Lecture 15

# COMPILER DESIGN

# Announcements

- **HW4**:  OAT v. 1.0
  - Parsing & basic code generation

# Why Inference Rules?

- Allow a compact, precise way of specifying language properties
  - About 20 pages for full Java, versus
  - Hundreds of pages of prose Java Language Specification

- Correspond closely to recursive AST traversal for implementing them

- Type checking/inference tries to prove a different judgment $\mathbf{C \vdash e : t}$
  - By searching backward through the rules

- Compiling is "interpreting" the type checking rules $[\![\mathbf{C \vdash e : t}]\!]$
  - Compilation follows the type checking judgment

- Strong mathematical foundations, e.g., "Curry-Howard correspondence"
  - Programming Language ~ Logic
  - Program ~ Proof
  - Type ~ Proposition

# OPTIMIZING CONTROL

# Standard Evaluation

- Consider compiling the following program fragment

```
if (x & !y | !w)
   z = 3;
else
   z = 4;
return z;
```

```
    %tmp1 = icmp Eq ⟦y⟧, 0        ; !y
    %tmp2 = and ⟦x⟧ %tmp1
    %tmp3 = icmp Eq ⟦w⟧, 0
    %tmp4 = or %tmp2, %tmp3
    %tmp5 = icmp Eq %tmp4, 0
    br %tmp5, label %else, label %then

then:
    store ⟦z⟧, 3
    br %merge

else:
    store ⟦z⟧, 4
    br %merge

merge:
    %tmp6 = load ⟦z⟧
    ret %tmp6
```

# Observation

- Usually, we want the translation $[\![e]\!]$ to produce a value
  - $[\![C \vdash e : t]\!]$ = (ty, operand, stream)
  - e.g. $[\![C \vdash e_1 + e_2 : int]\!]$ = (i64, `%tmp`, [`%tmp = add` $[\![e_1]\!]$ $[\![e_2]\!]$])
- But, when the compiled expression <u>appears in a test</u>
  - The program jumps to one label or another after the comparison
  - Otherwise, it never uses the value
- In many cases, we can avoid "materializing" the value (i.e. storing it in a temporary) and thus produce better code
  - This idea also lets us implement different functionality too: e.g. short-circuiting Boolean expressions

# Idea: Use a different translation for tests

- Usual Expression translation: **$[\![C \vdash e : t]\!]$ = (ty, operand, stream)**
- Conditional branch translation of Booleans, without materializing value
  **$[\![C \vdash e : \text{bool@}]\!]$ ltrue lfalse = stream**

$[\![C, rt \vdash \text{if (e) then s1 else s2} \Rightarrow C']\!] = \quad [\![C']\!],$

```
    insns₃
then:
     ⟦s1⟧
     br %merge
else:
     ⟦s₂⟧
     br %merge
merge:
```

where

$[\![C, rt \vdash s_1 \Rightarrow C']\!] = [\![C']\!], \text{insns}_1$

$[\![C, rt \vdash s_2 \Rightarrow C']\!] = [\![C']\!], \text{insns}_2$

$[\![C \vdash e : \text{bool@}]\!]$ `then` `else` $= \text{insns}_3$

Notes

- Two extra arguments
  - "true" branch label
  - "false" branch label
- Doesn't "return a value"

- Aside: this is a form of continuation-passing translation

# Short Circuit Compilation: Expressions

- **$[\![C \vdash e : bool@]\!]$ ltrue lfalse = insns**

$$\frac{}{[\![C \vdash \text{false} : \text{bool@}]\!]\ \text{ltrue}\ \text{lfalse} = \texttt{[br \%lfalse]}}\ \text{FALSE}$$

$$\frac{}{[\![C \vdash \text{true} : \text{bool@}]\!]\ \text{ltrue}\ \text{lfalse} = \texttt{[br \%ltrue]}}\ \text{TRUE}$$

$$\frac{[\![C \vdash e : \text{bool@}]\!]\ \text{lfalse}\ \text{ltrue} = \text{insns}}{[\![C \vdash !e : \text{bool@}]\!]\ \text{ltrue}\ \text{lfalse} = \text{insns}}\ \text{NOT}$$

# Short Circuit Evaluation

Idea: build the logic into the translation

$$\frac{[\![C \vdash e1 : bool@]\!]\ \text{ltrue right} = \text{insns}_1 \qquad [\![C \vdash e2 : bool@]\!]\ \text{ltrue lfalse} = \text{insns}_2}{[\![C \vdash e1 \,|\, e2 : bool@]\!]\ \text{ltrue lfalse} = \quad \begin{array}{l} \quad\texttt{insns}_1 \\ \texttt{right:} \\ \quad\texttt{insn}_2 \end{array}}$$

$$\frac{[\![C \vdash e1 : bool@]\!]\ \text{right lfalse} = \text{insns}_1 \qquad [\![C \vdash e2 : bool@]\!]\ \text{ltrue lfalse} = \text{insns}_2}{[\![C \vdash e1 \,\&\, e2 : bool@]\!]\ \text{ltrue lfalse} = \quad \begin{array}{l} \quad\texttt{insns}_1 \\ \texttt{right:} \\ \quad\texttt{insn}_2 \end{array}}$$

where `right` is a fresh label

# Short-Circuit Evaluation

- Consider compiling the following program fragment

```
if (x & !y | !w)
  z = 3;
else
  z = 4;
return z;
```

```
    %tmp1 = icmp Eq ⟦x⟧, 0
    br %tmp1, label %right2, label %right1

right1:
    %tmp2 = icmp Eq ⟦y⟧, 0
    br %tmp2, label %then, label %right2

right2:
    %tmp3 = icmp Eq ⟦w⟧, 0
    br %tmp3, label %then, label %else

then:
    store ⟦z⟧, 3
    br %merge

else:
    store ⟦z⟧, 4
    br %merge

merge:
    %tmp5 = load ⟦z⟧
    ret %tmp5
```

Compiling lambda calculus to straight-line code.

Representing evaluation environments at runtime.

# CLOSURE CONVERSION

# "Functional" languages

- Languages like ML, Haskell, Scheme, Python, C#, Java 8, Swift
  - Functions can be passed as arguments (e.g. map or fold)
  - Functions can be returned as values (e.g. compose)
  - Function nest:

    Inner function can refer to variables bound in the outer function

```
let add = fun x -> fun y -> x + y
let inc = add 1
let dec = add -1

let compose = fun f -> fun g -> fun x -> f (g x)
let id = compose inc dec
```

- How do we implement such functions?
  - In an interpreter? In a compiled language?

# Compiling First-class Functions

- To implement first-class functions on a processor, there are 2 problems
  - Must implement substitution of free variables
  - Must separate "**code**" from "**data**"

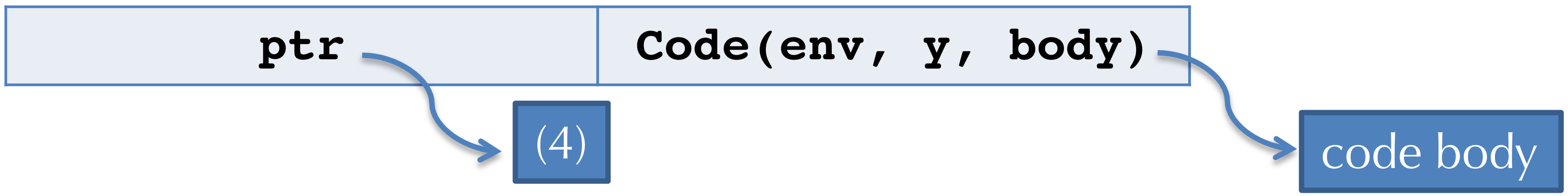
- Reify the substitution
  - Move substitution from the meta language to the object language by making the data structure & lookup operation explicit
  - The environment-based interpreter is one step in this direction
- Closure conversion
  - Eliminates free variables by packaging up the needed environment in the data structure
- Hoisting
  - Separates code from data, pulling closed code to the top level

# Example of Closure Creation

- Recall the "add" function
  `let add = fun x -> fun y -> x + y`

- Consider the inner function: `fun y -> x + y`

- When run the function application: `add 4`
  the program builds a closure and returns it
  - The closure is a pair of the <u>environment</u> and a <u>code pointer</u>




- The code pointer takes a pair of parameters: env and y
  - The function code is (essentially)
    `fun (env, y) -> let x = nth env 0 in x + y`

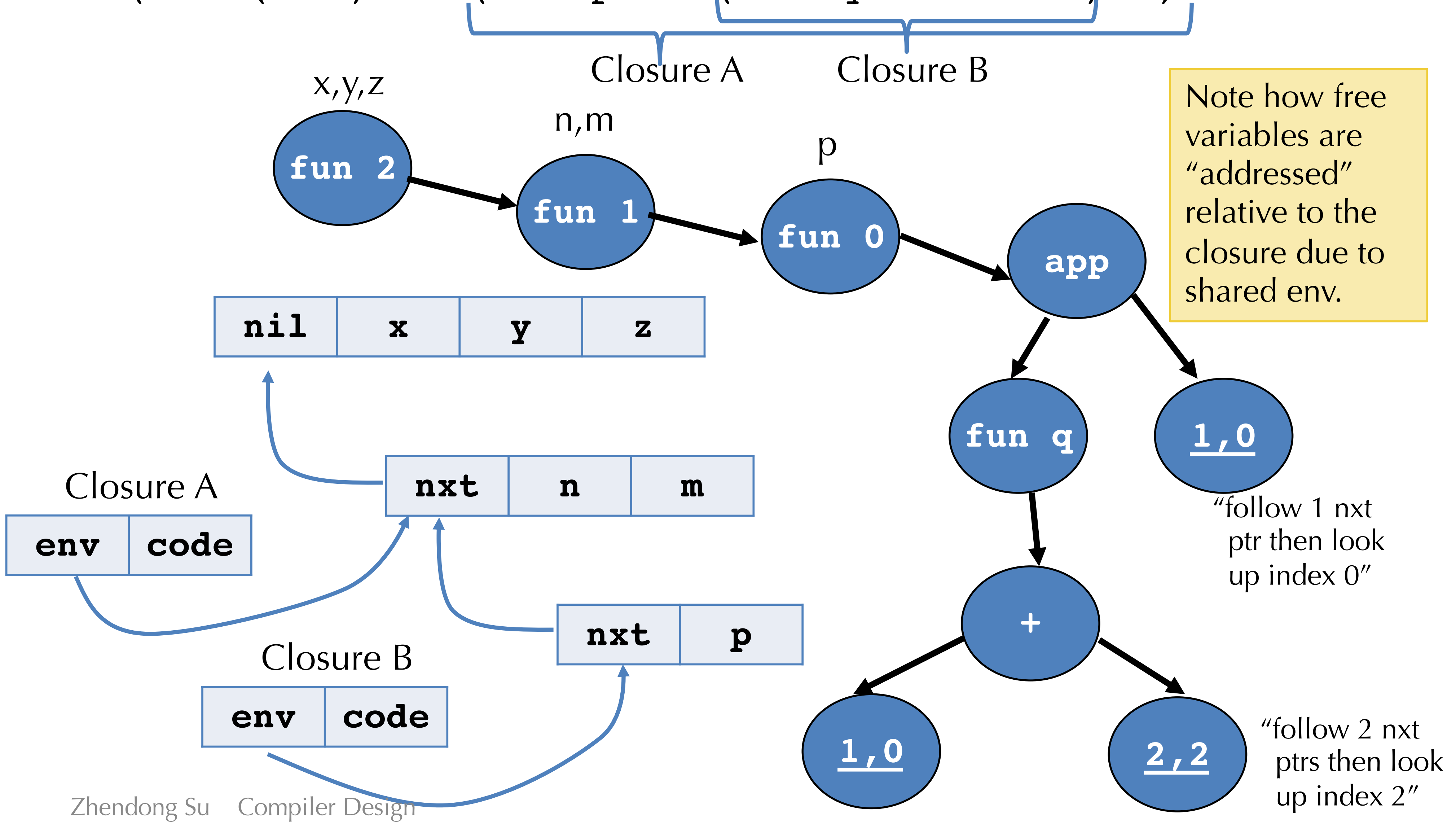
# Representing Closures

- The simple closure conversion doesn't generate very efficient code
  - It stores all the values for variables in the environment, even if they aren't needed by the function body
  - It copies the environment values each time a nested closure is created
  - It uses a linked-list data structure for tuples

- There are many options
  - Store only the values for free variables in the body of the closure
  - Share subcomponents of the environment to avoid copying
  - Use vectors or arrays rather than linked structures

# Array-based Closures with N-ary Functions

```
(fun (x y z) ->
   (fun (n m) -> (fun p -> (fun q -> n + z) x)
```



Scope, Types, and Context

# STATIC ANALYSIS

# Adding Integers to Lambda Calculus

```
exp ::=
      | …
      | n                          constant integers
      | exp1 + exp2                binary arithmetic operation

val ::=
      | fun x -> exp               functions are values
      | n                          integers are values

n{v/x}            = n                        constants have no free vars
(e1 + e2){v/x}    = (e1{v/x} + e2{v/x})      substitute everywhere
```

$$\frac{exp_1 \Downarrow n_1 \quad exp2 \Downarrow n_2}{exp_1 + exp_2 \;\Downarrow (n1 \; [\![+]\!] \; n2)}$$

object-level '+'

meta-level '+'

**NOTE:** there are no rules for the case where exp1 or exp2 evaluate to functions! The semantics is *undefined* in those cases.

# Variable Scoping

- Problem: How to determine whether a declared variable is in scope?

- Issues
  - Q1: Which variables are available at a given program location?
  - Q2: Can the same identifier be reused (i.e., shadowing), or it is an error?

- Example:  code below is syntactically correct, but not well-formed
  - y and **q** are used without being defined anywhere

```
int fact(int x) {
  var acc = 1;
  while (x > 0) {
    acc = acc * y;
    x = q - 1;
  }
  return acc;
}
```

Q: Can we solve this problem by changing the parser to rule out such programs?

# Type Checking / Static Analysis

The interpreter from the Eval3 module (fun.ml, Lec 13)

```
let rec eval env e =
  match e with
   | …
   | Add (e1, e2) ->
      (match (eval env e1, eval env e2) with
          | (IntV i1, IntV i2) -> IntV (i1 + i2)
          | _ -> failwith "tried to add non-integers")
   | …
```

- The interpreter might fail at runtime
  - Not all operations are defined for all values (e.g. 3/0,  3 + true, …)
- A compiler can't generate sensible code for this case
  - A naïve implementation might "add" an integer and a function pointer

See tc.ml (lec13.zip)

# STATICALLY RULING OUT PARTIALITY: TYPE CHECKING

# Notes about this Typechecker

- The interpreter only evaluates the body of a function when it's applied

- Typechecker always check function's body (even if it's never applied)
  - *Assume* the input has some type (say $t_1$)
  - Reflect this information in the type of the function ($t_1$ **->** $t_2$)

- Dually, at a call site ($e_1$ $e_2$), we don't know what *closure* we'll get, but
  - Can calculate $e_1$'s type
  - Check $e_2$ is an argument of the right type
  - Determine what type $e_1$ will return

(Q1) Why is this an approximation?

(Q2) What if `well_typed` always returns `false`?

# Contexts and Inference Rules

- Need to keep track of contextual information
  - What variables are in scope?
  - What are their types?
  - What information do we have about each syntactic construct?

- What relationships are there among the syntactic objects?
  - e.g. is one type a subtype of another?

- How do we describe this information?
  - In the compiler, there's a mapping from variables to information we know about them – the "context"
  - The compiler has a collection of (mutually recursive) functions that follow the structure of the syntax

# Type Judgments

- In the judgment: $E \vdash e : t$
  - E is a *typing environment* or a *type context*
  - E maps variables to types and is simply a set of bindings of the form: $x_1 : t_1, x_2 : t_2, \ldots, x_n : t_n$
- For example: $x : \text{int}, b : \text{bool} \vdash \text{if (b) 3 else x} : \text{int}$
- What do we need to know to decide whether “if (b) 3 else x” has type int in the environment x : int, b : bool?
  - b must be a bool i.e. $x : \text{int}, b : \text{bool} \vdash b : \text{bool}$
  - 3 must be an int i.e. $x : \text{int}, b : \text{bool} \vdash 3 : \text{int}$
  - x must be an int i.e. $x : \text{int}, b : \text{bool} \vdash x : \text{int}$

# Simply-typed Lambda Calculus

For the language in "tc.ml", we have five inference rules

**INT**

$$\frac{}{E \vdash i : int}$$

**VAR**

$$\frac{x : T \in E}{E \vdash x : T}$$

**ADD**

$$\frac{E \vdash e_1 : int \qquad E \vdash e_2 : int}{E \vdash e_1 + e_2 : int}$$

**FUN**

$$\frac{E, x : T \vdash e : S}{E \vdash \text{fun } (x{:}T) \text{ -> } e \; : T \text{ -> } S}$$

**APP**

$$\frac{E \vdash e_1 : T \text{ -> } S \qquad E \vdash e_2 : T}{E \vdash e_1 \; e_2 : S}$$

Note how these rules correspond to the code

# Type Checking Derivations

- *Derivation* or *proof tree*
  - Nodes: judgments
  - Edges: connect premises to a conclusion (according to inference rules)
- Leaves of the tree are *axioms* (i.e., rules with no premises)
  - Example: the INT rule is an axiom
- Goal of the type checker: verify that such a tree exists

- Ex: Find a tree for the following code using the given inference rules

$\vdash$ (fun (x:int) -> x + 3) 5 : int

# Example Derivation Tree

$$
\text{APP}\ \dfrac{\text{FUN}\ \dfrac{\text{ADD}\ \dfrac{\text{VAR}\ \dfrac{x : \text{int} \in x : \text{int}}{x : \text{int} \vdash x : \text{int}} \qquad \text{INT}\ \dfrac{}{x : \text{int} \vdash 3 : \text{int}}}{x : \text{int} \vdash x + 3 : \text{int}}}{\vdash (\text{fun } (x{:}\text{int}) \text{ -> } x + 3) : \text{int -> int}} \qquad \text{INT}\ \dfrac{}{\vdash 5 : \text{int}}}{\vdash (\text{fun } (x{:}\text{int}) \text{ -> } x + 3)\ 5 : \text{int}}
$$

Note

- The OCaml function `typecheck` verifies the existence of this tree
- Recursive calls for running `typecheck` follow the same shape as this tree
- **x : int ∈ E** is implemented by the function `lookup`

# Type Safety

***"Well typed programs do not go wrong."***

– Robin Milner, 1978

**Theorem:** (simply typed lambda calculus with integers)

If $\vdash e : t$, then there exists a value $v$ such that $e \Downarrow v$

- Note: This is a *very* strong property
  - Well-typed programs never executes undefined code like 3 + (fun x -> 2)
  - Simply-typed lambda calculus terminates (i.e., not Turing complete)

# Type Safety For General Languages

**Theorem: (Type Safety)**

If $\vdash P : t$ is a well-typed program, then either
(a) the program terminates in a well-defined way, or
(b) the program continues computing forever

- Well-defined termination could include
  - halting with a return value
  - raising an exception
- Type safety rules out undefined behavior
  - abusing "unsafe" casts: converting pointers to integers, etc.
  - treating non-code values as code (and vice-versa)
  - breaking the type abstractions of the language
- What is "defined" depends on the language semantics

# Arrays

- Array constructs are not hard
- First: add a new type constructor: **T[]**

NEW

$$\frac{E \vdash e_1 : \text{int} \qquad E \vdash e_2 : T}{E \vdash \text{new } T[e_1](e_2) \ : T[]}$$

$e_1$: size of newly alloc. array
$e_2$: initializes the array

INDEX

$$\frac{E \vdash e_1 : T[] \qquad E \vdash e_2 : \text{int}}{E \vdash e_1[e_2] \ : T}$$

UPDATE

$$\frac{E \vdash e_1 : T[] \qquad E \vdash e_2 : \text{int} \quad E \vdash e_3 : T}{E \vdash e_1[e_2] = e_3 \text{ ok}}$$

Note: These rules don't ensure array indices are within bounds, which should be checked *dynamically*

# Tuples

- ML-style tuples with statically known number of products
- First: add a new type constructor: $T_1 * \ldots * T_n$

TUPLE

$$\frac{E \vdash e_1 : T_1 \quad \ldots \quad E \vdash e_n : T_n}{E \vdash (e_1, \ldots, e_n) : T_1 * \ldots * T_n}$$

PROJ

$$\frac{E \vdash e : T_1 * \ldots * T_n \quad 1 \leq i \leq n}{E \vdash \#i\ e \ : \ T_i}$$

# References

- ML-style references (note that ML uses only expressions)
- First, add a new type constructor: **T ref**

REF

$$\frac{E \vdash e : T}{E \vdash \text{ref } e : T \text{ ref}}$$

DEREF

$$\frac{E \vdash e : T \text{ ref}}{E \vdash !e \; : T}$$

ASSIGN

$$\frac{E \vdash e_1 : T \text{ ref} \qquad E \vdash e_2 : T}{E \vdash e_1 := e_2 \; : \text{unit}}$$

Note the similarity with the rules for arrays

Beyond describing “structure”… describing “properties”

Types as sets

Subsumption

# TYPES, MORE GENERALLY

# What are types, anyway?

- A *type* is just a predicate on the set of values in a system
  - E.g., the type "int" can be thought of as a boolean function that returns "true" on integers and "false" otherwise
  - Equivalently, we can think of a type as just a *subset* of all values

- For efficiency and tractability, the predicates are usually very simple
  - Types are an *abstraction* mechanism

- We can easily add new types that distinguish different subsets of values

```
type tp =
    | IntT                      (* type of integers *)
    | PosT | NegT | ZeroT       (* refinements of ints *)
    | BoolT                     (* type of booleans *)
    | TrueT | FalseT            (* subsets of booleans *)
    | AnyT                      (* any value *)
```

# Modifying the typing rules

- We need to refine the typing rules too
- Some easy cases
  - Just split up the integers into their more refined cases

P-INT

$$\frac{i > 0}{E \vdash i : Pos}$$

N-INT

$$\frac{i < 0}{E \vdash i : Neg}$$

ZERO

$$\frac{}{E \vdash 0 : Zero}$$

- Same for booleans

TRUE

$$\frac{}{E \vdash true : True}$$

FALSE

$$\frac{}{E \vdash false : False}$$

# What about "if"?

- Two cases are easy

IF-T
$$\frac{E \vdash e_1 : \text{True} \qquad E \vdash e_2 : T}{E \vdash \text{if } (e_1)\ e_2 \text{ else } e_3 : T}$$

IF-F
$$\frac{E \vdash e_1 : \text{False} \qquad E \vdash e_3 : T}{E \vdash \text{if } (e_1)\ e_2 \text{ else } e_3 : T}$$

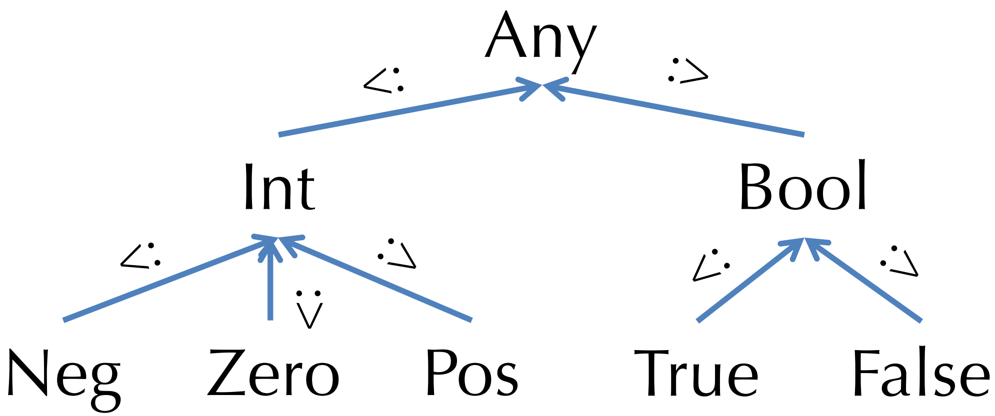
- What if we don't know statically which branch will be taken?
- Consider the typechecking problem

$$x\text{:bool} \vdash \text{if } (x)\ 3 \text{ else } \text{-}1 : \mathbf{?}$$

- The true branch has type Pos, while the false branch has type Neg
  - What should be the result type of the whole if?

# Subtyping and Upper Bounds

- If we view types as sets of values, there is a natural inclusion relation
  **Pos ⊆ Int**
- This subset relation gives rise to a ***subtype*** relation: **Pos <: Int**
- Such inclusions give rise to a ***subtyping hierarchy***




- For types $T_1$,$T_2$, define their ***least upper bound*** (LUB) wrt the hierarchy
  - Example: LUB(True, False) = Bool, LUB(Int, Bool) = Any
  - Note: may want to add types for “NonZero”, “NonNegative”, “NonPositive”, so that set union on values corresponds to taking LUBs on types

# "If" Typing Rule Revisited

- For statically unknown conditionals, we want the return value to be the LUB of the types of the branches

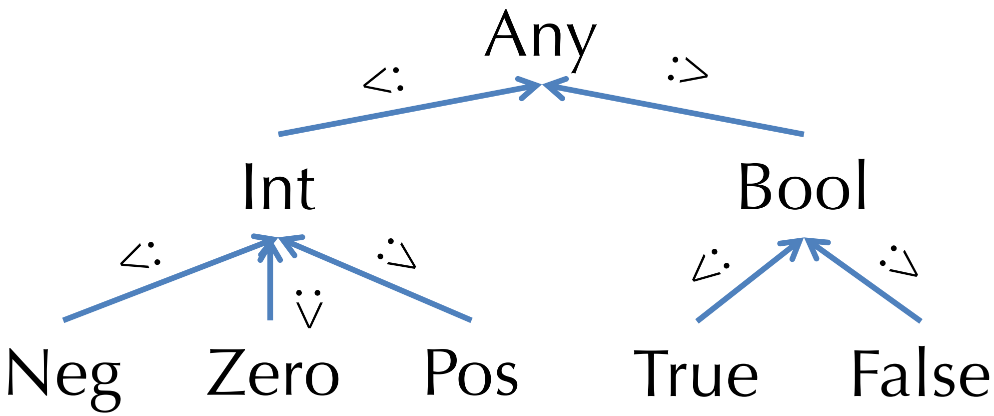
IF-BOOL

$$\frac{E \vdash e_1 : \text{bool} \quad E \vdash e_2 : T_1 \qquad E \vdash e_3 : T_2}{E \vdash \text{if } (e_1)\ e_2 \text{ else } e_3 : \mathbf{LUB(T_1,T_2)}}$$

- Note LUB($T_1$, $T_2$) is the most precise type (wrt the hierarchy) that is able to describe any value that has either type $T_1$ or type $T_2$
- In math notation, LUB(T1, T2) is sometimes written $T_1 \vee T_2$
- LUB is also called the *join* operation

# Subtyping Hierarchy

- A *subtyping hierarchy*




- The subtyping relation is a *partial order*
  - Reflexive: $T <: T$ for any type $T$
  - Transitive: $T_1 <: T_2$ and $T_2 <: T_3$ then $T_1 <: T_3$
  - Antisymmetric: It $T_1 <: T_2$ and $T_2 <: T_1$ then $T_1 = T_2$

# Soundness of Subtyping Relations

- We don't have to treat *every* subset of the integers as a type
  - e.g., we left out the type NonNeg

- A subtyping relation $T_1 <: T_2$ is *sound* if it approximates the underlying semantic subset relation
- Formally: write $[\![T]\!]$ for the subset of (closed) values of type T
  - i.e. $[\![T]\!] = \{v \mid \vdash v : T\}$
  - e.g. $[\![\text{Zero}]\!] = \{0\}$, $[\![\text{Pos}]\!] = \{1, 2, 3, \ldots\}$

- If $T_1 <: T_2$ implies $[\![T_1]\!] \subseteq [\![T_2]\!]$, then $T_1 <: T_2$ is sound
  - Ex 1: Pos <: Int is sound, since $\{1,2,3,\ldots\} \subseteq \{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$
  - Ex 2: Int <: Pos is not sound, since it is ***not*** the case that

    $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\} \subseteq \{1,2,3,\ldots\}$

# Soundness of LUBs

- Whenever we have a sound subtyping relation, it follows that
  $[\![\mathrm{LUB}(T_1, T_2)]\!] \supseteq [\![T_1]\!] \cup [\![T_2]\!]$
  - Note that the LUB is an over approximation of the "semantic union"
  - Example: $[\![\mathrm{LUB}(\mathrm{Zero}, \mathrm{Pos})]\!] = [\![\mathrm{Int}]\!] = \{\ldots,-3,-2,-1,0,1,2,3,\ldots\} \supseteq$
    $\{0,1,2,3,\ldots\} = \{0\} \cup \{1,2,3,\ldots\} = [\![\mathrm{Zero}]\!] \cup [\![\mathrm{Pos}]\!]$

- Using LUBs in the typing rules yields sound approximations of the program behavior (as if the IF-B rule)
- It just so happens that LUBs on types <: Int correspond to +

ADD

$$\frac{E \vdash e_1 : T_1 \quad E \vdash e_2 : T_2 \qquad T_1 <: \mathrm{Int} \quad T_2 <: \mathrm{Int}}{E \vdash e_1 + e_2 : T_1 \vee T_2}$$

# Subsumption Rule

- When we add subtyping judgments of the form $T <: S$ we can uniformly integrate it into the type system generically

$$\boxed{\text{SUBSUMPTION}} \quad \frac{E \vdash e : T \qquad T <: S}{E \vdash e : S}$$

- Subsumption allows any value of type T to be treated as an S whenever $T <: S$
- Adding this rule makes the search for typing derivations more difficult – this rule can be applied anywhere, since $T <: T$
  - But careful engineering of the typing system can incorporate the subsumption rule into a deterministic algorithm
  - See for example the OAT type system

# Downcasting

- What happens if we have an Int, but need something of type Pos?
  - At compile time, we don't know whether the Int is greater than zero
  - At run time, we do

- Add a "checked downcast"

$$\frac{E \vdash e_1 : \text{Int} \qquad E, x : \text{Pos} \vdash e_2 : T_2 \qquad E \vdash e_3 : T_3}{E \vdash \text{ifPos } (x = e_1)\ e_2 \text{ else } e_3 : T_2 \vee T_3}$$

- At runtime, ifPos checks whether $e_1$ is > 0. If so, branches to $e_2$ and otherwise branches to $e_3$
- Inside the expression $e_2$, x is the name for $e_1$'s value, which is known to be strictly positive because of the dynamic check
- Note such rules force the programmer to add the appropriate checks
  - We could give integer division the type: Int -> NonZero -> Int