Exercise 1

This exercise requires some knowledge of MATLAB.

1. Determine the linear interpolant $\pi f \in \mathbb{P}_1(I)$ on the single interval $I = [0, 1]$ of the functions

   (a) $f(x) = x^2$.

   (b) $f(x) = 3\sin(2\pi x)$.

   Make plots of f and πf in the same figure.

   Hint: In MATLAB: `vq=interp1(x,v,xq)` returns interpolated values of a 1-D function at specific query points using linear interpolation. Vector x contains the sample points, and v contains the corresponding values, v(x). Vector xq contains the coordinates of the query points.

2. Reproduce Figure 1.6 from the book by Larson & Bengzon, see also page 26 of the slides. Write functions to assemble the mass matrix and the load vector. Use `L2Projector1D` below and compute the L2 projection $P_h f$ of the following function

   $$f(x) = 2x\sin(2\pi x) + 3$$

   Use a uniform mesh I of the interval $I = [0, 1]$ with $n = 5$ subintervals.
   `foo` in `L2Projector1D` is a function handle.

   ```
   function L2Projector1D(foo)
   n = 5; % number of subintervals
   h = 1/n; % mesh size
   x = 0:h:1; % mesh
   M = MassAssembler1D(x); % assemble mass
   b = LoadAssembler1D(x,foo); % assemble load
   Pf = M\b; % solve linear system
   plot(x,Pf,'o',x,Pf,0:h/10:1,foo(0:h/10:1)) % plot L^2 projection
   ```

3. Let us try to interpolate a function that is not continuous. To be specific, let

   $$f(x) = \begin{cases} 1, & 0 \le x \le \frac{1}{2}, \\ 0, & \frac{1}{2} < x \le 1. \end{cases}$$

   We interpolate f by a *continuous* piecewise linear polynomial πf, choosing grid points $x_i = ih \equiv i/n$, $i = 0, \dots, n$. We assume that n is even such that $\frac{1}{2}$ is among the grid points.

   Compute $\|f - \pi f\|_{L^2(I)}$ and $\|(f - \pi f)'\|_{L^2(I)}$.

   Is the theorem on Slide I-24 wrong?

Please submit your solution via e-mail to Peter Arbenz (`arbenz@inf.ethz.ch`) by September 29, 2017. (12:00). Please specify the tag **FEM17** in the subject field.