



Randomized and Systematic Testing of Software

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automated testing

- Oracle generation
 - ◆ How to define test oracles?
- Test input generation
 - ◆ How to synthesize test inputs?

Two instances

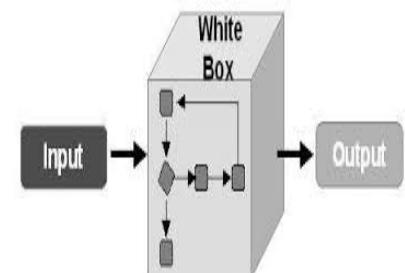
- Validate production compilers

- ◆ Black-box analysis



- Analyze floating-point software

- ◆ Dynamic analysis



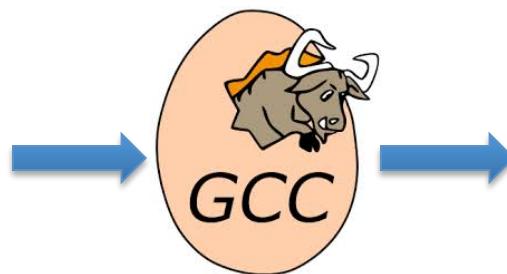
Validate Production Compilers



black box

compilers

```
int main ()  
{  
    // do something  
    ...  
    return 0;  
}
```



```
...  
call puts  
movl $0, %eax  
popq %rbp  
ret
```

Developers' belief:
Compilers are *faithful* translators

let's reflect on this trust

How trustworthy are compilers?

How they impact security & reliability?

In 2014, GCC 4.9 miscompiled the Linux kernel!

*“Ok, so I’m looking at the code generation and your compiler is pure and utter ***.”*

.....

*Adding Jakub to the cc, because gcc-4.9.0 seems to be **terminally broken.**”*

-- Linus Torvalds

Type #1 bugs

llvm bug 14972

```
struct tiny { char c; char d; char e; };

void foo(struct tiny x) {
    if (x.c != 1) abort();
    if (x.e != 1) abort();
}

int main() {
    struct tiny s;
    s.c = 1; s.d = 1; s.e = 1;
    foo(s);
    return 0;
}
```

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
Aborted (core dumped)
```

developer comment

*“... very, very concerning when I
got to the root cause, and very
annoying to fix ...”*

http://llvm.org/bugs/show_bug.cgi?id=14972

Type #2 “bugs”

gcc “bug” 8537

```
#include <string>
using std::string;
#include <memory>

// The specifics of this function are not important for
// demonstrating this bug.
const string getPasswordFromUser() const;

bool isPasswordCorrect() {
    bool isPasswordCorrect = false;
    string Password("password");

    if(Password == getPasswordFromUser()) {
        isPasswordCorrect = true;
    }

    // Removed from the optimized code although it secures
    // the code by wiping the password from memory
    memset(Password, 0, sizeof(Password));
}

return isPasswordCorrect;
}
```

debatable ...

From: "Joseph D. Wagner" <wagnerjd@prodigy.net>
To: <fw@gcc.gnu.org>, gcc-bugs@gcc.gnu.org, <gcc-prs@gcc.gnu.org>,
nobody@gcc.gnu.org, wagnerjd@prodigy.net, <gcc-gnats@gcc.gnu.org>

Cc:

Subject: RE: optimization/8537: Optimizer Removes Code Necessary for Security

Date: Sun, 17 Nov 2002 08:59:53 -0600

Direct quote from:

<http://gcc.gnu.org/onlinedocs/gcc-3.2/gcc/Bug-Criteria.html>

"If the compiler produces valid assembly code that does not correctly execute the input source code, that is a compiler bug."

So to all you naysayers out there who claim this is a programming error or poor coding, YES, IT IS A BUG!

Type #3 “bugs”

how about this one?

```
char *buf = ...;
char *buf_end = ...;
unsigned int len = ...;

if (buf + len >= buf_end)
    return; // len too large

if (buf + len < buf)
    return; //overflow, buf+len wrapped around

// write to buf[0..len-1]
...
```

even more debatable ...

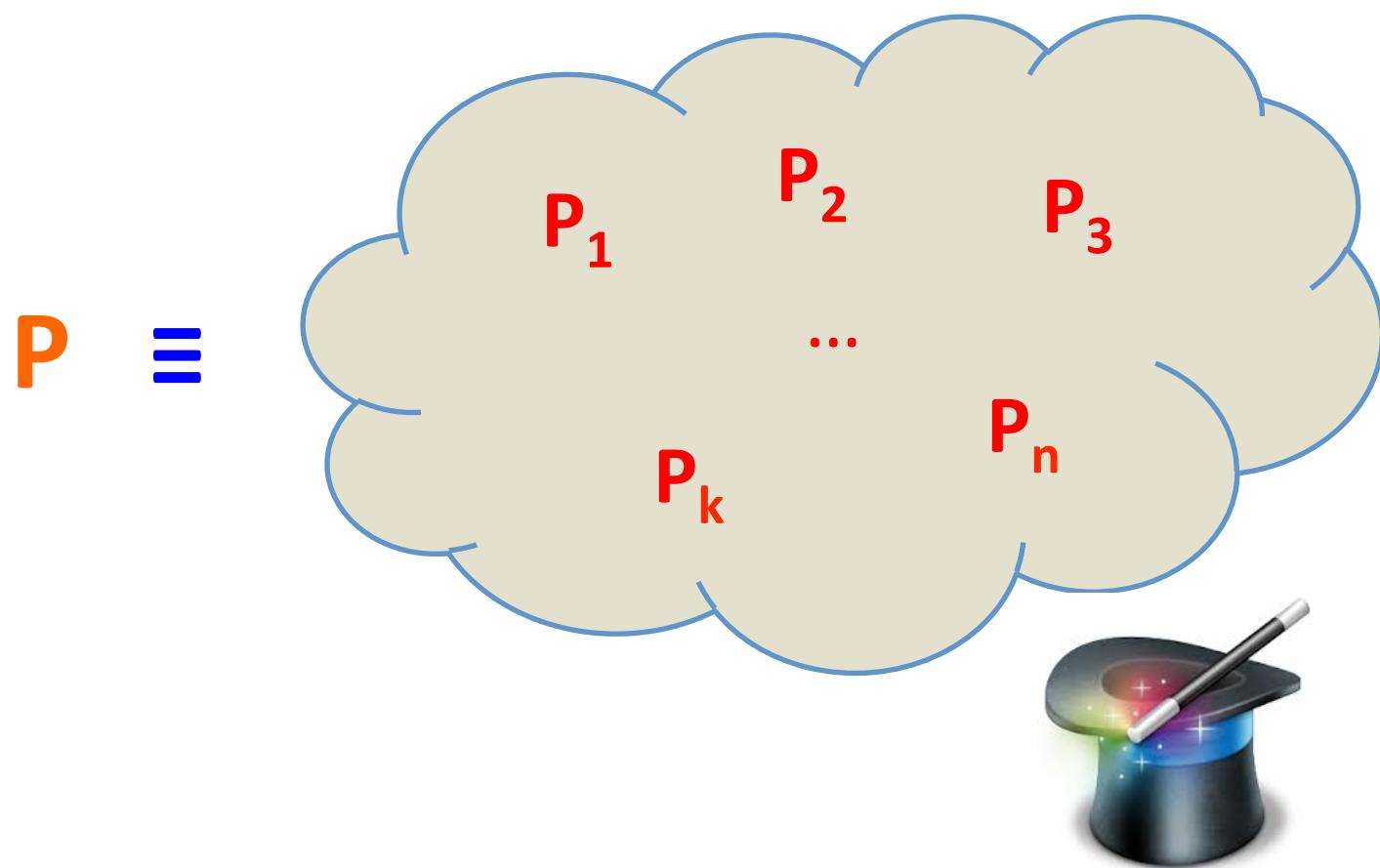
- Pointer overflow is **undefined behavior**
 - ◆ Compiler assumes code has no undefined behavior
 - ◆ Thus compiler assumes: **buf + len** cannot overflow
 - ◆ Then compiler infers: **if (buf + len < buf) ⇒ if (0)**
- But this is a **security threat**
 - ◆ Use a large **len** to trigger **buffer overflow**

Type #1 bugs: EMI Testing

How to find 1600+ GCC/LLVM bugs in 5 years?

real

vision



key challenges

- Generation

- ◆ How to generate **different**, yet **equivalent** tests?

- Validation

- ◆ How to check that tests are **indeed equivalent**?

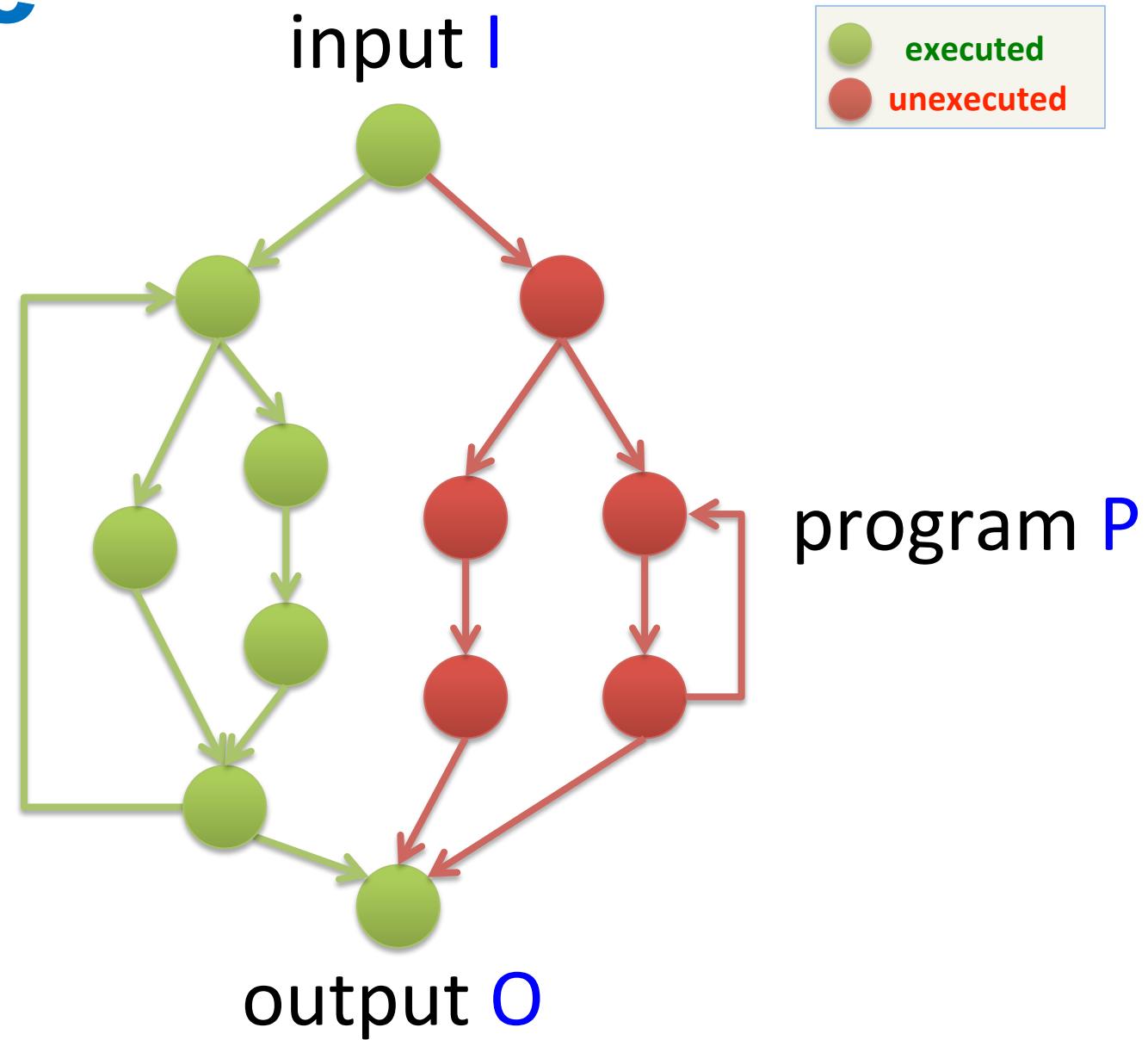
- Both are long-standing hard issues

equiv. modulo inputs

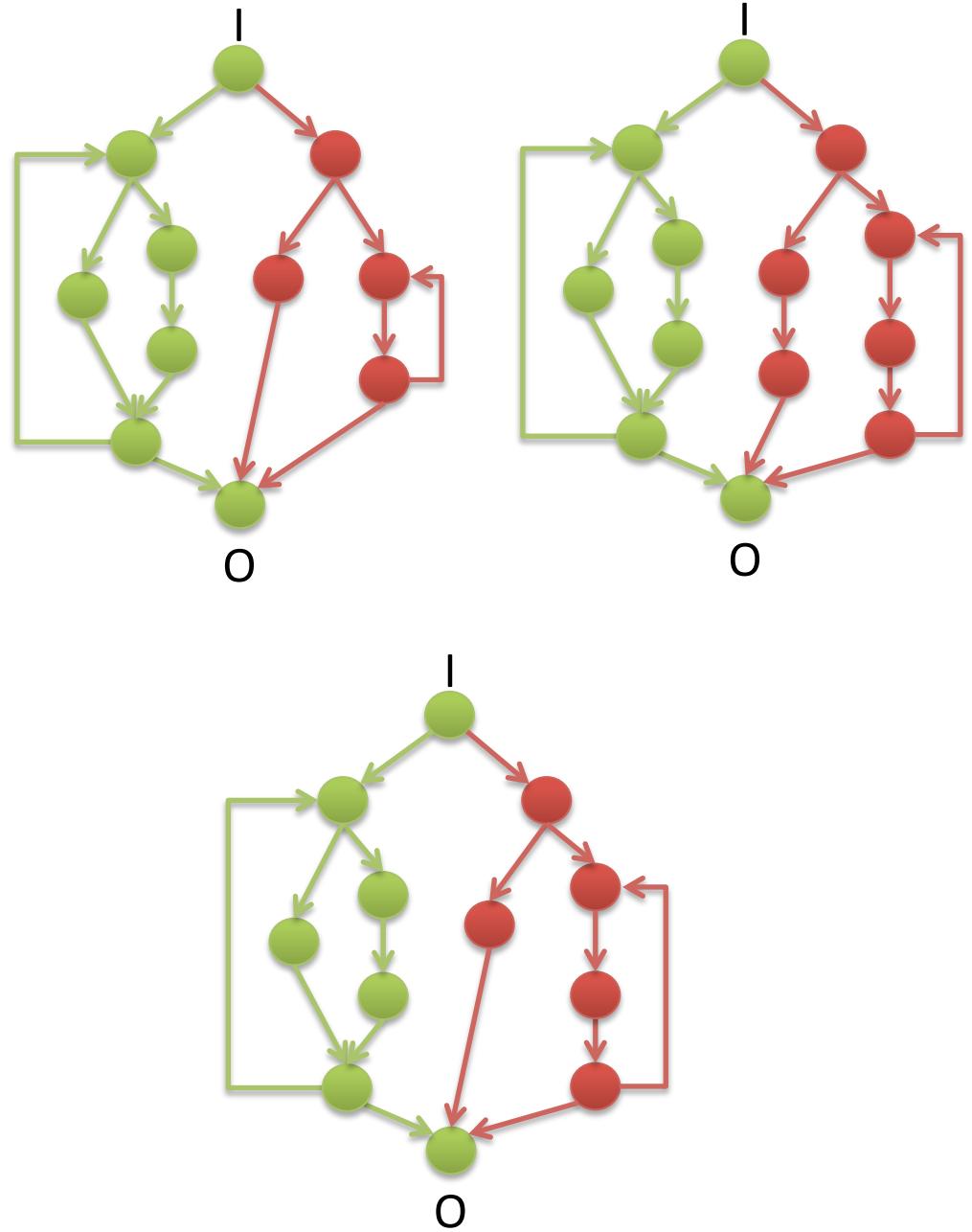
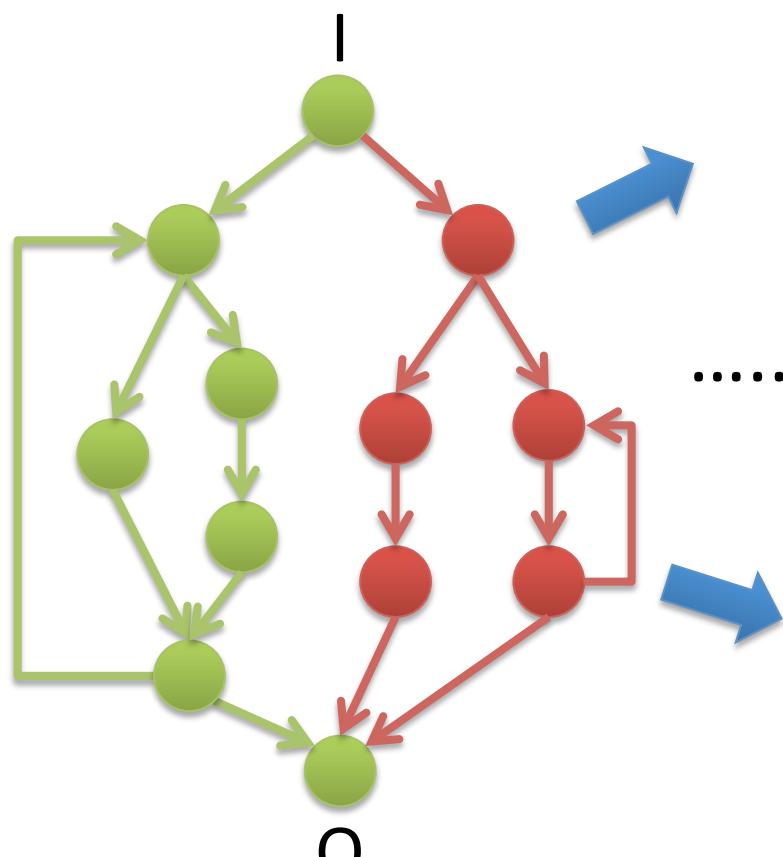


- Relax equiv. wrt a **given input i**
 - ◆ Must: $P(i) = P_k(i)$ on input i
 - ◆ Okay: $P(j) \neq P_k(j)$ on all input $j \neq i$
- Exploit close **interplay** between
 - ◆ **Dynamic** program execution on **some input**
 - ◆ **Static** compilation for **all input**

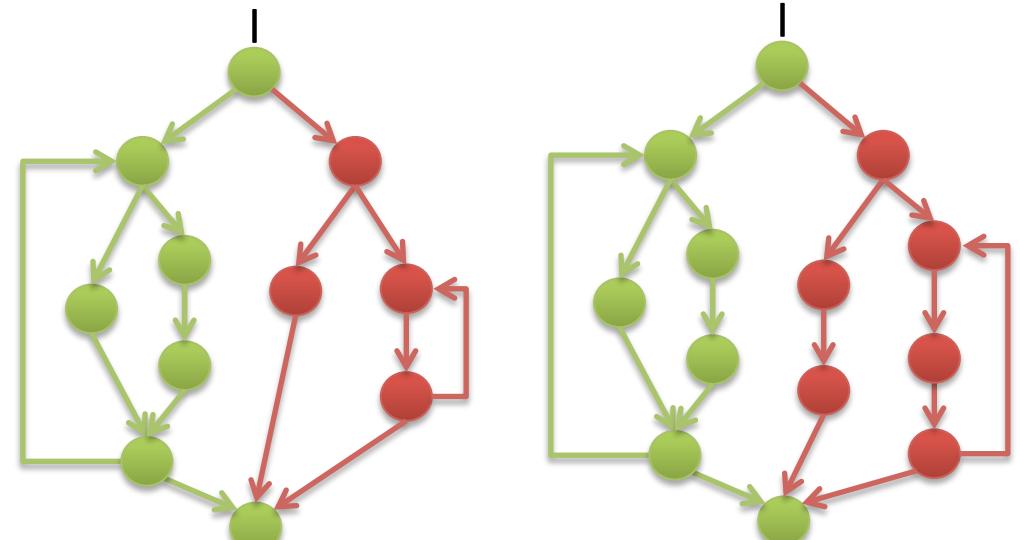
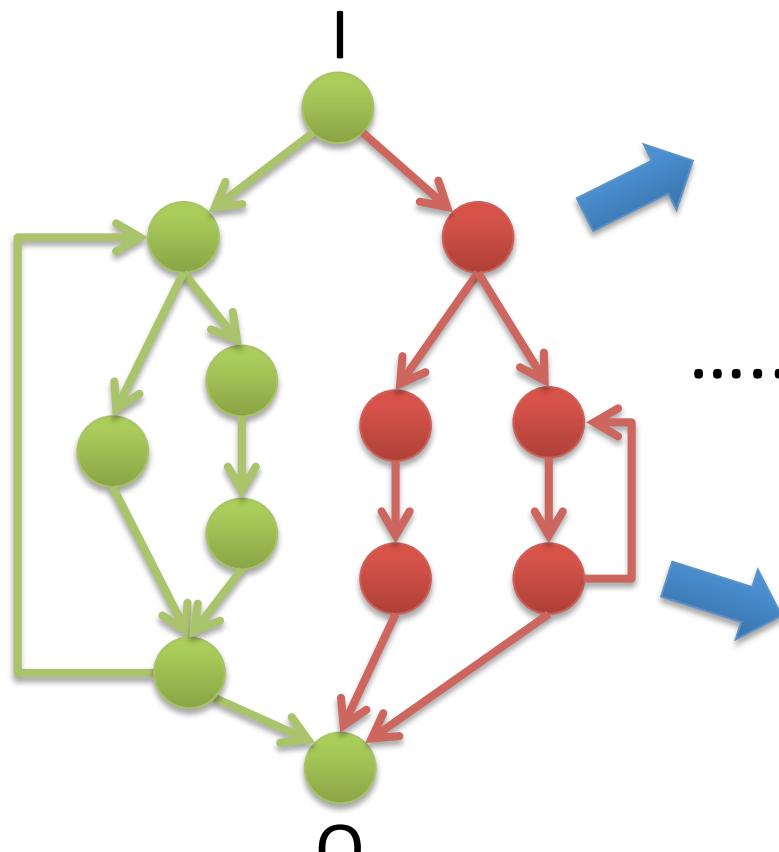
profile



mutate

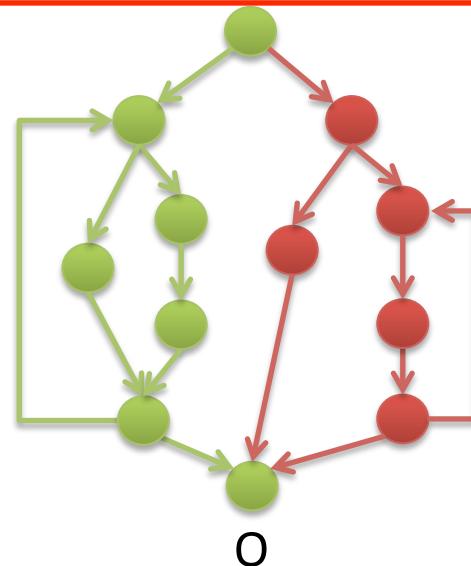


mutate

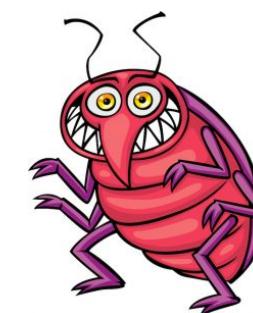
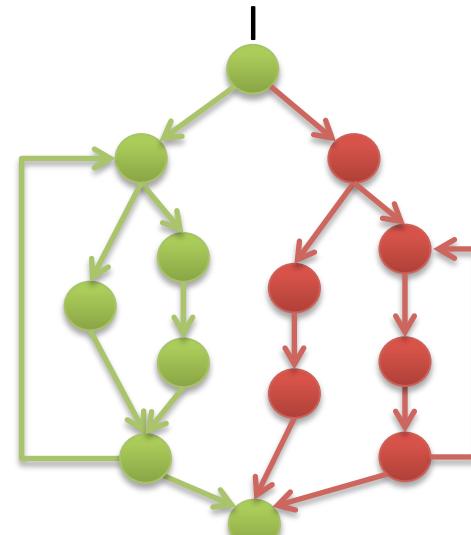
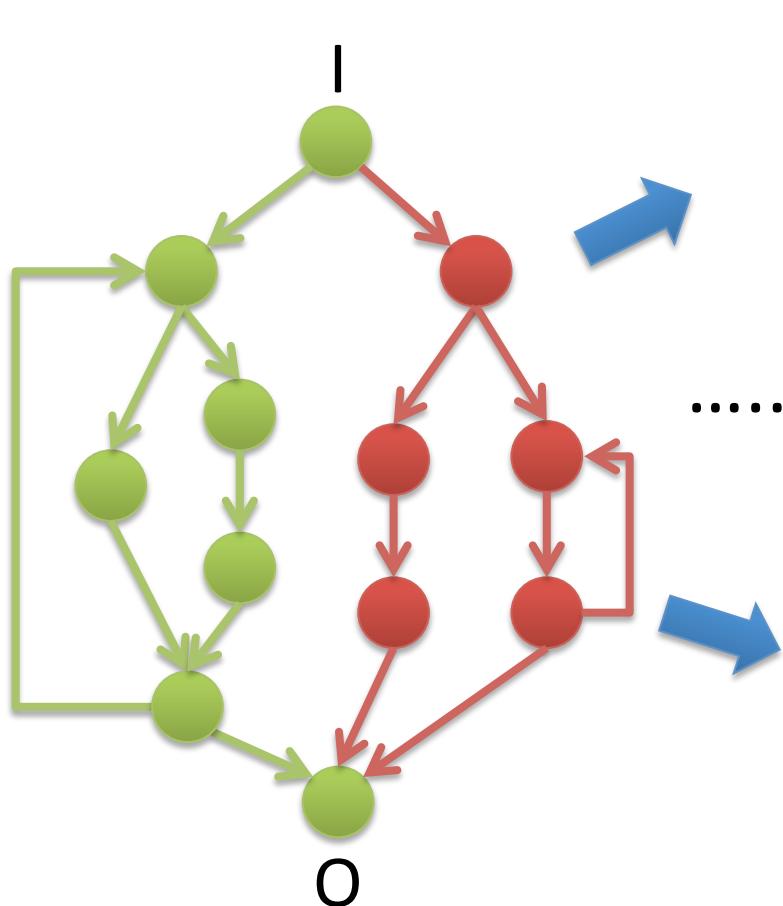


.....

equivalent wrt I



Find bugs



revisit challenges

- Generation (**easy**)
 - ◆ How to generate **different**, yet **equivalent** tests?
- Validation (**easy**)
 - ◆ How to check that tests are **indeed equivalent**?
- Both are long-standing hard issues

llvm bug 14972

```
struct tiny { char c; char d; char e; };

void foo(struct tiny x) {
    if (x.c != 1) abort();
    if (x.e != 1) abort();
}

int main() {
    struct tiny s;
    s.c = 1; s.d = 1; s.e = 1;
    foo(s);
    return 0;
}
```

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
Aborted (core dumped)
```

seed file

```
struct tiny { char c; char d; char e; };
f(int n, struct tiny x, struct tiny y,
   struct tiny z, long l) {
    if (x.c != 10) abort();
    if (x.d != 20) abort();
    if (x.e != 30) abort();
    if (y.c != 11) abort();
    if (y.d != 21) abort();
    if (y.e != 31) abort();
    if (z.c != 12) abort();
    if (z.d != 22) abort();
    if (z.e != 32) abort();
    if (l != 123) abort();
}
main() {
    struct tiny x[3];
    x[0].c = 10;
    x[1].c = 11;
    x[2].c = 12;
    x[0].d = 20;
    x[1].d = 21;
    x[2].d = 22;
    x[0].e = 30;
    x[1].e = 31;
    x[2].e = 32;
    f(3, x[0], x[1], x[2], (long)123);
    exit(0);
}
```

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
```

seed file

```
struct tiny { char c; char d; char e; };
f(int n, struct tiny x, struct tiny y,
   struct tiny z, long l) {
    if (x.c != 10) abort();
    if (x.d != 20) abort();
    if (x.e != 30) abort();
    if (y.c != 11) abort();
    if (y.d != 21) abort();
    if (y.e != 31) abort();
    if (z.c != 12) abort();
    if (z.d != 22) abort();
    if (z.e != 32) abort();
    if (l != 123) abort();
}
main() {
    struct tiny x[3];
    x[0].c = 10;
    x[1].c = 11;
    x[2].c = 12;
    x[0].d = 20;
    x[1].d = 21;
    x[2].d = 22;
    x[0].e = 30;
    x[1].e = 31;
    x[2].e = 32;
    f(3, x[0], x[1], x[2], (long)123);
    exit(0);
}
```

← unexecuted

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
```

transformed file

```
struct tiny { char c; char d; char e; };
f(int n, struct tiny x, struct tiny y,
   struct tiny z, long l) {
    if (x.c != 10) /* deleted */;
    if (x.d != 20) abort();
    if (x.e != 30) /* deleted */;
    if (y.c != 11) abort();
    if (y.d != 21) abort();
    if (y.e != 31) /* deleted */;
    if (z.c != 12) abort();
    if (z.d != 22) /* deleted */;
    if (z.e != 32) abort();
    if (l != 123) /* deleted */;
}
main() {
    struct tiny x[3];
    x[0].c = 10;
    x[1].c = 11;
    x[2].c = 12;
    x[0].d = 20;
    x[1].d = 21;
    x[2].d = 22;
    x[0].e = 30;
    x[1].e = 31;
    x[2].e = 32;
    f(3, x[0], x[1], x[2], (long)123);
    exit(0);
}
```

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
Aborted (core dumped)
```

reduced file

```
struct tiny { char c; char d; char e; };

void foo(struct tiny x) {
    if (x.c != 1) abort();
    if (x.e != 1) abort();
}

int main() {
    struct tiny s;
    s.c = 1; s.d = 1; s.e = 1;
    foo(s);
    return 0;
}
```

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
Aborted (core dumped)
```

llvm bug autopsy

```
struct tiny { char c; char d; char e; };
```

```
void foo(struct tiny x) {
    if (x.c != 1) abort();
    if (x.e != 1) abort();
}
```

GVN: load struct
using 32-bit load

```
int main() {
    struct tiny s;
    s.c = 1; s.d = 1; s.e = 1;
    foo(s);
    return 0;
}
```

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
Aborted (core dumped)
```

Ilvm bug autopsy

```
struct tiny { char c; char d; char e; };
```

```
void foo(struct tiny x) {
    if (x.c != 1) abort();
    if (x.e != 1) abort();
}
```

```
int main() {
    struct tiny s;
    s.c = 1; s.d = 1; s.e = 1;
    foo(s);
    return 0;
}
```

GVN: load struct
using 32-bit load

SRoA: read past
the struct's end

undefined
behavior

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
Aborted (core dumped)
```

llvm bug autopsy

```
struct tiny { char c; char d; char e; };
```

```
void foo(struct tiny x) {
    if (x.c != 1) abort();
    if (x.e != 1) abort();
}
```

```
int main() {
    struct tiny s;
    s.c = 1; s.d = 1; s.e = 1;
    foo(s);
    return 0;
}
```

GVN: load struct
using 32-bit load

SRoA: read past
the struct's end

remove

undefined
behavior

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
Aborted (core dumped)
```

seed file

```
struct tiny { char c; char d; char e; };
f(int n, struct tiny x, struct tiny y,
   struct tiny z, long l) {
    if (x.c != 10) abort();
    if (x.d != 20) abort();
    if (x.e != 30) abort();
    if (y.c != 11) abort();
    if (y.d != 21) abort();
    if (y.e != 31) abort();
    if (z.c != 12) abort();
    if (z.d != 22) abort();
    if (z.e != 32) abort();
    if (l != 123) abort();
}
main() {
    struct tiny x[3];
    x[0].c = 10;
    x[1].c = 11;
    x[2].c = 12;
    x[0].d = 20;
    x[1].d = 21;
    x[2].d = 22;
    x[0].e = 30;
    x[1].e = 31;
    x[2].e = 32;
    f(3, x[0], x[1], x[2], (long)123);
    exit(0);
}
```

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
```

transformed file

```
struct tiny { char c; char d; char e; };
f(int n, struct tiny x, struct tiny y,
   struct tiny z, long l) {
    if (x.c != 10) /* deleted */;
    if (x.d != 20) abort();
    if (x.e != 30) /* deleted */;
    if (y.c != 11) abort();
    if (y.d != 21) abort();
    if (y.e != 31) /* deleted */;
    if (z.c != 12) abort();
    if (z.d != 22) /* deleted */;
    if (z.e != 32) abort();
    if (l != 123) /* deleted */;
}
main() {
    struct tiny x[3];
    x[0].c = 10;
    x[1].c = 11;
    x[2].c = 12;
    x[0].d = 20;
    x[1].d = 21;
    x[2].d = 22;
    x[0].e = 30;
    x[1].e = 31;
    x[2].e = 32;
    f(3, x[0], x[1], x[2], (long)123);
    exit(0);
}
```

```
$ clang -m32 -O0 test.c ; ./a.out
$ clang -m32 -O1 test.c ; ./a.out
Aborted (core dumped)
```

gcc bug 58731

```
int a, b, c, d, e;
int main() {
    for (b = 4; b > -30; b--)
        for (; c++)
            for (;;) {
                e = a > 2147483647 - b;
                if (d) break;
            }
    return 0;
}
```

```
$ gcc -O0 test.c ; ./a.out
$ gcc -O3 test.c ; ./a.out
^C
```

gcc bug autopsy

```
int a, b, c, d, e;
int main() {
    for (b = 4; b > -30; b--)
        for (; c++)
            for (;;) {
                e = a > 2147483647 - b;
                if (d) break;
            }
    return 0;
}
```

PRE: loop invariant

```
$ gcc -O0 test.c ; ./a.out
$ gcc -O3 test.c ; ./a.out
^C
```

gcc bug autopsy

```
int a, b, c, d, e;
int main() {
    for (b = 4; b > -30; b--)
        int f = 2147483647 - b;
    for (; c;)
        for (;;) {
            e = a > f;
            if (d) break;
        }
    return 0;
}
```

```
$ gcc -O0 test.c ; ./a.out
$ gcc -O3 test.c ; ./a.out
^C
```

gcc bug autopsy

```
int a, b, c, d, e;
int main() {
    for (b = 4; b > -30; b--)
        int f = 2147483647 - b;
    for (; c;)
        for (;;) {
            e = a > f; integer overflow
            if (d) break;
        }
    return 0;
}
$ gcc -O0 test.c ; ./a.out
$ gcc -O3 test.c ; ./a.out
^C
```

seed program

```
int a, b, c, d, e;
int main() {
    for (b = 4; b > -30; b--)
        for (; c++)
            for (;;) {
                b++;
                e = a > 2147483647 - b;
                if (d) break;
            }
    return 0;
}
```

no longer a loop invariant

```
$ gcc -O0 test.c ; ./a.out
$ gcc -O3 test.c ; ./a.out
```

orion

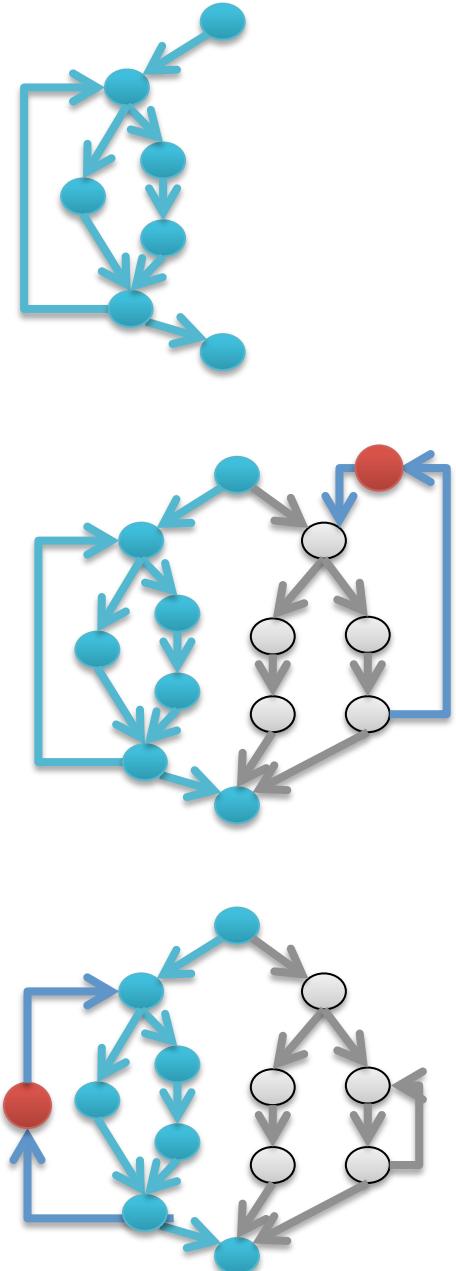
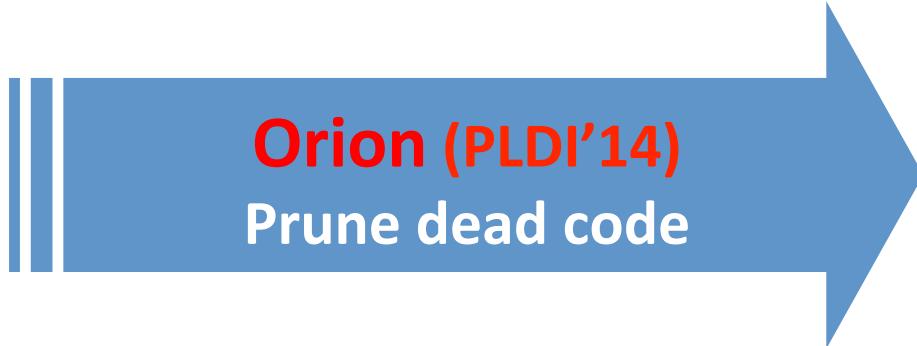
- First and simplest EMI realization
- Targeting C compilers
 - ◆ Randomly **prune** unexecuted code
 - ◆ Very **effective**

bug counts (till 03/2014)

	GCC	LLVM	TOTAL
Reported	111	84	195
Marked Duplicate	28	7	35
Confirmed	79	68	147
Fixed	56	54	110

bug types

	GCC	LLVM	TOTAL
Wrong code	46	49	95
Crash	23	10	33
Performance	10	9	19



Athena: gcc bug 61383

Seed Program P

```
int a, c, d, e = 1, f;
int fn1 () {
    int h;
    for (; d < 1; d = e) {
        h = (f == 0) ? 0 : 1 % f;
        if (f < 1) c = 0;
        else C = 1;
    }
}
int main () {
    fn1 ();
    return 0;
}
```

Athena

Bug-triggering Variant

```
int a, c, d, e = 1, f;
int fn1 () {
    int h;
    for (; d < 1; d = e) {
        h = (f == 0) ? 0 : 1 % f;
        if (f < 1) c = 0;
        else if (h) break;
    }
}
int main () {
    fn1 ();
    return 0;
}
```

```
$ gcc -O0 test.c ; ./a.out
$ gcc -O2 test.c ; ./a.out
Floating point exception (core dumped)
```

Current

```
int a, c, d, e = 1, f;
int fn1 () {
    int h;
    for (; d < 1; d = e) {
        h = (f == 0) ? 0 : 1 % f;
        if (f < 1) c = 0;
        else c = 1;
    }
}
int main () {
    fn1 ();
    return 0;
}
```

Current

```
int a, c, d, e = 1, f;
int fn1 () {
    int h;
    for (; d < 1; d = e) {
        h = (f == 0) ? 0 : 1 % f;
        if (f < 1) c = 0;
        else c = 1;
    }
}
int main () {
    fn1 ();
    return 0;
}
```

Proposal

```
int a, c, d, e = 1, f;
int fn1 () {
    int h;
    for (; d < 1; d = e) {
        h = (f == 0) ? 0 : 1 % f;
        if (f < 1) c = 0;
        else:
    }
}
int main () {
    fn1 ();
    return 0;
}
```

Delete “c = 1”

Current

```
int a, c, d, e = 1, f;
int fn1 () {
    int h;
    for (; d < 1; d = e) {
        h = (f == 0) ? 0 : 1 % f;
        if (f < 1) c = 0;
        else;
    }
}
int main () {
    fn1 ();
    return 0;
}
```

Insert the statement below

Proposal

```
int a, c, d, e = 1, f;
int fn1 () {
    int h;
    for (; d < 1; d = e) {
        h = (f == 0) ? 0 : 1 % f;
        if (f < 1) c = 0;
        else if (h) break;
    }
}
int main () {
    fn1 ();
    return 0;
}
```

Context

1. Requires loop

2. `int i;`

Statement

```
if (i)
break;
```

Bug-triggering Variant

```
int a, c, d, e = 1, f;
int fn1 () {
    int h;
    for (; d < 1; d = e) {
        h = (f == 0) ? 0 : 1 % f;
        if (f < 1) c = 0;
        else if (h) break;
    }
}
int main () {
    fn1 ();
    return 0;
}
```

loop invariant
hoisted

Miscompiled Executable

```
int a, c, d, e = 1, f;
int fn1 () {
    int h;
int g = 1 % f;
    for (; d < 1; d = e) {
        h = (f == 0) ? 0 : g;
        if (f < 1) c = 0;
        else if (h) break;
    }
}
int main () {
    fn1 ();
    return 0;
}
```

Hermes

- **Profile and record variable values**
- Synthesize code snippets
 - Ensure **no undefined behavior**
 - Ensure **EMI property locally**
 - Maintain same program state at entry & exit

Hermes: llvm 26266

Seed Program P

```
char a;
int b, c = 9, d, e;

void fn1() {
    unsigned f = 1;
    int g = 8, h = 5;
    for (; a != 6; a--) {
        int *i = &h, *j;
        for (;;) {
// b=0,c=9,e=0,f=1,g=8,h=5
            if (d <= 8) break;
            *i = 0;
            for (; *j <= 0;)
}
}
int main() {fn1(); return 0;}
```

EMI Variant

```
int *i = &h, *j;
for (;;) {
// b=0,c=9,e=0,f=1,g=8,h=5
int backup_g = e, backup_f = ~1;
if (g && h) {
    backup_g = g;
    backup_f = f;
    f = -(~(c && b)|~(e*~backup_f));
    if (c < f) abort();
}
g = backup_g;
f = backup_f;

    if (d <= 8) break;
    *i = 0;
    for (; *j <= 0;)
}
```

```
$ clang -m32 -O0 test.c  
$ ./a.out  
$ clang -m32 -O1 test.c  
$ ./a.out  
Aborted (core dumped)
```

EMI Variant

```
int *i = &h, *j;  
for (;;) {  
// b=0,c=9,e=0,f=1,g=8,h=5  
    int backup_g = e, backup_f = ~1;  
    if (g && h) {  
        backup_g = g;  
        backup_f = f;  
        f = -(~(c && b) | -~(e*~backup_f));  
        if (C < f) abort();  
    }  
    g = backup_g;  
    f = backup_f;  
    if (d <= 8) break;  
    *i = 0;  
    for (; *j <= 0;);  
}
```

Clang (mistakenly) deems
this predicate always **true**

bug counts

	GCC	LLVM	TOTAL
Reported	841	781	1622
Fixed	612	419	1031

- **ISSTA'15**: Stress-testing link-time optimization
- **ICSE'16**: Analyzing compilers' diagnostic support
- **PLDI'17**: Skeletal program enumeration (**SPE**)

LLVM 3.9 & 4.0 Release Notes

“... thanks to Zhendong Su and his team
whose fuzz testing **prevented many bugs**
going into the release ...”

GCC's list of contributors

<https://gcc.gnu.org/onlinedocs//gcc/Contributors.html>

“Zhendong Su … for reporting numerous bugs”

“Chengnian Sun … for reporting numerous bugs”

“Qirun Zhang … for reporting numerous bugs”

program enumeration

- **Vision:** Bounded compiler verification
- **Goal:** Program enumeration
 - ◆ **Exhaustive** (all small test programs)
 - ◆ **Practical**
- **Idea:**
 - ◆ Context-free grammar to token sequences
 - ◆ Token sequences to programs (**SPE**)

SPE (PLDI'17)

$a := 10;$
 $b := 1;$
while(a) do
 $a := a - b;$

(a) Program P

$\square := 10;$
 $\square := 1;$
while(\square) do
 $\square := \square - \square;$

(b) Skeleton \mathbb{P}

$b := 10;$
 $a := 1;$
while(b) do
 $b := b - a;$

(c) Program P_1

$a := 10;$
 $b := 1;$
while(b) do
 $b := a - b;$

(d) Program P_2

example 1: wrong code

```
1 int a = 0;
```

Marek Polacek 2016-02-25 09:13:03 UTC

[Comment 1](#)

Hm, this really seems like a wrong code. Started a looooong time ago, before [r104500](#).

Richard Biener 2016-02-25 10:22:43 UTC

[Comment 2](#)

We have a duplicate for this - basically (most) of alias analysis doesn't handle aliases (hah).

```
11 return a; // Bug: the program exits with 1
12 }
```

example 2: gcc crash

```
1 struct s { char c[1]; };
2 struct s a, b, c;
3 int d; int e;
4
5 void bar (void)
6 {
7 //e ? (d=e==0 ? b : c).c : (e==0 ? b : c).c;
8   e ? (d=e==0 ? b : c).c : (d==0 ? b : c).c;
9 }
```

Release blocking

example 3: clang crash

```
1 int a;
2 double b, *c;
3
4 void fn1(int p1) {
5     for (;;) p1-- {
6         a = p1;
7         for (; p1 >= a; a--)
8             b = c[p1];
9     }
10 }
```

general, effective

- Found bugs in **many compilers**
 - ◆ Scala, Rust, Go, ...
- Very **simple to apply**
- **No need to reduce**

how about CompCert?

```
// test.c
#include <stdio.h>
int main(){
    int i = '\214';
    printf("%d\n", i);
    return 0;
}
```

```
$ ccomp-2.4 test.c; ./a.out
$ -116
$ ccomp-2.0 test.c; ./a.out
$ 140
```

how about CompCert?

```
// test.c
volatile long long a;
unsigned b;
int main () {
    a = b;
    return 0;
}
```

```
$ ccomp-2.6 test.c
Fatal error: exception File "ia32/Asmexpand.ml", line 191, ...
...
$
```

how about CompCert?

```
// test.c
#include <stdio.h>
int main () {
    int t = printf ("0\n");
    printf ("%d\n", t);
    return 0;
}
```

```
$ ccomp-2.6 -interp -quiet test.c
0
0
$ ccomp-2.7 -interp -quiet test.c
0
2
$
```

how about CompCert?

```
// test.c
int a, b, c, d, e, f, g;
void fn1 () {
    int h, i, j;
    if (g) {
        g = 1;
        L1: if (1) ;
    }
    short k = ~j;
    int l = 1 / (h & e & ~d + ((k & ~h) - ((1 | i) & (a | c)))),
        m = ~~j / (~h | d + a);
    j = l & m | h;
    if (j) ;
    j = k;
    int n = ~i | f, o = ~b - 1 / -n * ~i, p = f;
    goto L1;
}
int main () {
if (a) fn1 ();
return 0;
}
```

```
$ ccomp-2.7 test.c
...
Fatal error: exception File "backend/Regalloc.ml", line 741, ...
...
$
```

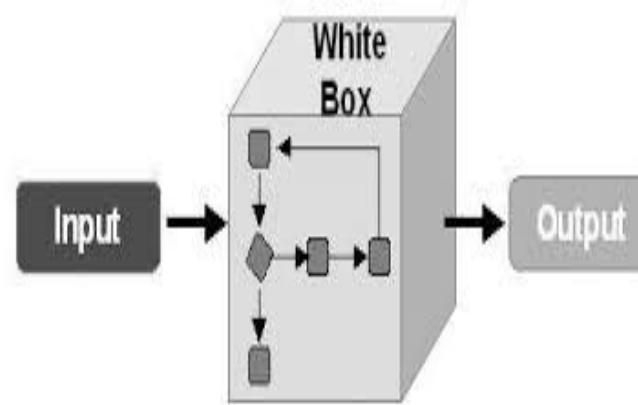
Relationship to MT

- ❑ Instance of metamorphic testing (MT)
 - ◆ Given input **i**
 - ◆ Constructed input **j**
 - ◆ Property **P** holds on **i, j**
- ❑ EMI is the most successful MT instance

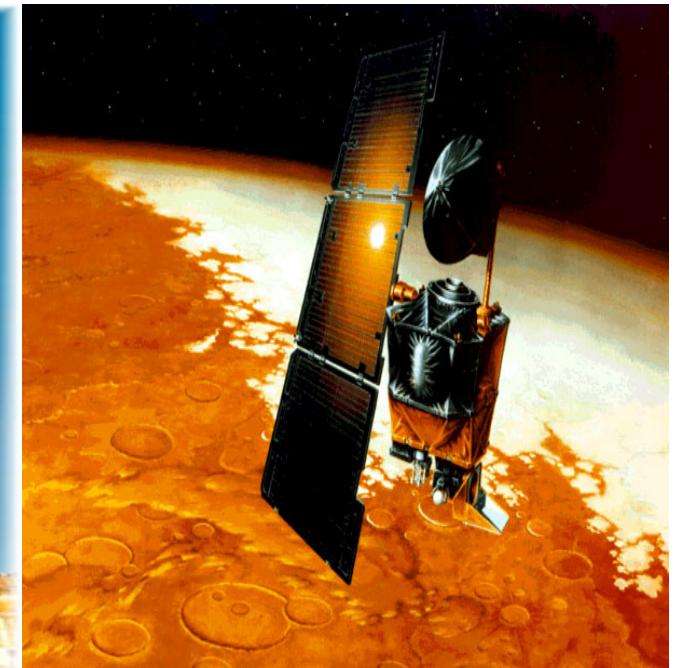
a few key problems

- How to select **good** seed inputs?
- How to decide **where** & **how** to mutate?
- How to generate inputs that **satisfy** a property **P**?
- How to find **missed optimization** opportunities?
- How to **synthesize optimizations**?

Analyze Floating-Point Software



Floating-point code



- **Important:** bugs can lead to disasters
- **Challenging:** hard to get right

Why difficult?

- FP Math ≠ Real Math

- Non-linear relations

- Transcendental functions

sin, log, exp, ...

```
1 double foo(double x){  
2     if (x<=1.0)  
3         x++;  
4  
5     y = x*x;  
6     if (y<=4.0)  
7         x--;  
8     return x;  
9 }
```

Challenging for all known approaches

New perspective: ME

(p, ϕ)

Analyzing numerical programs

- Coverage-based testing
- Boundary value analysis
- Numerical exception detection

Floating-point constraint solving



Mathematical Execution (ME)

r

Mathematical optimization (MO)

input x drives p to satisfy $\phi \leftrightarrow x$ minimizes r

FP constraints

Solving the floating-point constraint π

$$(SIN(x) == x) \wedge (x \geq 10^{-10})$$

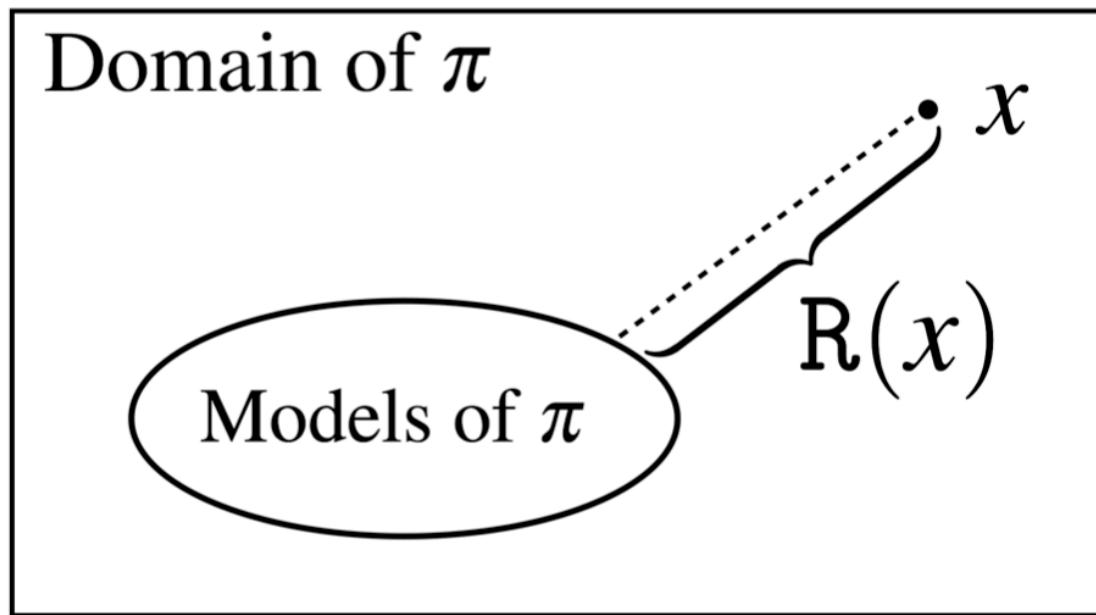
- ▶ Satisfiable if x is floating-point

For $x \in \mathbb{F}$, $SIN(x) = x \Leftrightarrow x \simeq 0$

- ▶ Unsatisfiable if x is real

For $x \in \mathbb{R}$, $SIN(x) = x \Leftrightarrow x = 0$

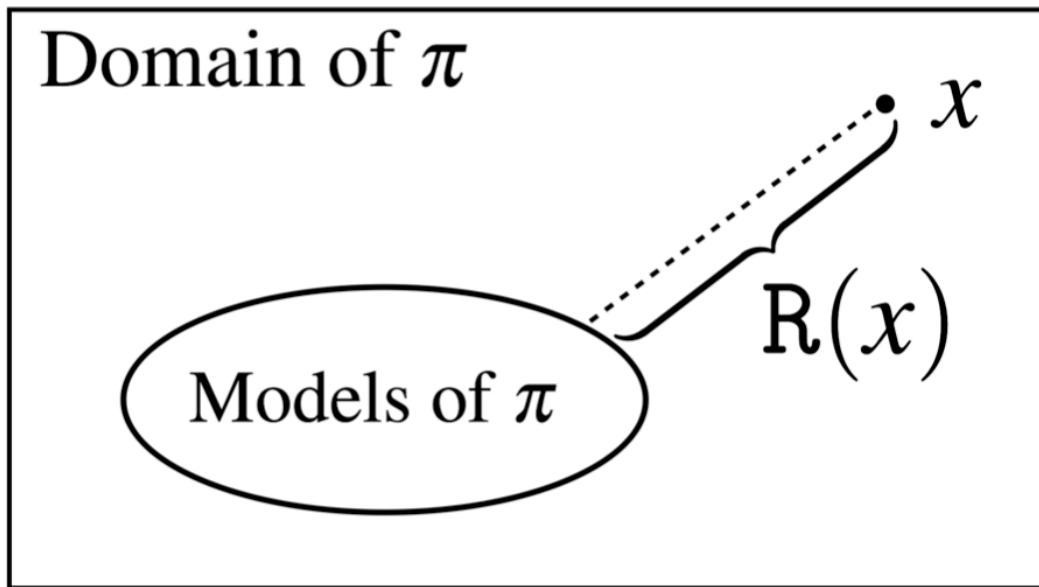
Step 1



Simulate π with a floating-point program R

- ▶ $R(x) \geq 0$ for all x
- ▶ $R(x) = 0 \Leftrightarrow x \models \pi$

Step 2



Minimize R as if it is a mathematical function

- ▶ Let x^* be the minimum point

$$\pi \text{ satisfiable} \Leftrightarrow R(x^*) = 0$$

Construct R

Necessary Conditions to meet :

1. $R(x) \geq 0$ for all x
2. $R(x) = 0 \Leftrightarrow x \models \pi$

How?

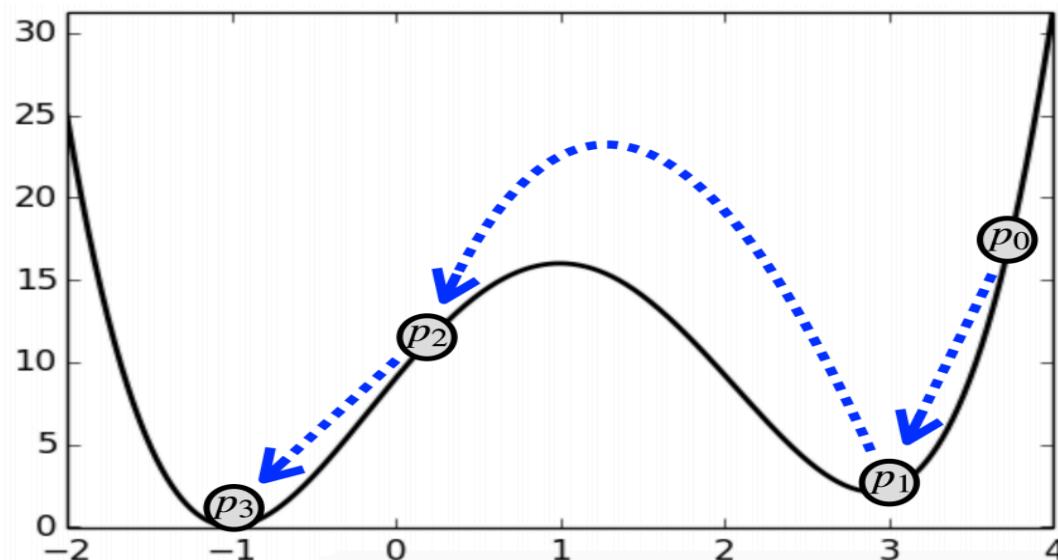
Constraint π	Program R
$x == y$	$(x - y)^2$
$x \leq y$	$x \leq y ? 0 : (x - y)^2$
$\pi_1 \wedge \pi_2$	$R_1 + R_2$
$\pi_1 \vee \pi_2$	$R_1 * R_2$

R can be constructed from a CNF form

Minimize R

Unconstrained programming techniques:

- ▶ Local optimization
- ▶ Monte Carlo Markov Chain (MCMC)
- ▶ We use them as black-box
- ▶ Do not analyze π ; execute R



Theoretical guarantees

Let R satisfy (1) $R(x) \geq 0$, and (2) $R(x) = 0 \Leftrightarrow x \models \pi$, and x^* be a minimum point of R . Then

$$\pi \text{ satisfiable} \Leftrightarrow R(x^*) = 0.$$

Threats

- ▶ Floating-point inaccuracy when calculating with R
- ▶ Sub-optimal x^*

Example

$$(SIN(x) == x) \wedge (x \geq 10^{-10})$$



$$(SIN(x) - x)^2 + \begin{cases} 0 & \text{if } x \geq 10^{-10} \\ (x - 10^{-10})^2 & \text{otherwise} \end{cases}$$



$$x^* = 9.0 * 10^{-9} \text{ (can be others)}$$

XSat & results

- Developed the ME-based XSat tool
- Evaluated against **MathSat** and **Z3**
- Used **SMT-Comp 2015 FP benchmarks**
- Result summary
 - **100% consistent results**
 - **700+X faster than MathSat**
 - **800+X faster than Z3**

Generalizations

- ❑ Coverage-based testing of FP code
- ❑ Boundary value analysis
- ❑ FP exception detection
- ❑ Path divergence detection

Coverage-based testing

Goal

To generate test inputs to cover all branches of a program like this:

- pointer operations: &, *
- type casting: (int*), (unsigned)
- bit operations ^, &, >>
- floating-point comparison

```
double __ieee754_fmod(double x, double y){  
    ...  
    Zero[] = {0.0, -0.0,};  
    hx = *(1+(int*)&x);  
    lx = *(int*)&x;  
    hy = *(1+(int*)&y);  
    ly = *(int*)&y;  
    sx = hx&0x80000000;  
    hx ^=sx;  
    hy &= 0xffffffff;  
  
    if((hy|ly)==0||(hx>=0x7ff00000)||  
        ((hy|((ly|-ly)>>31))>0x7ff00000))  
        return (x*y)/(x*y);  
    if(hx<=hy) {  
        if((hx<hy)||!(lx<ly)) return x;  
        if(lx==ly)  
            return Zero[(unsigned)sx>>31];  
    }  
  
    if(hx<0x00100000) {  
        if(hx==0) {
```

State-of-the-art & Challenges

Symbolic execution

- Path explosion
- Constraint solving

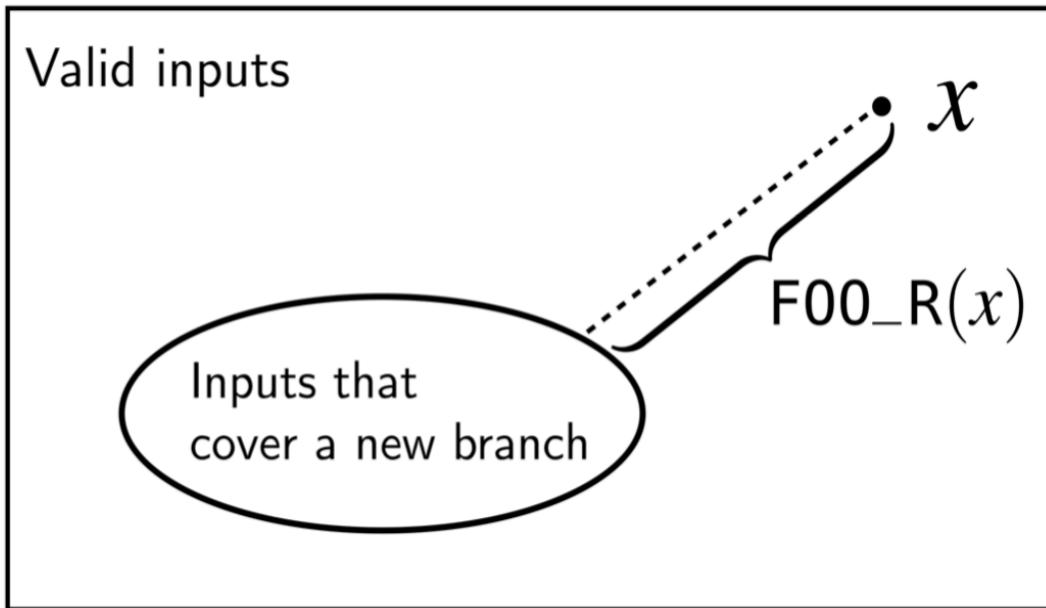
Search-based testing

- Fitness function
- Search strategies

Our approach

- No path issues
- No need to solve constraints
- Effective for FP programs

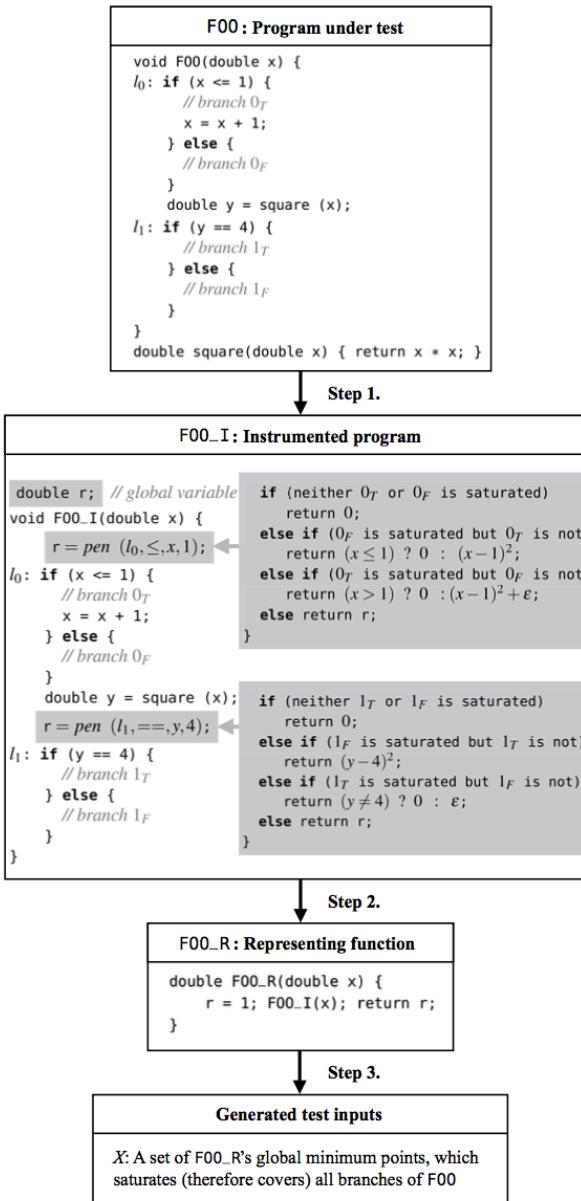
Our approach



Step 1: Derive a program $F00_R$ from $F00$ s.t.

- $F00_R(x) \geq 0$ for all x , and
- $F00_R(x) = 0 \Leftrightarrow x$ covers a new branch

Step 2: Repeatedly minimize $F00_R$ until > 0



FOO : Program under test

```

void FOO(double x) {
    l0: if (x <= 1) {
        // branch 0T
        x = x + 1;
    } else {
        // branch 0F
    }
    double y = square (x);
    l1: if (y == 4) {
        // branch 1T
    } else {
        // branch 1F
    }
}
double square(double x) { return x * x; }

```

FOO_I: Instrumented program

```
double r; // global variable
void FOO_I(double x) {
    r = pen (l0,≤,x,1); ←
l0: if (x <= 1) {
    // branch 0T
    x = x + 1;
} else {
    // branch 0F
}
double y = square (x);
r = pen (l1,==,y,4); ←
l1: if (y == 4) {
    // branch 1T
} else {
    // branch 1F
}
}

if (neither 0T or 0F is saturated)
    return 0;
else if (0F is saturated but 0T is not)
    return (x ≤ 1) ? 0 : (x - 1)2;
else if (0T is saturated but 0F is not)
    return (x > 1) ? 0 : (x - 1)2 + ε;
else return r;

if (neither 1T or 1F is saturated)
    return 0;
else if (1F is saturated but 1T is not)
    return (y - 4)2;
else if (1T is saturated but 1F is not)
    return (y ≠ 4) ? 0 : ε;
else return r;
```

FOO_I: Instrumented program

```
double r; // global variable
void FOO_I(double x) {
    r = pen (l0,≤,x,1); ←
    l0: if (x <= 1) {
        // branch 0T
        x = x + 1;
    } else {
        // branch 0F
    }
    double y = square (x);
    r = pen (l1,==,y,4); ←
    l1: if (y == 4) {
        // branch 1T
    } else {
        // branch 1F
    }
}

if (neither 0T or 0F is saturated)
    return 0;
else if (0F is saturated but 0T is not)
    return (x ≤ 1) ? 0 : (x - 1)2;
else if (0T is saturated but 0F is not)
    return (x > 1) ? 0 : (x - 1)2 + ε;
else return r;

if (neither 1T or 1F is saturated)
    return 0;
else if (1F is saturated but 1T is not)
    return (y - 4)2;
else if (1T is saturated but 1F is not)
    return (y ≠ 4) ? 0 : ε;
else return r;
```

FOO_R: Representing function

```
double FOO_R(double x) {
    r = 1; FOO_I(x); return r;
}
```

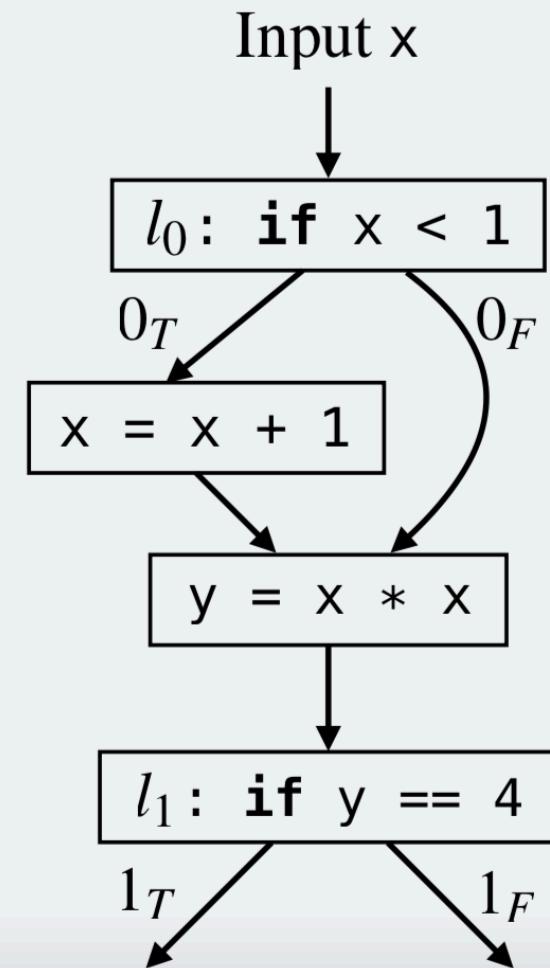
Generated test inputs

X : A set of FOO_R's global minimum points, which saturates (therefore covers) all branches of FOO

Example

Generate an input set to cover $\{0_T, 0_F, 1_T, 1_F\}$

```
void FOO (double x){  
l0: if (x < 1)  
    x++;  
    double y = x * x;  
l1: if (y == 4)  
    ...  
}
```

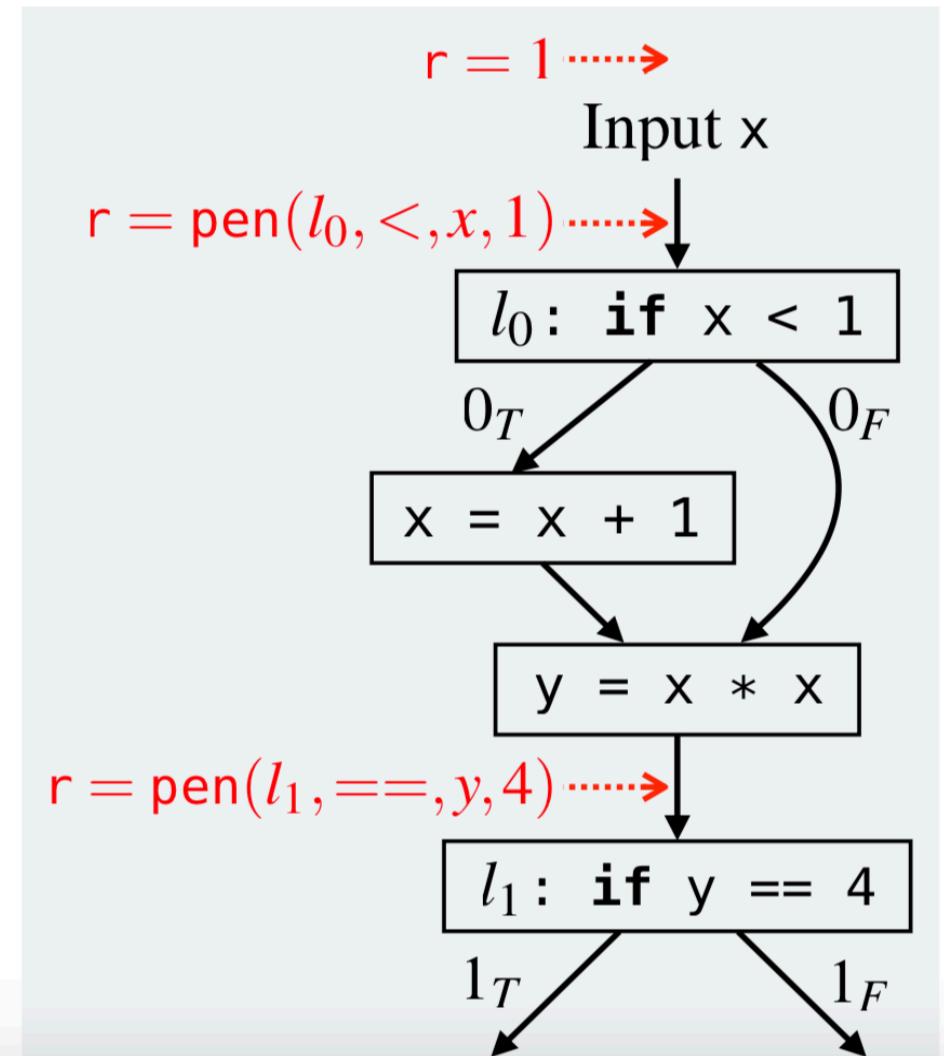


Step 1: Construct F00_R

covered at l_i $\text{pen}(l_i, op, a, b)$

\emptyset	0
$\{i_F\}$	$R_{a \text{ op } b}$
$\{i_T\}$	$R_{\neg(a \text{ op } b)}$
$\{i_T, i_F\}$	r

- r: global variable
- F00_R : $x \rightarrow r$
- $R_{a \text{ op } b}$: Branch distance



Branch distance $R_{a \text{ op } b}$

A helper function to quantify how far a and b are from attaining branch $a \text{ op } b$.

$R_{a==b}$ defined as $(a - b)^2$

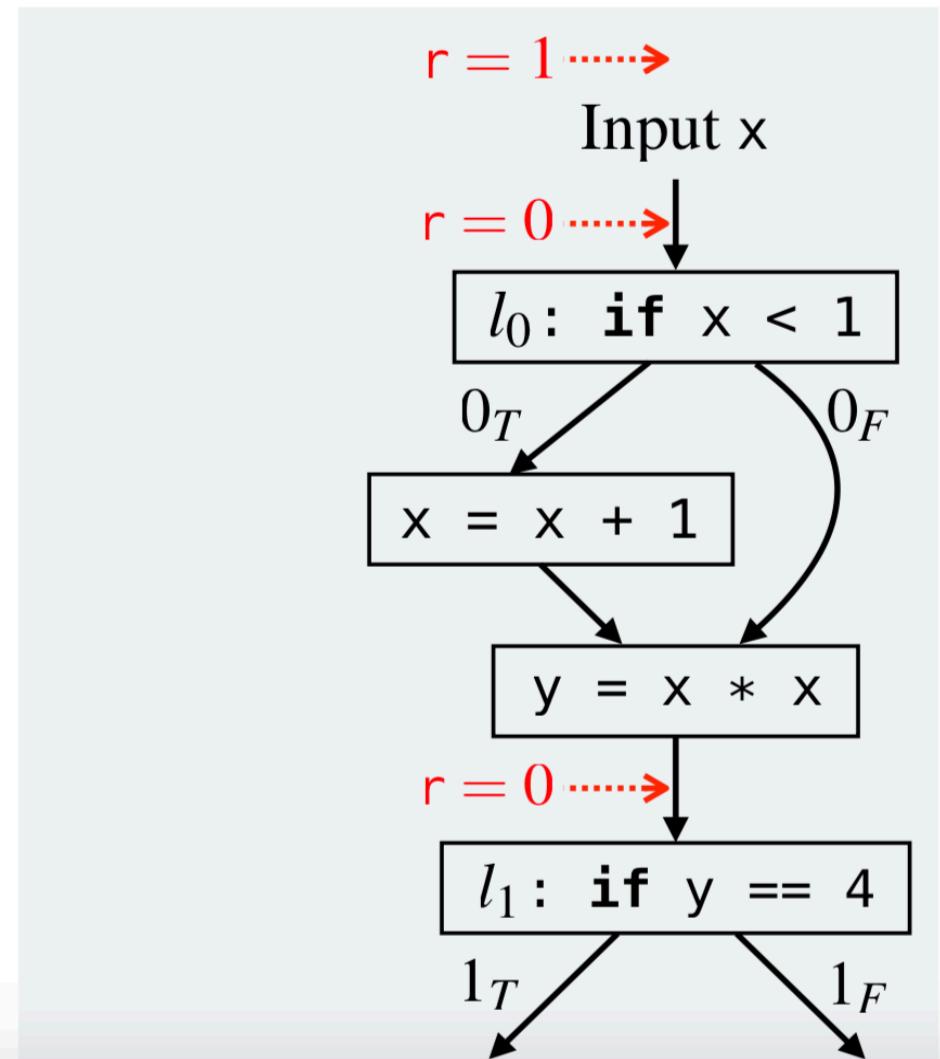
$R_{a \geq b}$ defined as $(a \geq b) ? 0 : (a - b)^2$

Step 2: Minimize F00_R

covered at l_i $pen(l_i, op, a, b)$

\emptyset	0
$\{i_F\}$	$R_a \text{ op } b$
$\{i_T\}$	$R_{\neg(a \text{ op } b)}$
$\{i_T, i_F\}$	r

- No branch is covered
- Any input is a minimum point
- Assume $x^* = 0.7$

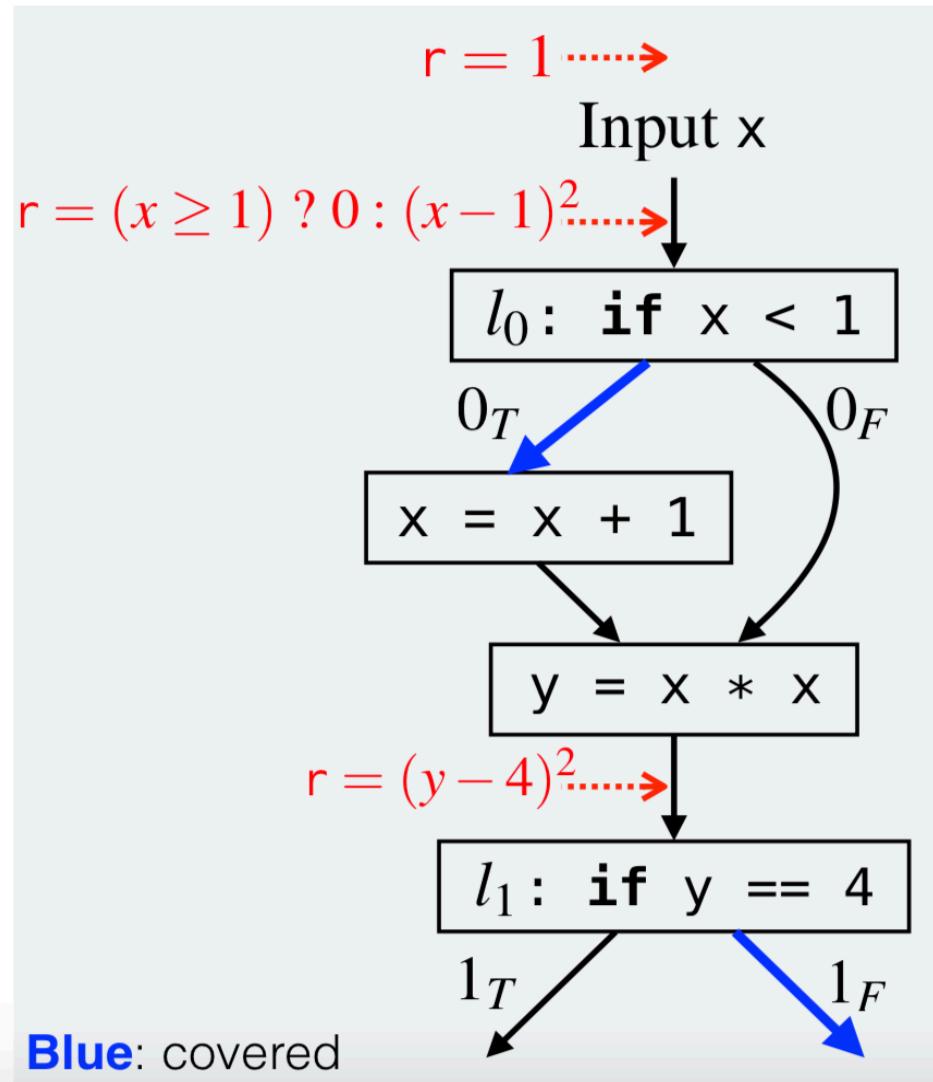


Step 2: Minimize F00_R

covered at l_i $pen(l_i, op, a, b)$

\emptyset	0
$\{i_F\}$	$R_{a \ op \ b}$
$\{i_T\}$	$R_{\neg(a \ op \ b)}$
$\{i_T, i_F\}$	r

- $1_F, 0_T$ are covered
- F00_R attains minimum at -3 or 2
- Assume $x^* = -3$

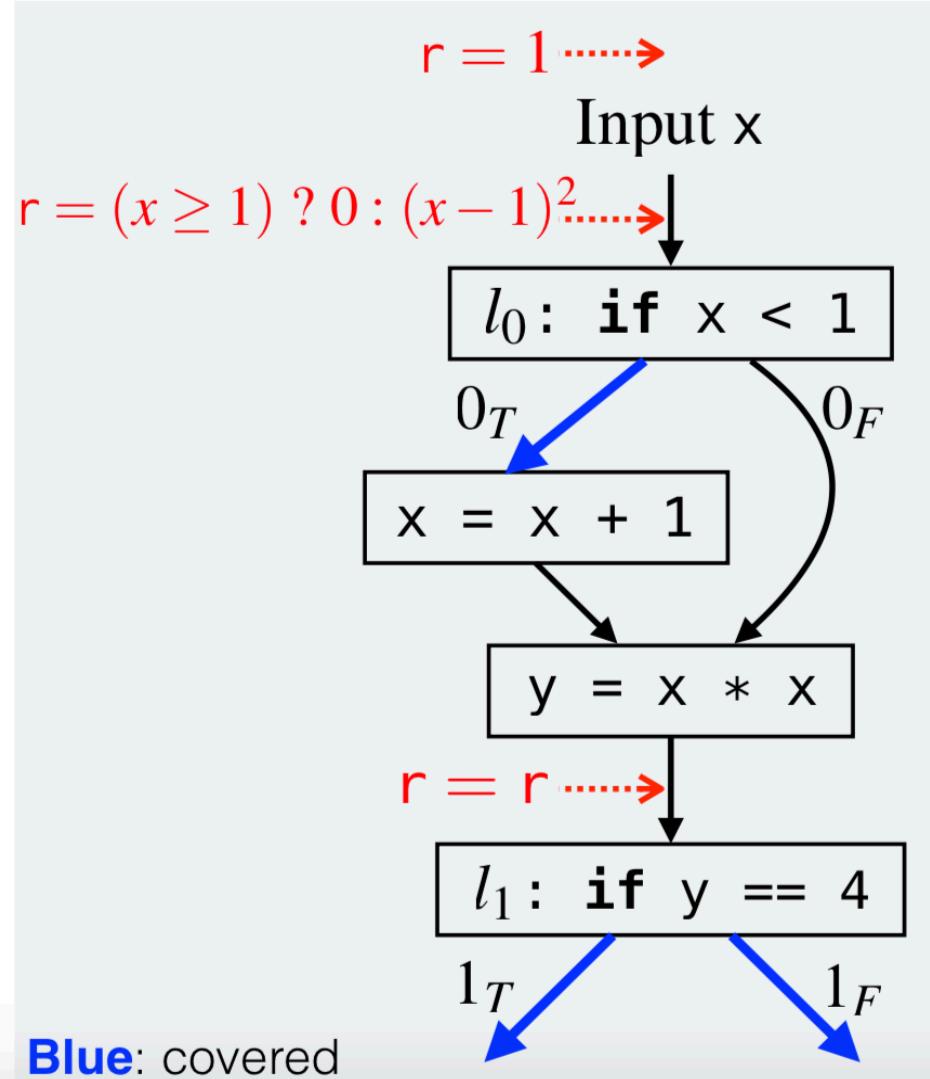


Step 2: Minimize FOO_R

covered at $l_i \ pen(l_i, op, a, b)$

\emptyset	0
$\{i_F\}$	$R_a \ op \ b$
$\{i_T\}$	$R_{\neg(a \ op \ b)}$
$\{i_T, i_F\}$	r

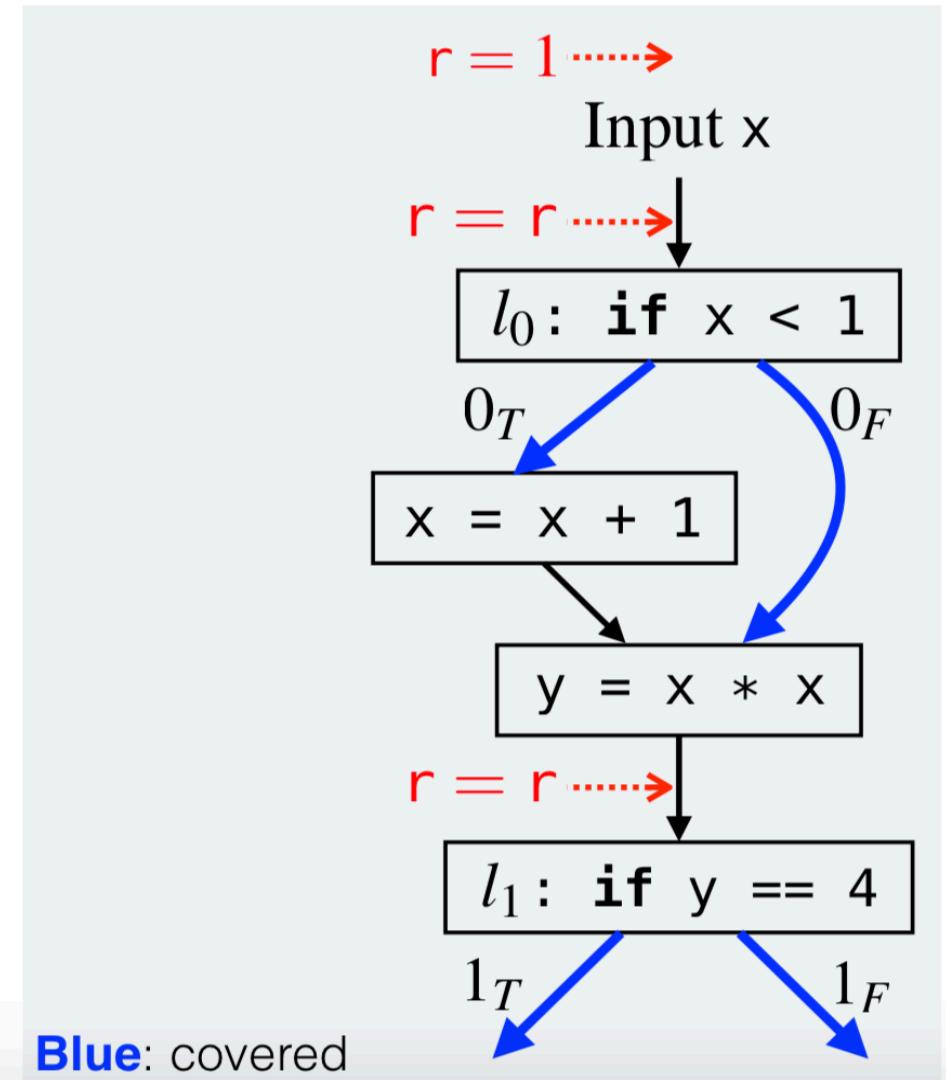
- $1_F, 1_T, 0_T$ are covered
- FOO_R attains minimum at ≥ 1
- Assume $x^* = 5.1$



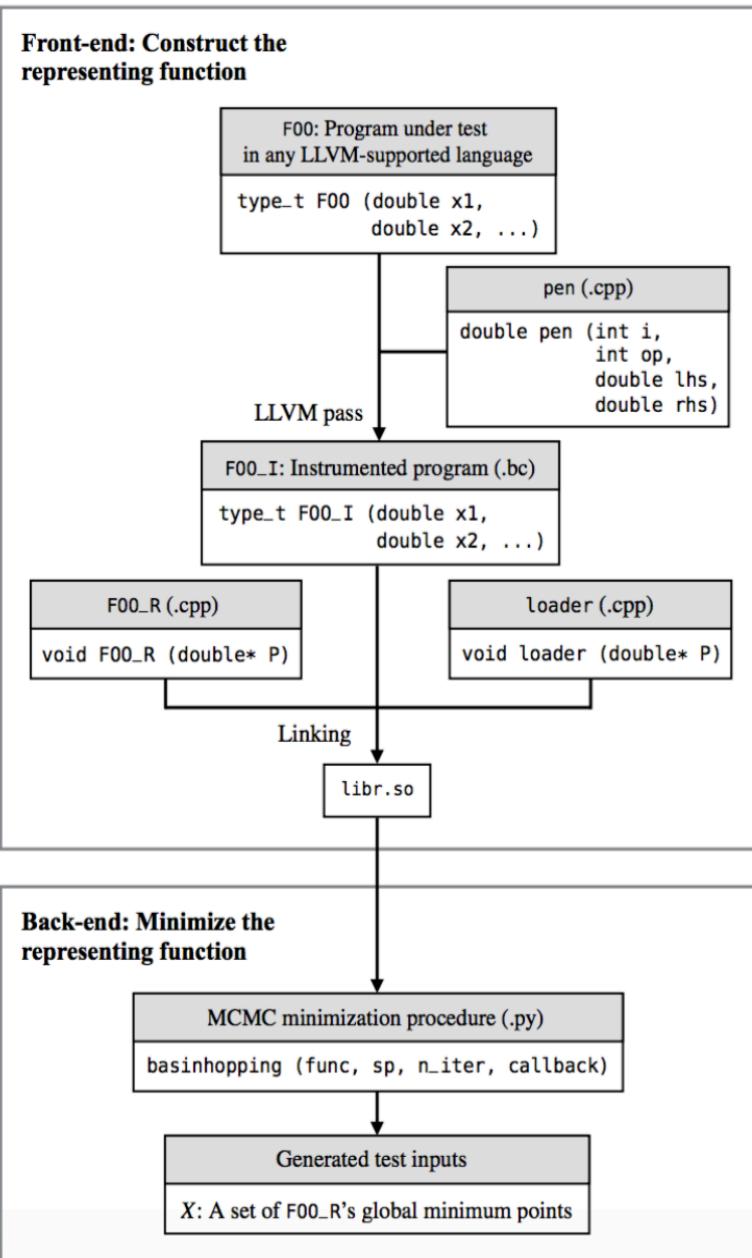
Step 2: Minimize FOO_R

covered at l_i $\text{pen}(l_i, op, a, b)$	
\emptyset	0
$\{i_F\}$	$R_{a \text{ op } b}$
$\{i_T\}$	$R_{\neg(a \text{ op } b)}$
$\{i_T, i_F\}$	r

- All branches are covered
- $\forall x, \text{FOO_R}(x) = 1$
- Termination



Our implementation CoverMe



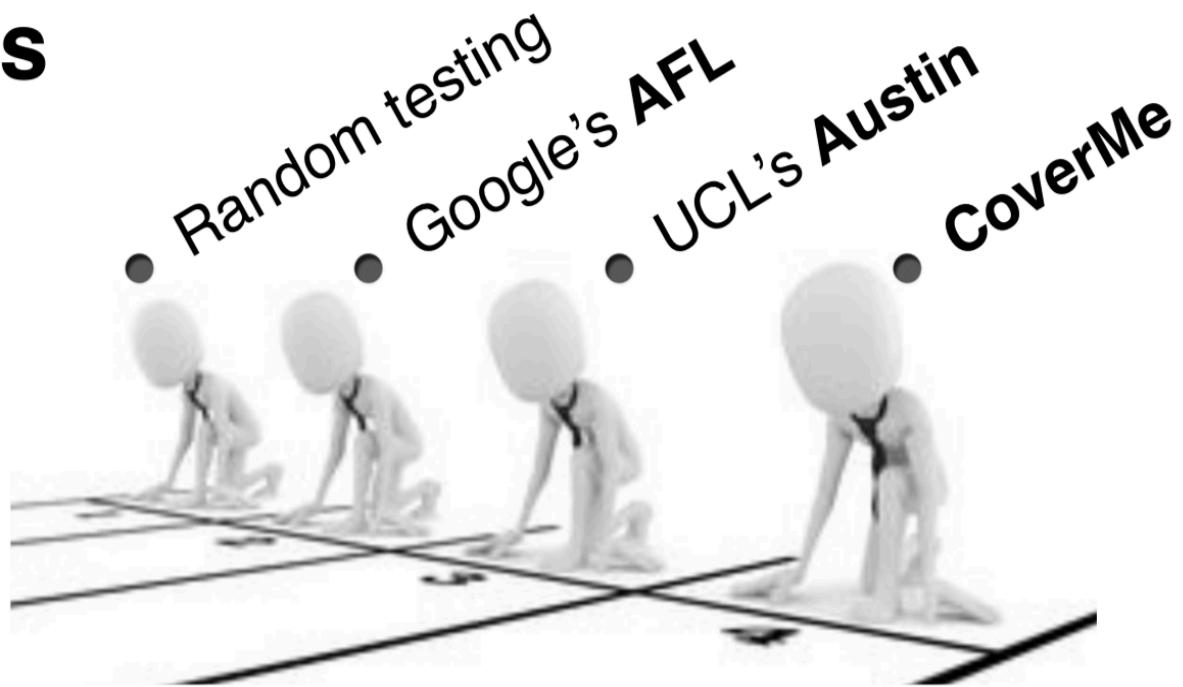
- **Front-end:** Program transformation in LLVM
- **Back-end:** Basin hopping

Experiments

Benchmarks: **Fdlibm**

- Sun's math library
- Reference for Java SE 8's math library
- Used in Matlab, JavaScript and Android
- Heavy on branches (max=114, avg=23)

Results



CoverMe covers

- $\approx 90\%$ branches in **7** seconds
- $\approx 18\%$ more branches than AFL with **1/10** time
- $\approx 40\%$ more branches than Austin with
speedups of several orders of magnitudes

ME in the long run

- Offers a new general analysis paradigm
- Complements existing approaches
 - ◆ Random concrete execution (CE)
 - ◆ Symbolic execution (SE)
 - ◆ Abstract execution/interpretation (AE)

Thank you!

- Feedback always welcomed
- Bachelor's thesis topics available
 - ◆ Programming languages & compilers
 - ◆ Software engineering
 - ◆ Software security
 - ◆ Machine learning & AI
 - ◆ Education technologies (EdTech)