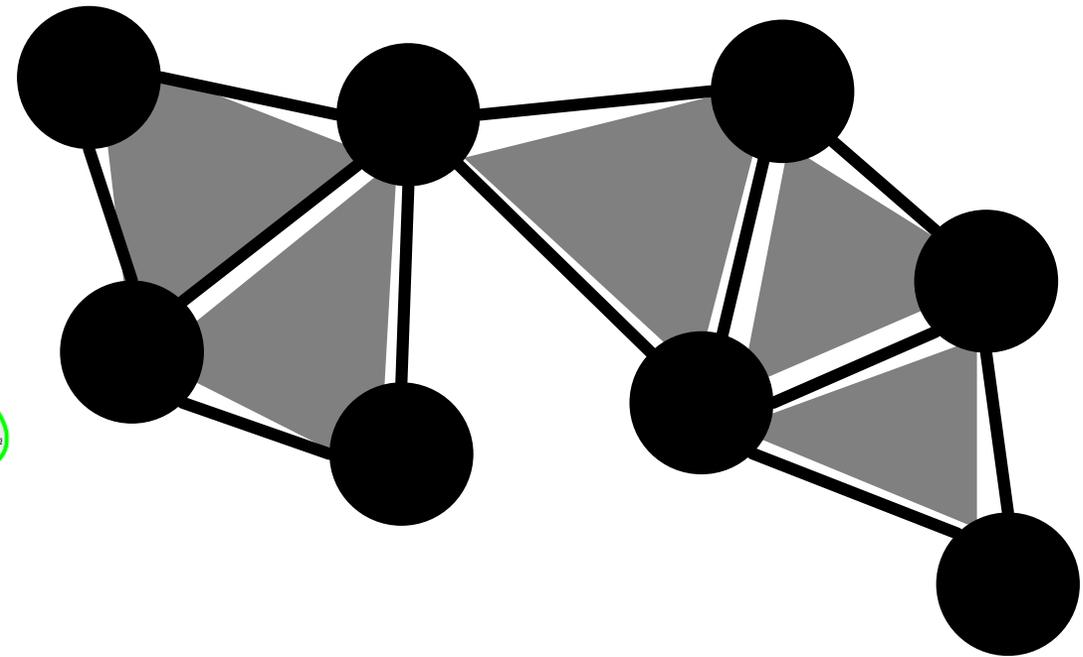
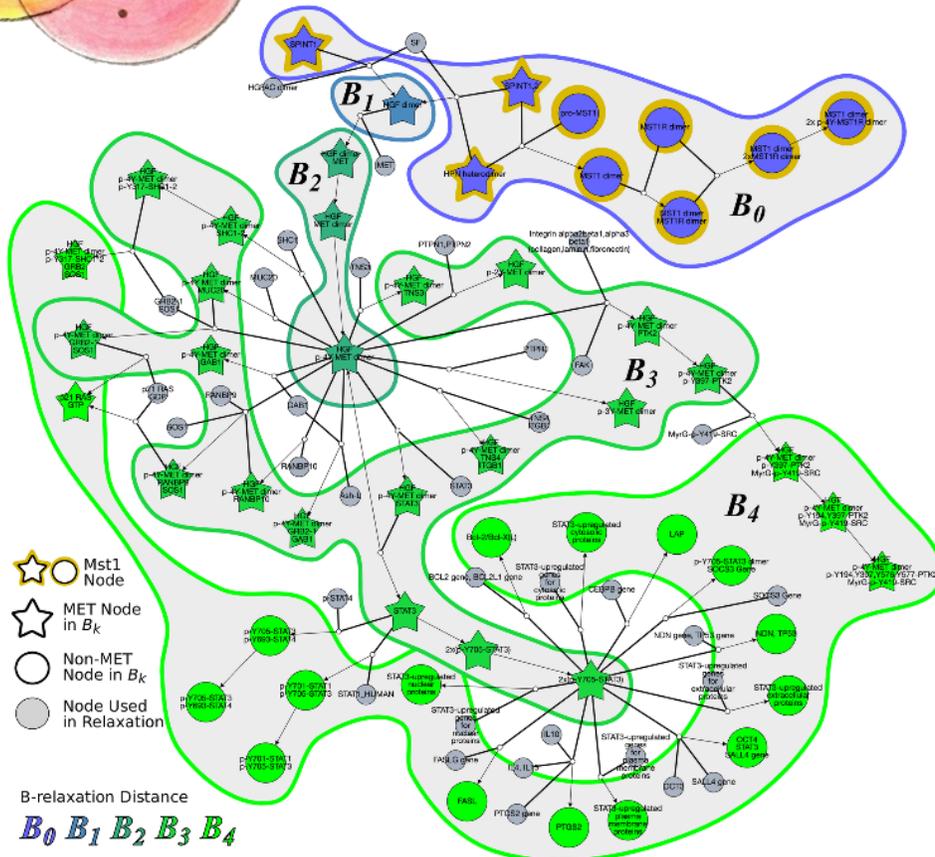
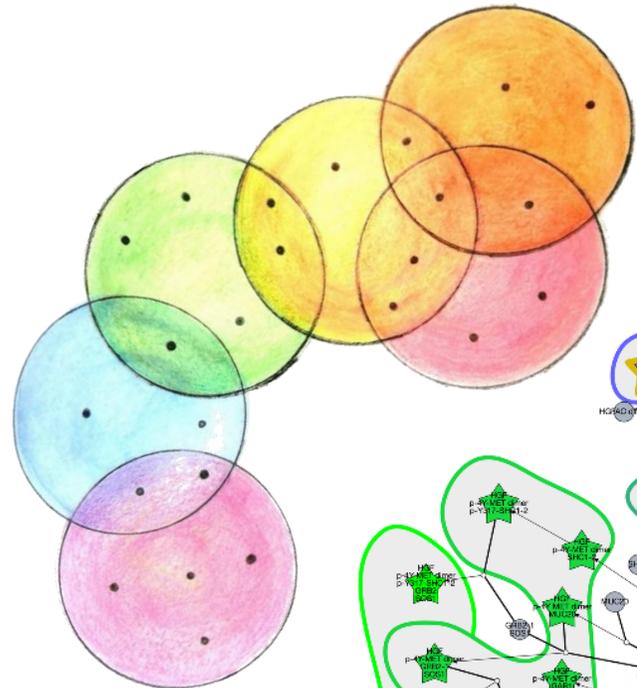
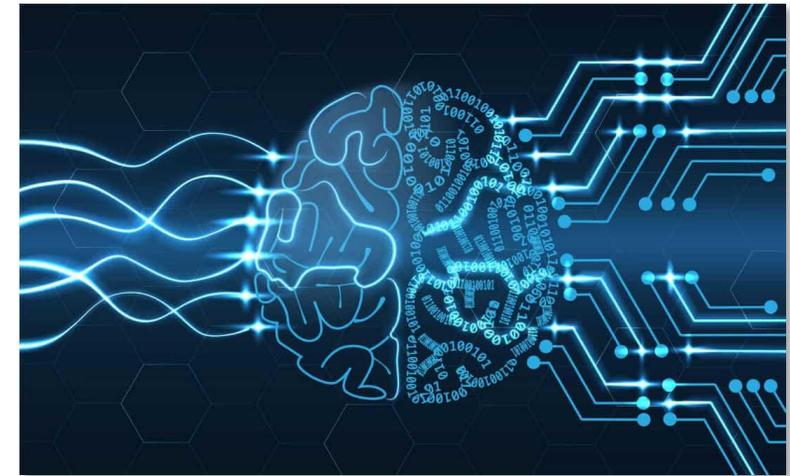


Use Case 2: Clique Counting



Use Case 2: Clique Counting

Learning over higher-order networks

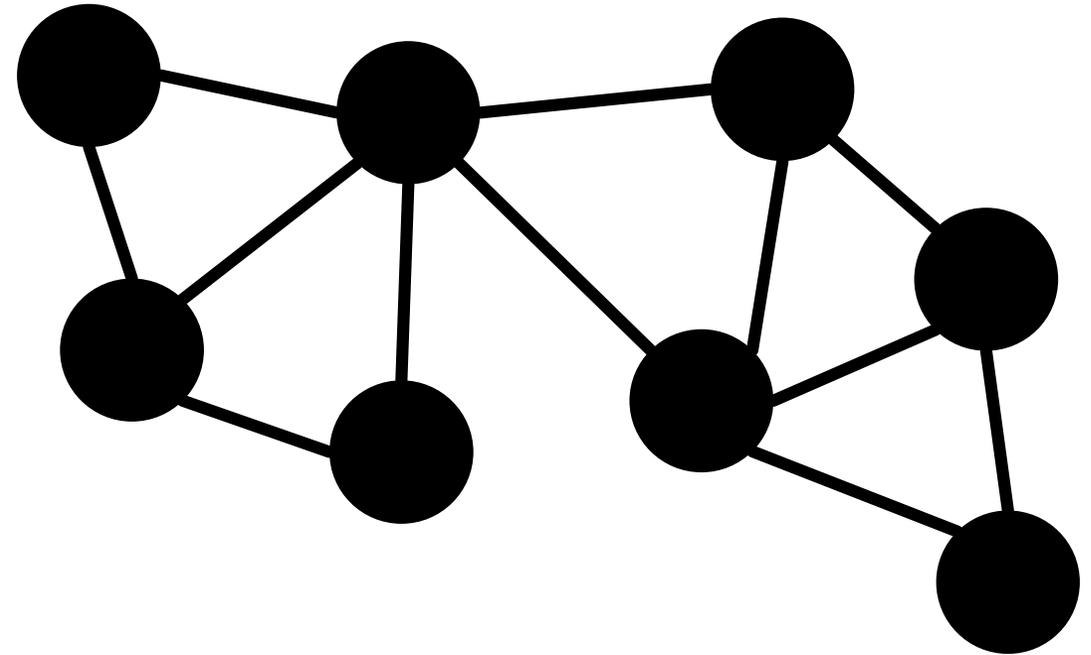


Use Case 3: Clustering

Clusters?



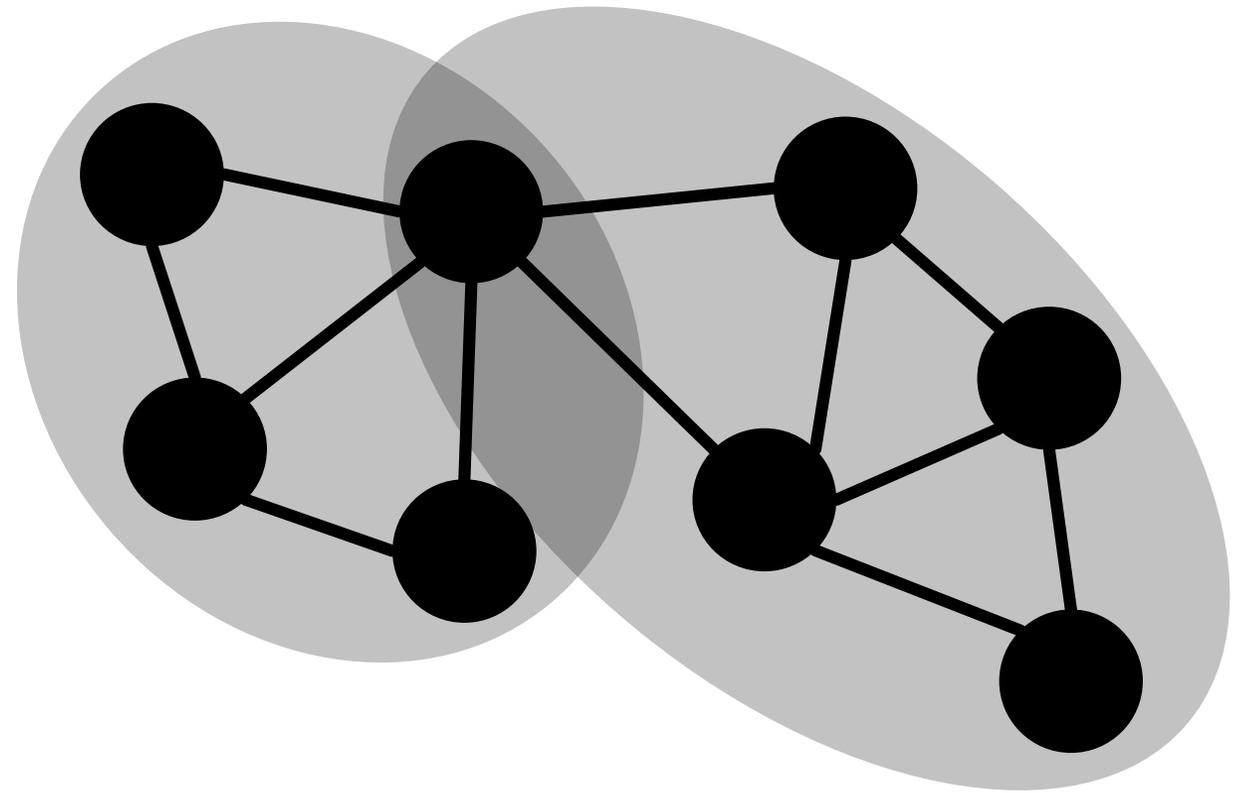
Structure of clusters?



Use Case 3: Clustering

Clusters? 

Structure of clusters? 

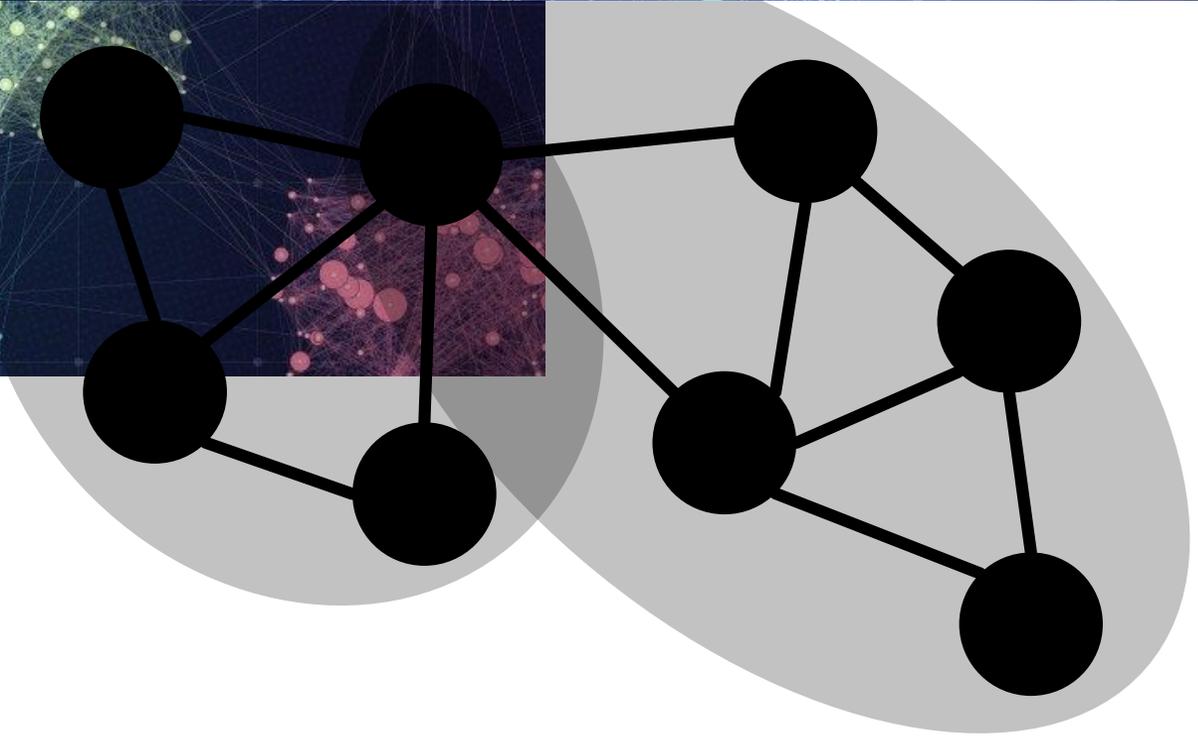
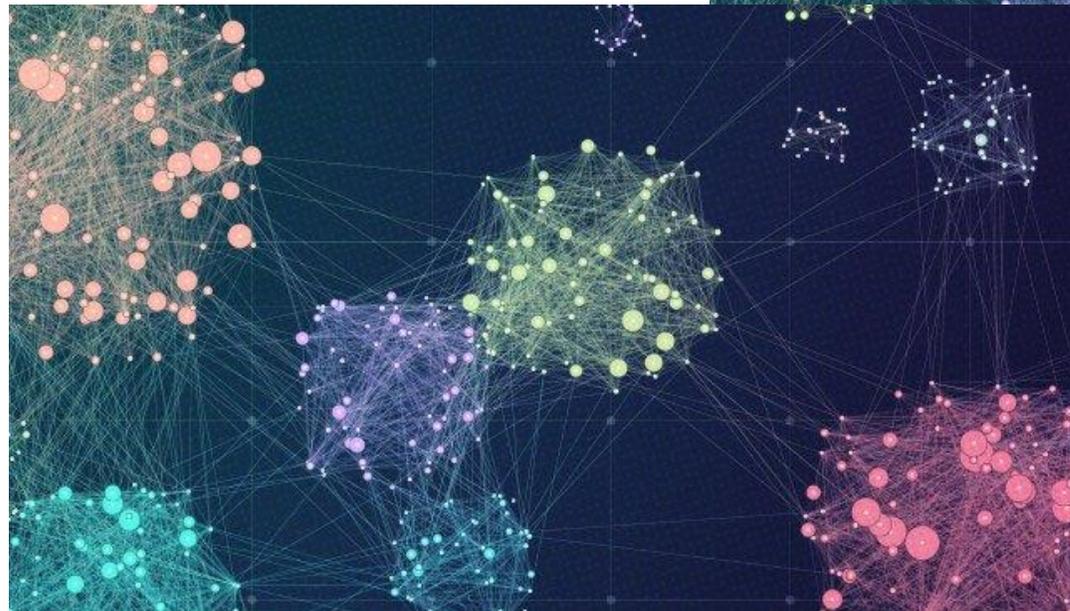


Use Case 3: Clustering

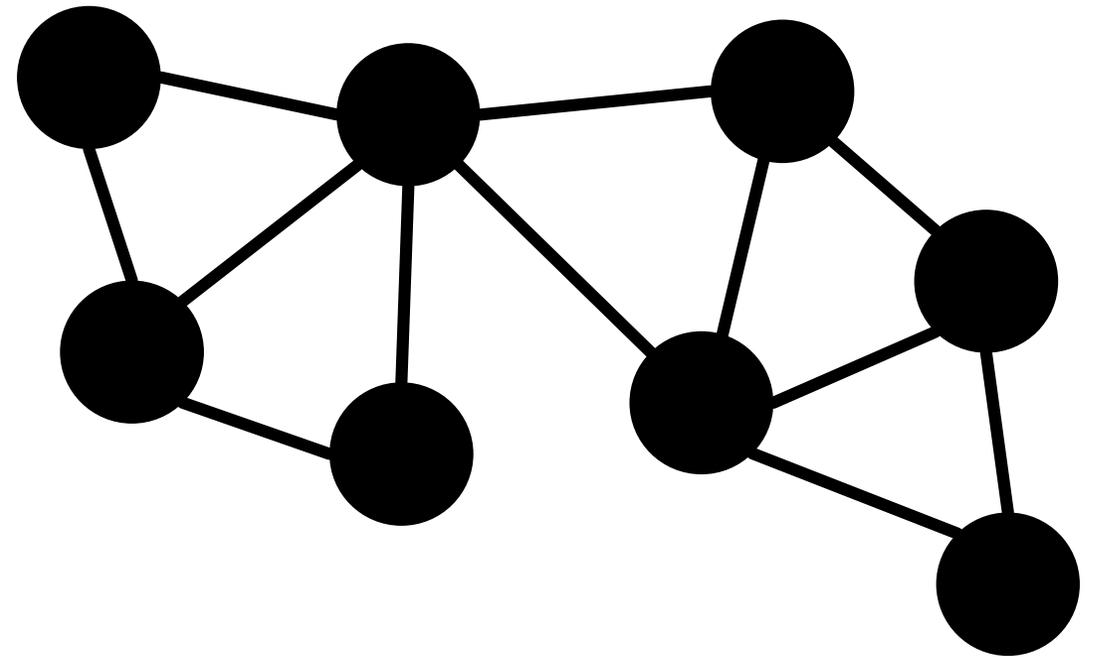
Clusters? 

Structure of clusters? 

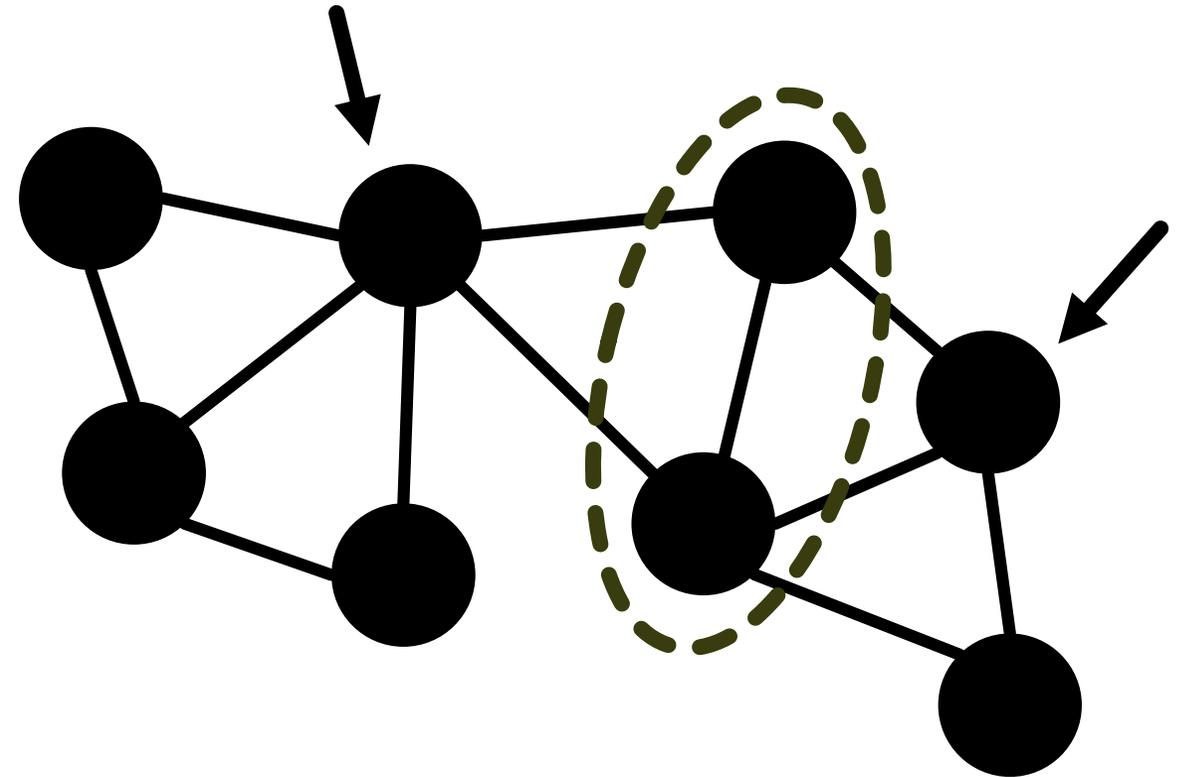
Minibatch selection in Graph Neural Networks



Use Case 4: Vertex Similarity

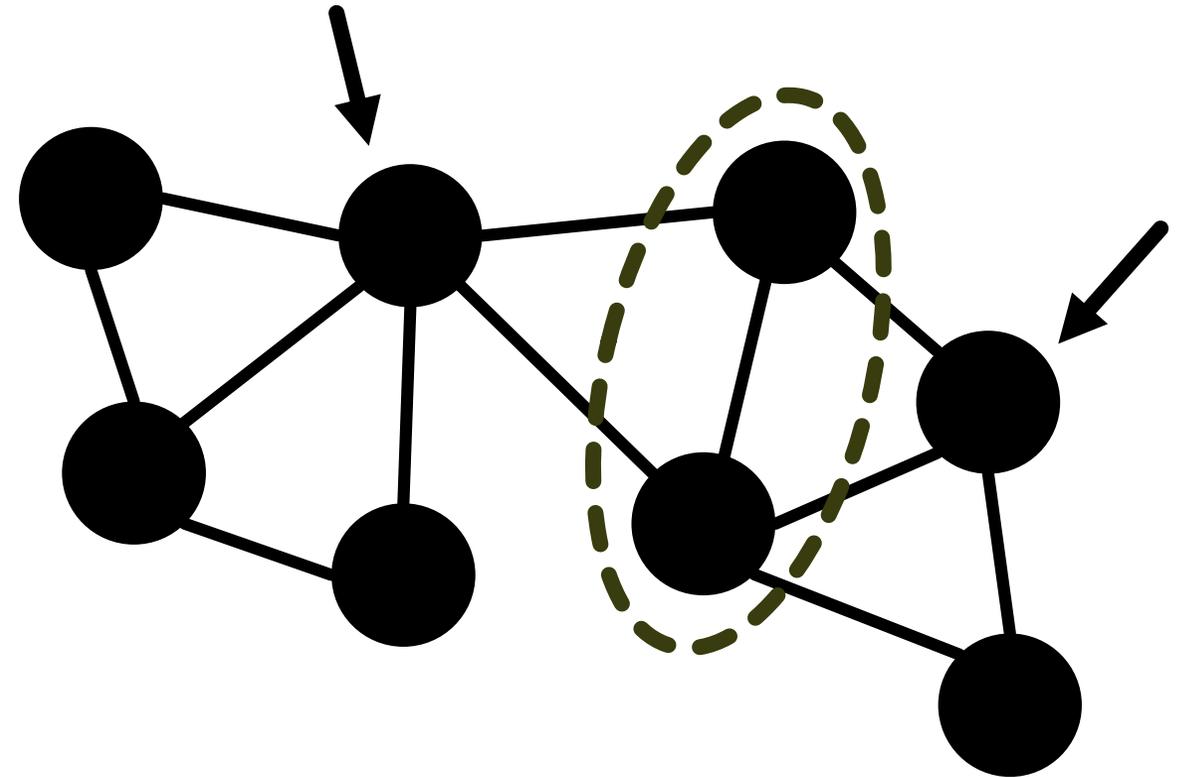
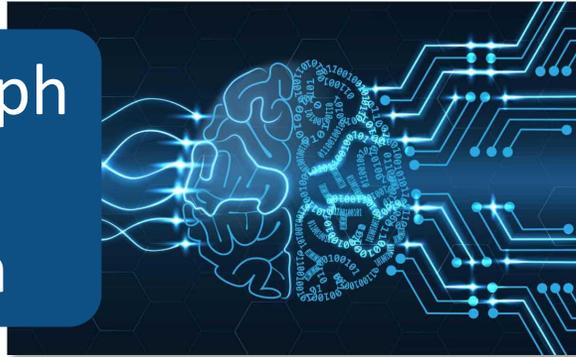


Use Case 4: Vertex Similarity



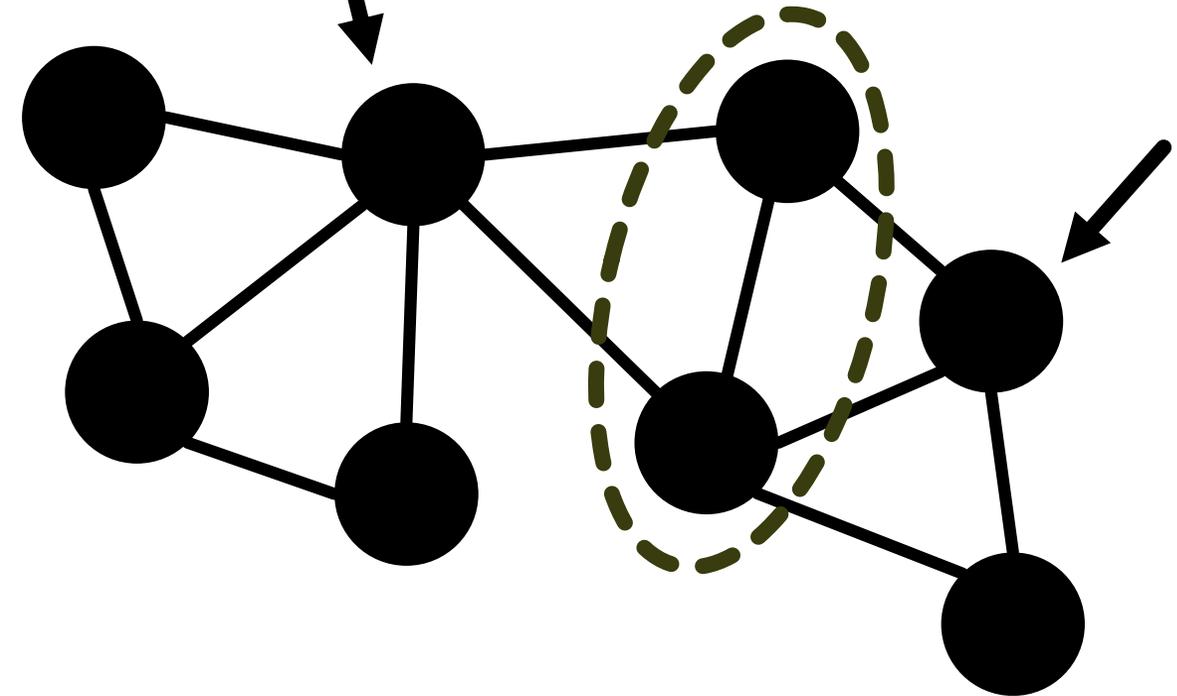
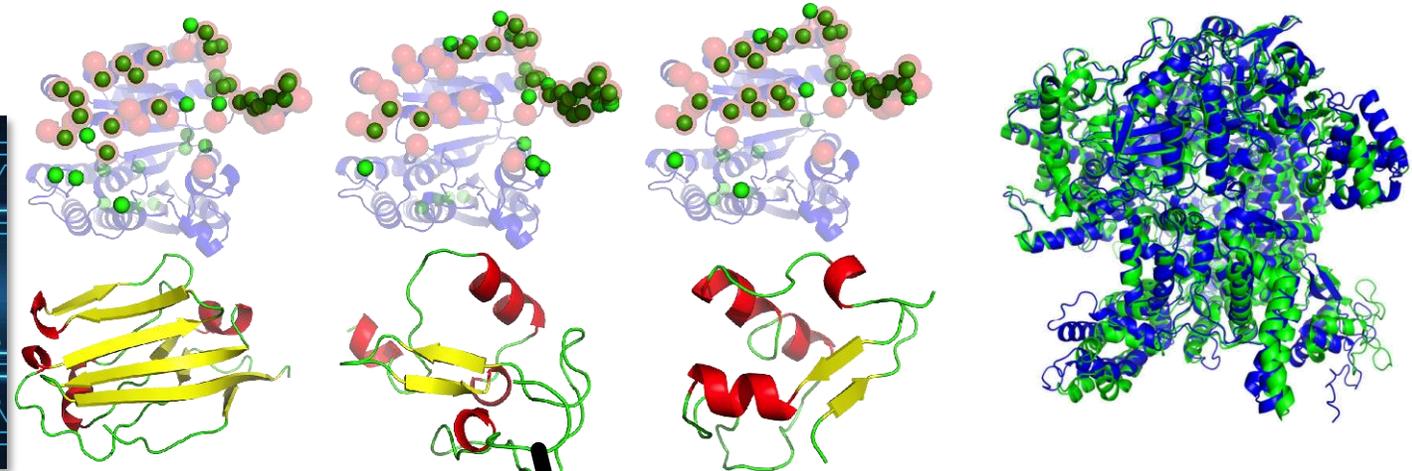
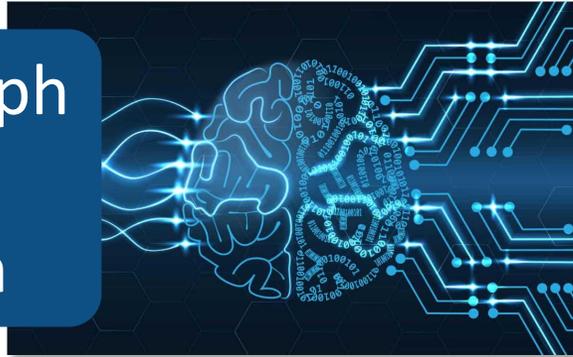
Use Case 4: Vertex Similarity

Enhancing graph embedding construction



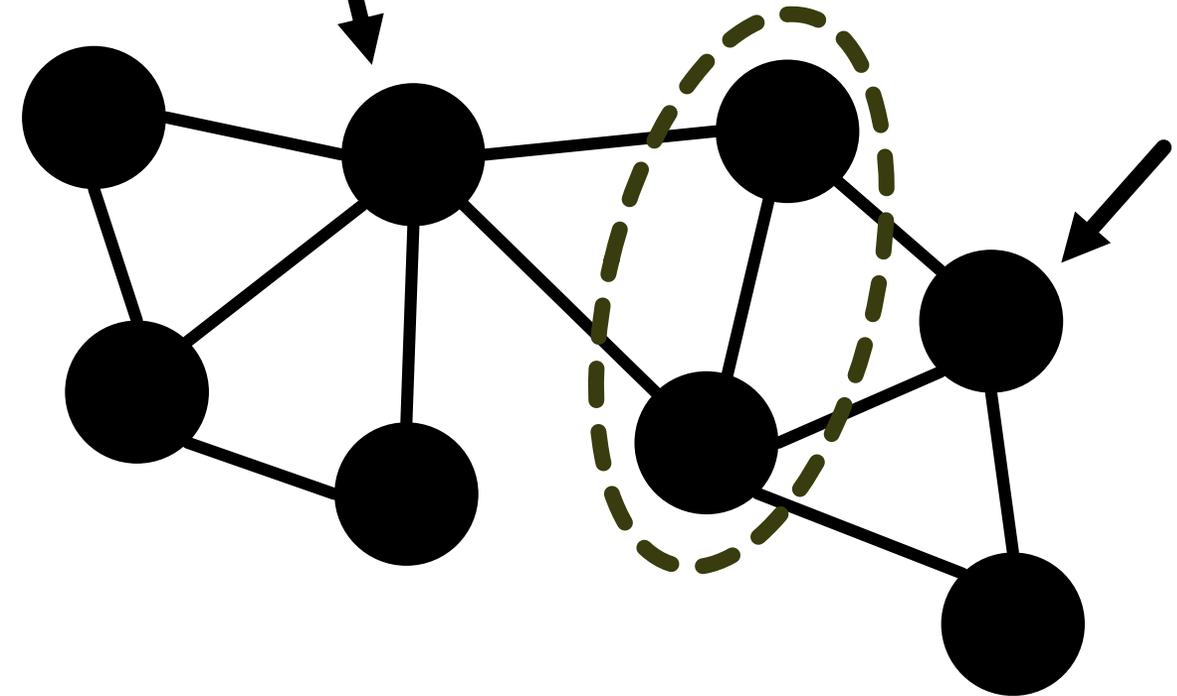
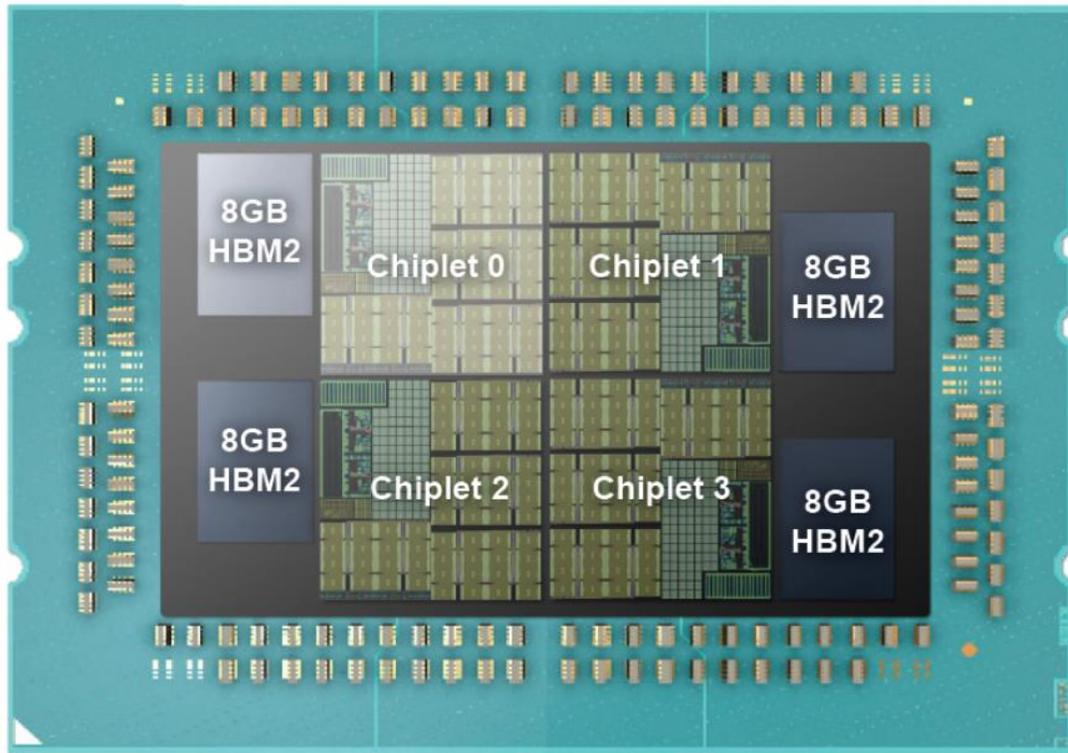
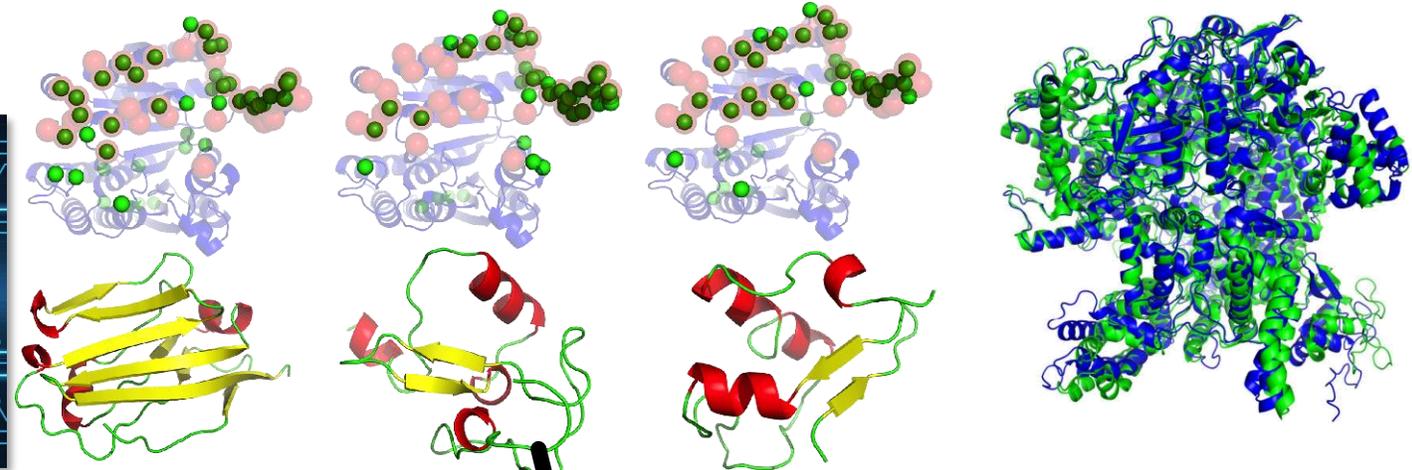
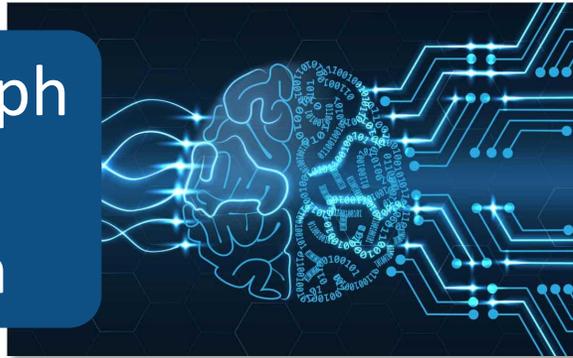
Use Case 4: Vertex Similarity

Enhancing graph embedding construction

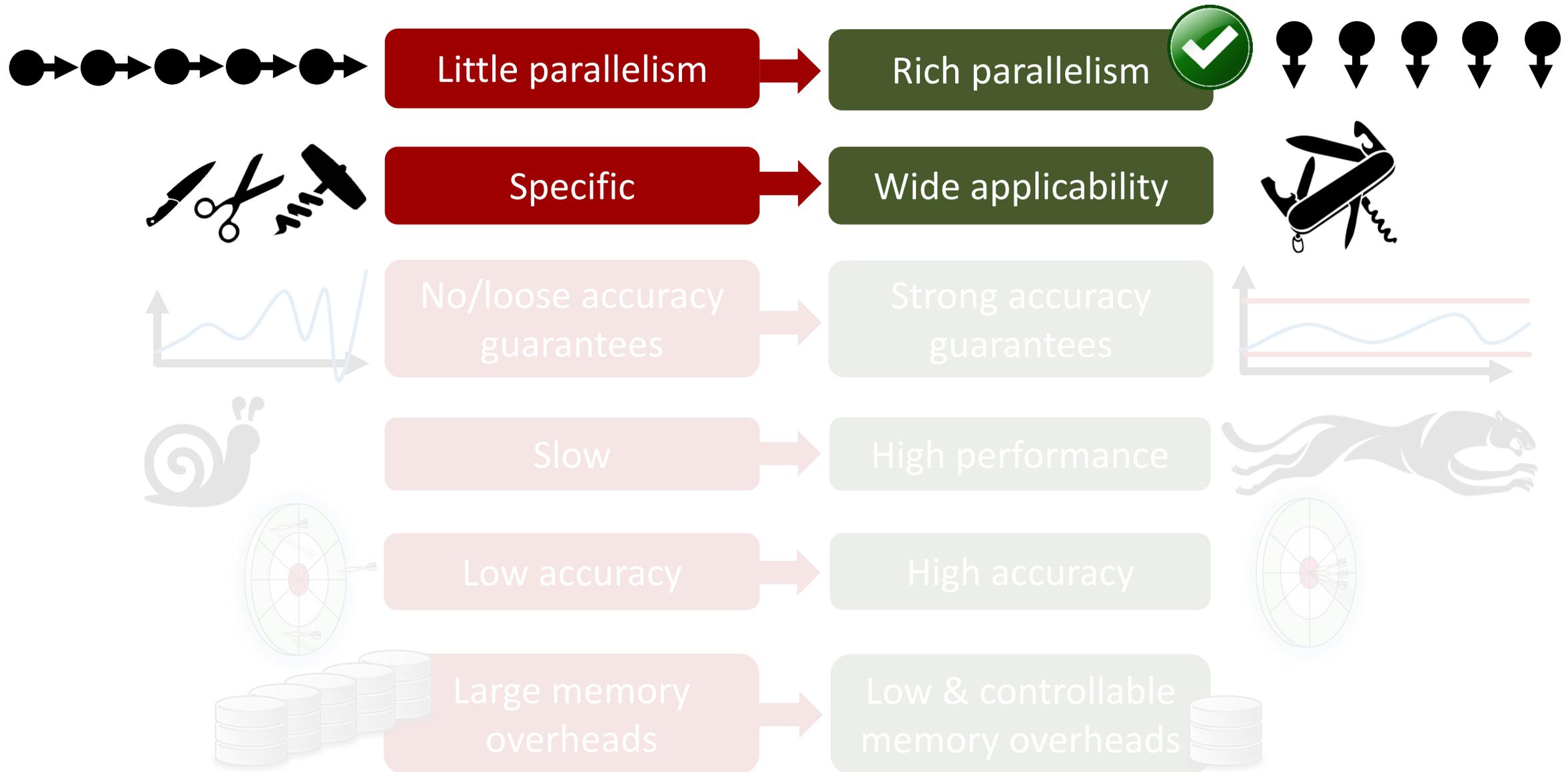


Use Case 4: Vertex Similarity

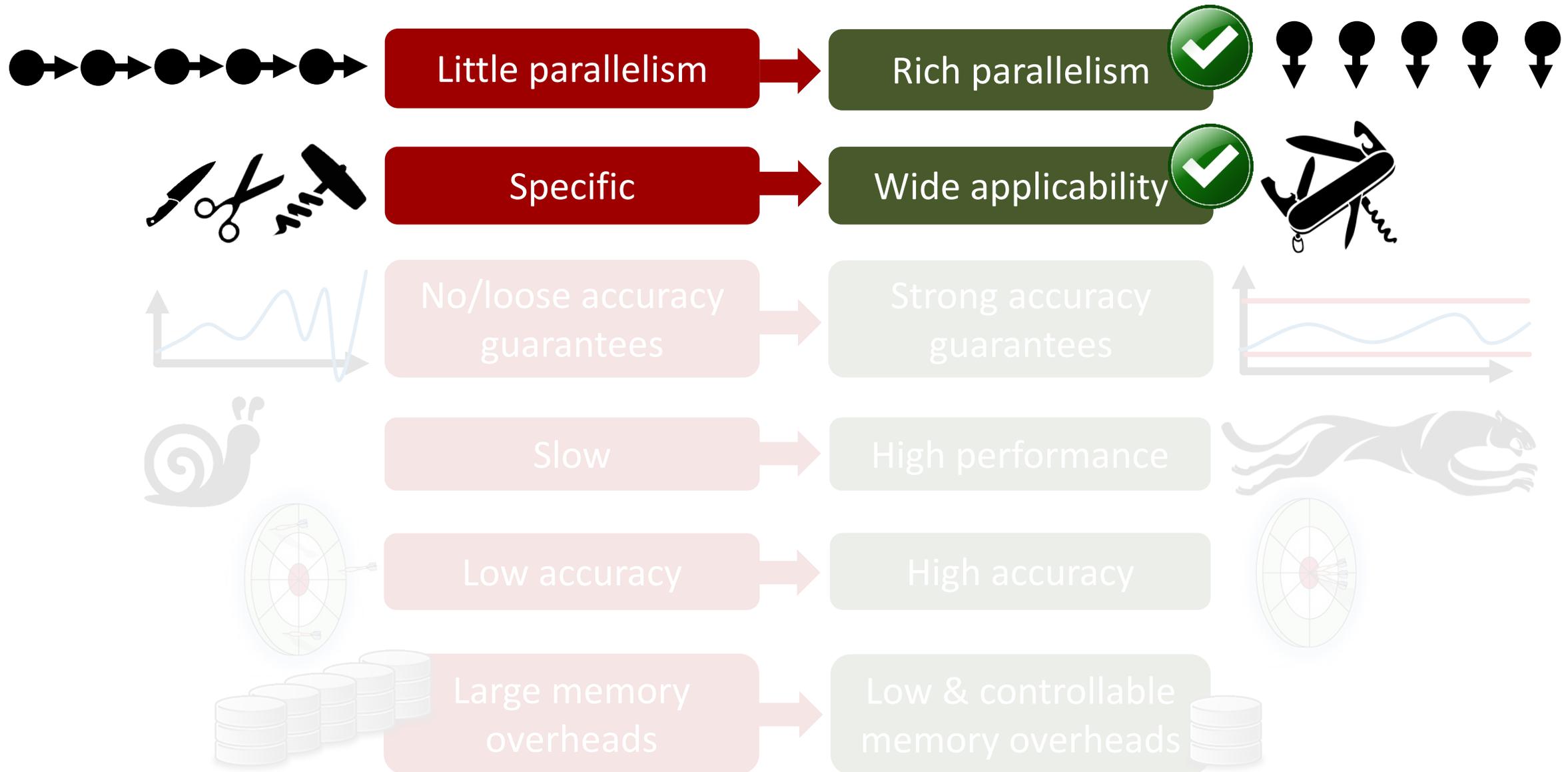
Enhancing graph embedding construction



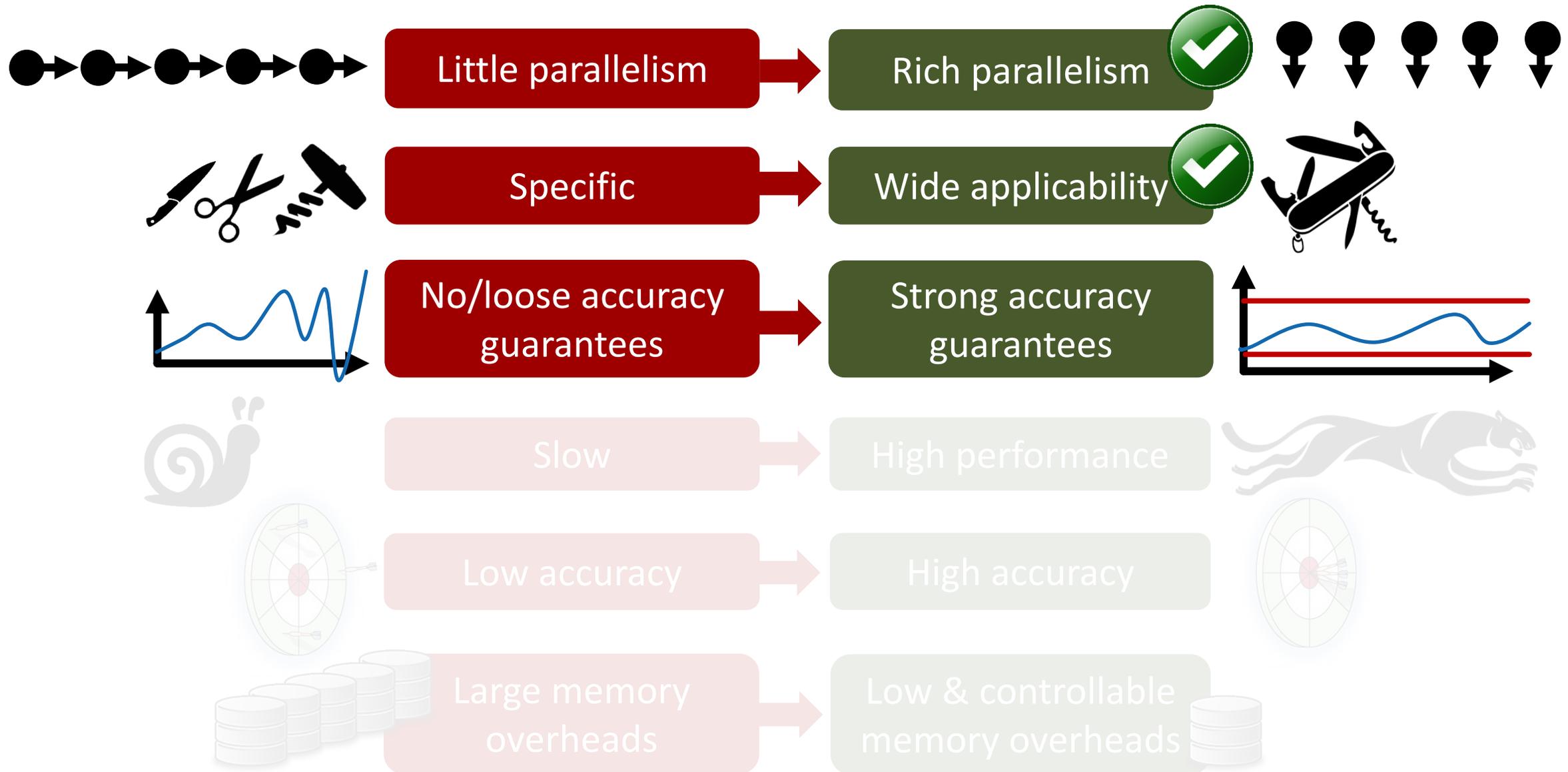
Approximate Graph Processing: Our Objectives



Approximate Graph Processing: Our Objectives



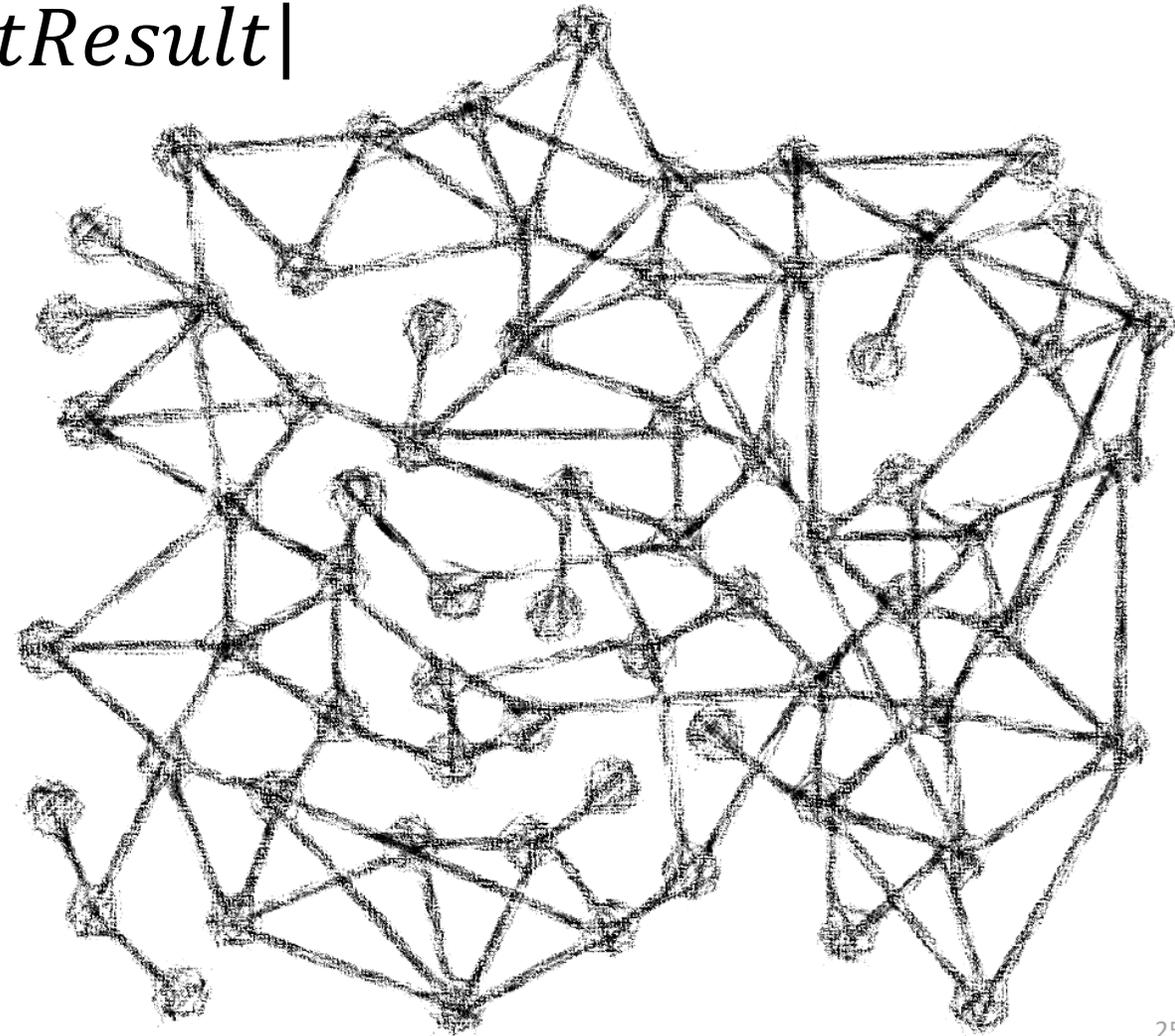
Approximate Graph Processing: Our Objectives



ProbGraph: Summary of Theoretical Results

ProbGraph: Summary of Theoretical Results

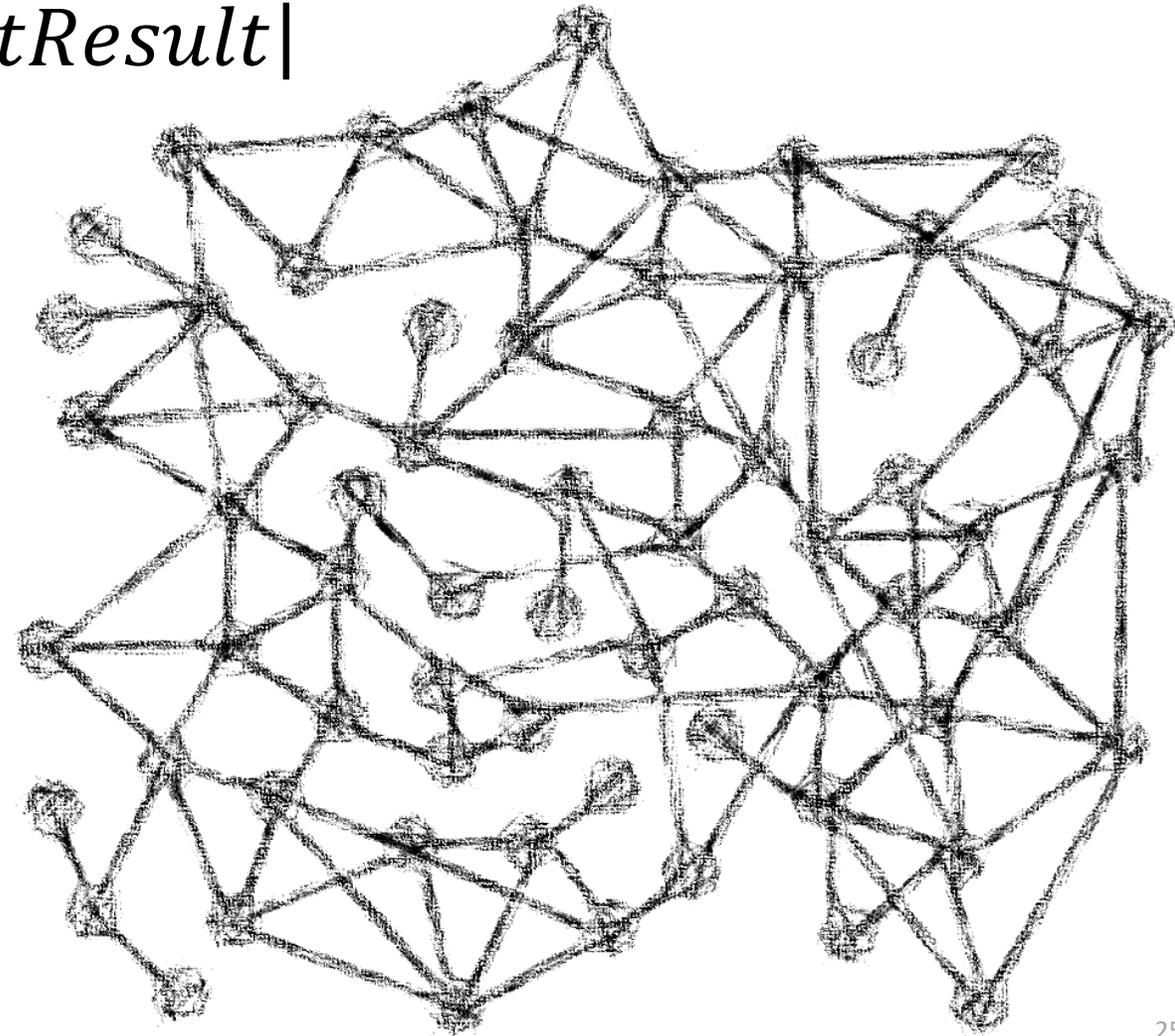
We want guarantees for
 $|ProbGraphEstimate - exactResult|$



ProbGraph: Summary of Theoretical Results

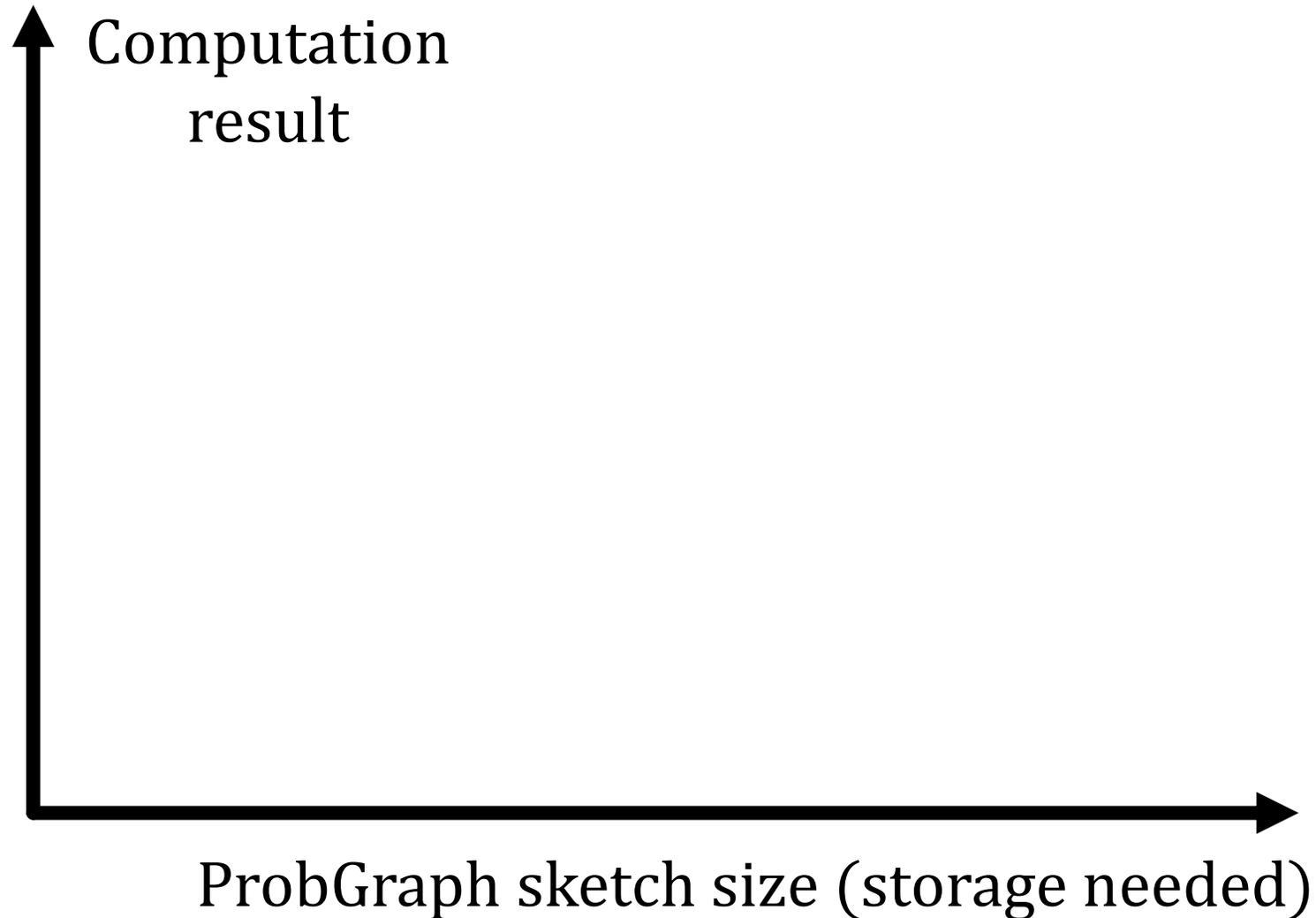
We want guarantees for
 $|ProbGraphEstimate - exactResult|$

We incorporate
statistical theory of
estimators

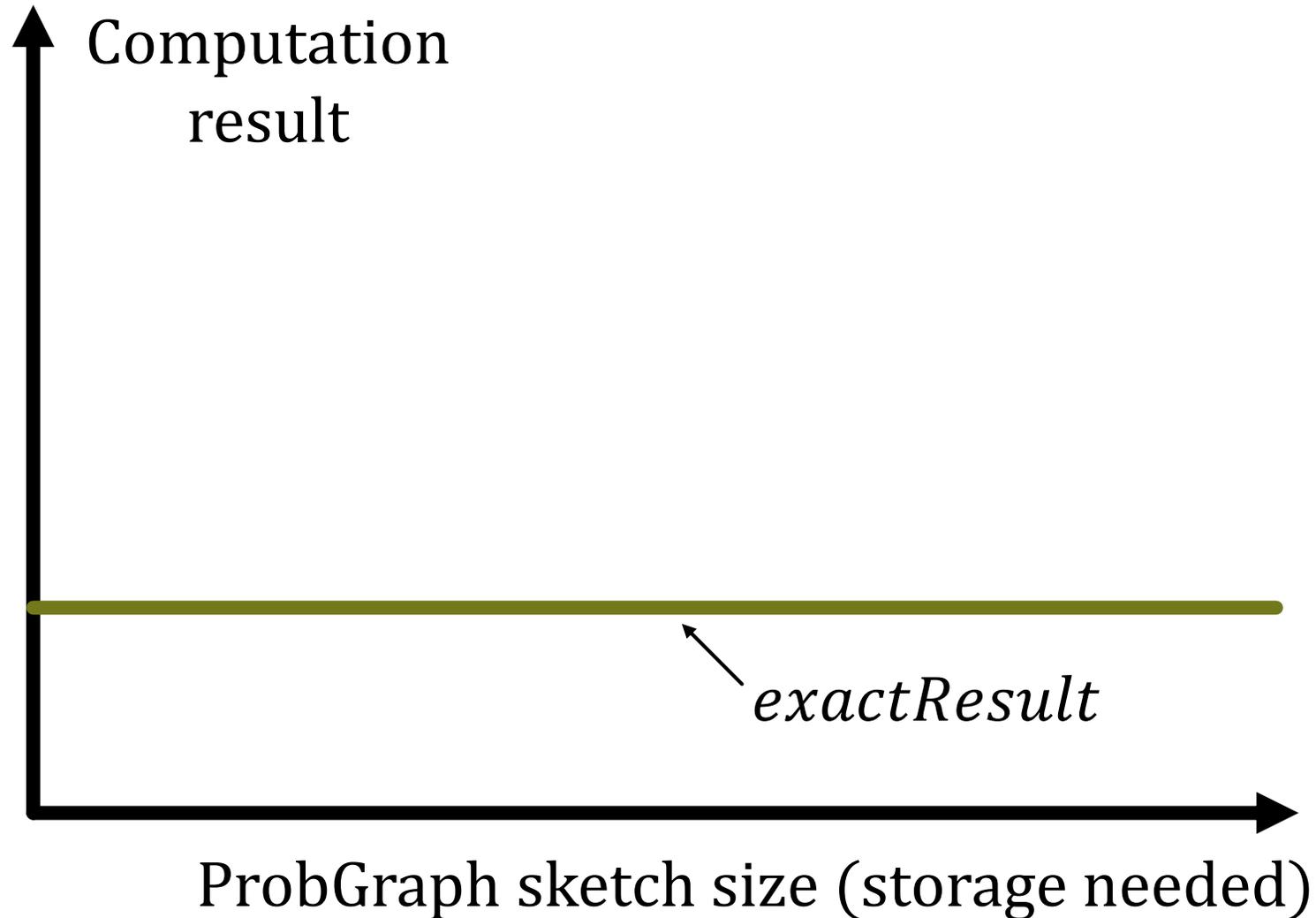


ProbGraph is asymptotically unbiased

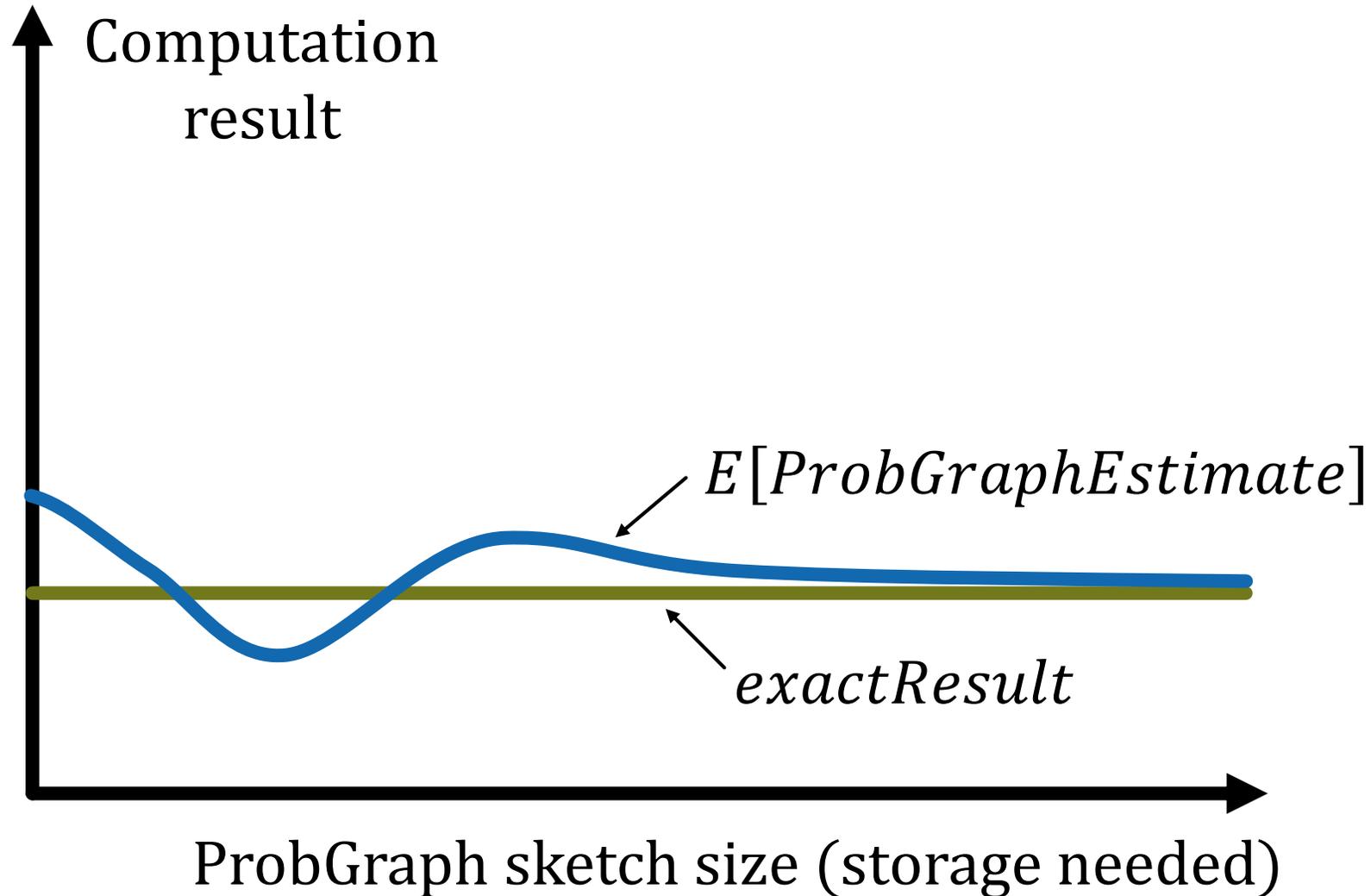
ProbGraph is asymptotically unbiased



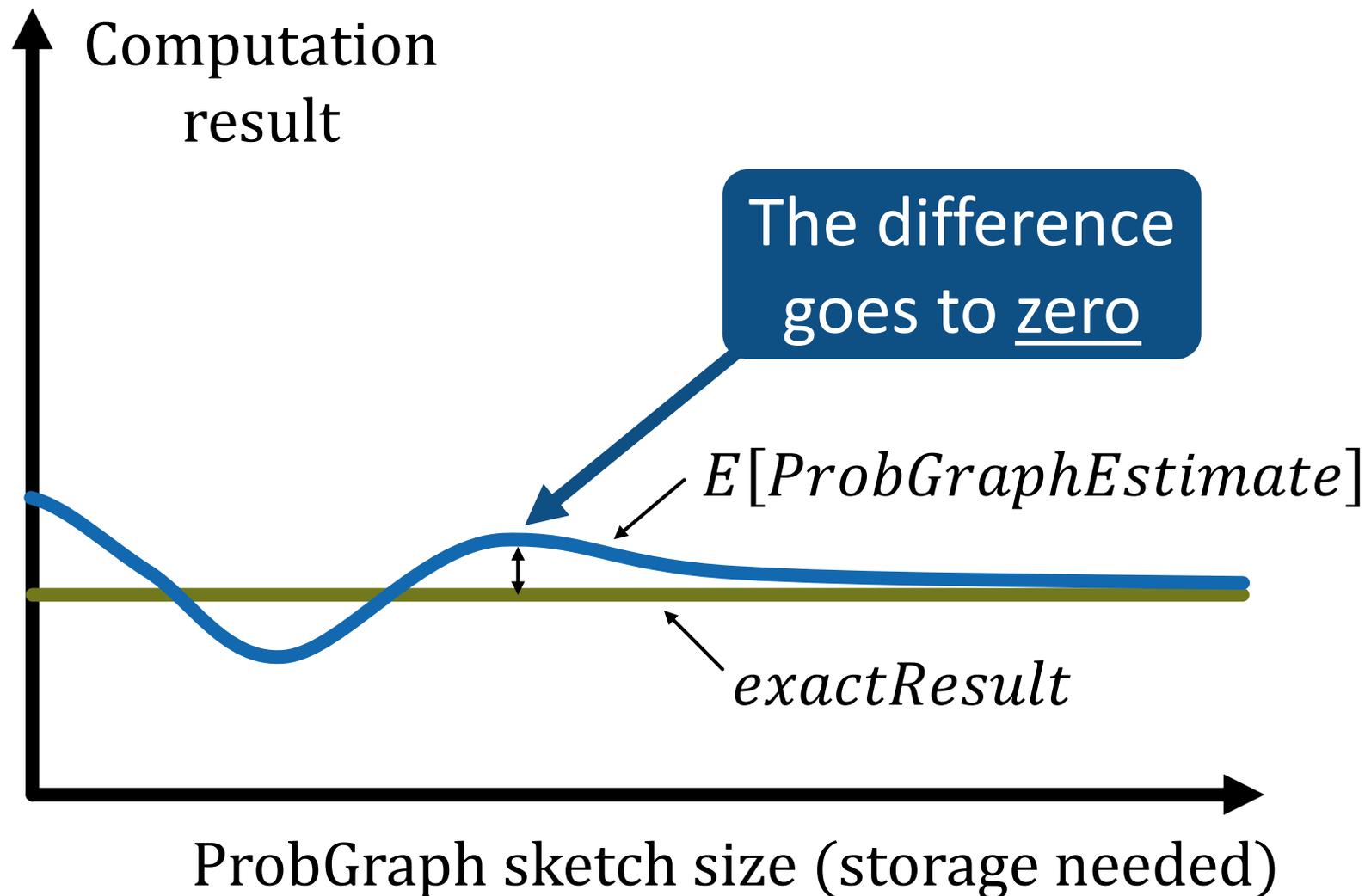
ProbGraph is asymptotically unbiased



ProbGraph is asymptotically unbiased

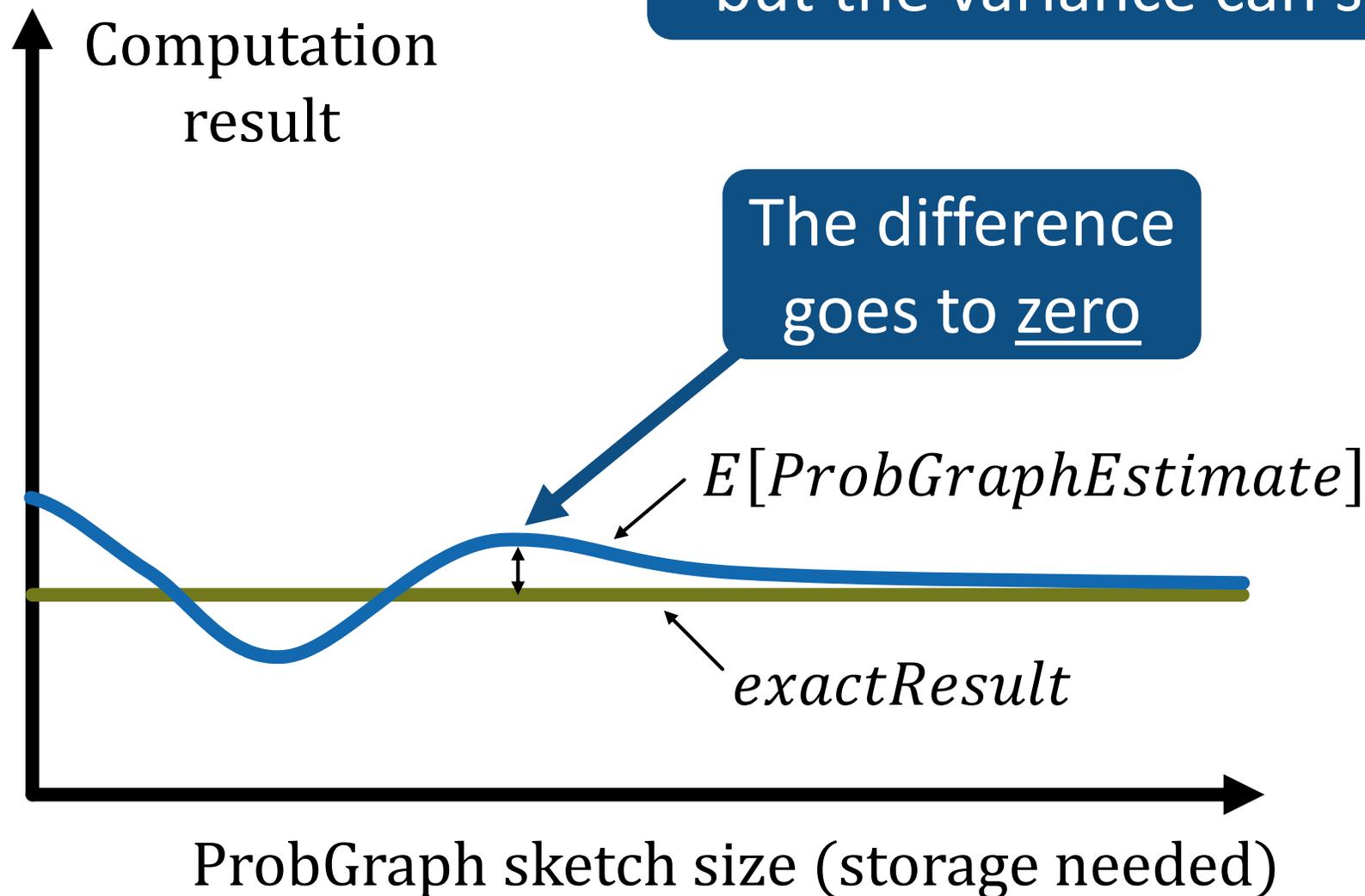


ProbGraph is asymptotically unbiased

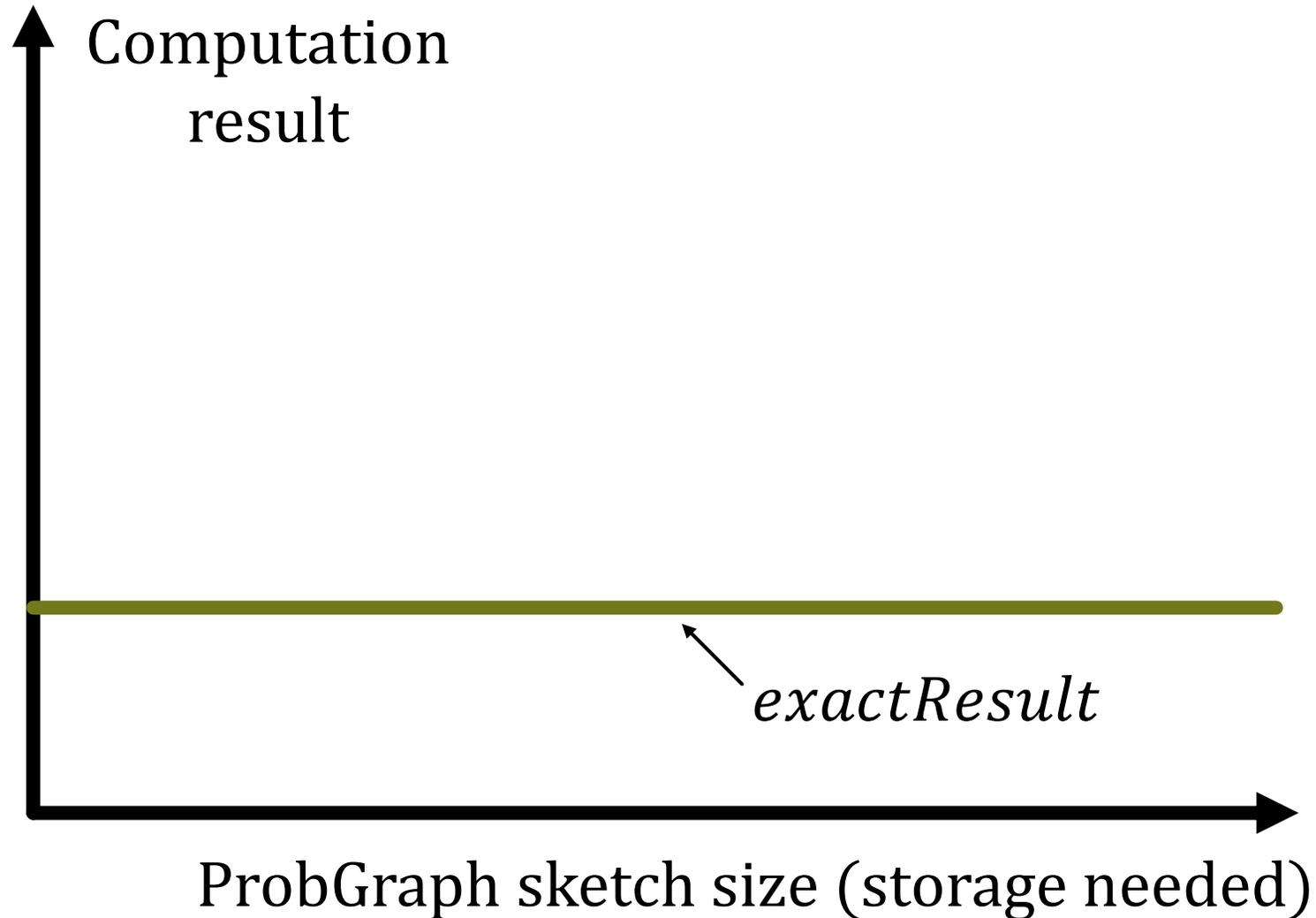


ProbGraph is asymptotically unbiased

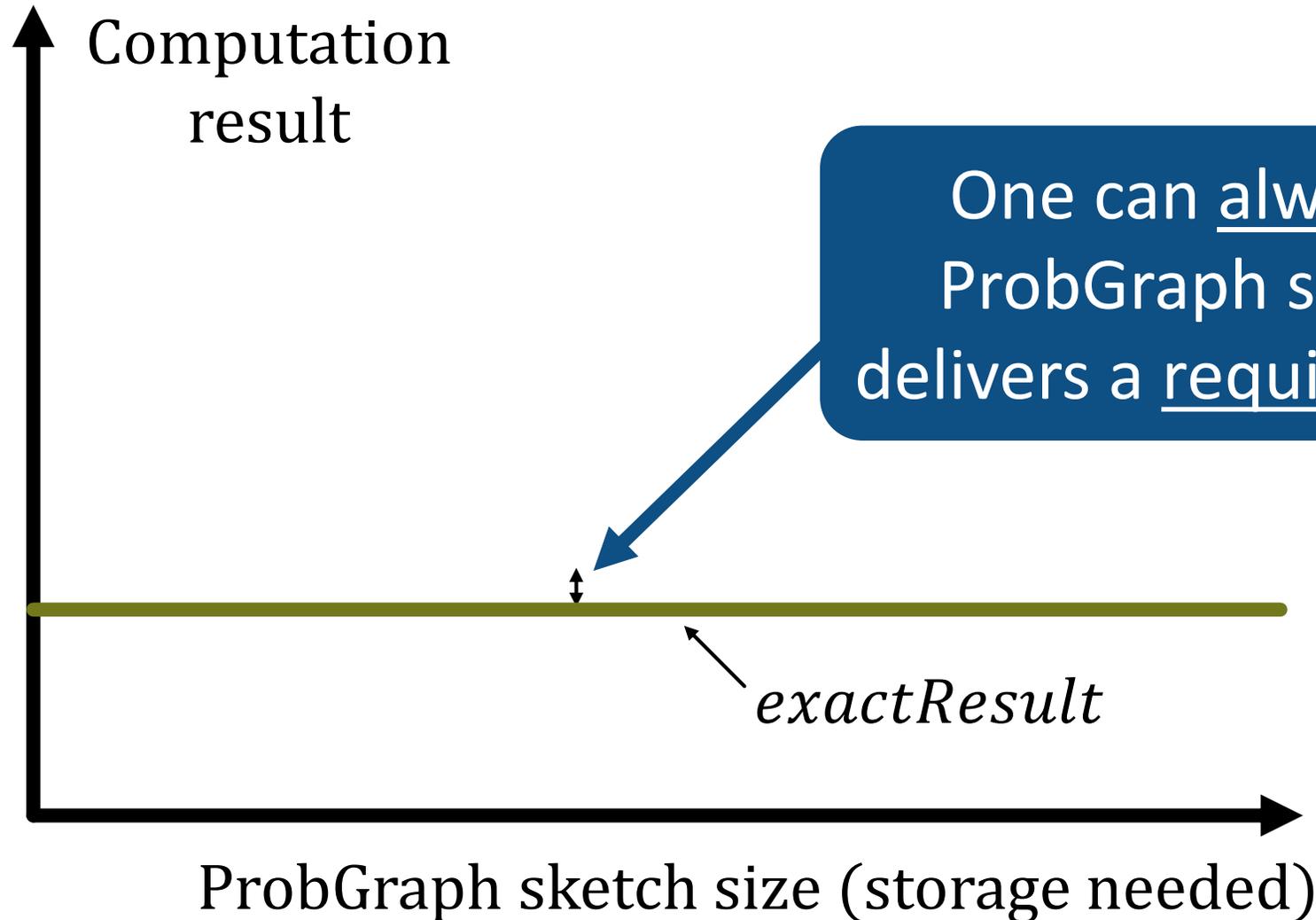
Zero average error at some point...
but the variance can still go wild



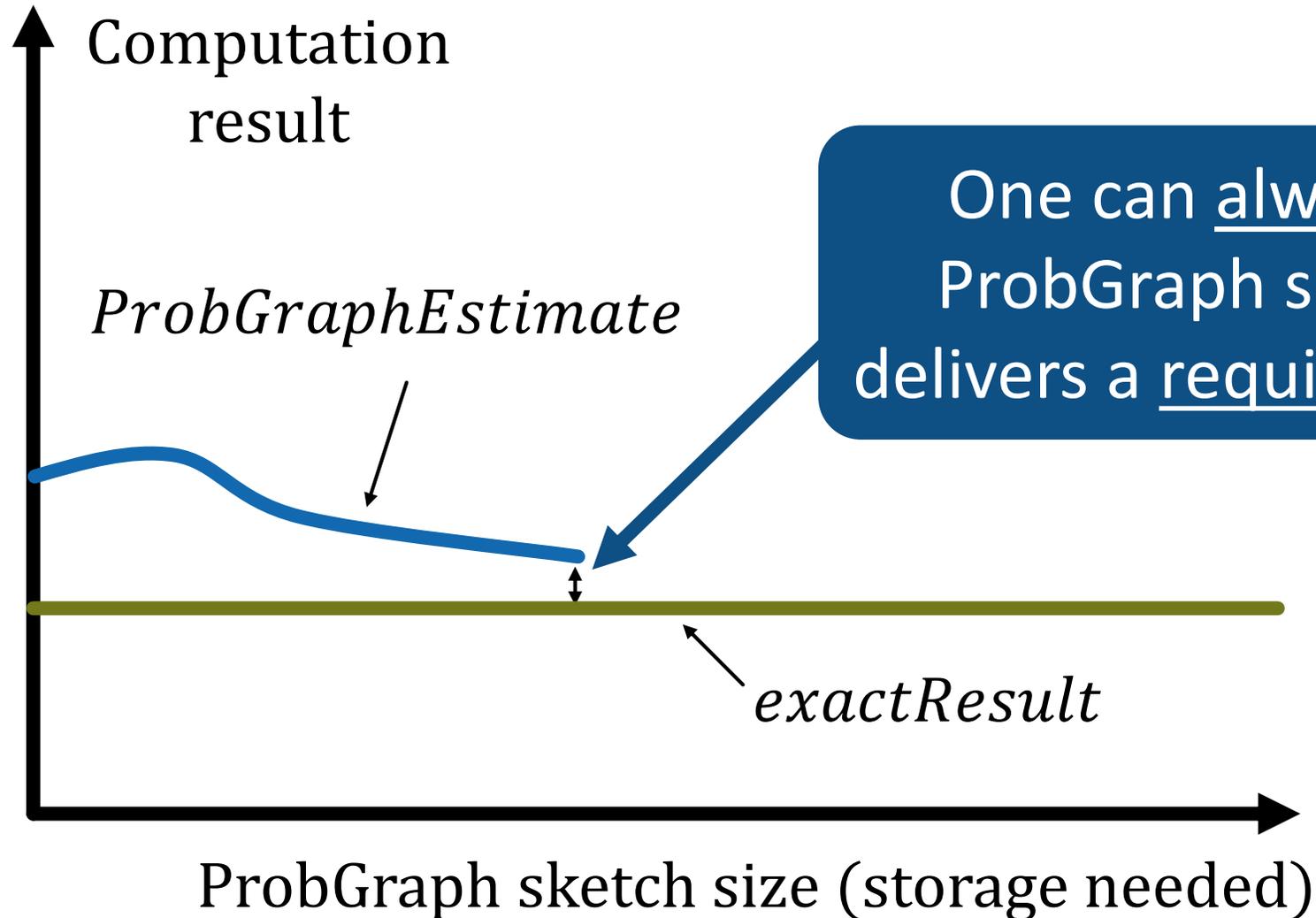
ProbGraph is consistent



ProbGraph is consistent

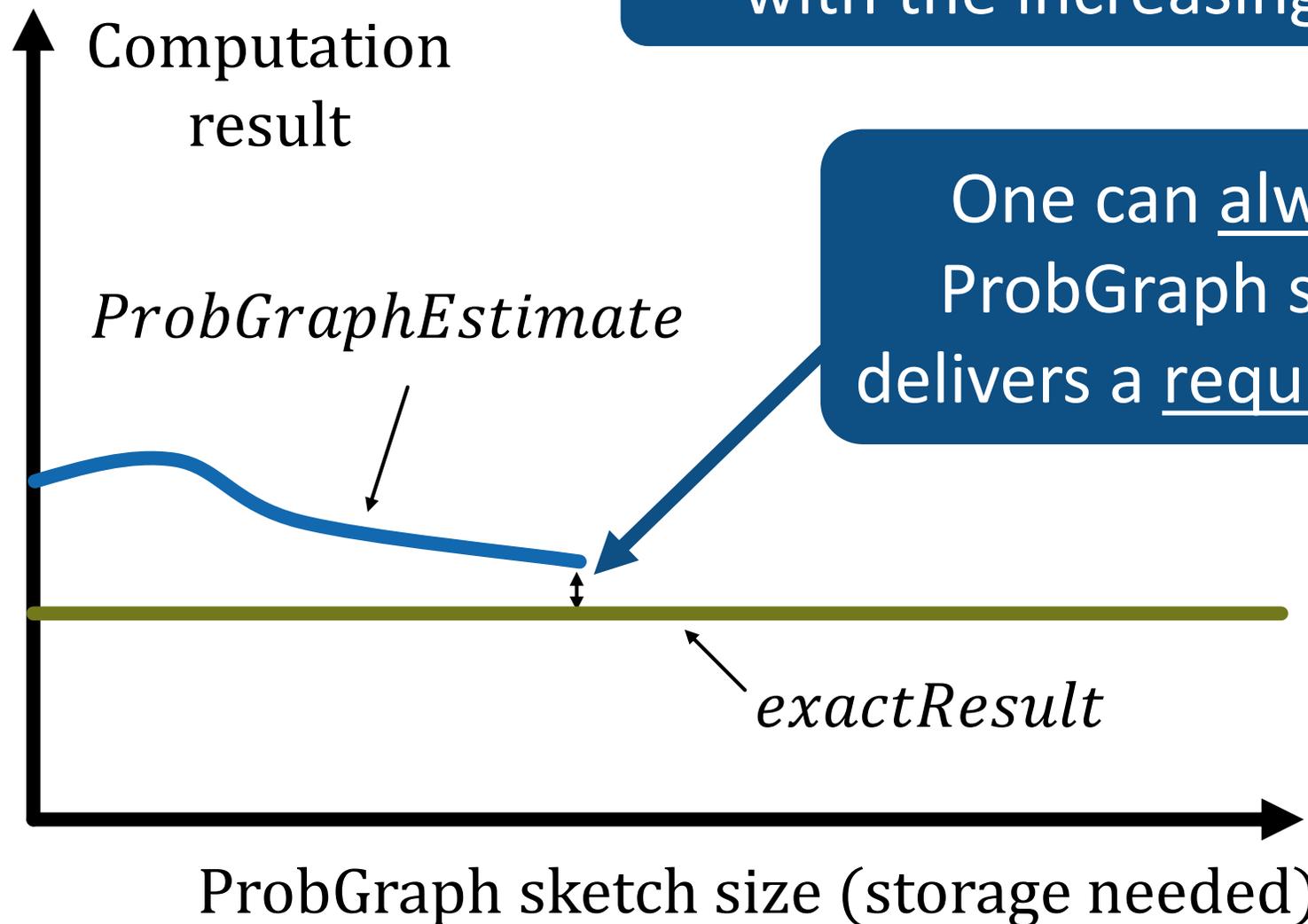


ProbGraph is consistent



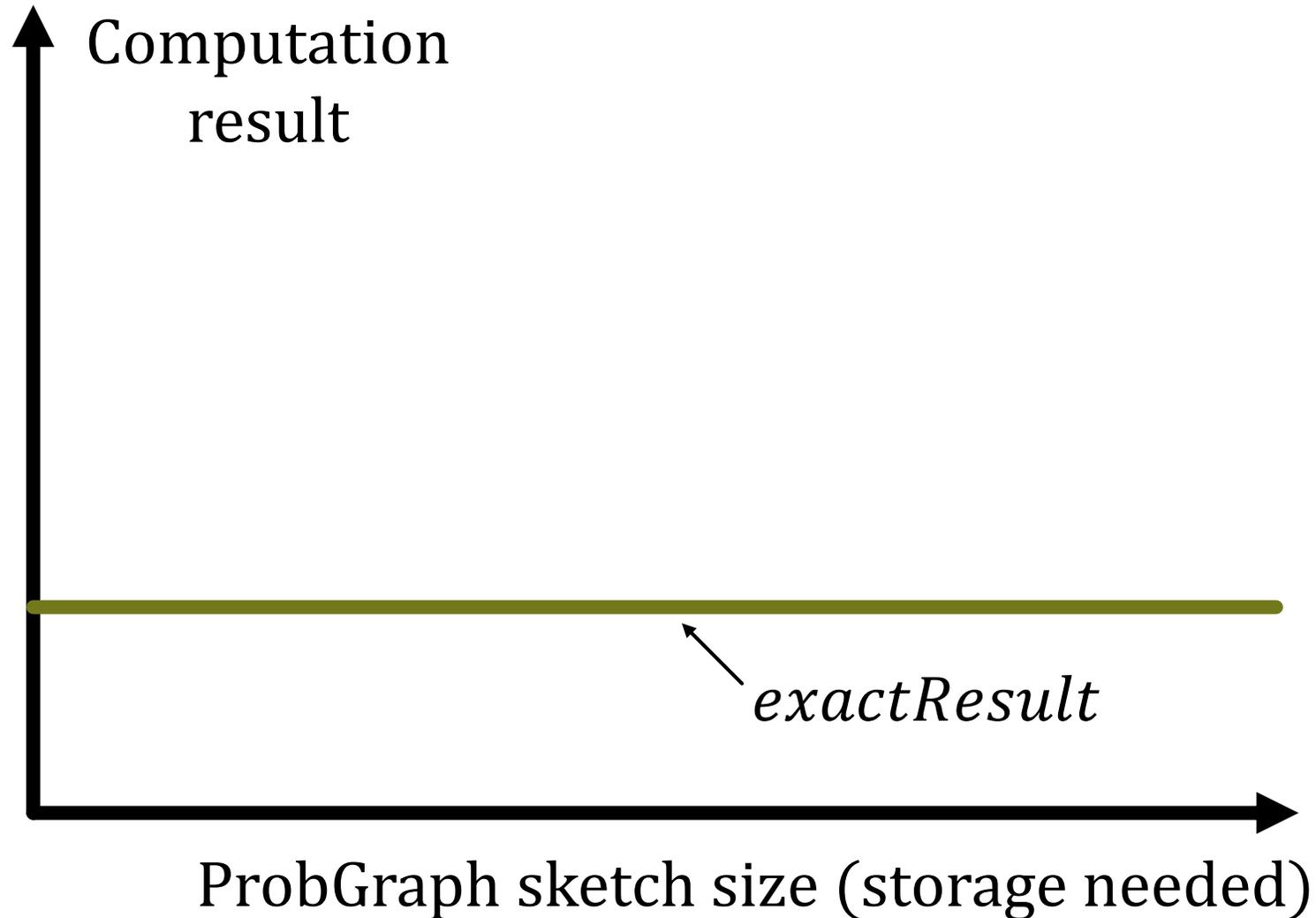
ProbGraph is consistent

The variance also converges to zero with the increasing sketch size

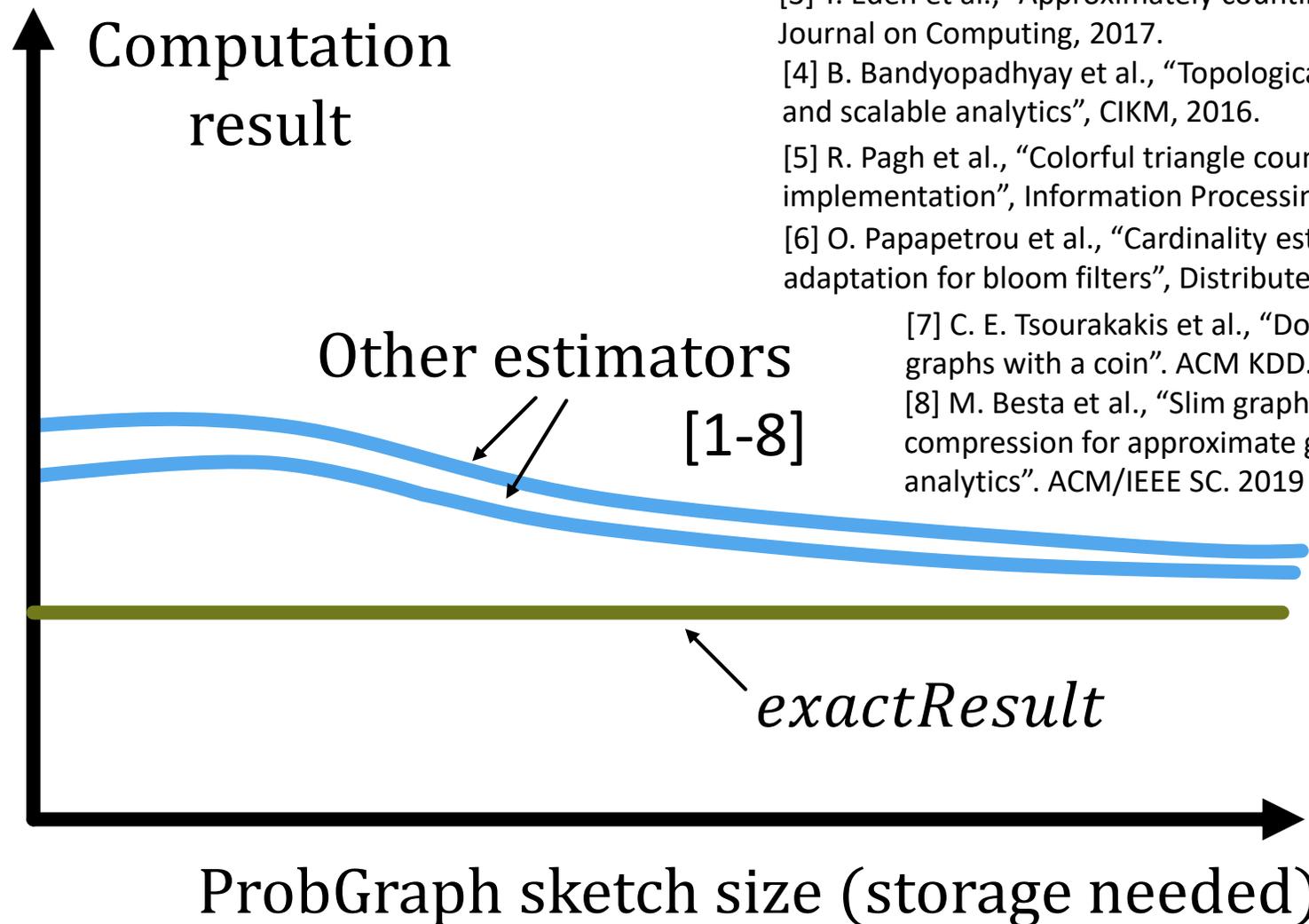


One can always find a ProbGraph sketch that delivers a required accuracy

ProbGraph is asymptotically efficient



ProbGraph is asymptotically efficient



[1] J. Tetek, "Approximate triangle counting via sampling and fast matrix multiplication", arXiv 2021.

[2] S. Assadi et al., "A simple sublinear-time algorithm for counting arbitrary subgraphs via edge sampling", arXiv 2018.

[3] T. Eden et al., "Approximately counting triangles in sublinear time", SIAM Journal on Computing, 2017.

[4] B. Bandyopadhyay et al., "Topological graph sketching for incremental and scalable analytics", CIKM, 2016.

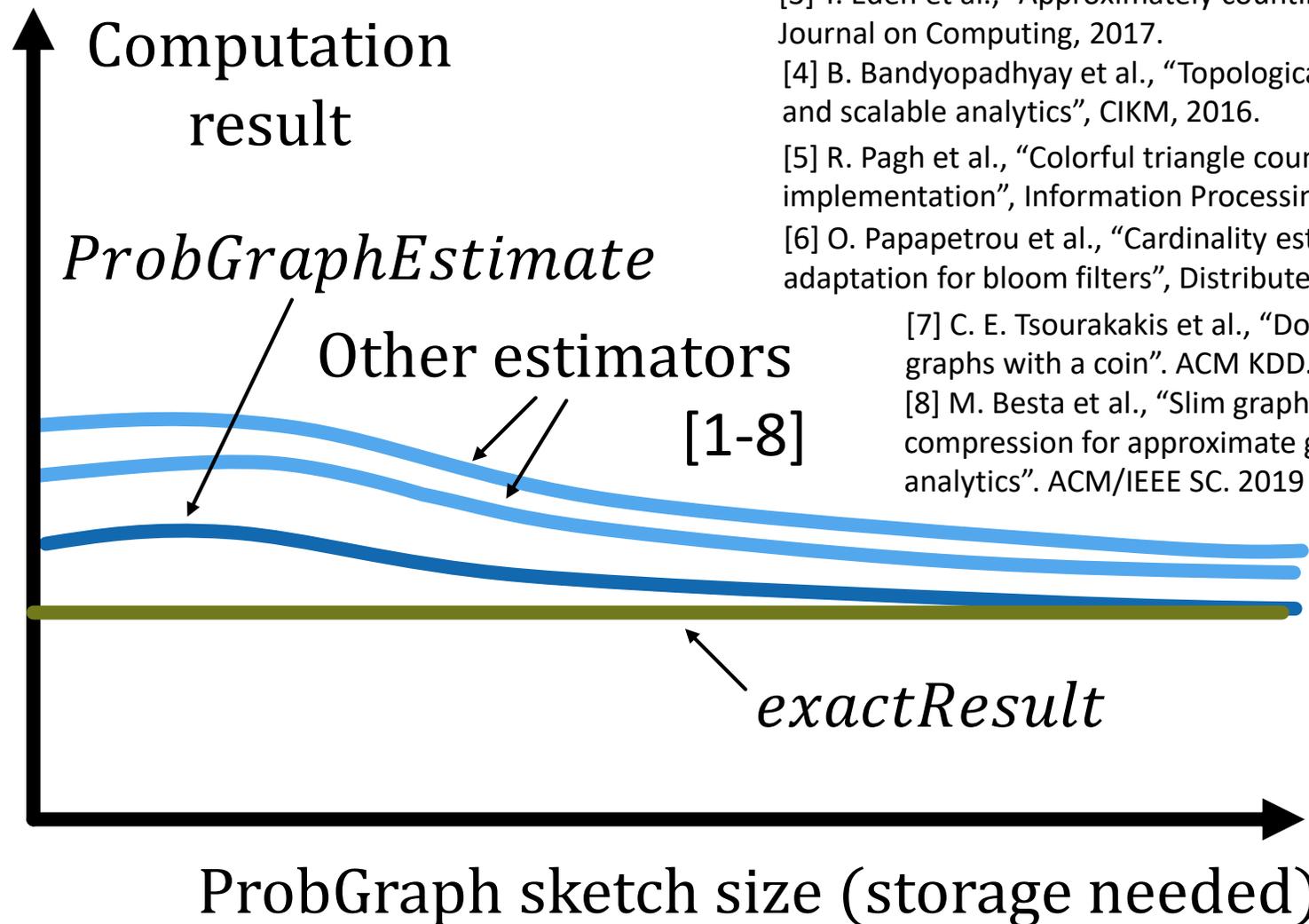
[5] R. Pagh et al., "Colorful triangle counting and a MapReduce implementation", Information Processing Letters, 2012.

[6] O. Papapetrou et al., "Cardinality estimation and dynamic length adaptation for bloom filters", Distributed and Parallel Databases, 2010.

[7] C. E. Tsourakakis et al., "Doulion: counting triangles in massive graphs with a coin". ACM KDD. 2009.

[8] M. Besta et al., "Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics". ACM/IEEE SC. 2019

ProbGraph is asymptotically efficient



[1] J. Tetek, "Approximate triangle counting via sampling and fast matrix multiplication", arXiv 2021.

[2] S. Assadi et al., "A simple sublinear-time algorithm for counting arbitrary subgraphs via edge sampling", arXiv 2018.

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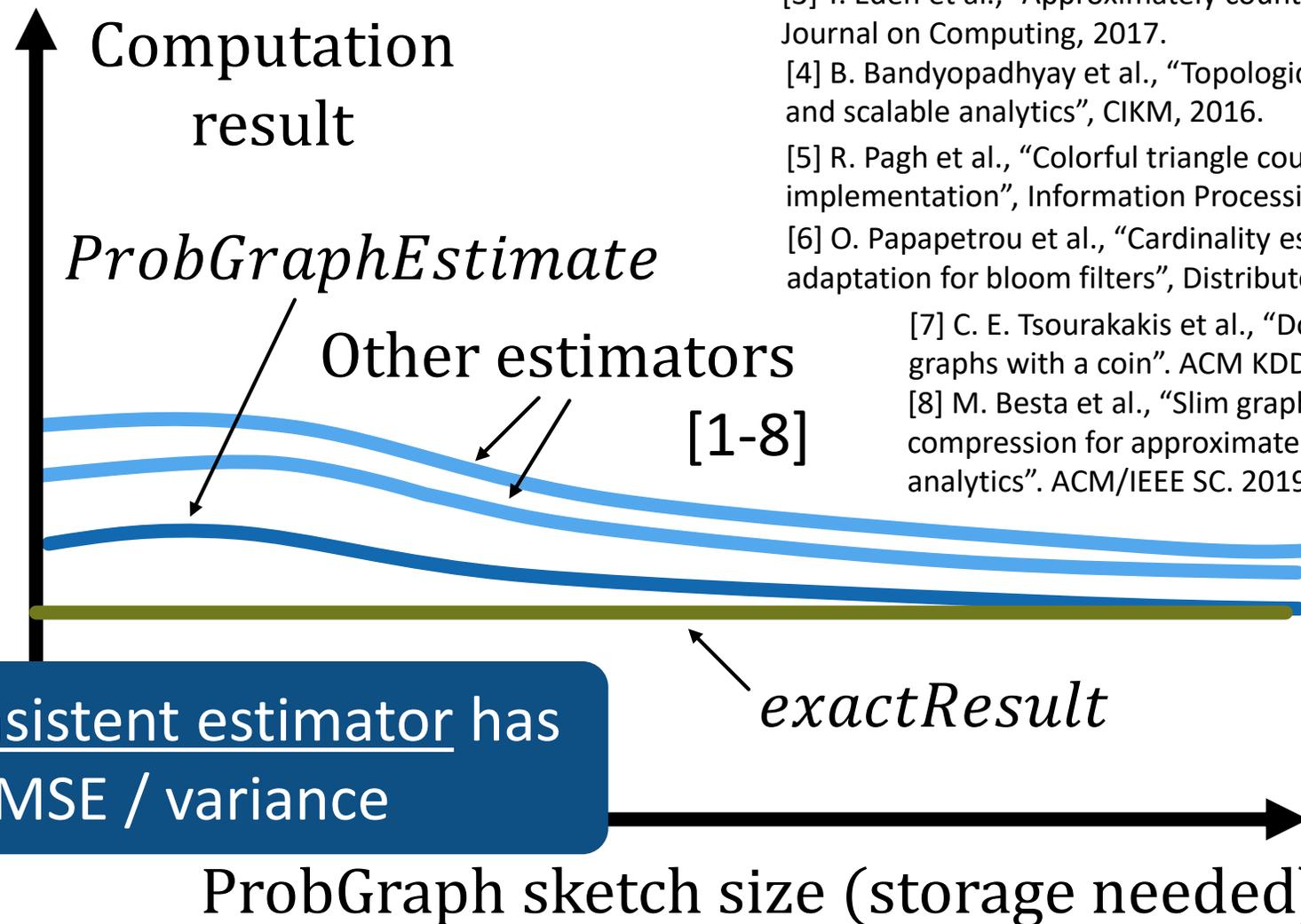
[5] R. Pagh et al., "Colorful triangle counting and a MapReduce implementation", Information Processing Letters, 2012.

[6] O. Papapetrou et al., "Cardinality estimation and dynamic length adaptation for bloom filters", Distributed and Parallel Databases, 2010.

[7] C. E. Tsourakakis et al., "Doulion: counting triangles in massive graphs with a coin". ACM KDD. 2009.

[8] M. Besta et al., "Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics". ACM/IEEE SC. 2019

ProbGraph is asymptotically efficient



[1] J. Tetek, "Approximate triangle counting via sampling and fast matrix multiplication", arXiv 2021.

[2] S. Assadi et al., "A simple sublinear-time algorithm for counting arbitrary subgraphs via edge sampling", arXiv 2018.

[3] T. Eden et al., "Approximately counting triangles in sublinear time", SIAM Journal on Computing, 2017.

[4] B. Bandyopadhyay et al., "Topological graph sketching for incremental and scalable analytics", CIKM, 2016.

[5] R. Pagh et al., "Colorful triangle counting and a MapReduce implementation", Information Processing Letters, 2012.

[6] O. Papapetrou et al., "Cardinality estimation and dynamic length adaptation for bloom filters", Distributed and Parallel Databases, 2010.

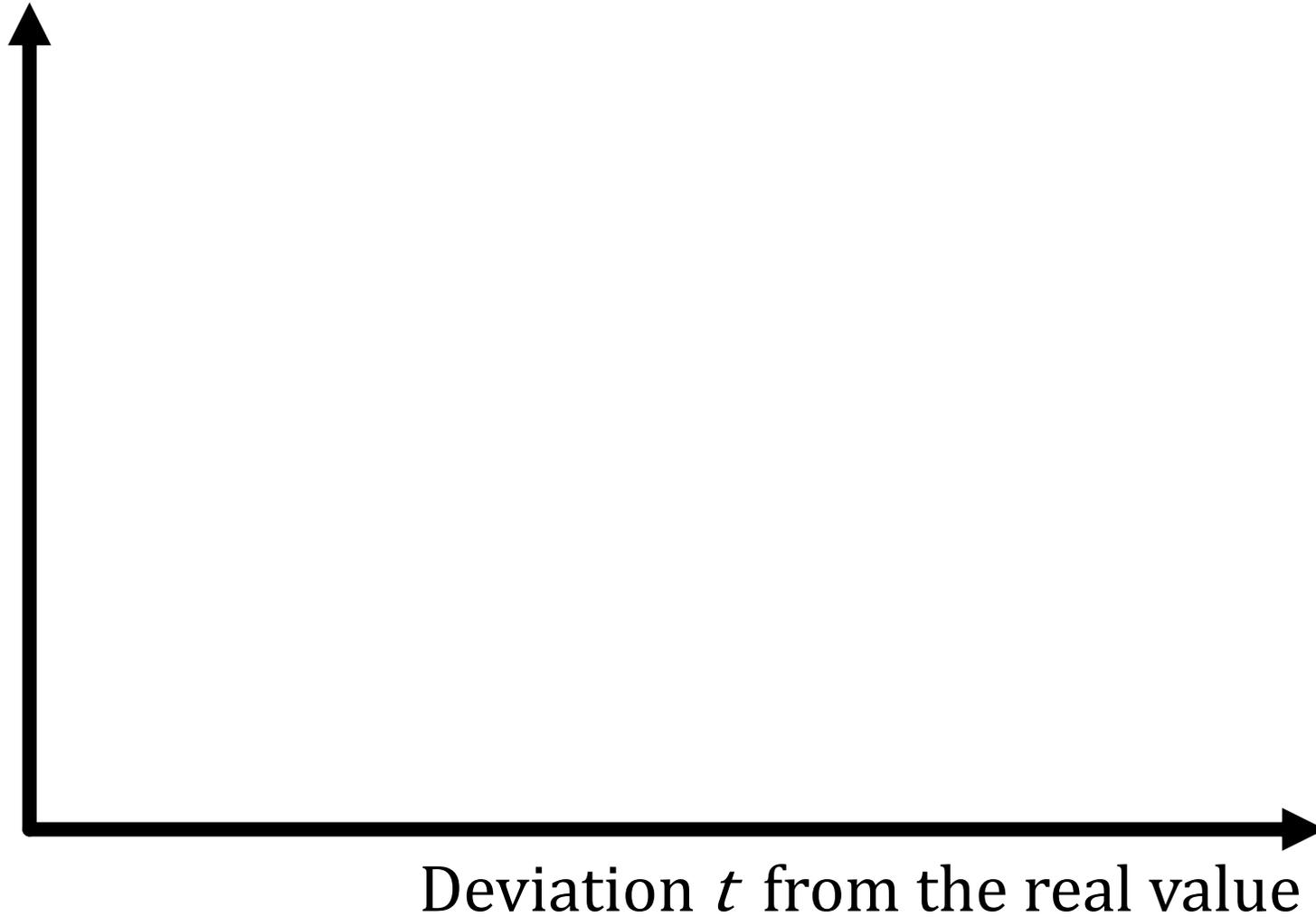
[7] C. E. Tsourakakis et al., "Doulion: counting triangles in massive graphs with a coin". ACM KDD. 2009.

[8] M. Besta et al., "Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics". ACM/IEEE SC. 2019

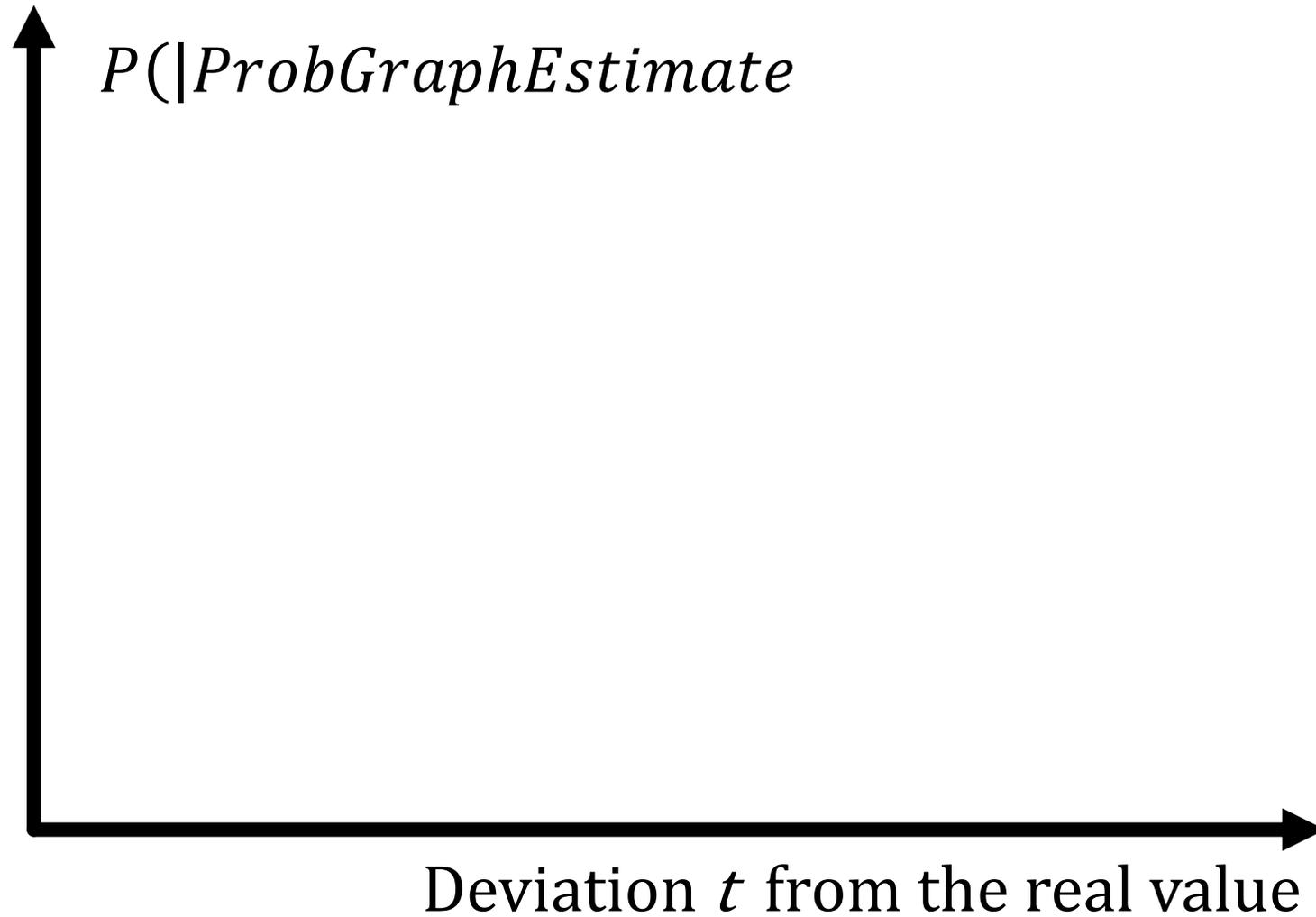
ProbGraph has strong concentration bounds



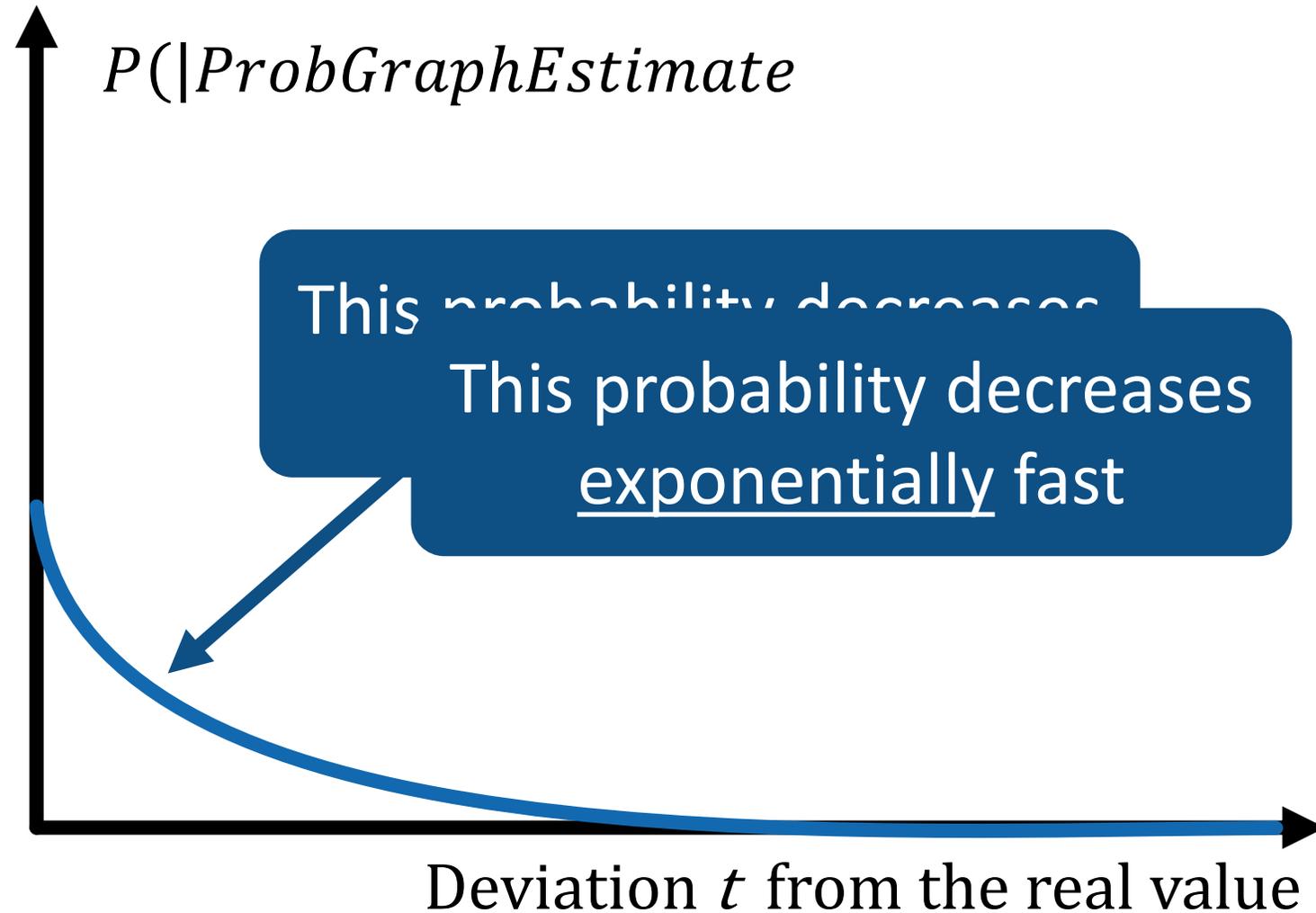
ProbGraph has strong concentration bounds



ProbGraph has strong concentration bounds

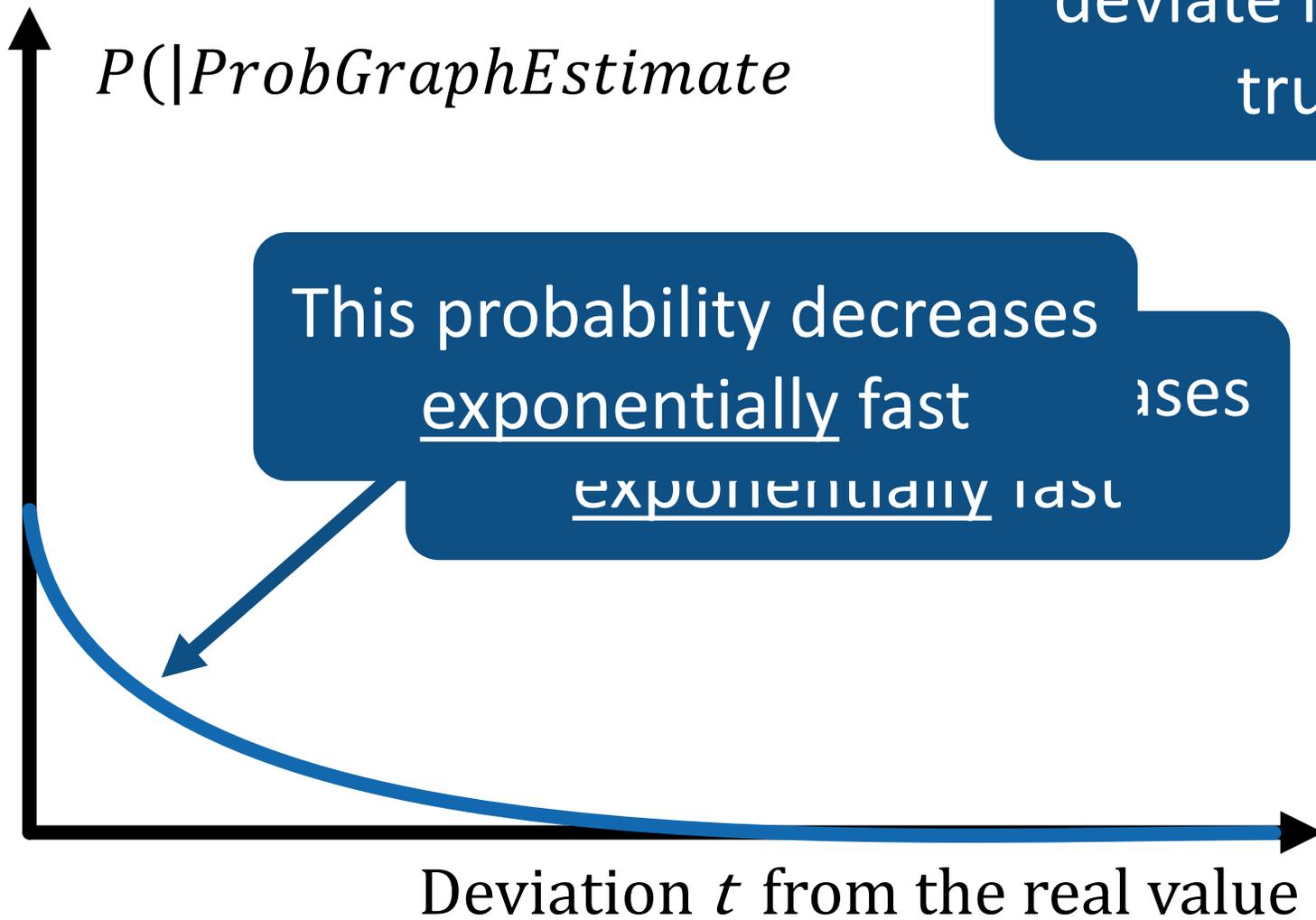


ProbGraph has strong concentration bounds

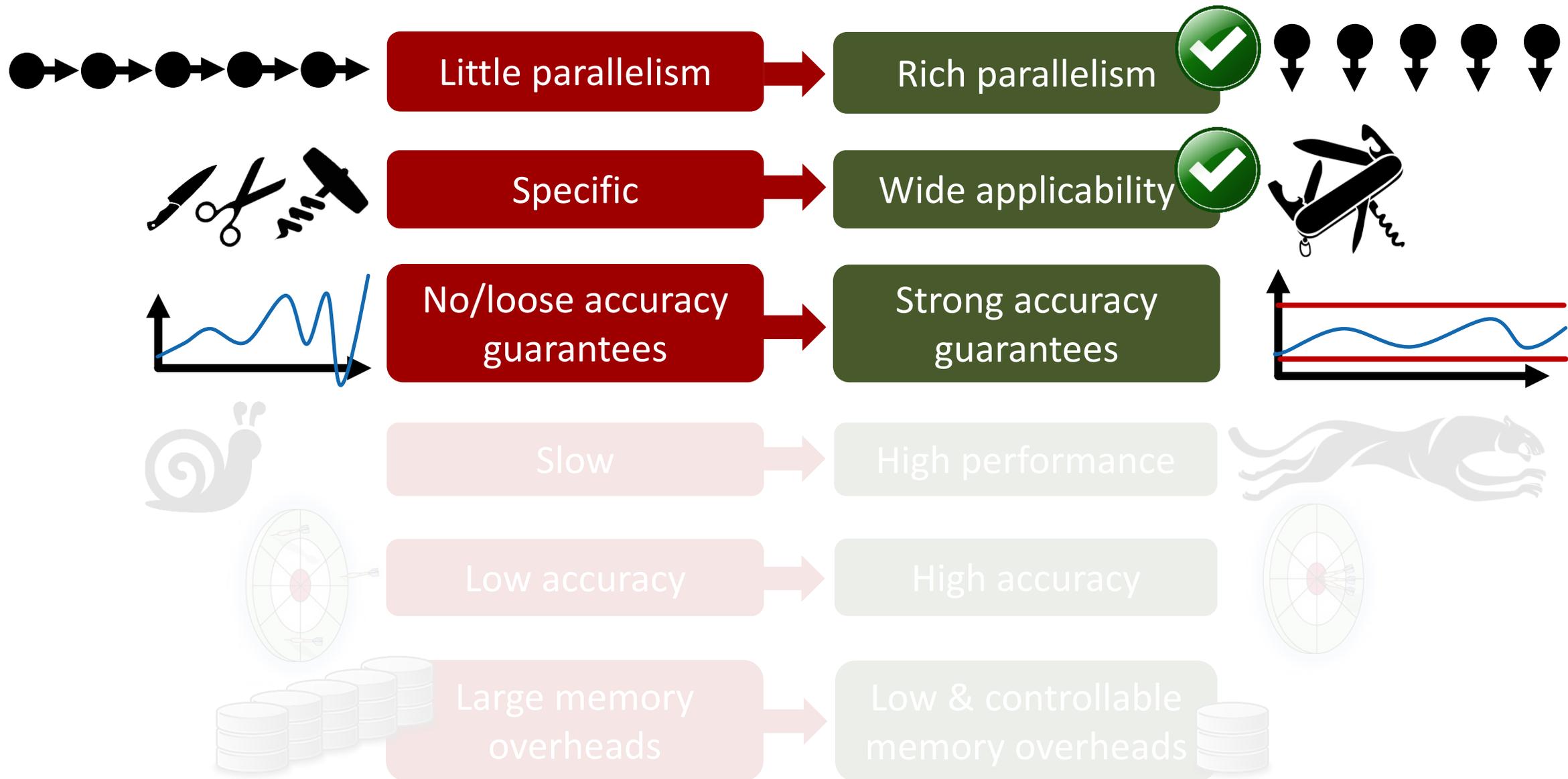


ProbGraph has strong concentration bounds

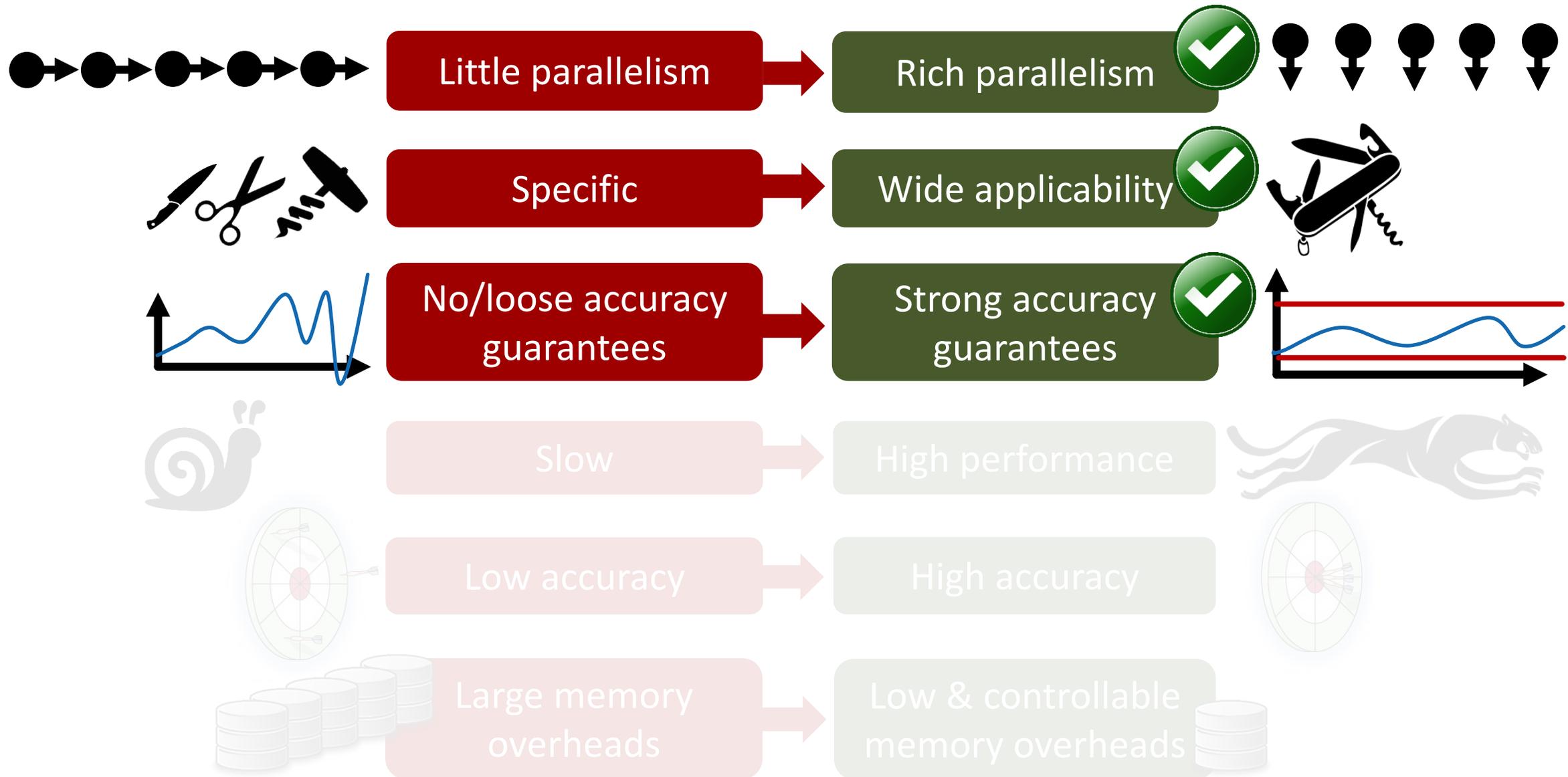
ProbGraph is unlikely to deviate much from the true values



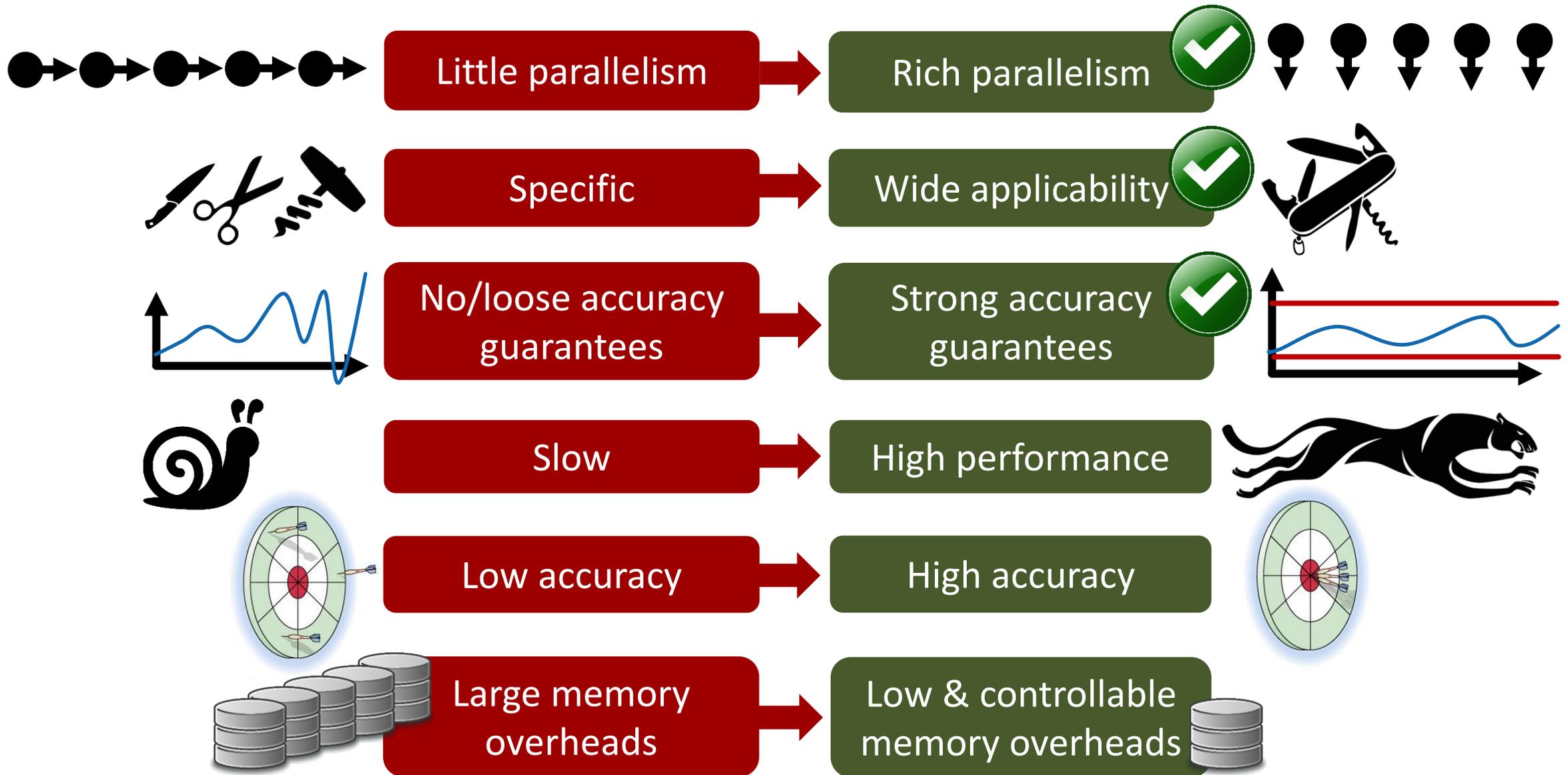
Approximate Graph Processing: Our Objectives



Approximate Graph Processing: Our Objectives



Approximate Graph Processing: Our Objectives



Evaluation: Used Machines & Objectives



Evaluation: Used Machines & Objectives



**CSCS Cray Piz Daint,
64 GB per compute node**

Evaluation: Used Machines & Objectives



Dell PowerEdge R910 server

CSCS Cray Piz Daint,
64 GB per compute node

Evaluation: Used Machines & Objectives

Goal: One design with...
large speedups +
small & controlled accuracy loss +
small & controlled memory requirements



Dell PowerEdge R910 server

CSCS Cray Piz Daint,
64 GB per compute node

Considered Graph Datasets

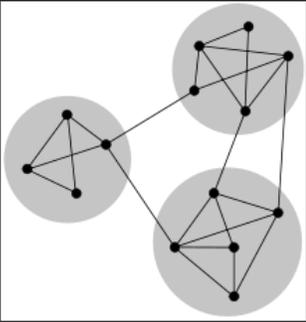
Considered Graph Datasets

67 graph datasets,
15 areas,
5 major graph
dataset repositories

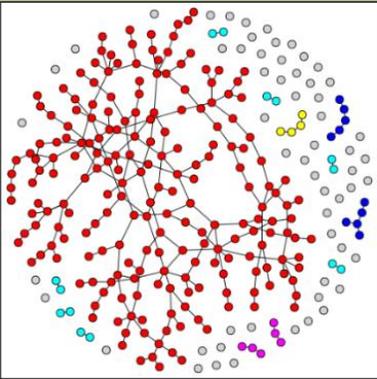
Considered Graph Datasets

67 graph datasets,
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Synthetic graphs



Kronecker [1]

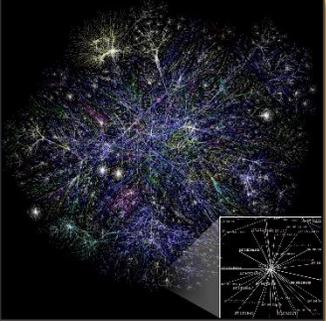


Erdős-Rényi [2]

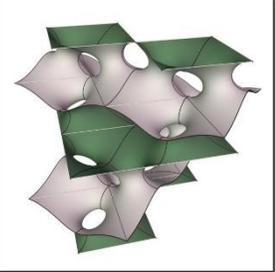
Real-world graphs



Social networks

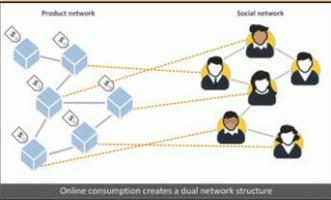


Web graphs



Mathematics

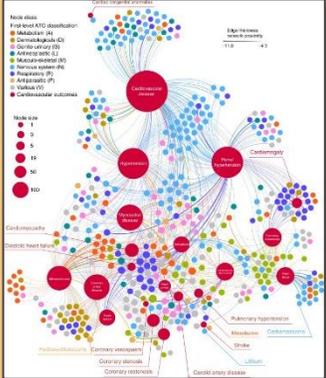
Purchases



Online consumption creates a dual network structure

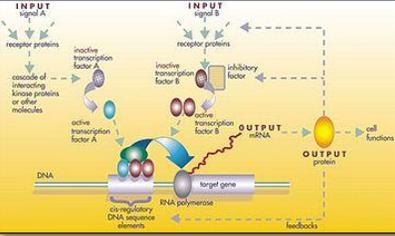


Road nets



Medicine

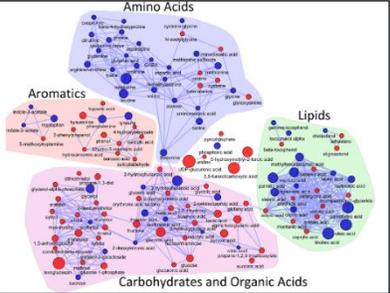
Gene functions



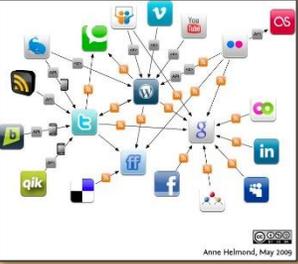
Brain structure



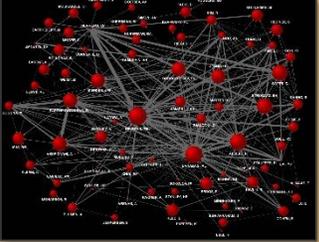
Chemistry



Chemistry



Communication



Citation graphs



Compute graphs



Economic nets

[1] J. Leskovec et al. Kronecker Graphs: An Approach to Modeling Networks. J. Mach. Learn. Research. 2010.

[2] P. Erdos and A. Renyi. On the evolution of random graphs. Pub. Math. Inst. Hun. A. Science. 1960.

Considered Graph Datasets

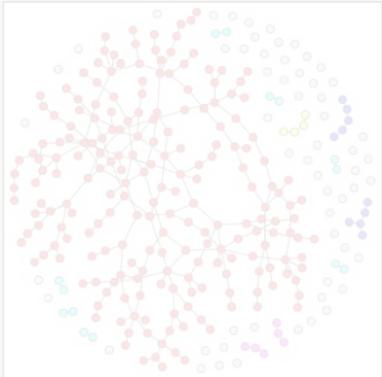
67
5
dataset repositories

Highly irregular data

Synthetic graphs



Kronecker [1]



Erdős-Rényi [2]

Real-world graphs

Purchases

Gene functions

Communication

Brain structure

Citation graphs

Compute graphs

Economic nets

Mathematics

Medicine

Chemistry

Lots of load imbalance

[1] J. Leskovec et al. Kronecker Graphs: An Approach to Modeling Networks. J. Mach. Learn. Research. 2010.
 [2] P. Erdos and A. Renyi. On the evolution of random graphs. Pub. Math. Inst. Hun. A. Science. 1960.

Triangle Counting

Cores/threads: 32

Max memory

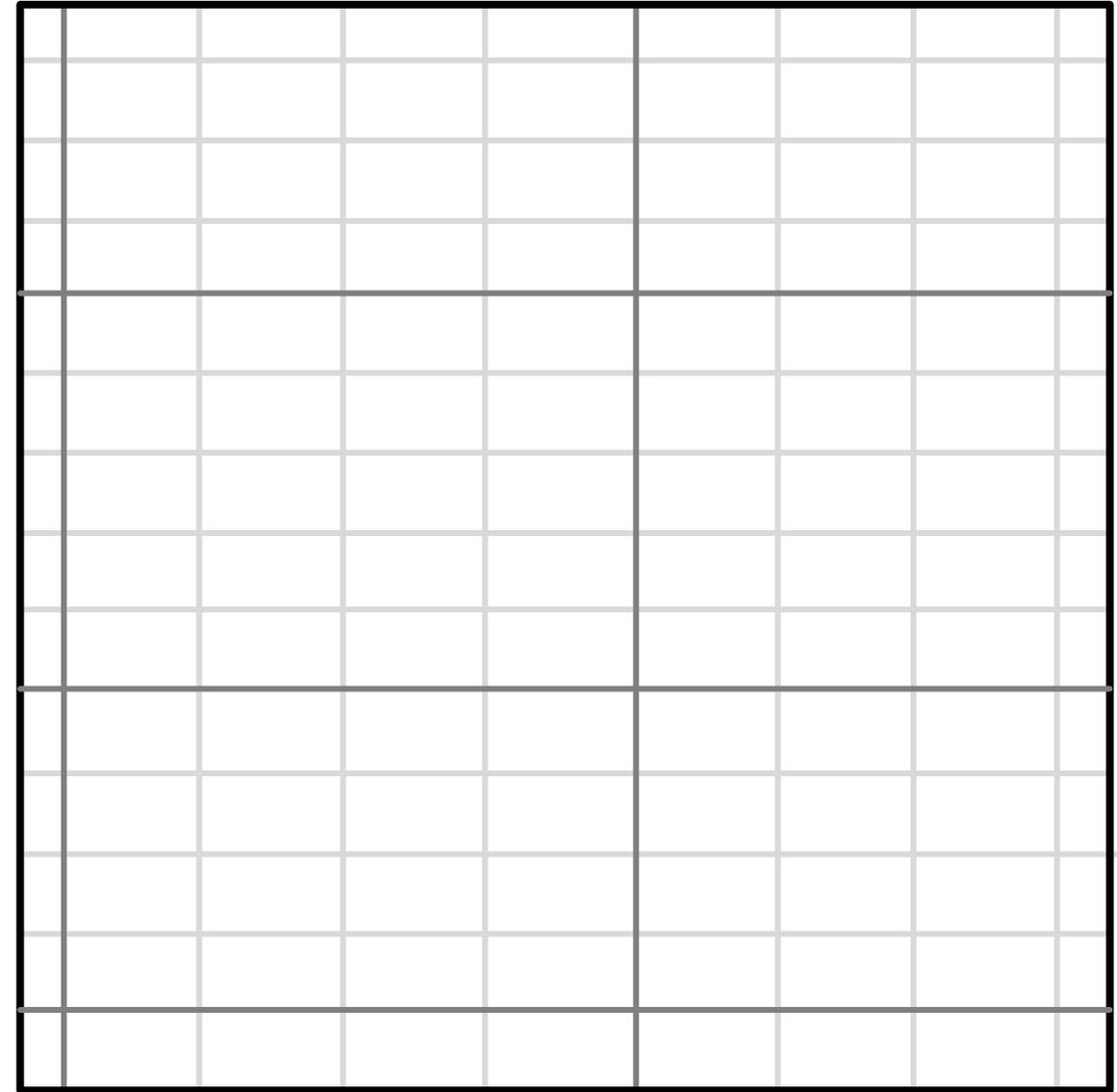
overhead: 20%

Triangle Counting

Cores/threads: 32

Max memory

overhead: 20%

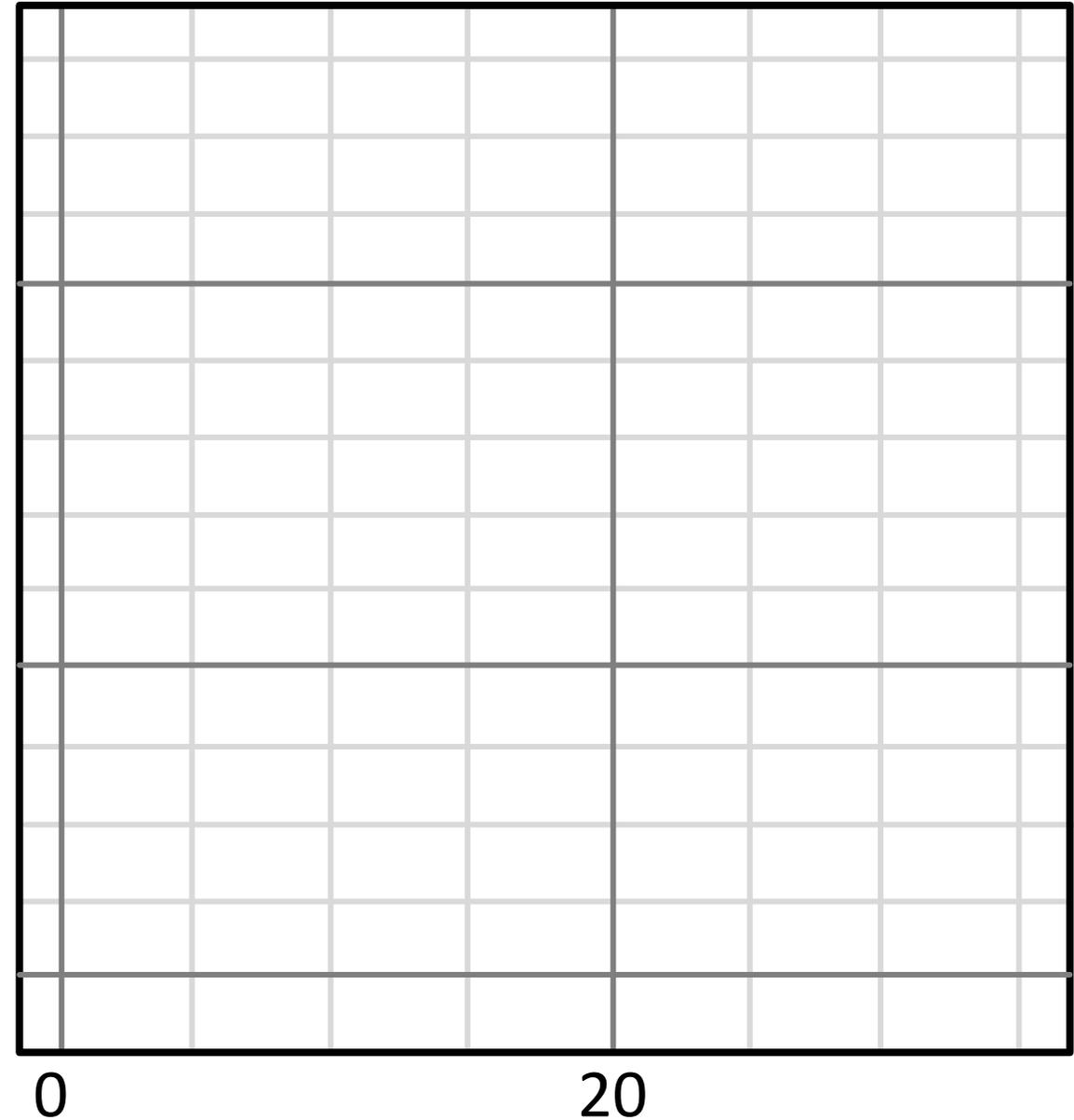


Triangle Counting

Cores/threads: 32

Max memory

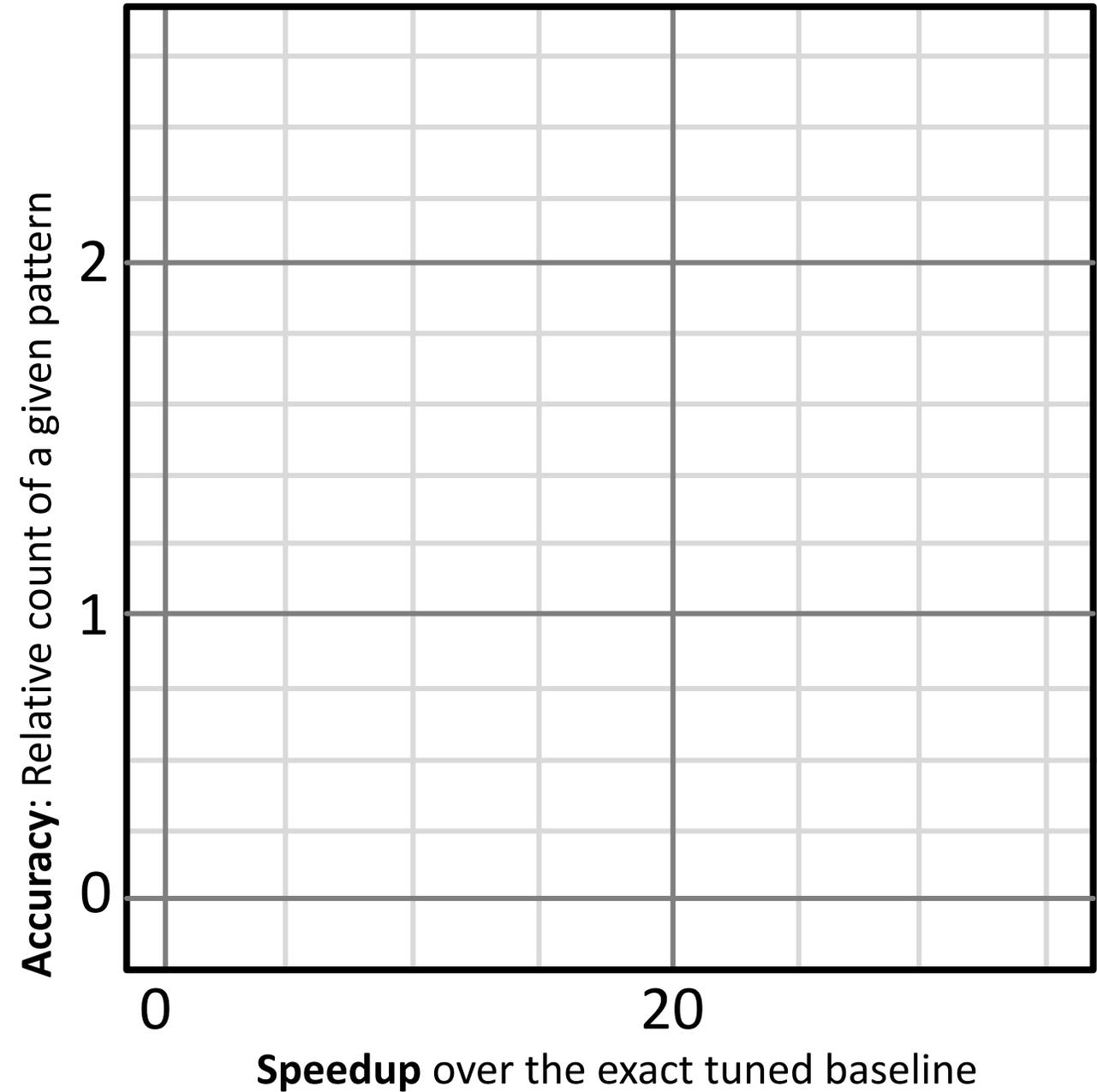
overhead: 20%



Speedup over the exact tuned baseline

Triangle Counting

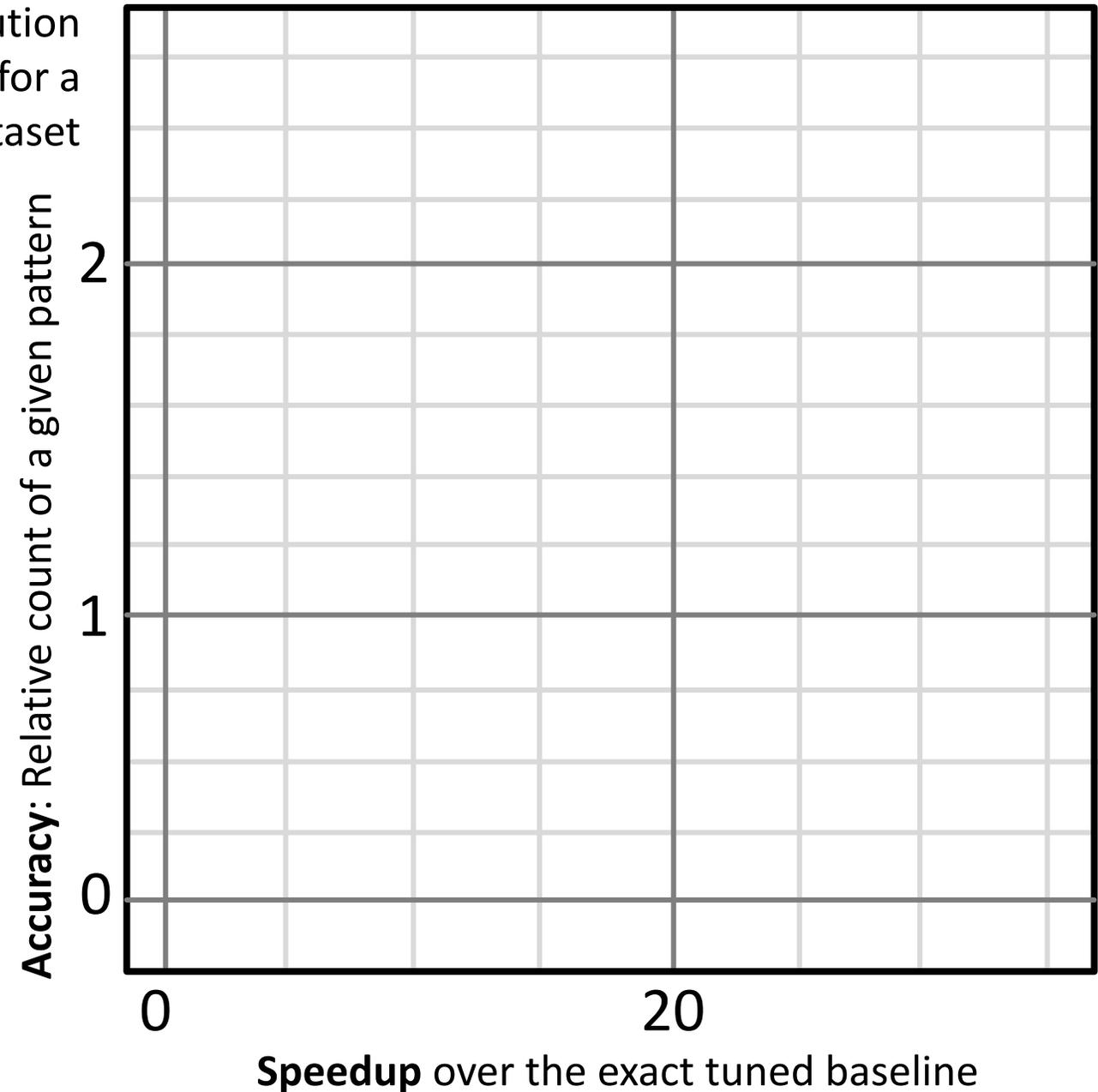
Cores/threads: 32
Max memory overhead: 20%



Triangle Counting

Cores/threads: 32
Max memory overhead: 20%

Each data point: the execution of a given scheme for a specific graph dataset



Triangle Counting

Cores/threads: 32

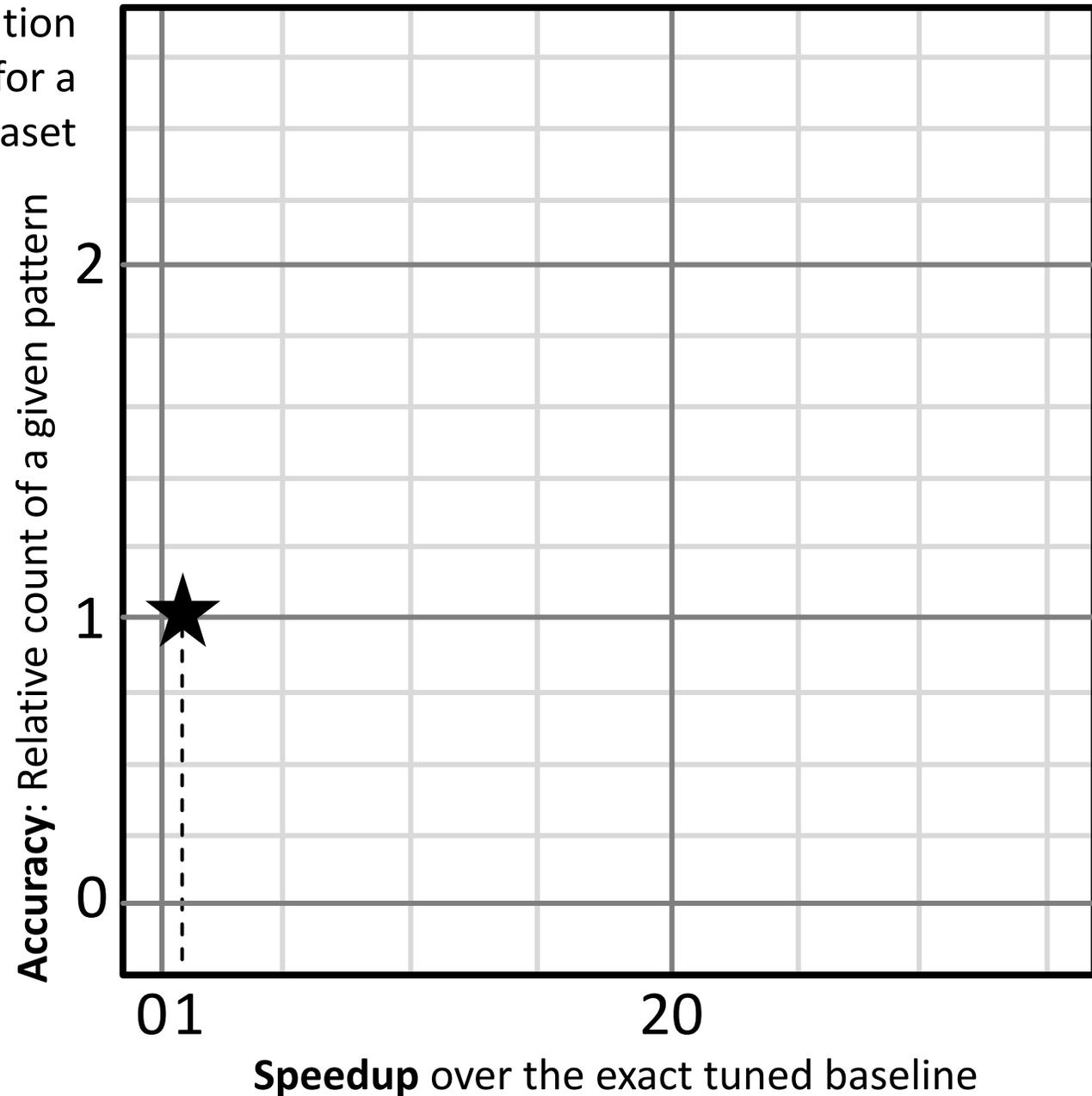
Max memory

overhead: 20%

Each data point: the execution of a given scheme for a specific graph dataset



Exact
baseline [1]

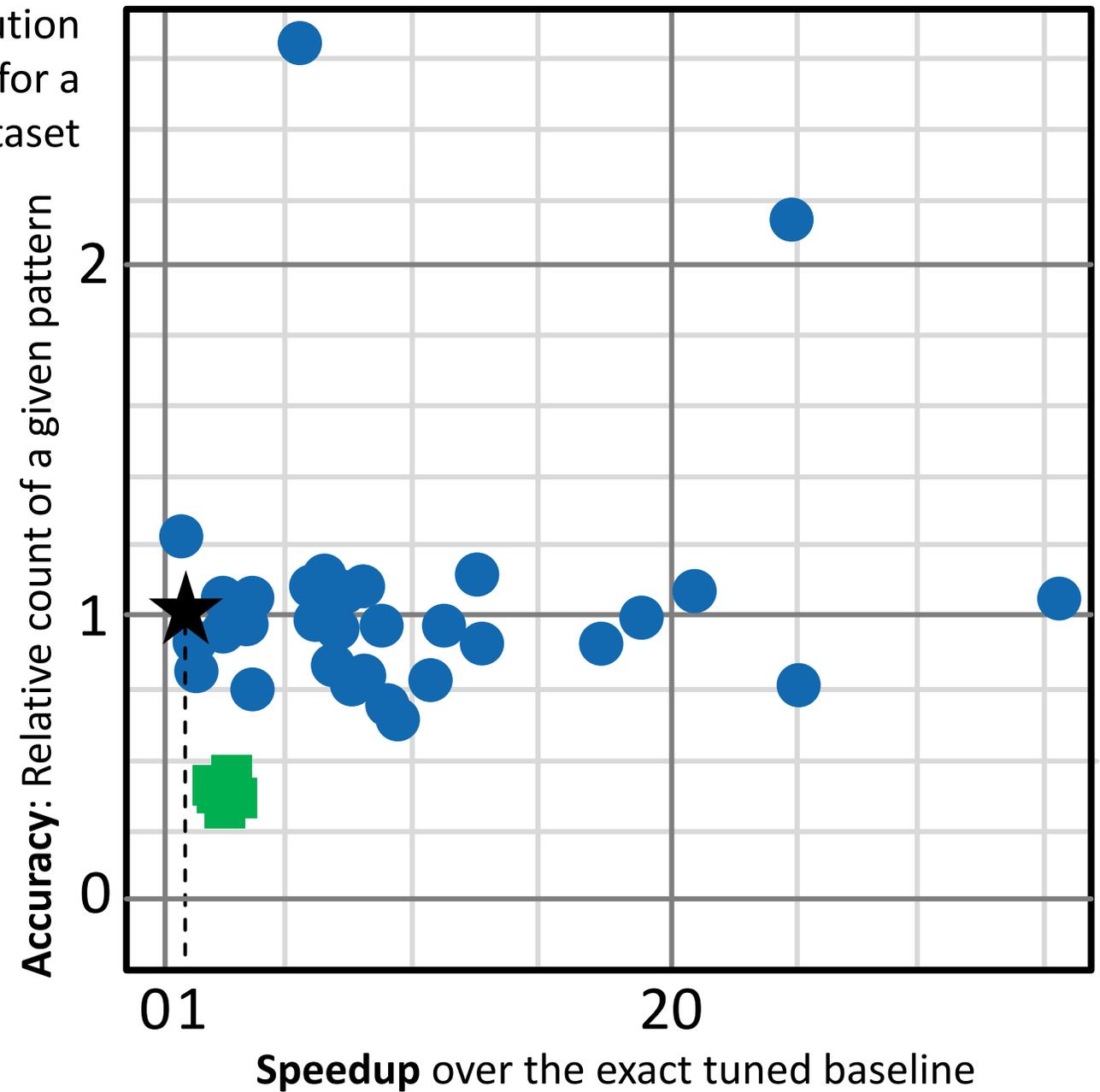
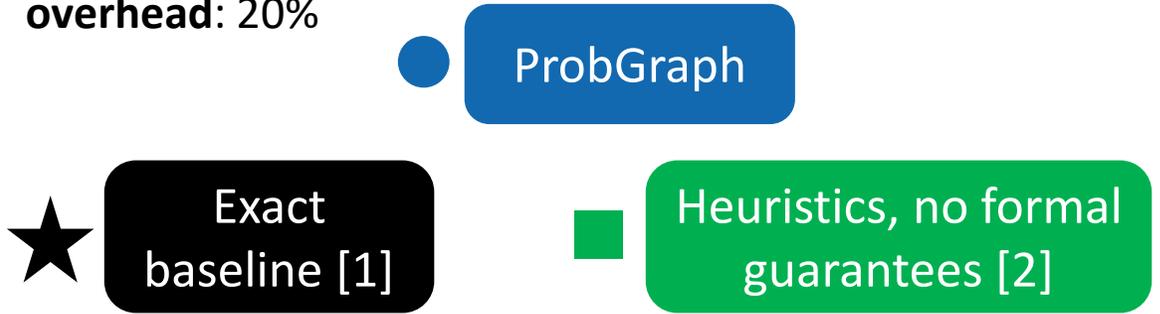


[1] S. Beamer et al., „The GAP Benchmark Suite“. 2015

Triangle Counting

Cores/threads: 32
Max memory overhead: 20%

Each data point: the execution of a given scheme for a specific graph dataset



[1] S. Beamer et al., „The GAP Benchmark Suite”. 2015
 [2] S. Singh et al., “Scalable and performant graph processing on GPUs using approximate computing”. IEEE TMSCS. 2018

Triangle Counting

Cores/threads: 32

Max memory overhead: 20%

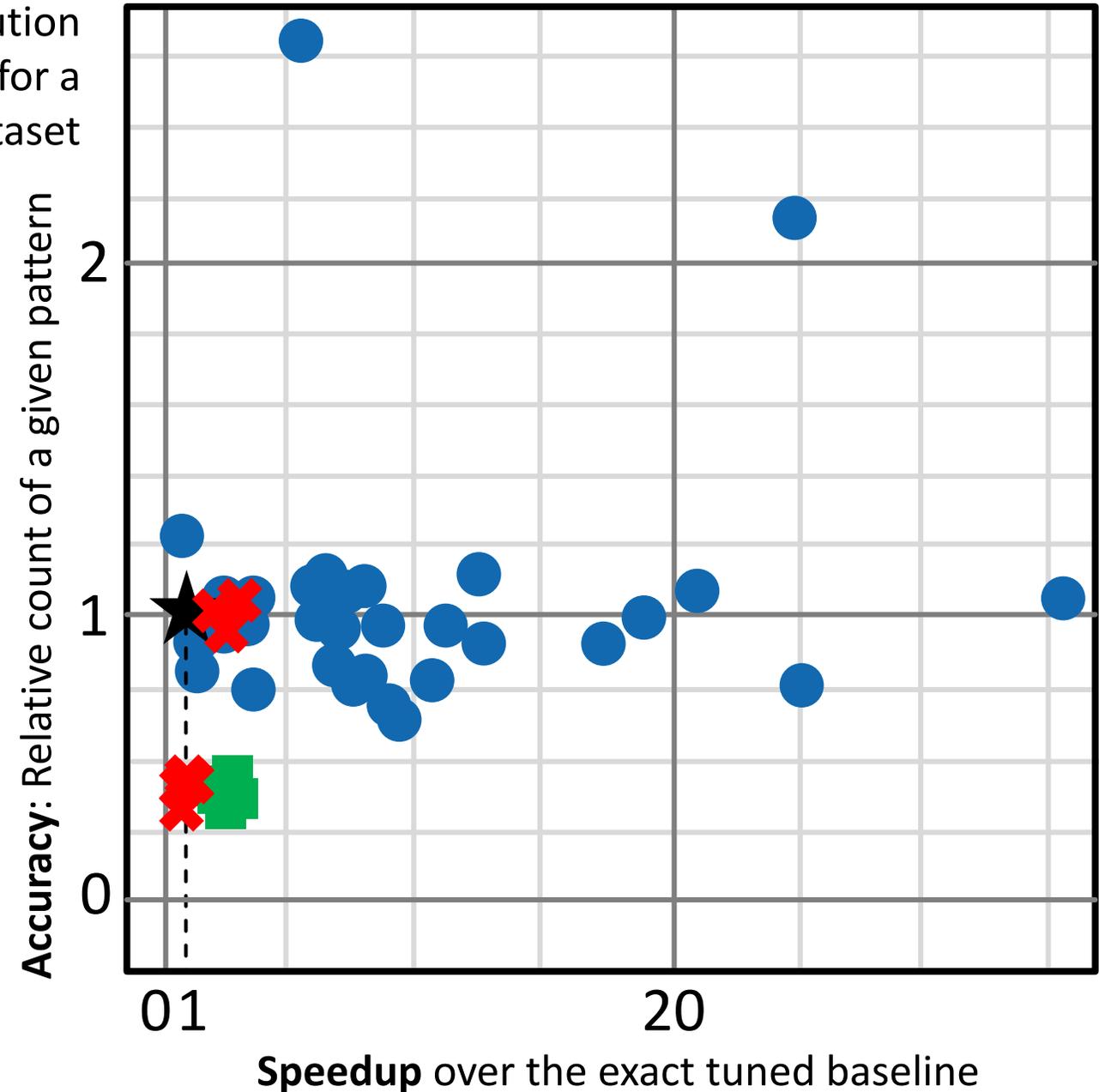
Each data point: the execution of a given scheme for a specific graph dataset

● ProbGraph

★ Exact baseline [1]

■ Heuristics, no formal guarantees [2]

✗ Heuristics, formal guarantees [3-4]



[1] S. Beamer et al., „The GAP Benchmark Suite”. 2015
 [2] S. Singh et al., “Scalable and performant graph processing on GPUs using approximate computing”. IEEE TMSCS. 2018
 [3] R. Pagh et al., “Colorful triangle counting and a mapreduce implementation”. Information Processing Letters. 2012
 [4] Z. Shang et al., “Auto-approximation of graph computing”. VLDB. 2014

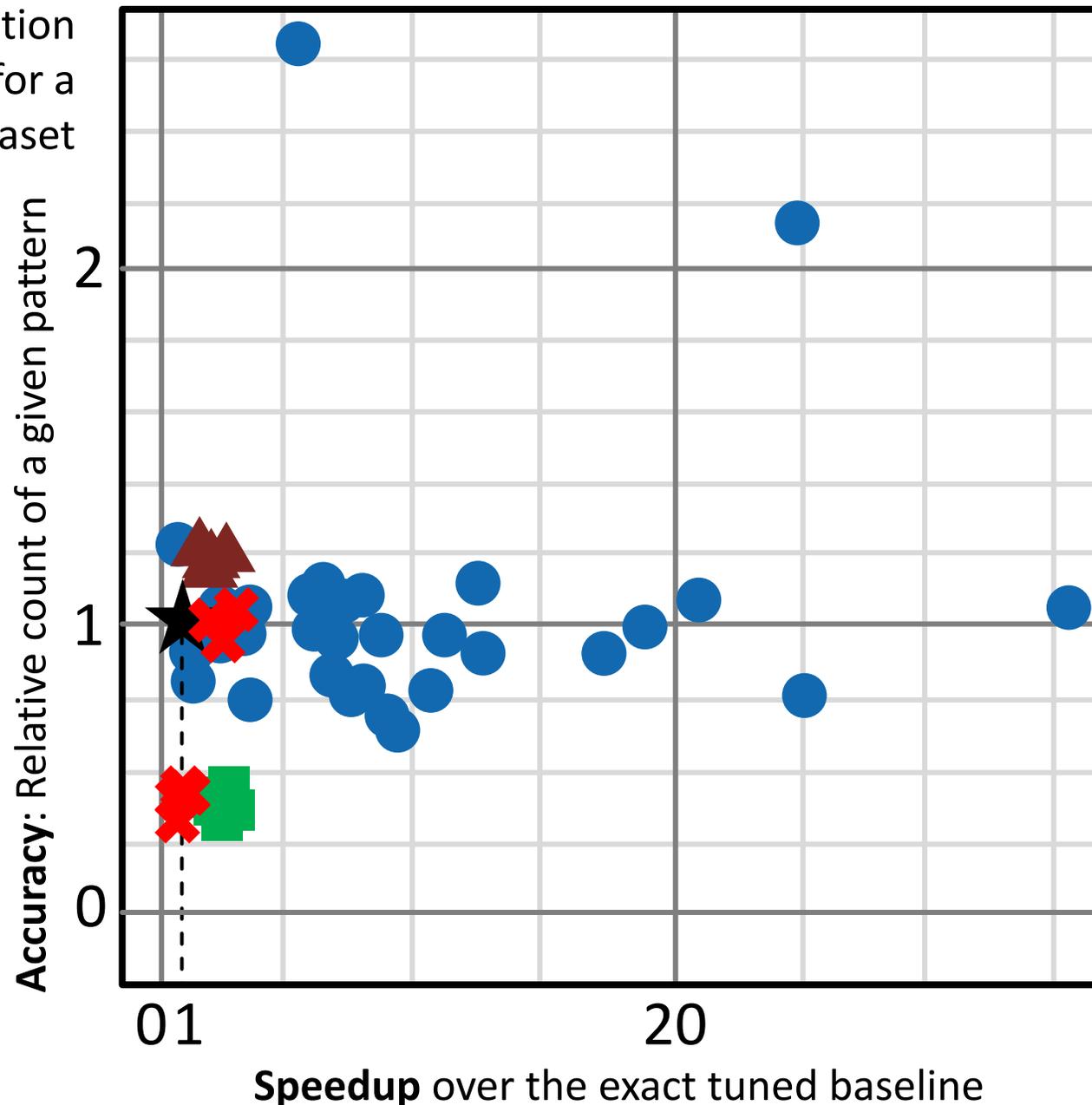
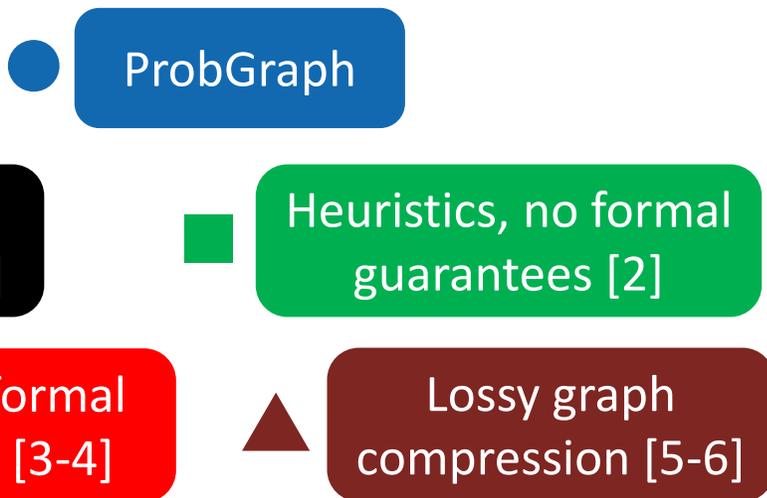
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Cores/threads: 32

Max memory

overhead: 20%

Each data point: the execution of a given scheme for a specific graph dataset



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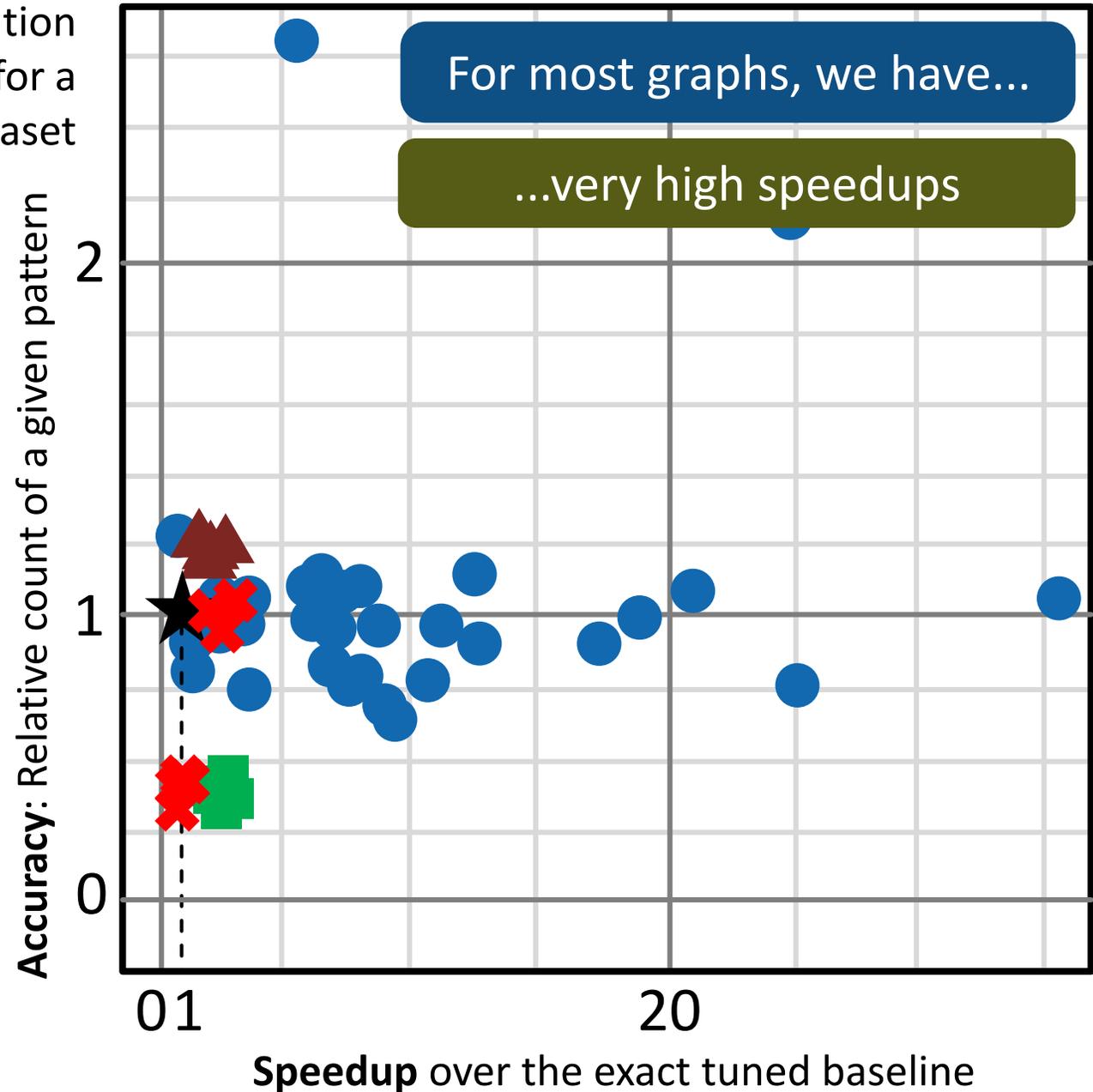
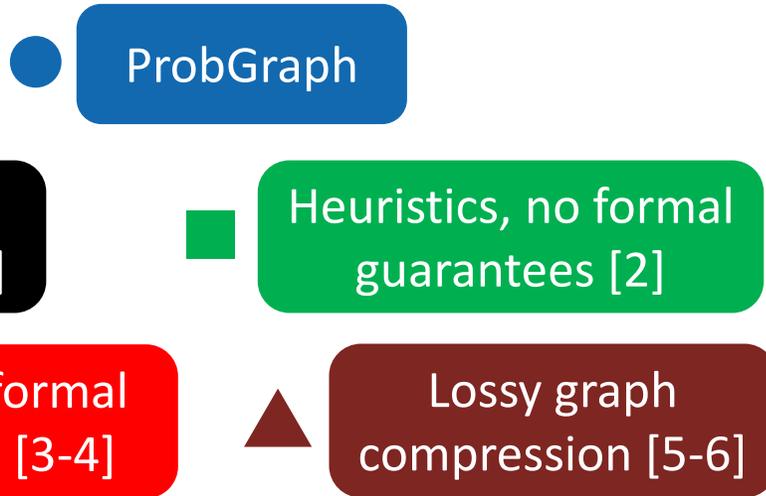
[5] C. E. Tsourakakis et al., “Doulion: counting triangles in massive graphs with a coin”. ACM KDD. 2009.

[6] M. Besta et al., “Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics”. ACM/IEEE SC. 2019

Triangle Counting

Each data point: the execution of a given scheme for a specific graph dataset

Cores/threads: 32
Max memory overhead: 20%



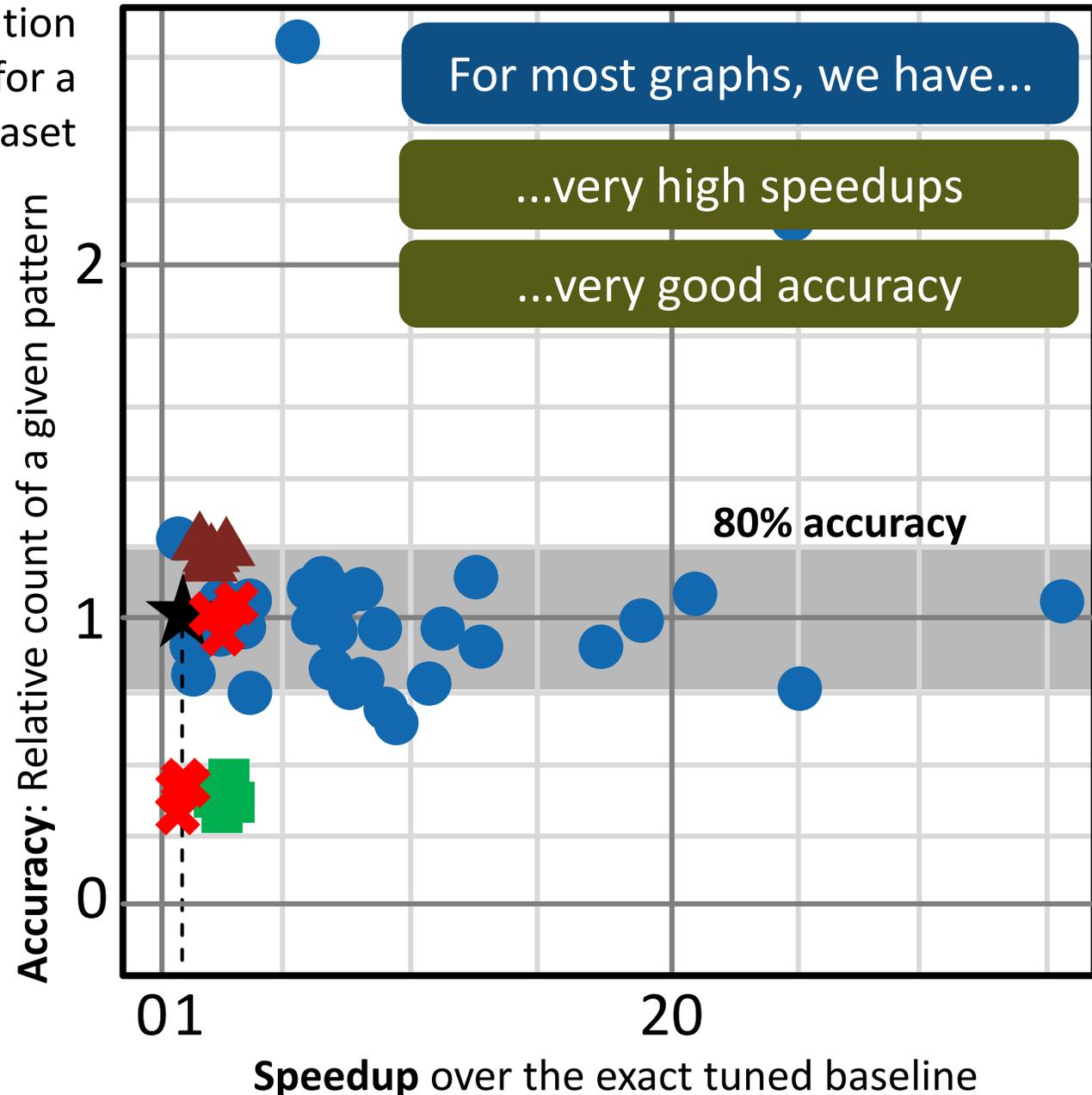
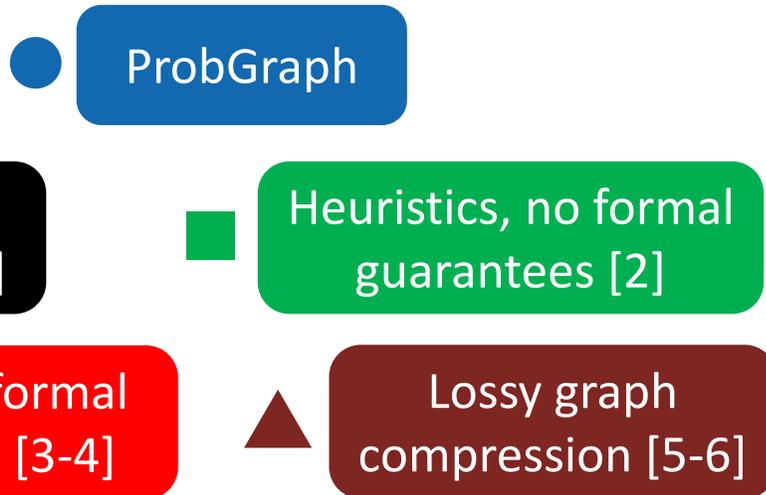
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Triangle Counting

Cores/threads: 32

Max memory overhead: 20%

Each data point: the execution of a given scheme for a specific graph dataset

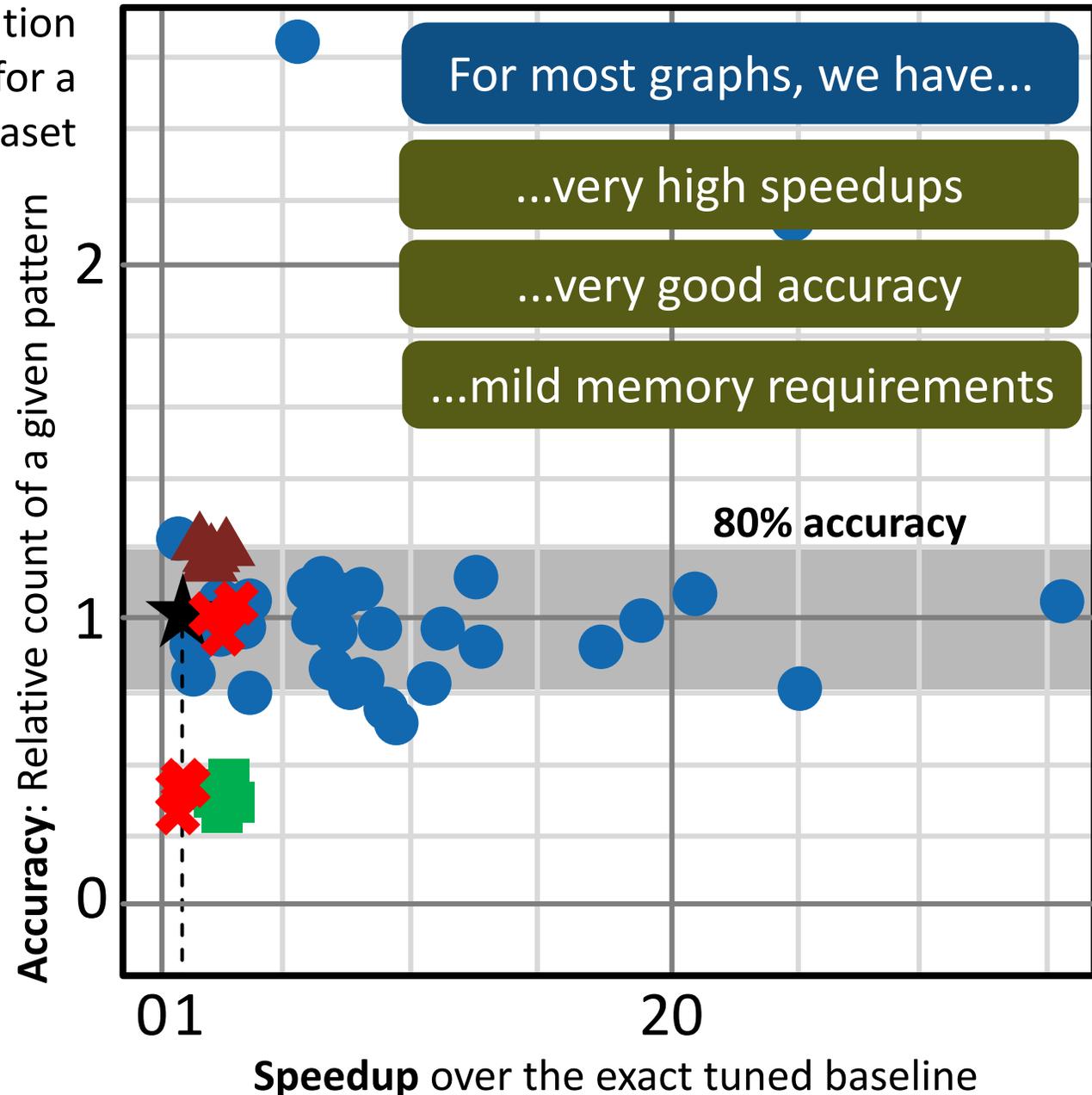
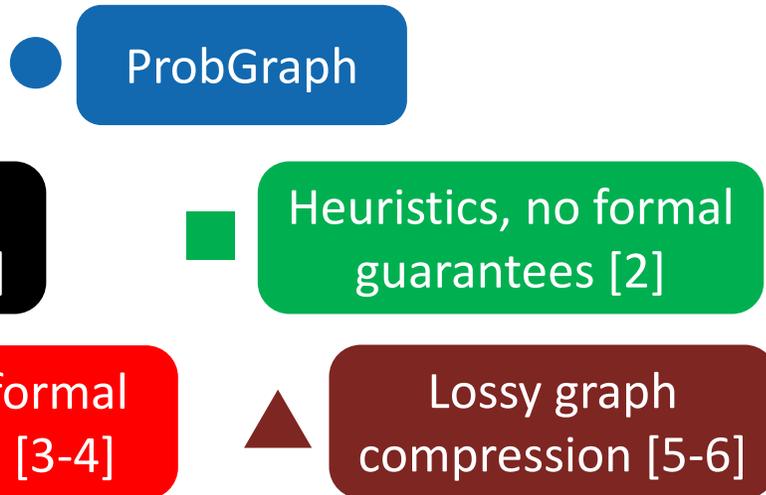


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Triangle Counting

Cores/threads: 32
Max memory overhead: 20%

Each data point: the execution of a given scheme for a specific graph dataset



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 [4] Z. Shang et al., “Auto-approximation of graph computing”. VLDB. 2014
 [5] C. E. Tsourakakis et al., “Doulion: counting triangles in massive graphs with a coin”. ACM KDD. 2009.
 [6] M. Besta et al., “Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics”. ACM/IEEE SC. 2019

4-Clique Counting

Each data point: the execution of a given scheme for a specific graph dataset

Cores/threads: 32

Max memory

overhead: 20%

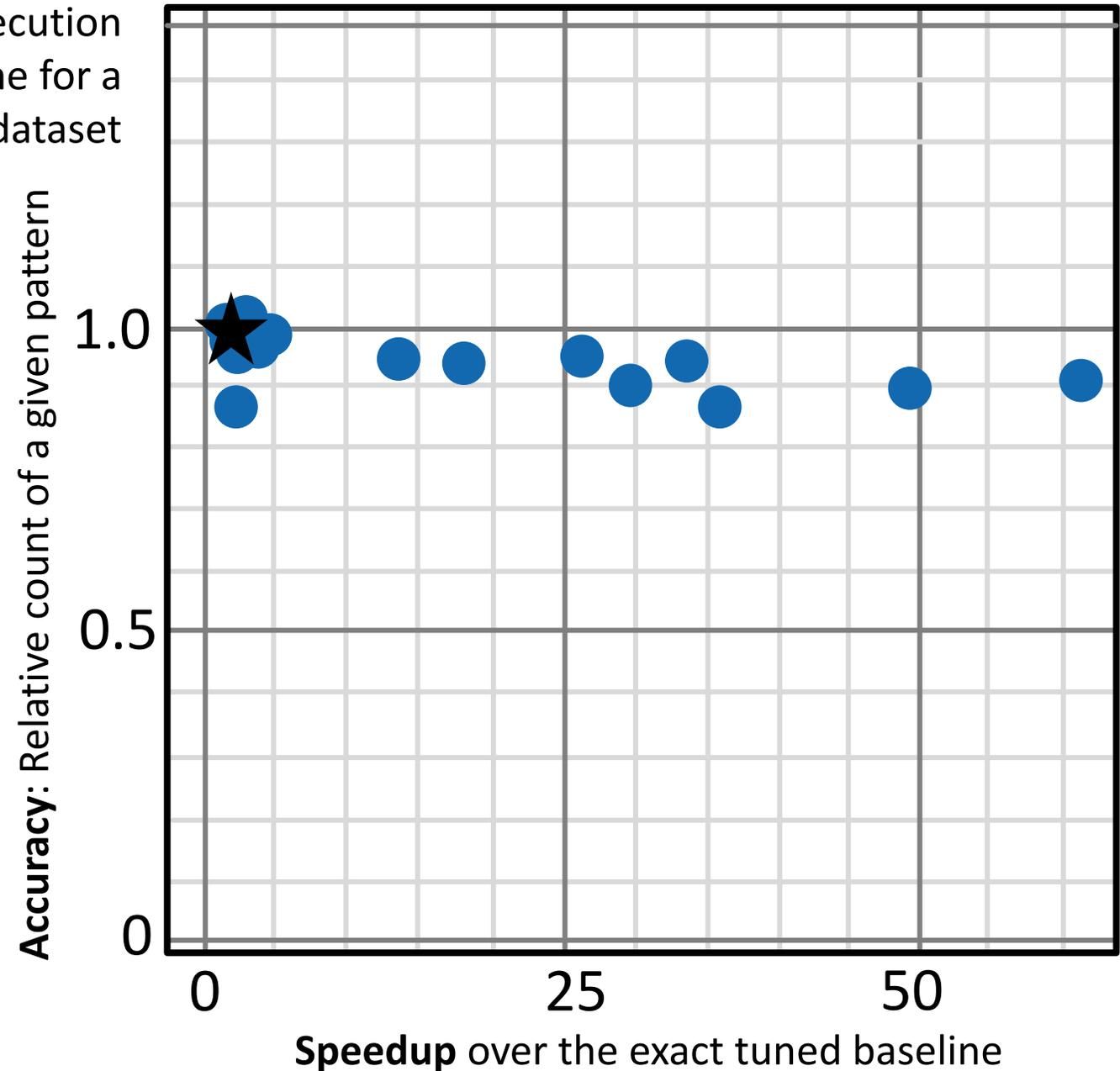


For most graphs, we have...

...very high speedups

...very good accuracy

...mild memory requirements



[1] based on S. Beamer et al., „The GAP Benchmark Suite“. 2015

4-Clique Counting

Each data point: the execution of a given scheme for a specific graph dataset

Cores/threads: 32

Max memory

overhead: 20%

★ Exact baseline [1]

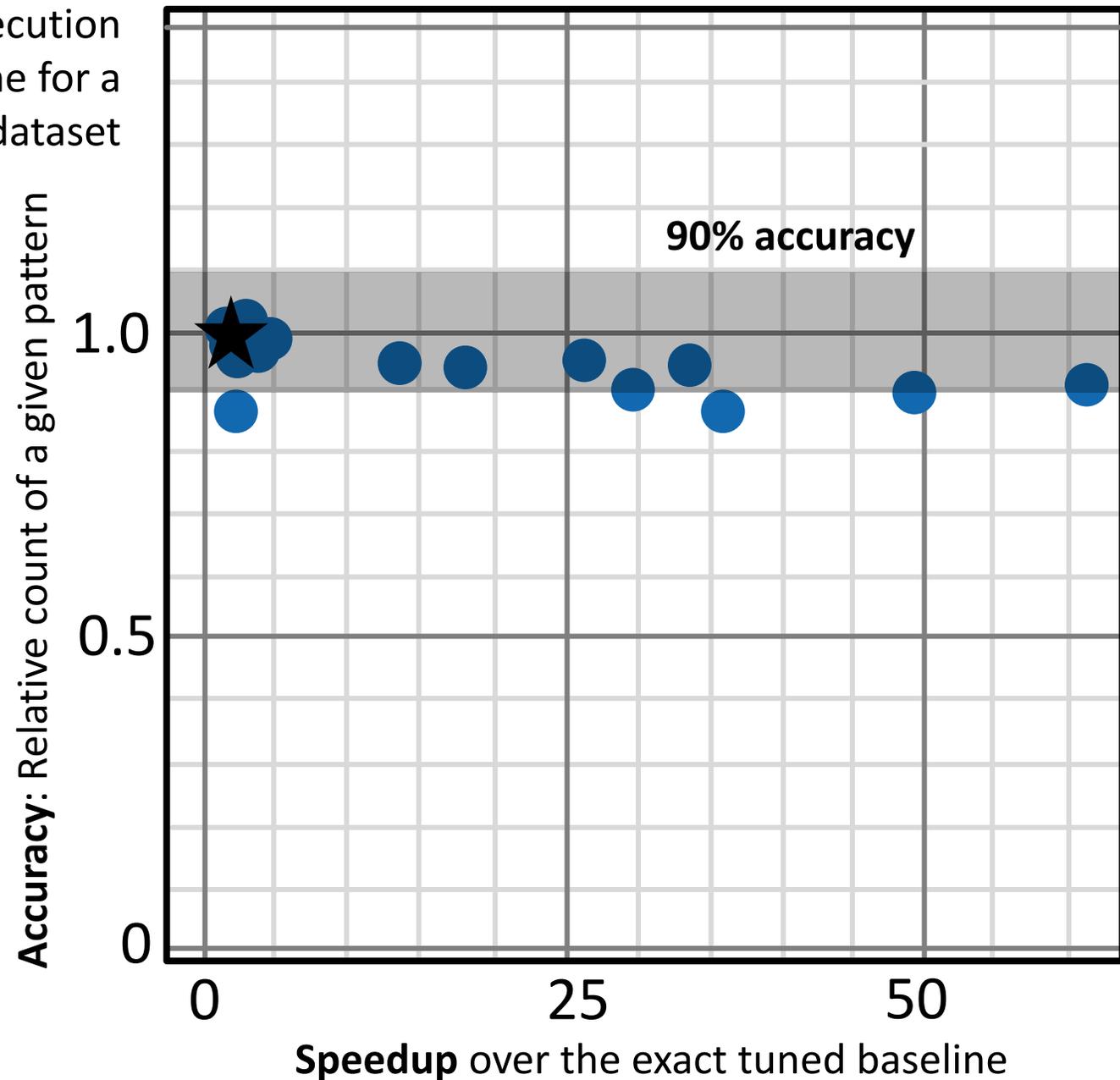
● ProbGraph

For most graphs, we have...

...very high speedups

...very good accuracy

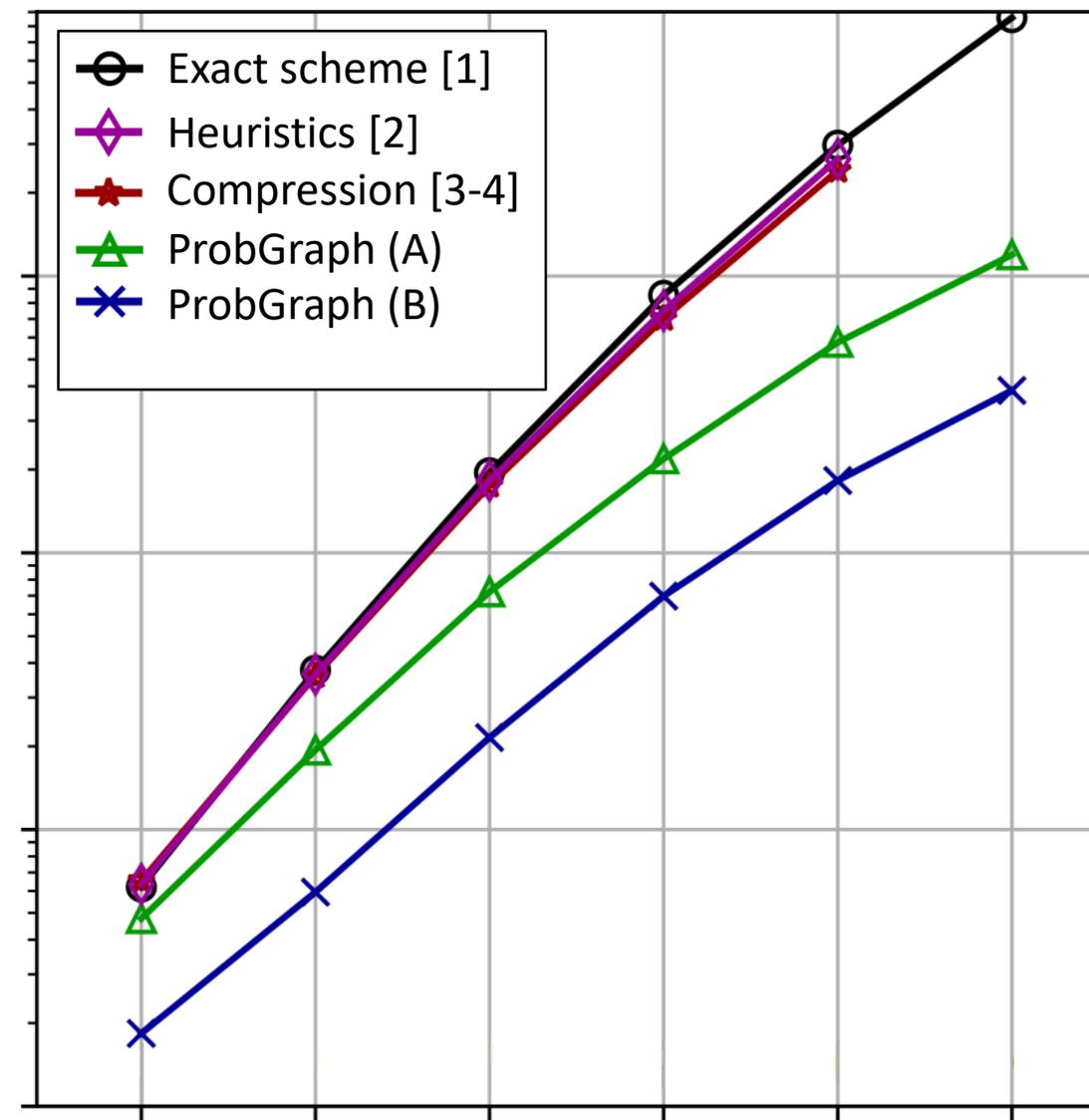
...mild memory requirements



[1] based on S. Beamer et al., „The GAP Benchmark Suite“. 2015

Clustering (Scaling)

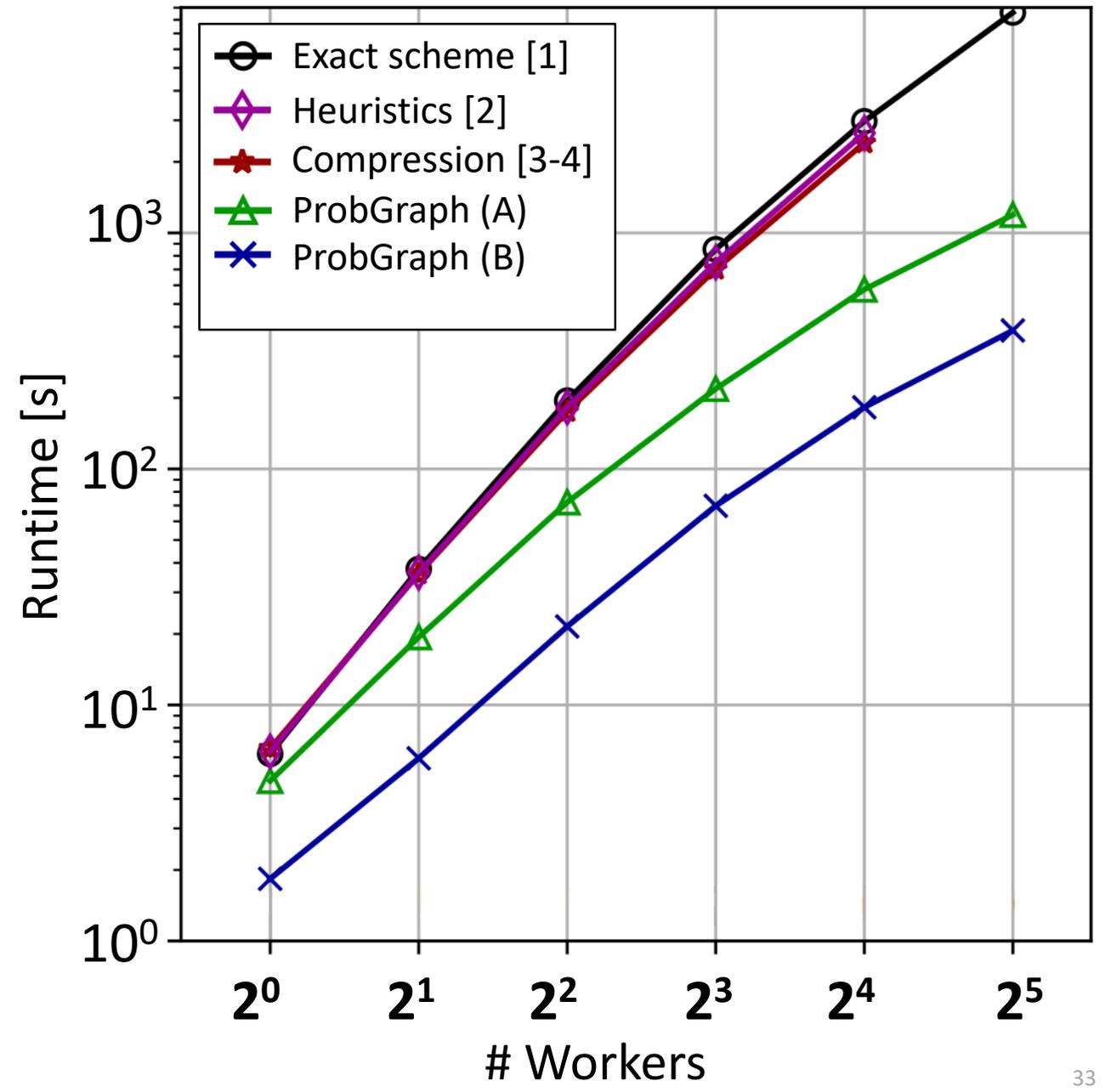
Max memory
overhead: 20%



- [1] S. Beamer et al., „The GAP Benchmark Suite”. 2015
- [2] R. Pagh et al., “Colorful triangle counting and a mapreduce implementation”. Information Processing Letters. 2012
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Clustering (Scaling)

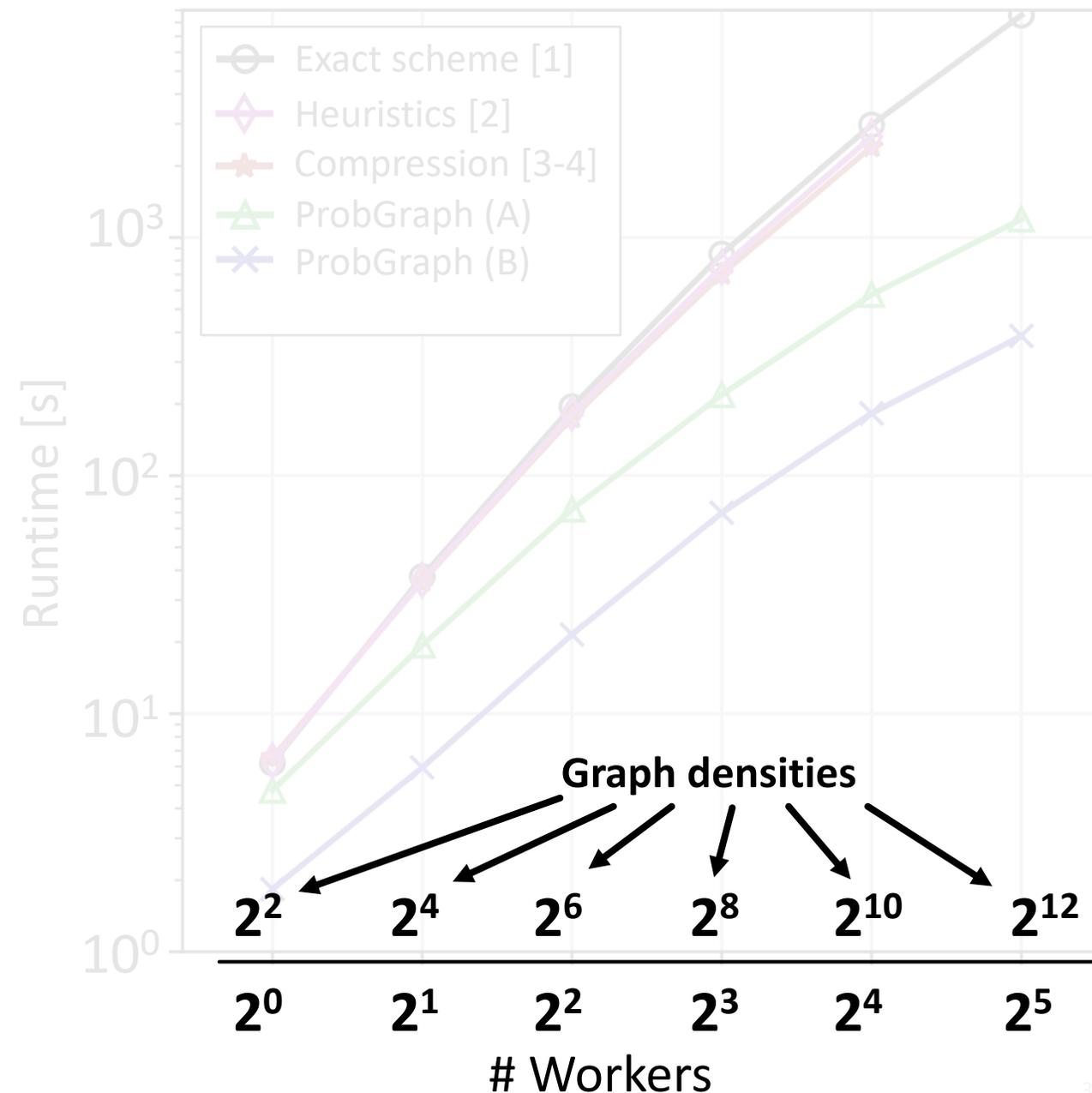
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Clustering (Scaling)

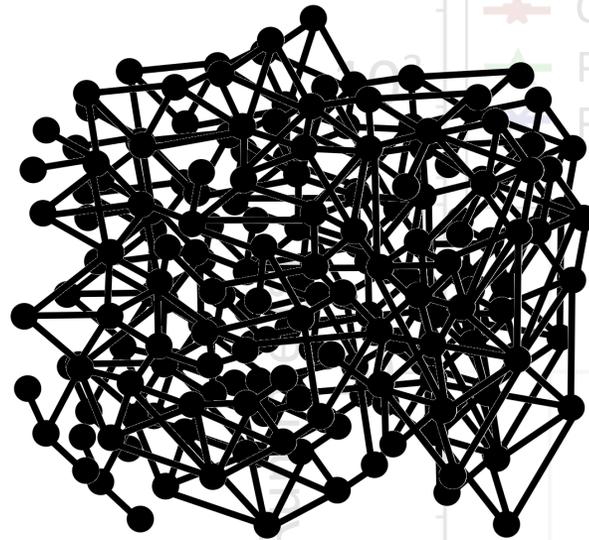
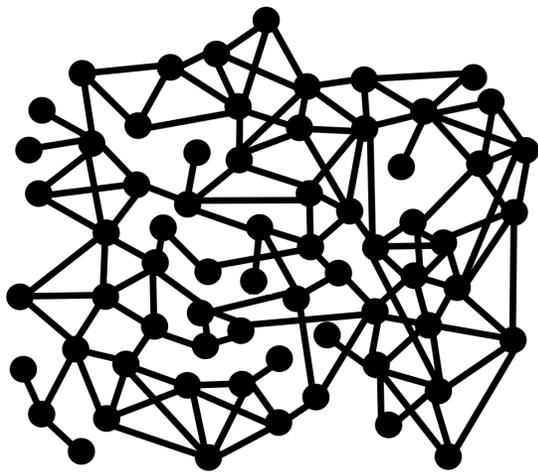
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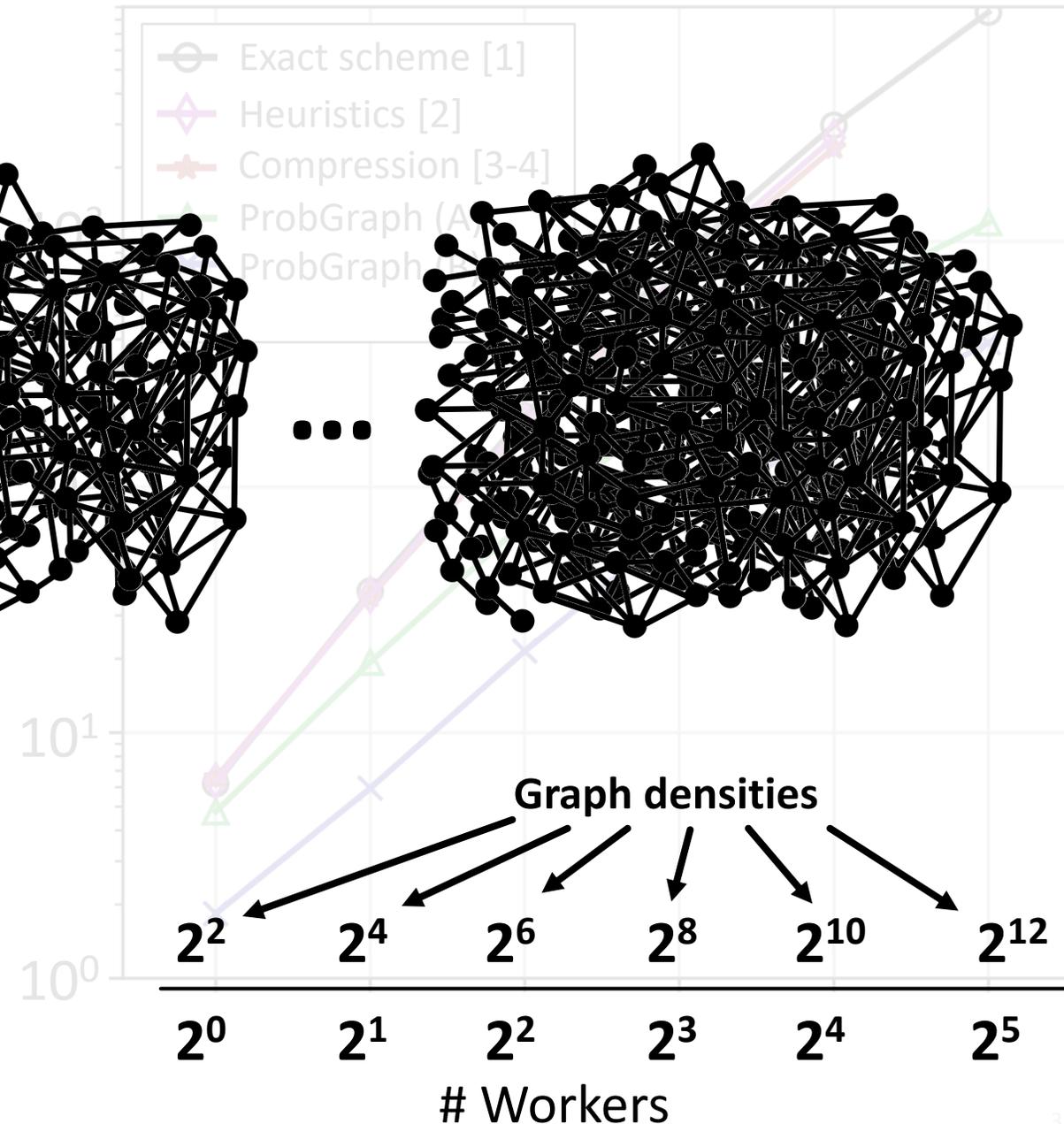
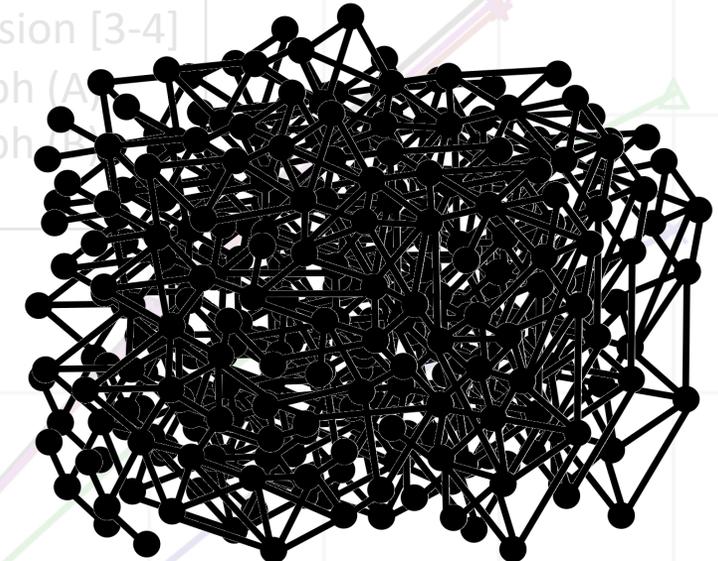
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Clustering (Scaling)

Max memory overhead: 20%



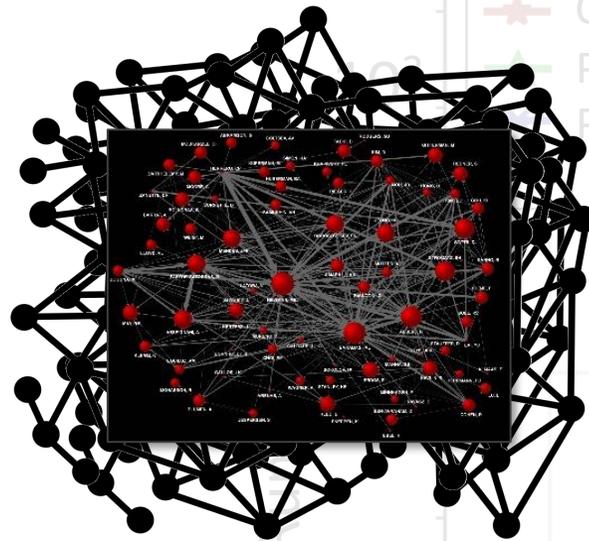
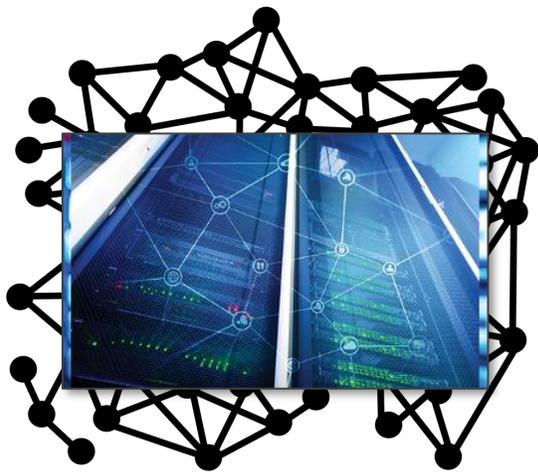
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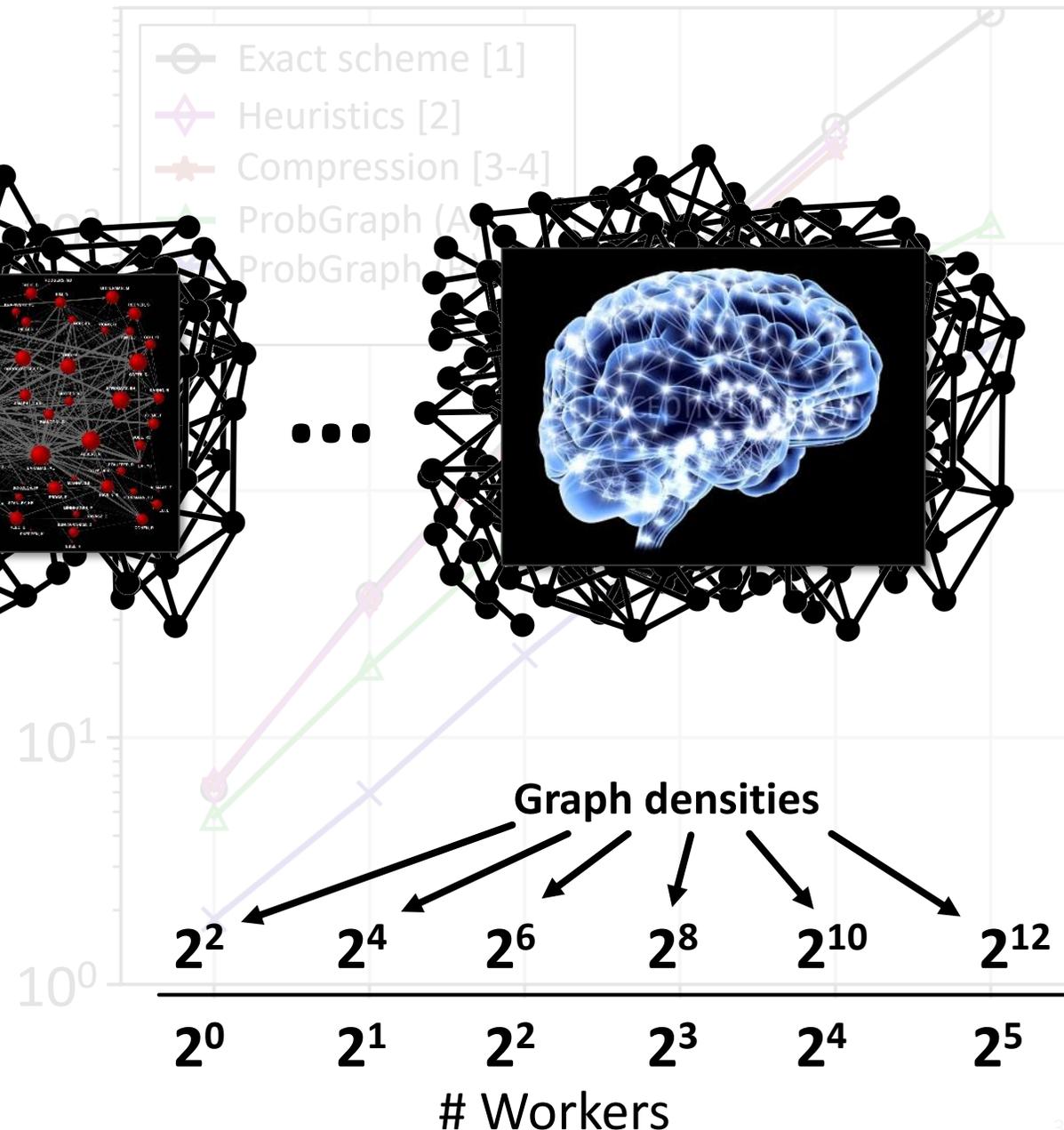
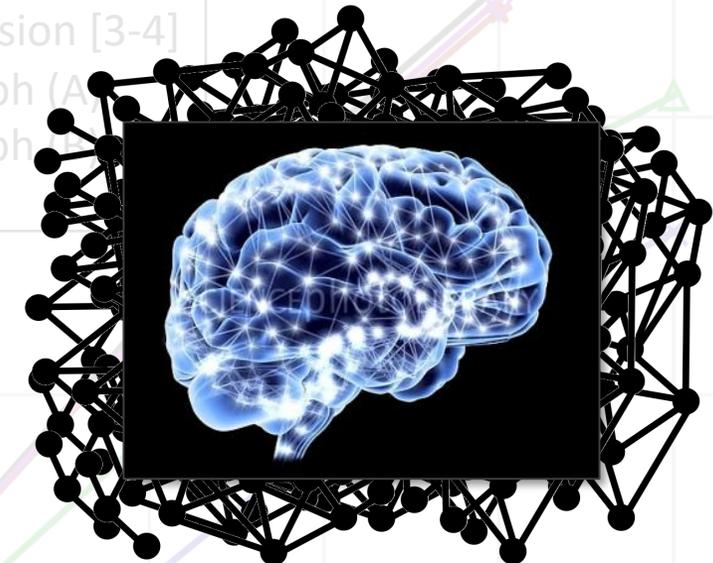
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Clustering (Scaling)

Max memory overhead: 20%



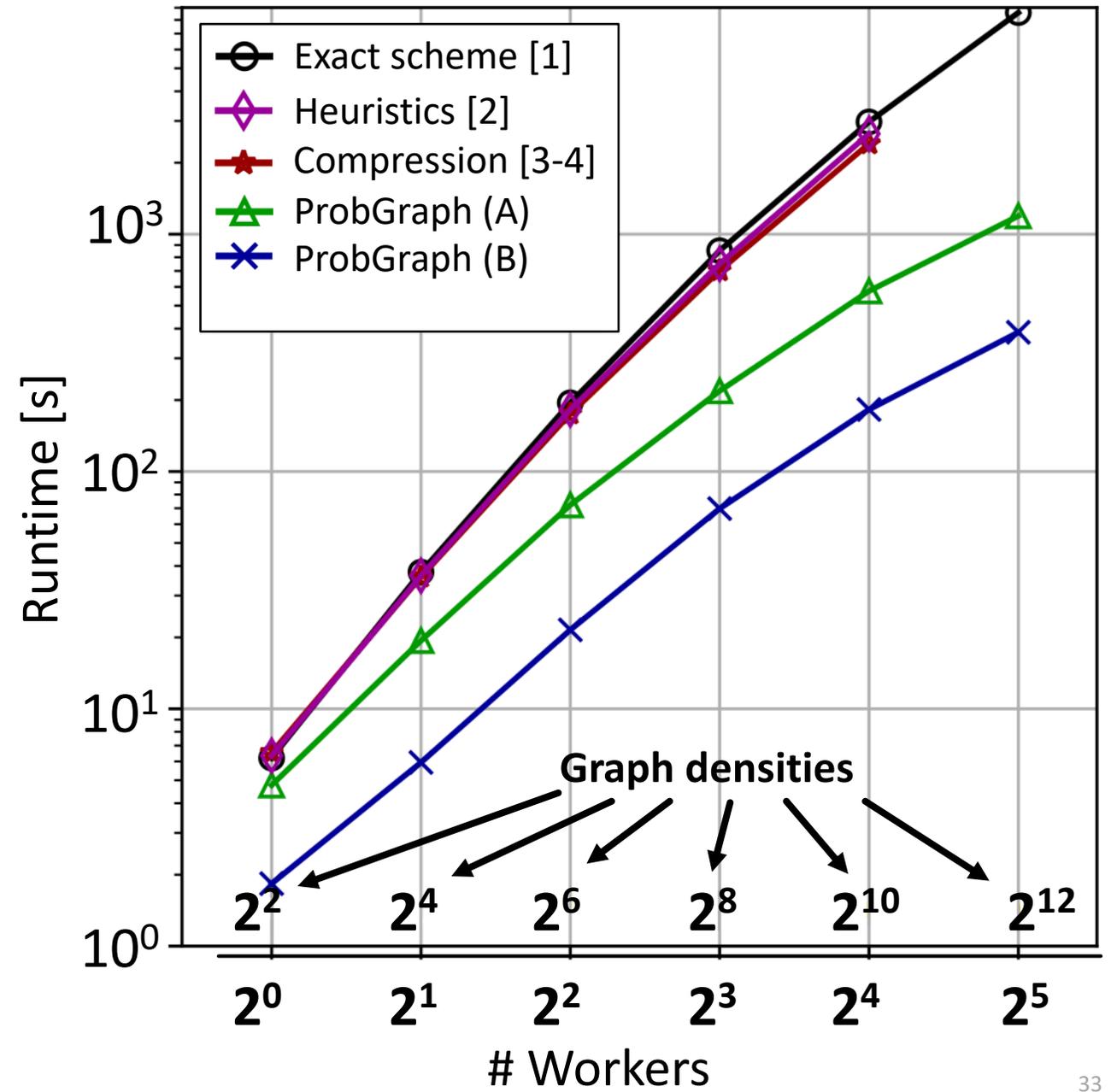
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Clustering (Scaling)

Max memory overhead: 20%

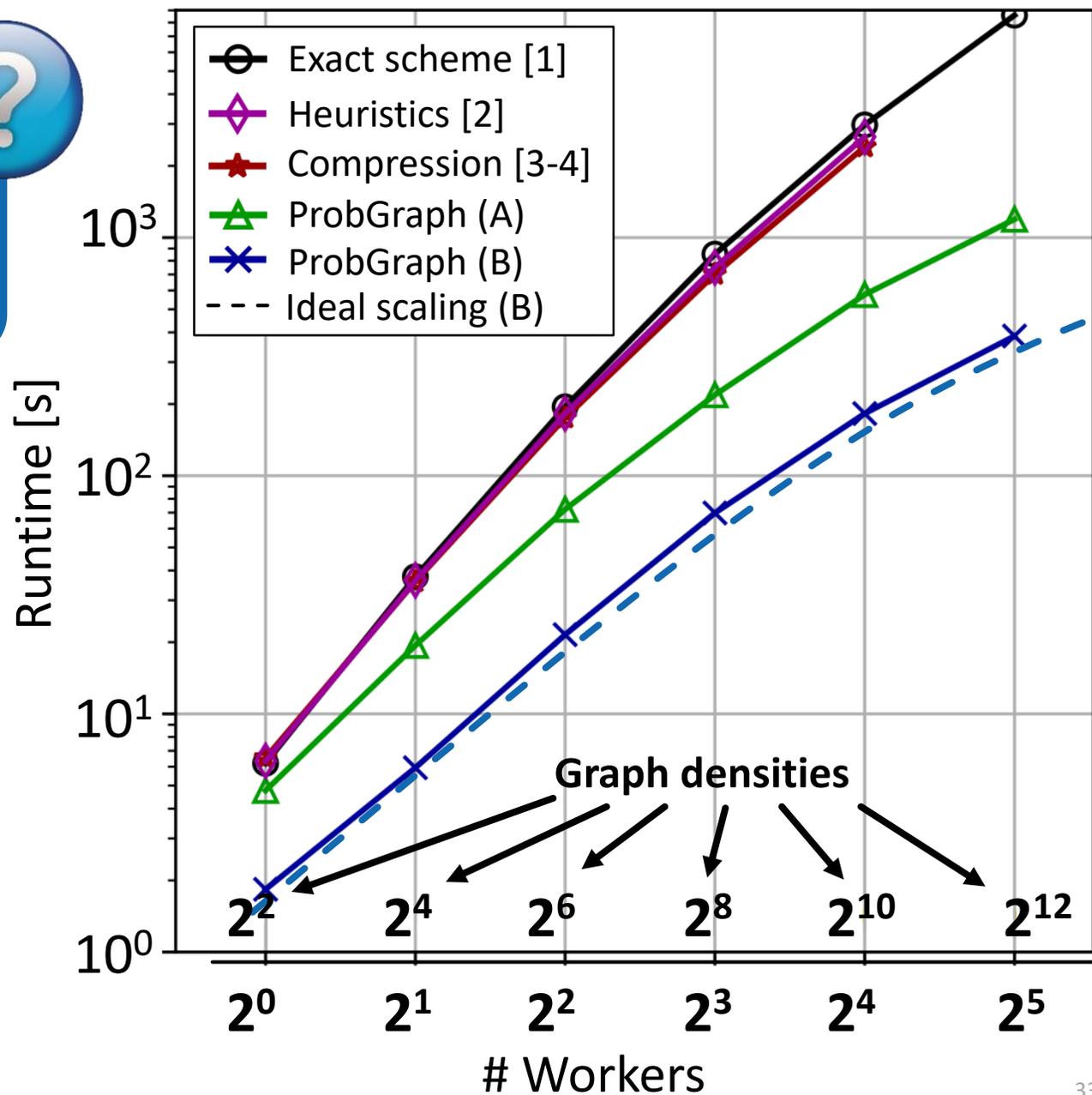


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Clustering (Scaling)

Max memory overhead: 20%

Why do we scale so well?



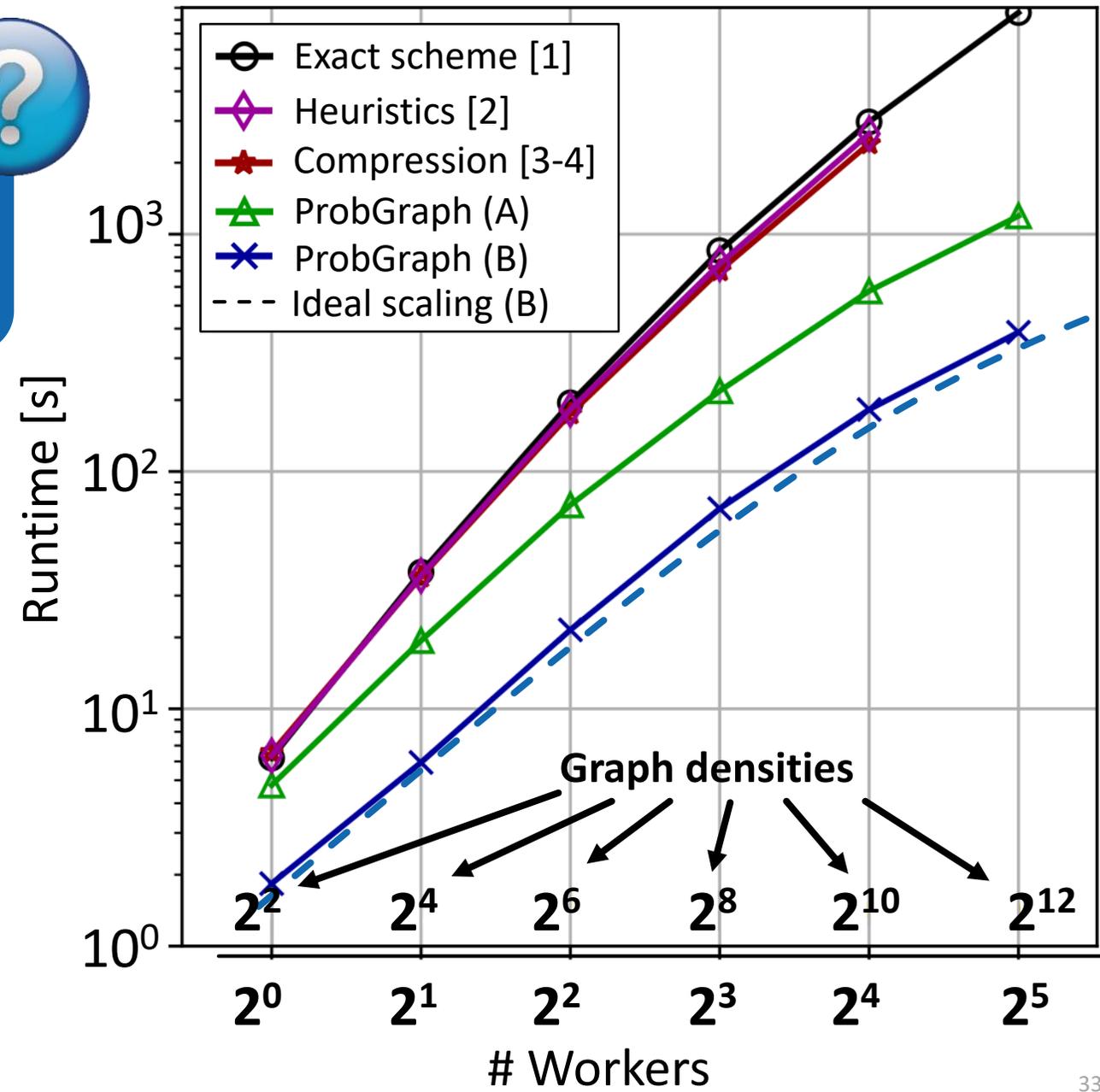
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 [2] R. Pagh et al., “Colorful triangle counting and a mapreduce implementation”. Information Processing Letters. 2012
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 [4] M. Besta et al., “Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics”. ACM/IEEE SC. 2019

Clustering (Scaling)

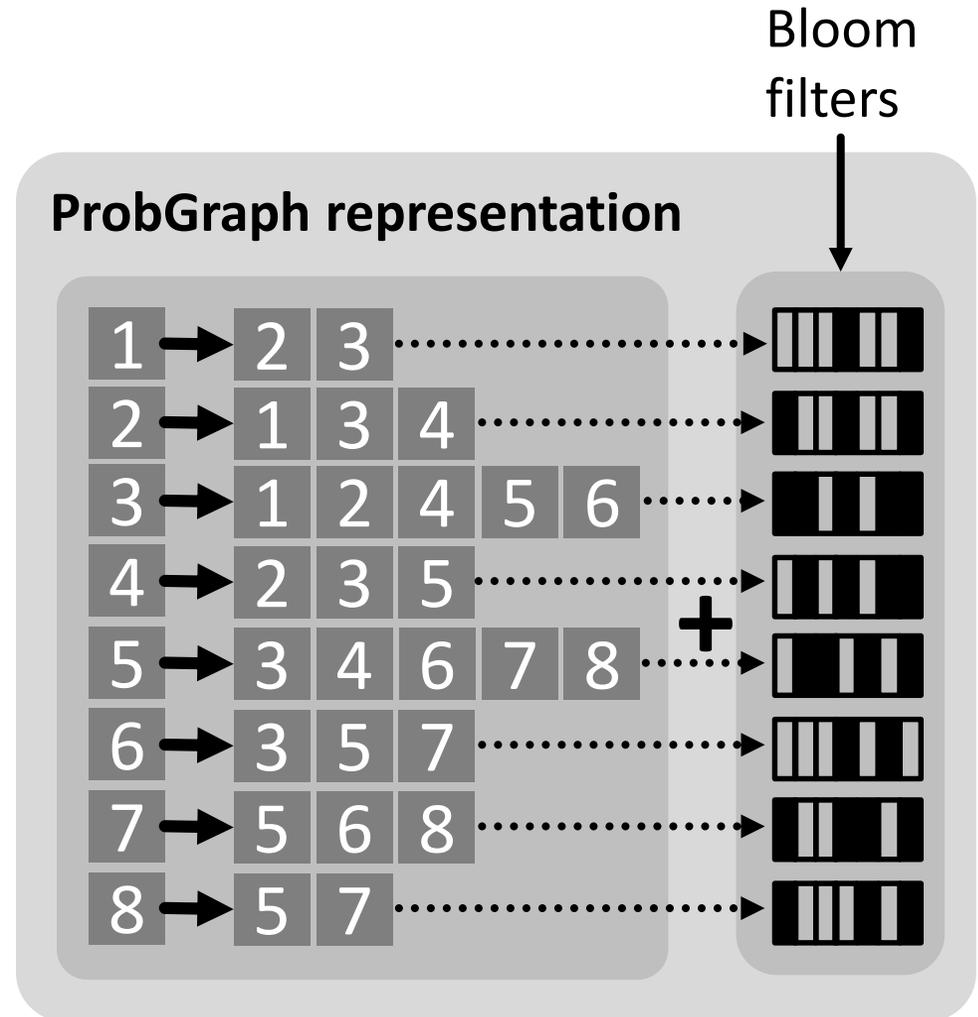
Max memory overhead: 20%

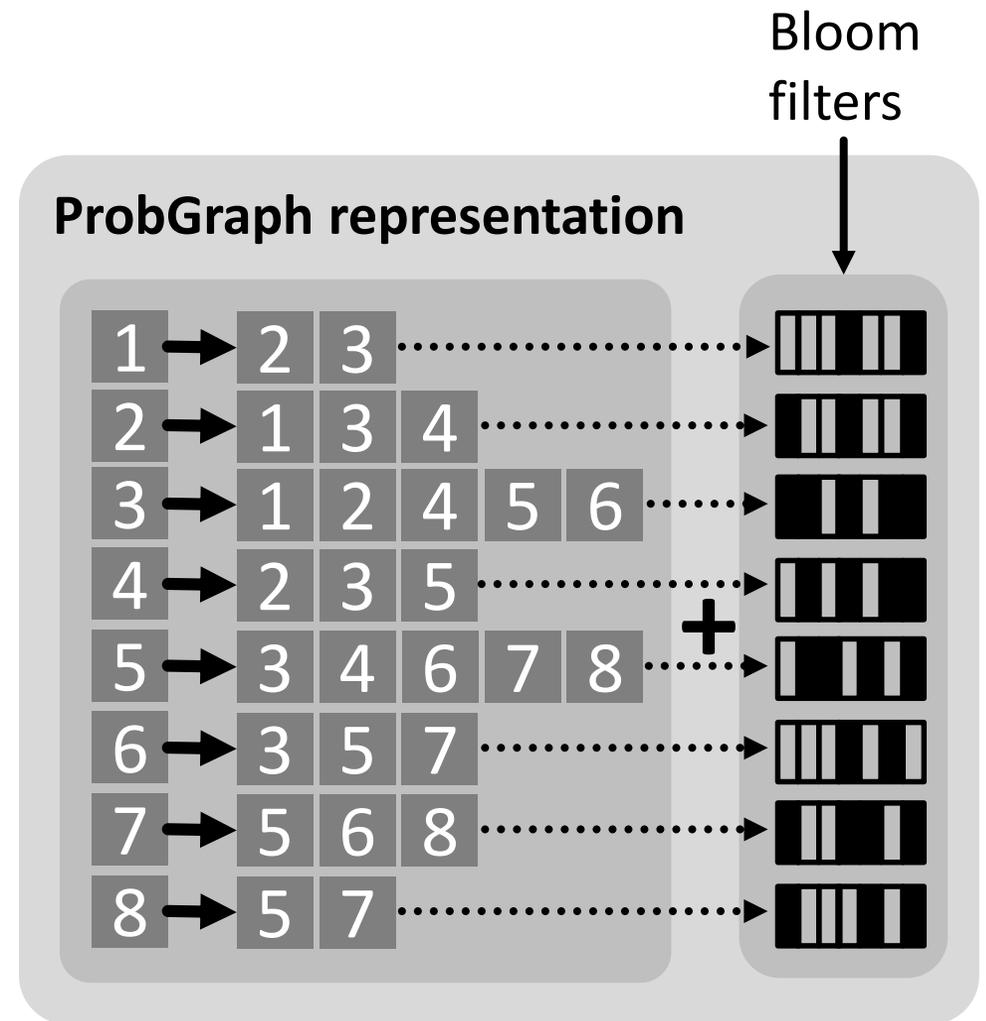
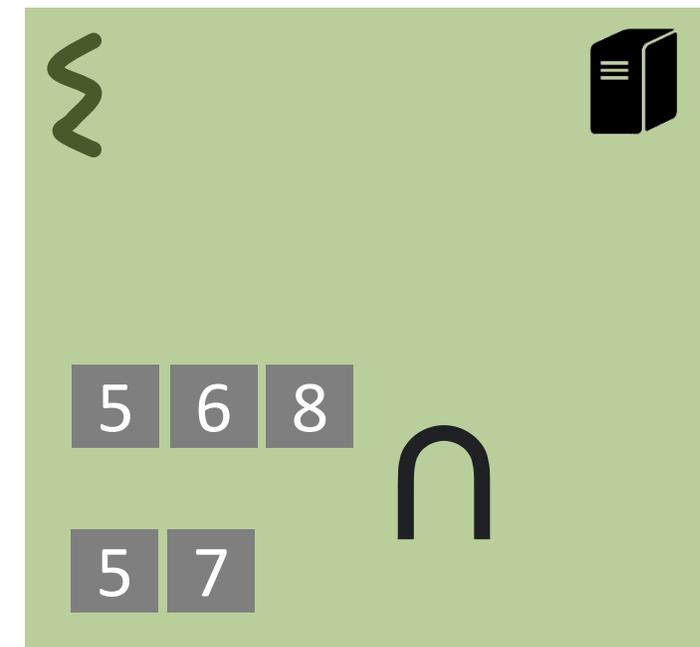
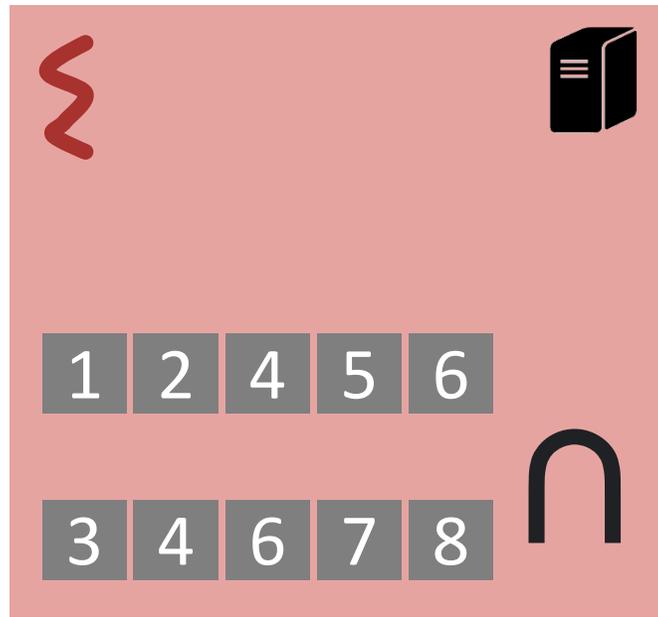
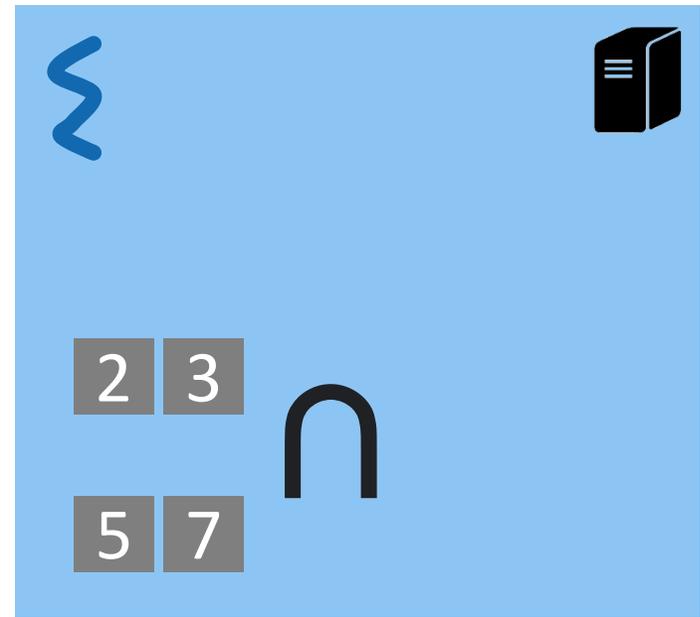
Why do we scale so well?

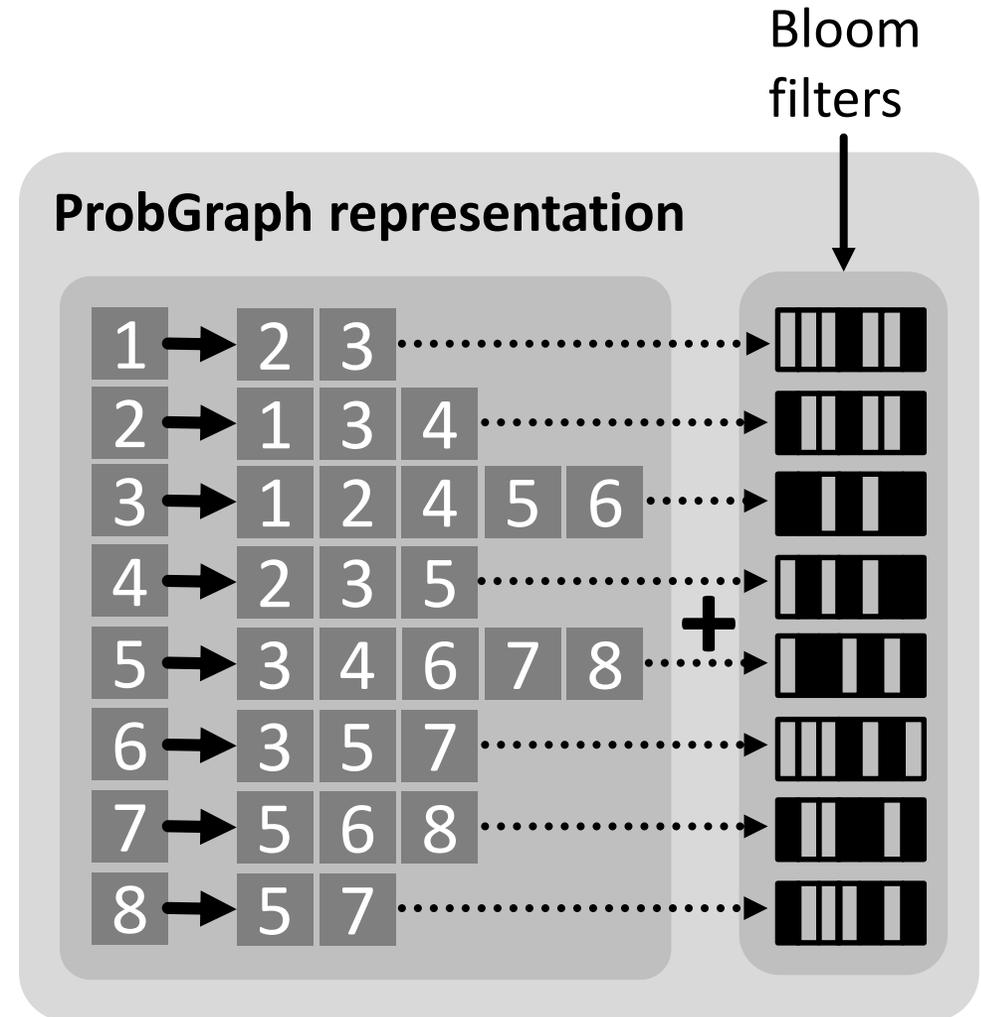
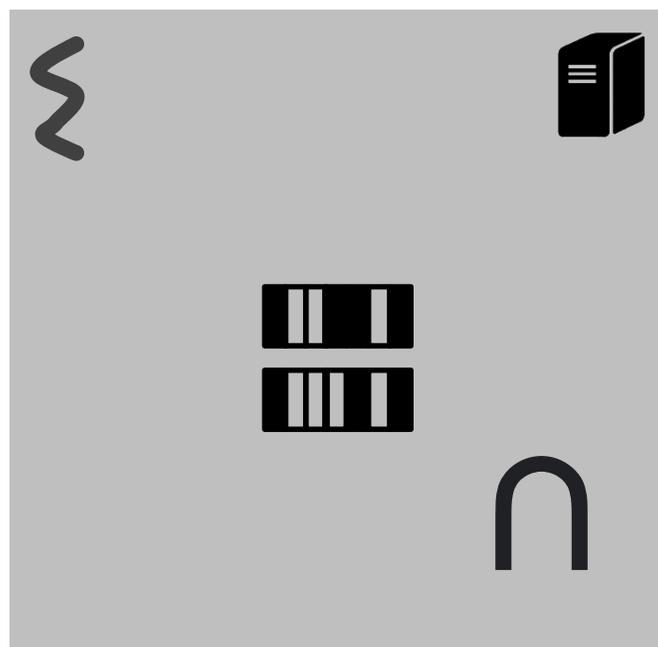
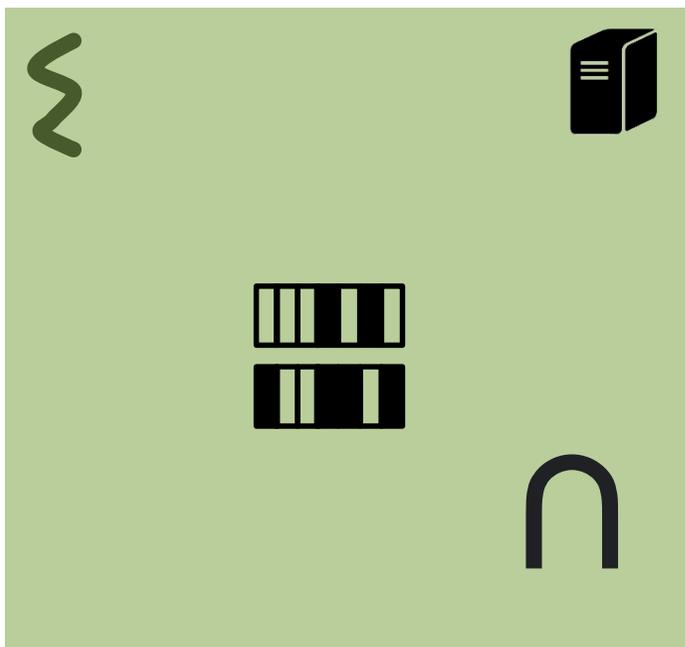
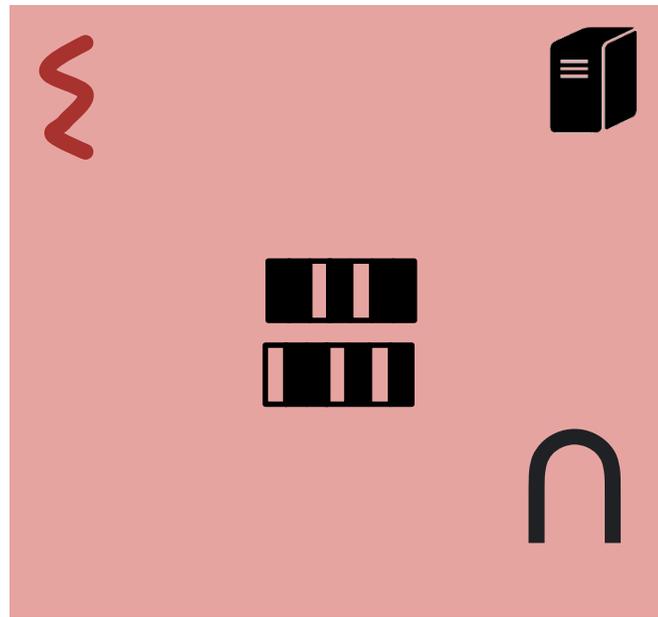
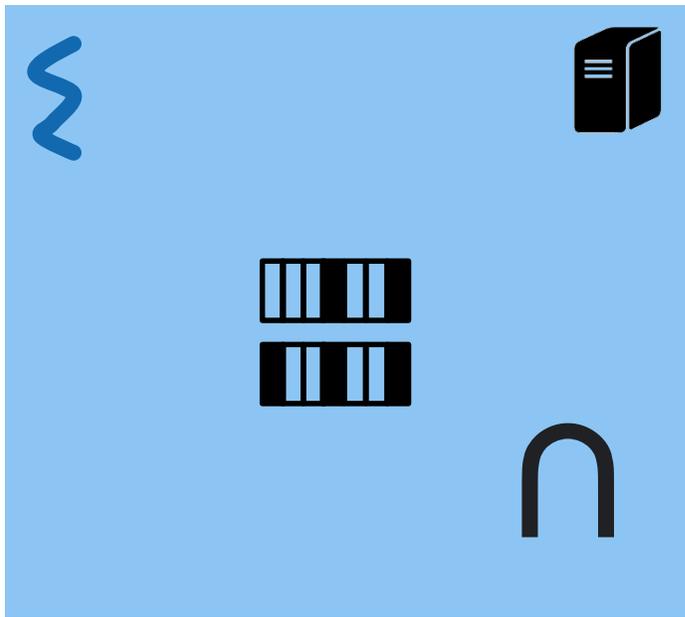
Great load balancing properties



[1] S. Beamer et al., „The GAP Benchmark Suite”. 2015
 [2] R. Pagh et al., “Colorful triangle counting and a mapreduce implementation”. Information Processing Letters. 2012
 [3] C. E. Tsourakakis et al., “Doulion: counting triangles in massive graphs with a coin”. ACM KDD. 2009.
 [4] M. Besta et al., “Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics”. ACM/IEEE SC. 2019







...Many more data & a lot of strong theory results!

$$P\left(\left|TC - \widehat{TC}_{1H}\right| \geq t\right) \leq 2 \exp\left(-\frac{18 k t^2}{\left(\sum_{v \in V} d(v)^2\right)^2}\right)$$

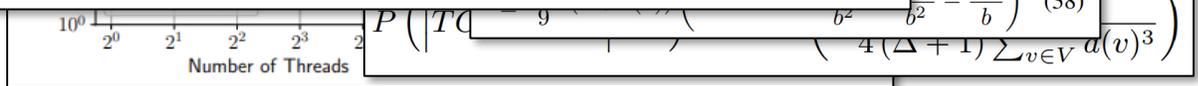
Result	Where	Class
$ \widehat{X} _S$	Eq. (1)	BF
$ \widehat{X} \cap Y $	Theorem A.6. Let $Y_1 = \mathcal{X}_1, \dots, \mathcal{X}_X$ such that estim independent. Then for any $S = \sum_{i=1}^n C_{i+1}^j$	
$ \widehat{X} \cap Y _{AN}$		
$ \widehat{X} \cap Y _L$		
$ \widehat{X} \cap Y _{kH}$		
$ \widehat{X} \cap Y _{1H}$	Eq. (7)	1-Hash

Reference	Constr. time	Memory used
Doulion [46]	$O(m)$	$O(pm)$
Colorful [47]	$O(m)$	$O(pm)$
Sketching [48]	$O(km)$	$O(kn)$
ASAP [49]	n/a	$O(n+m)$
GAP [50]	$O(m)^\dagger$	$O(m')^\dagger$
Slim Gr. [51]	$O(m)$	$O(pm)$
Eden et al. [52]	n/a	$O\left(\frac{n}{TC^{1/3}}\right)$
Assadi et al. [53]	n/a	$O(1)$
Tětek [54]	n/a	$\left(\frac{m^{1.41}}{TC^{0.82}}\right)$
\widehat{TC}_{AND} (BF)	$O(bm)$	$O(n+m)$
\widehat{TC}_{kH} (MH)	$O(km)$	$O(n+m)$
\widehat{TC}_{1H} (MH)	$O(km)$	$O(n+m)$

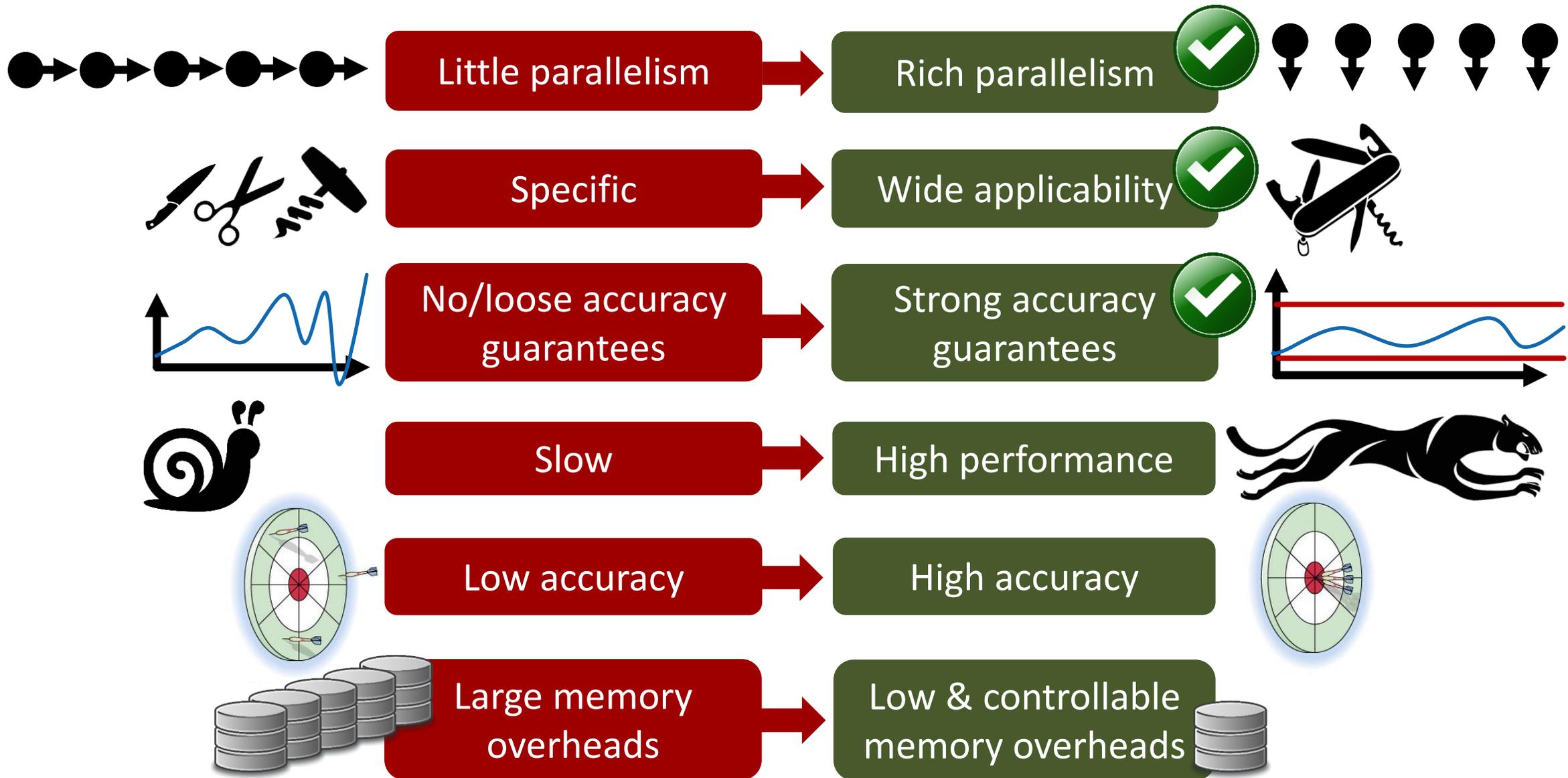
Work	Depth
$O(d_u + d_v)$	$O(\log(d_u + d_v))$

$$\begin{aligned}
 & E[(|\widehat{X}| - |X|)^2] \\
 &= E[(|\widehat{X}| - |X|)^2 | \mathcal{E}] P(\mathcal{E}) + E[(|\widehat{X}| - |X|)^2 | \neg \mathcal{E}] P(\neg \mathcal{E}) \\
 &\leq (1 + \varepsilon) E[(|\widehat{X}| - \kappa)^2 | \mathcal{E}] + \frac{1 + \varepsilon}{\varepsilon} E[(\kappa - |X|)^2 | \mathcal{E}] + O(B_X^2 \log^2 B_X) \cdot \exp(-B_X^{\Omega(1)}) \\
 &\leq \frac{(1 + \varepsilon) B_X^2}{b^2} E[(\log(B_{X,0}/B_X) - \log(1 - 1/B_X)^{|X|})^2 | \mathcal{E}] + O((\kappa - |X|)^2) + \exp(-B_X^{\Omega(1)}) \\
 &\leq \frac{(1 + \varepsilon) B_X^2}{b^2} E[(\log(B_{X,0}/B_X) - \log(1 - 1/B_X)^{|X|})^2 | \mathcal{E}] + O(|X|/B_X) \\
 &\leq \frac{(1 + \varepsilon)^2 B_X^2}{b^2} e^{2b|X|/B_X} E[(B_{X,0}/B_X - (1 - 1/B_X)^{|X|})^2 | \mathcal{E}] + O(|X|/B_X) \\
 &\leq \frac{(1 + \varepsilon)^2 B_X^2}{b^2} e^{2b|X|/(B_X-1)} \cdot E[(B_{X,0}/B_X - (1 - 1/B_X)^{|X|})^2] / P[\mathcal{E}] + O(|X|/B_X) \\
 &= ((1 + \varepsilon)^2 + o(1)) \frac{B_X^2}{b^2} e^{2b|X|/(B_X-1)} \cdot E[(B_{X,0}/B_X - (1 - 1/B_X)^{|X|})^2] + O(|X|/B_X) \\
 &= ((1 + \varepsilon)^2 + o(1)) \frac{e^{2b|X|/(B_X-1)}}{b^2} Var[B_{X,0}] + O(|X|/B_X) \\
 &\leq ((1 + \varepsilon)^2 + o(1)) e^{2b|X|/(B_X-1)} \cdot \left(e^{-\frac{b|X|}{B_X}} \frac{B_X}{b^2} - B_X/b^2 - |X|/b \right) + O(|X|/B_X) \\
 &\leq ((1 + \varepsilon)^2 + o(1)) \left(e^{|X|b/(B_X-1)} \frac{B_X}{b^2} - B_X/b^2 - |X|/b \right) + O(|X|/B_X) \\
 &\leq ((1 + \varepsilon)^2 + o(1)) \left(e^{|X|b/(B_X-1)} \frac{B_X}{b^2} - B_X/b^2 - |X|/b \right) \\
 &\leq ((1 + \varepsilon)^2 + o(1)) \left(e^{|X|b/(B_X-1)} \frac{B_X}{b^2} - B_X/b^2 - |X|/b \right)
 \end{aligned}$$

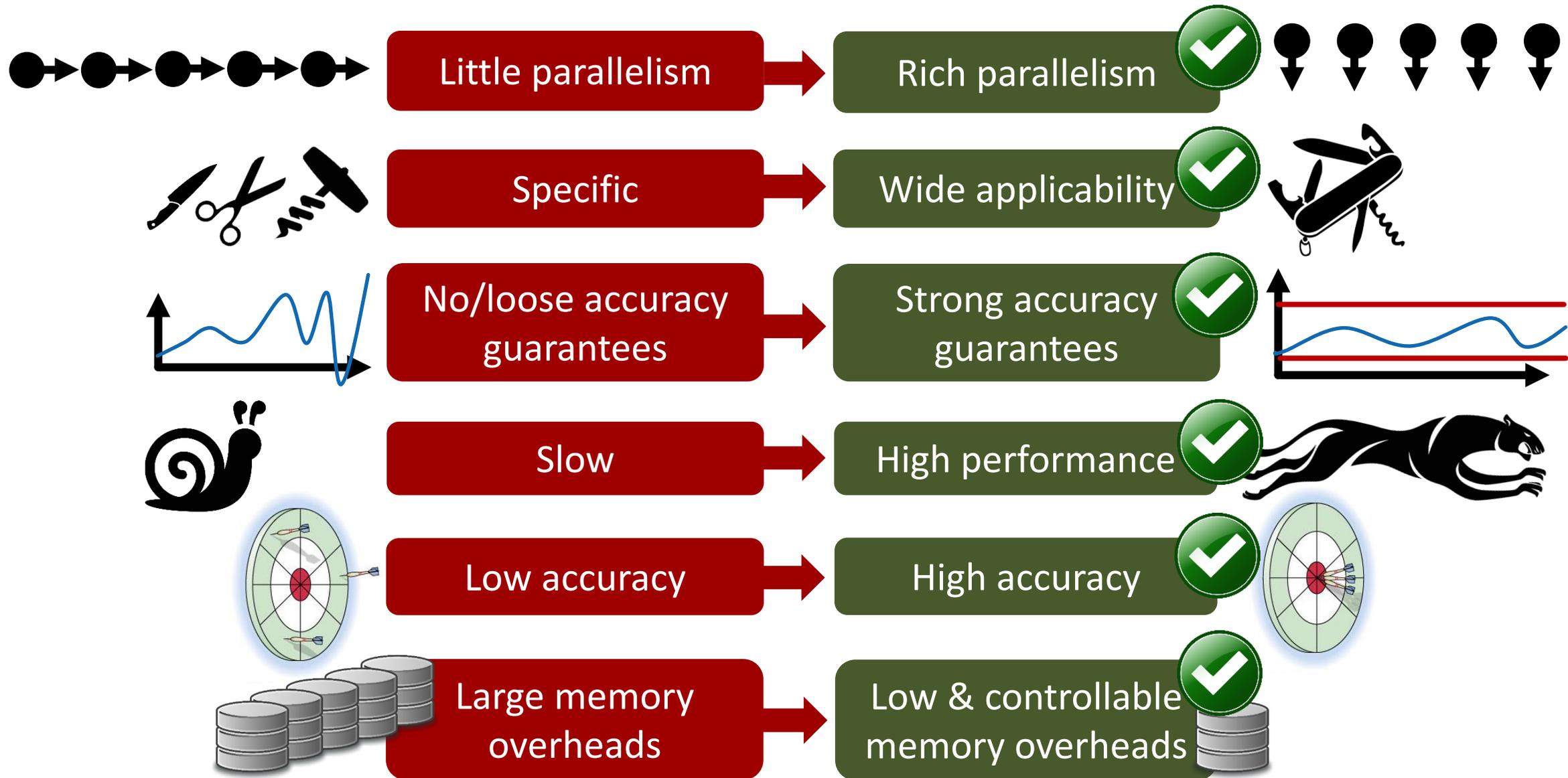
(work)	(depth)
$O(bd_v)$	$O(\log(bd_v))$
$O(kd_v)$	$O(\log d_v)$
PG (BF)	PG (MH)
$\frac{ndB_X}{W}$	$O(ndk)$
$\log\left(\frac{B_X}{W}\right)$	$O(\log k)$
$\frac{nd^2 B_X}{W}$	$O(nd^2 k)$
$\log d \log\left(\frac{B_X}{W}\right)$	$O(\log^2 k)$

$$\begin{aligned}
 & |X|_i | E[|\widehat{X}|_j] - |X|_j \Big| \Big\} \quad (31) \\
 & |X|_i \Big| \left| E[|\widehat{X}|_j] - |X|_j \right| \Big\} \quad (32) \\
 & \left| E[|\widehat{X}|_j] - |X|_j \right|^2 \quad (33) \\
 & \left(\frac{2\Delta}{b} \right) \quad (34) \\
 & \left(\frac{B_X}{b^2} - \frac{2\Delta}{b} \right) \quad (35) \\
 & \left(\frac{B_X}{b^2} - \frac{2\Delta}{b} \right) \quad (36) \\
 & \left(\frac{B_X}{b^2} - \frac{2\Delta}{b} \right) \quad (37) \\
 & \left(\frac{B_X}{b^2} - \frac{2\Delta}{b} \right) \quad (38)
 \end{aligned}$$


Approximate Graph Processing: Our Objectives



Approximate Graph Processing: Our Objectives

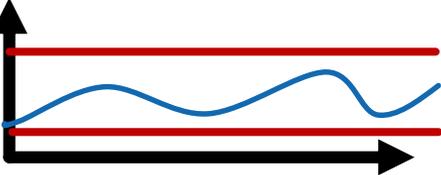


Conclusion: ProbGraph Enables Approximate Graph Mining with...

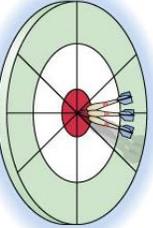
- ✓


Rich parallelism
- ✓


Wide applicability
- ✓

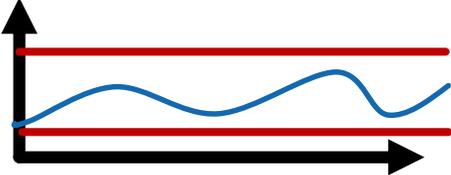
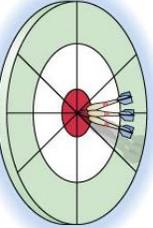

Strong accuracy guarantees
- ✓


High performance
- ✓


High accuracy
- ✓

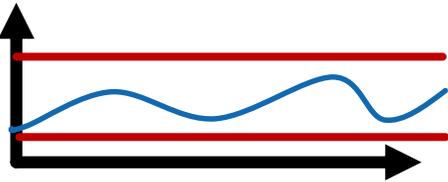
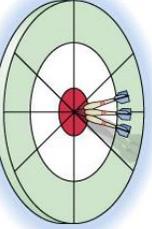

Low & controllable memory overheads

Conclusion: ProbGraph Enables Approximate Graph Mining with...

- Rich parallelism**  
- Wide applicability**  
- Strong accuracy guarantees**  
- High performance**  
- High accuracy**  
- Low & controllable memory overheads**  

Conclusion: ProbGraph Enables Approximate Graph Mining with...

- Rich parallelism  
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- High performance  
- High accuracy  
- Low & controllable memory overheads  

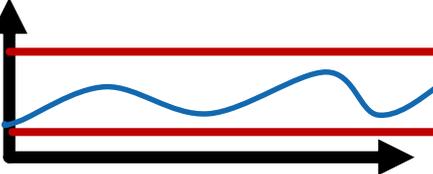
Thank you



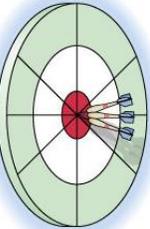
Conclusion: ProbGraph Enables Approximate Graph Mining with...

Rich parallelism  

Wide applicability  

Strong accuracy guarantees  

High performance  

High accuracy  

Low & controllable memory overheads  



Thank you

Want to know more?

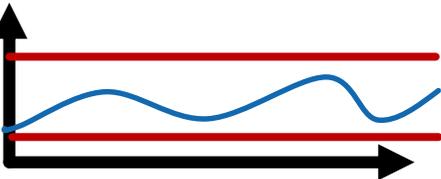
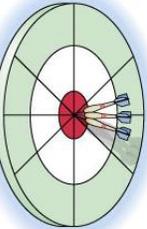
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Conclusion: ProbGraph Enables Approximate Graph Mining with...

- Rich parallelism  
- Wide applicability  
- Strong accuracy guarantees  
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Backup slides

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