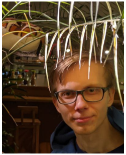
Polylog-Competitive Deterministic Local Routing and Scheduling


Bernhard[1,2] Haeupler


Shyamal[3] Patel


Antti[2] **Roeyskoe**


Cliff[3] Stein

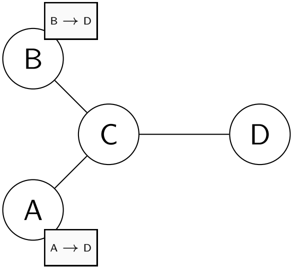

Goran[4] Zuzic

1: INSAIT, 2: ETH Zürich, 3: Columbia University, 4: Google Research

Packet Routing

Packet Routing Problem

- Undirected graph
- Packets with set start and destination vertices



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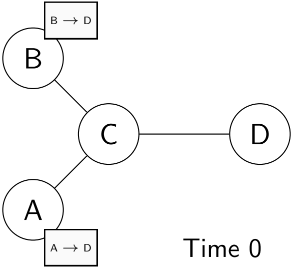

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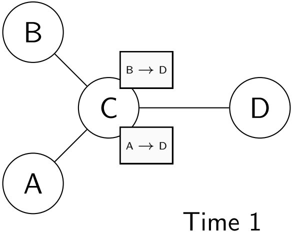

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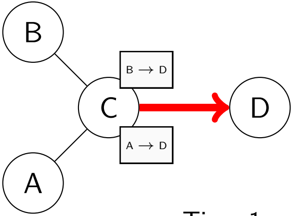

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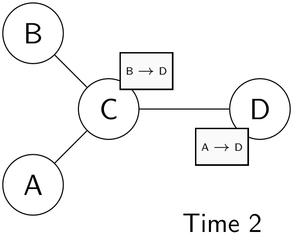

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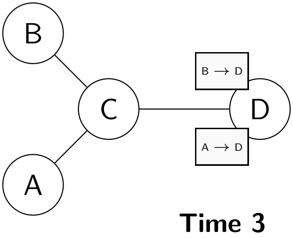
- Goal: minimize time to *deliver* all packets (*completion time*)



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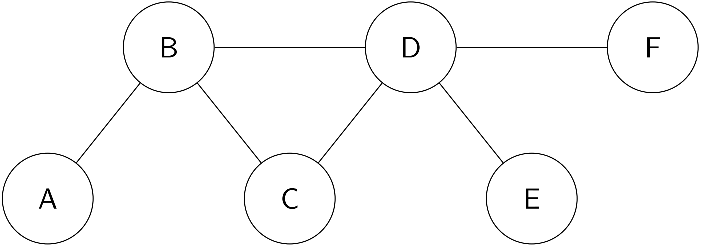
That solve packet routing *competitively*

- C-competitive: for **every** packet routing instance, the forwarding rules achieve completion time $C \cdot \mathrm{OPT}_{\mathrm{global}}$

Example



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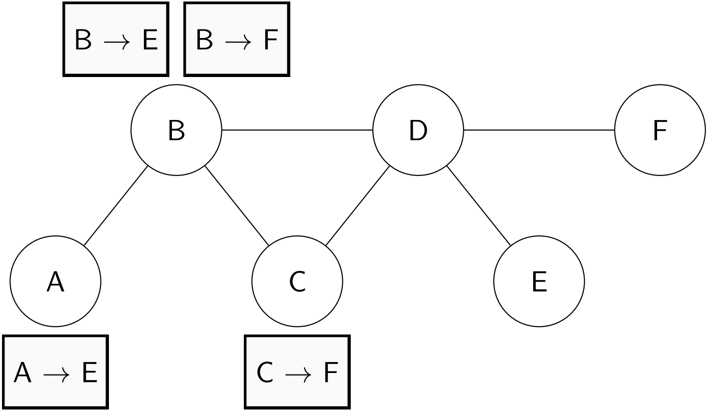

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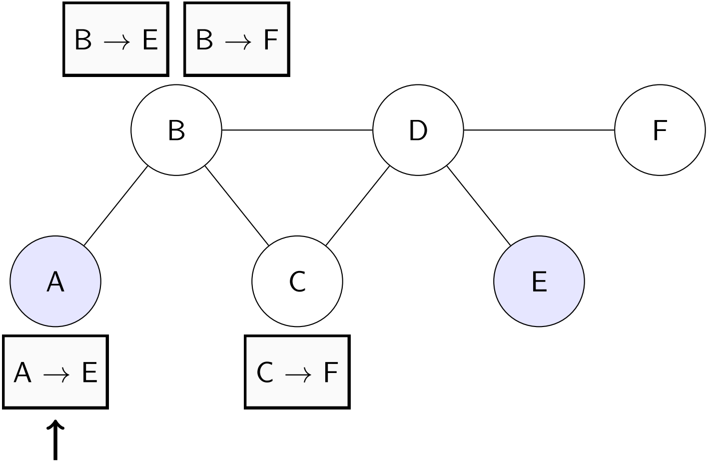

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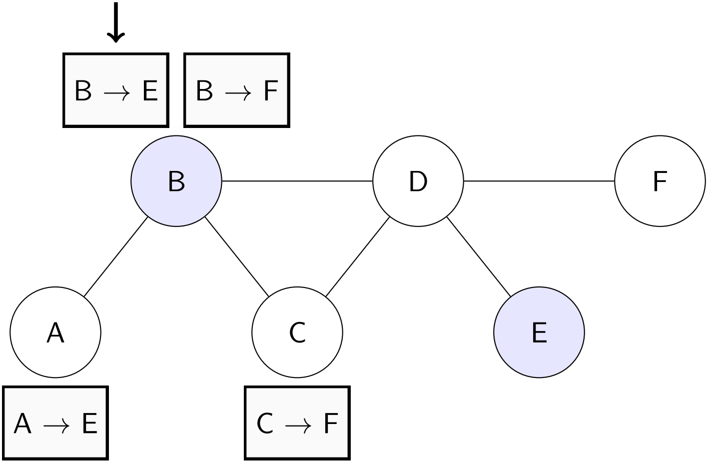

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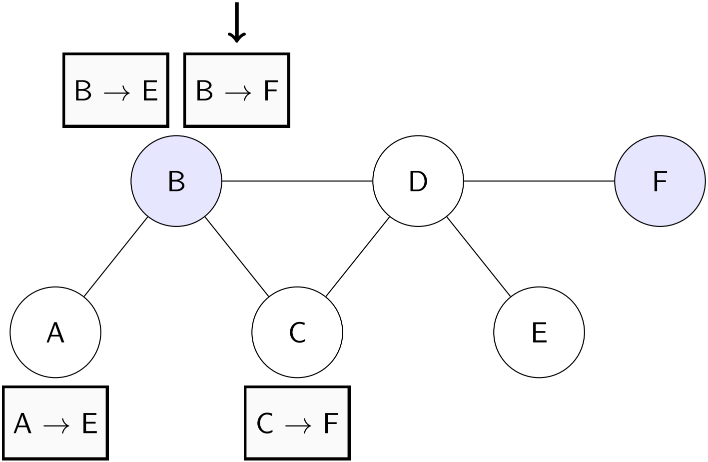

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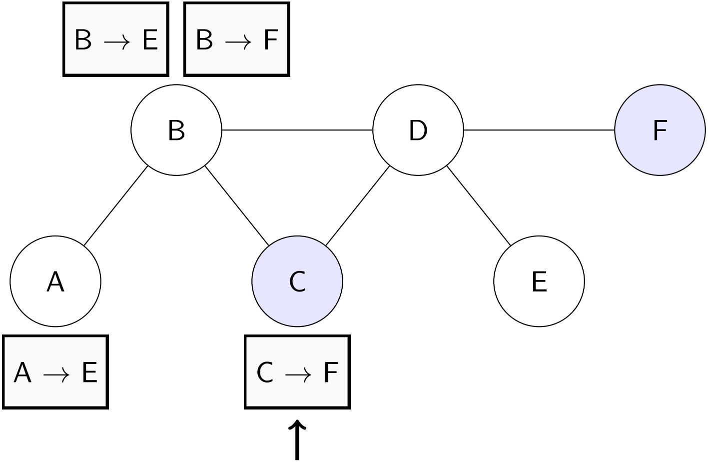

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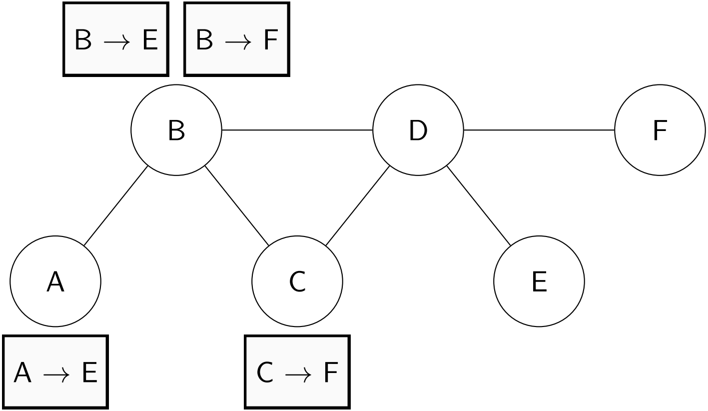

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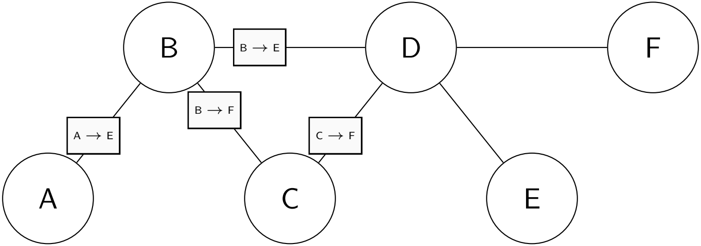

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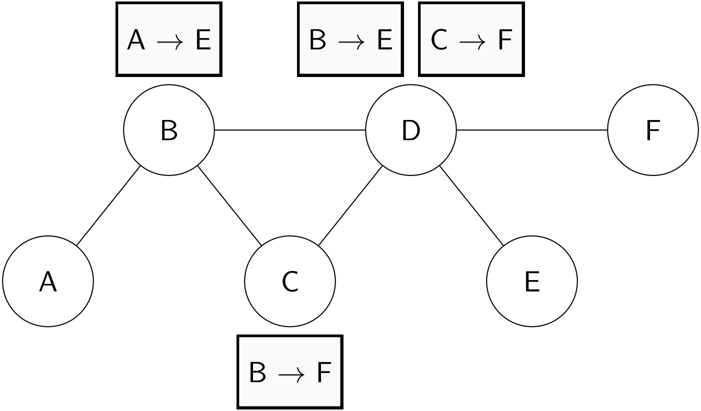

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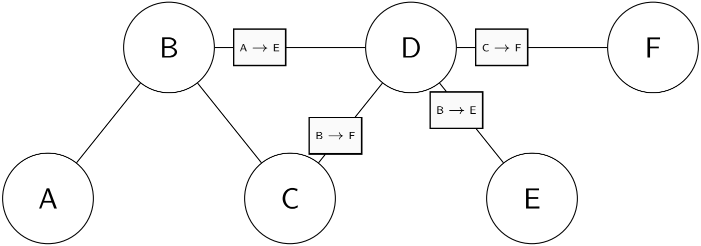

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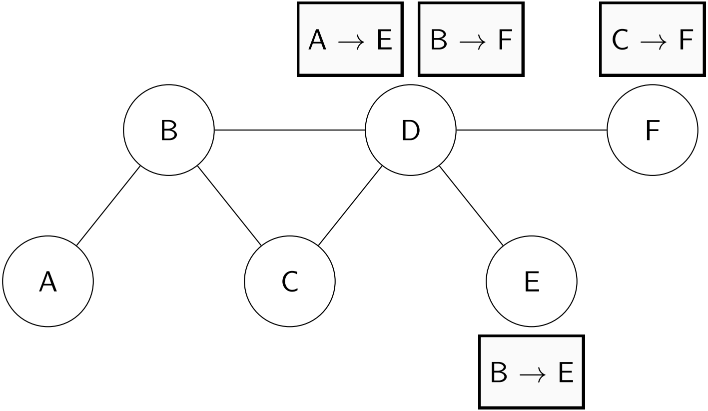

Time 2

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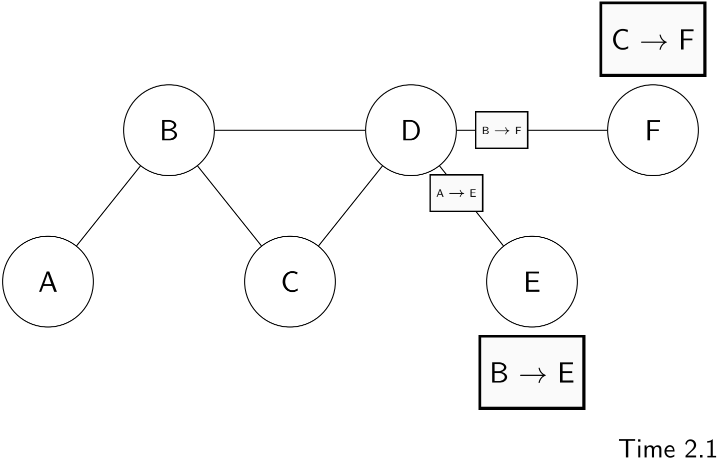

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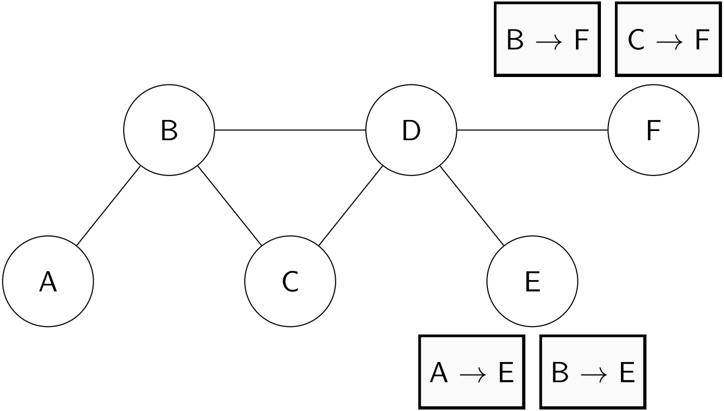
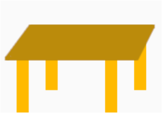
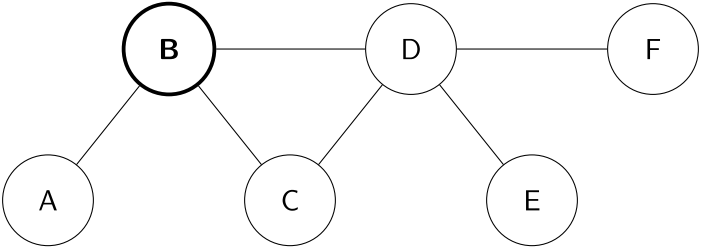


Time 3

Example: B's Perspective





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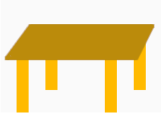

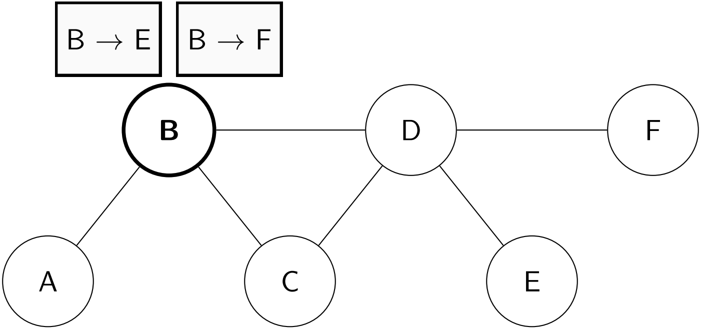

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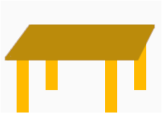
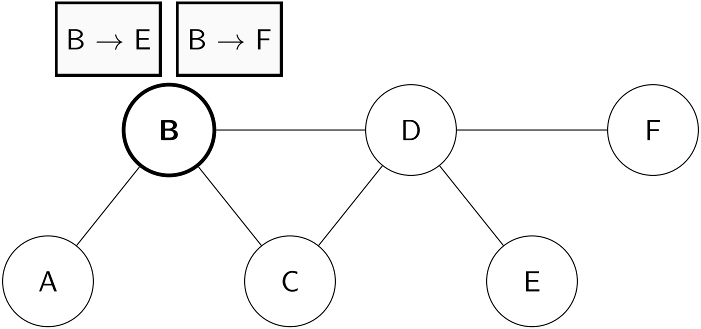


Time 0
([B $\to$ F], [B $\to$ E])



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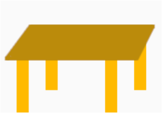

→

[B → F]: Forward to D
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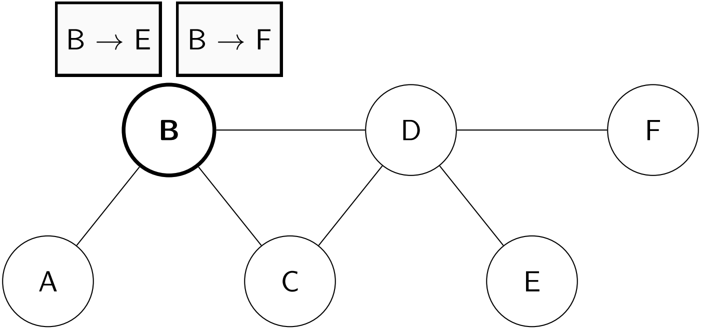

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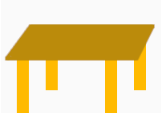

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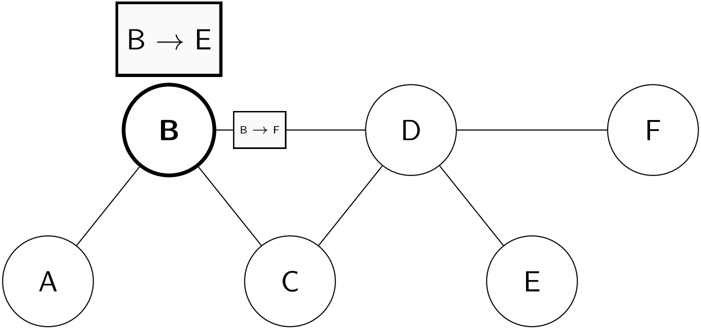

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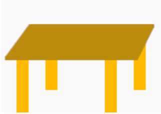
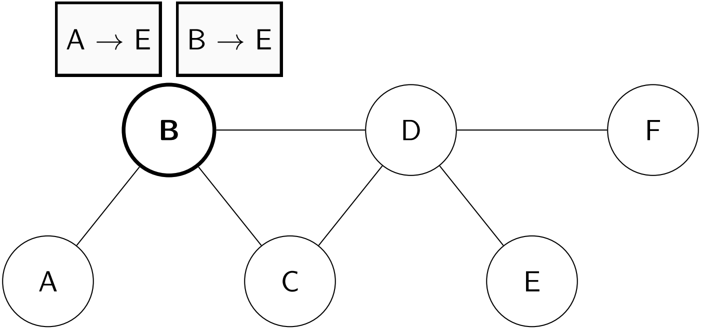
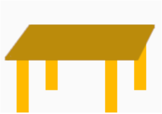




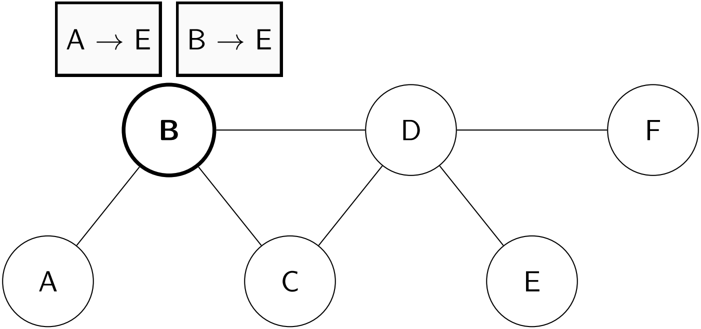
Time 1

Example: B's Perspective



Time 1

([A → E], [B → E])



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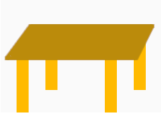

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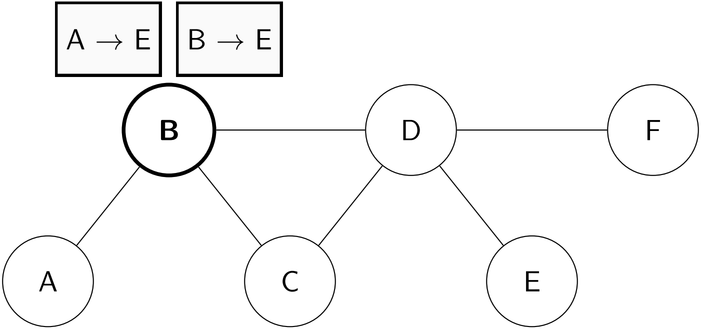

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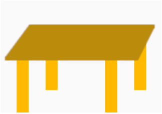

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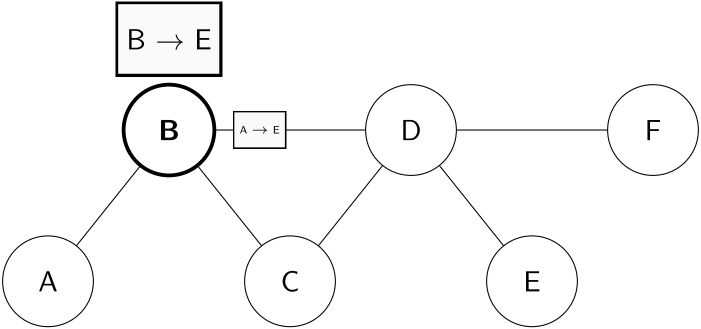

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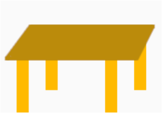

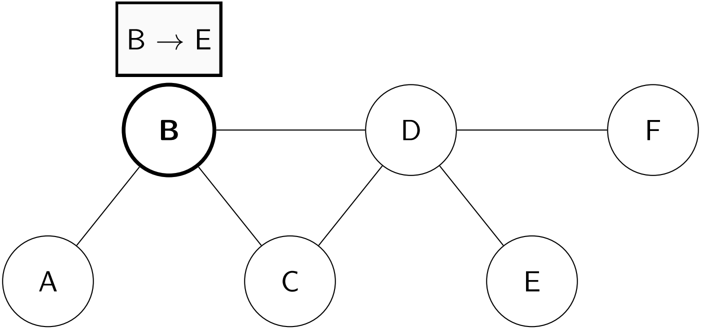

Time 2

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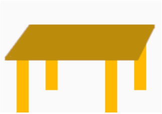

Time 2
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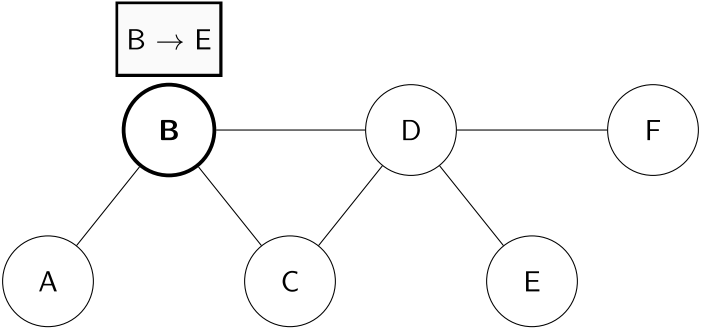

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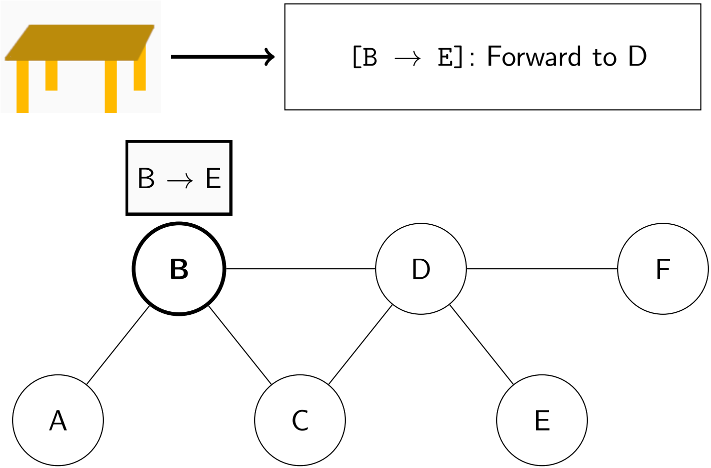

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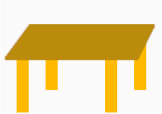

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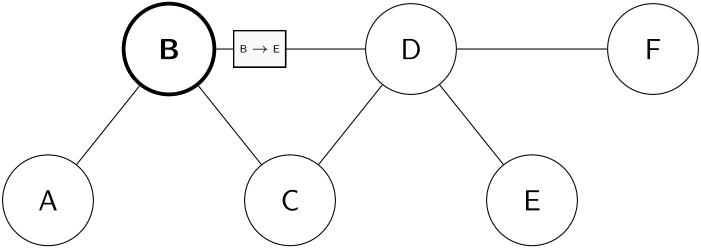

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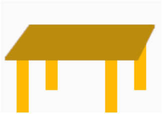

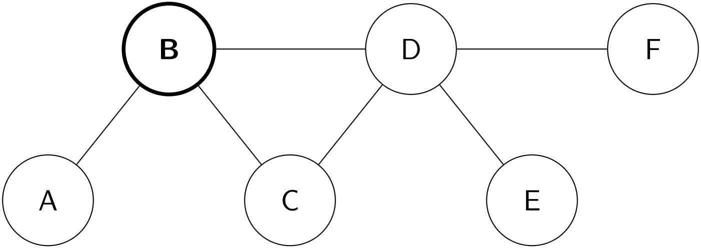

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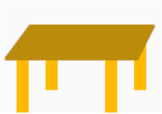

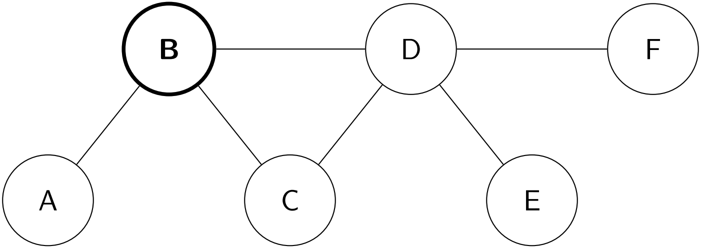

Time 4

Our Results

Main Result

For every graph, there exists *deterministic* $\mathrm{poly}(\log n)$-competitive routing tables.

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  - Requires specialized, *slow* hardware
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  - Requires specialized, *slow* hardware
  - Routers process *millions* of packets per second, and must be **fast**
- **Guaranteed** not to fail!
  - Even if the chance is low, the Internet going down is **extremely bad**

Talk Overview

So far:

- Problem definition
- Our result

Next:

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  - $\rightarrow$ Competitive **randomized** local routing rules

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- Our *deterministic* approach

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- **Deterministic**: nothing $o(CD)$ known

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For any graph, there exists a distribution of paths between every pair of vertices, such that for any set of packets, sampling paths from the distribution achieves **congestion** at most $\mathcal{O}(\log n)$ times the global optimum congestion.

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Randomized Approach: Summary

$$\text{Packet Routing} = \text{Oblivious Path Selection} \\ + \text{Local Scheduling}$$

- Oblivious path selection: **sample** from hop-bounded oblivious routing
- Local scheduling: **sample** a random delay

Deterministic Local Scheduling / Oblivious Path Selection

Deterministic local scheduling:

- **any** strategy achieves $\mathcal{O}(C \cdot D)$
- Nothing better is known

Deterministic oblivious path selection:

- I.E. single fixed path between every vertex pair
- $\Omega(\sqrt{n} \cdot \mathrm{OPT})$ lower bound [KKT90]
- Even on hypercubes!

are deterministic routing tables doomed?

Our Results (again)

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But How?

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Packet Routing = ~~Oblivious Path Selection~~

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But How?

Packet Routing = Sparse Semi-Oblivious Path Selection
+ Local Det. Noise-Tolerant Scheduling

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- "+": This talk [and paper]

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For every graph, there exists a $\beta = \mathrm{poly}(\log n)$-competitive $\alpha = \mathcal{O}(\log^2 n)$-sparse semi-oblivious routing.

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- Key issue: how to *locally* select correct path

  ... amongst the $\alpha - 1$ paths of *noise*

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- Here: domain path set = semi-oblivious routing (αn^2 paths!)

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Competitive Local Det. Noise-Tolerant Scheduling

For every graph and poly-size domain path set $\mathcal{P}$, there exists a *local* and *deterministic* Noise-Tolerant Scheduling algorithm that uses $\alpha T \cdot \mathrm{poly}(\log n)$ time steps.

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        i. For each (s, t), set (s, t)-packets' paths to $P(s, t)_i$
        ii. Run $(2\alpha, T)$-local det. noise-tolerant scheduling *with return*

End of Talk

Questions?

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