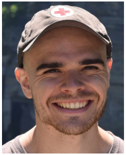
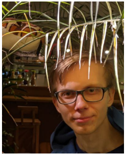
Low-Step Multi-Commodity Flow Emulators

Bernhard[1,2] Haeupler



D Ellis[3] Hershkowitz

Jason[4] Li



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- Goal: maximize $|F|$

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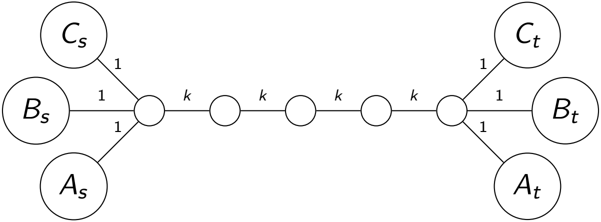

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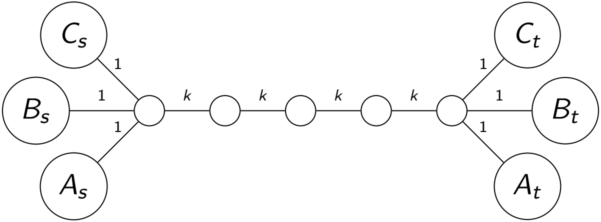

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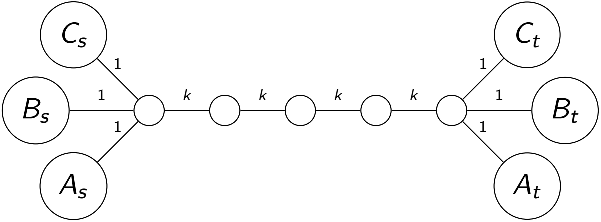

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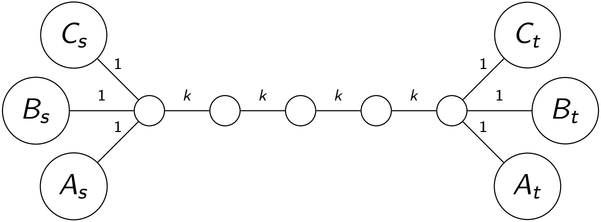

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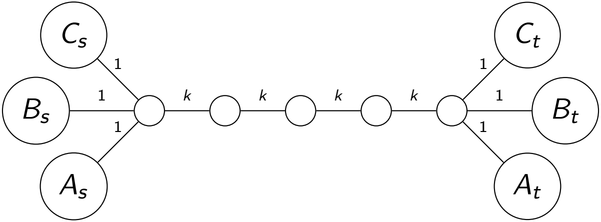

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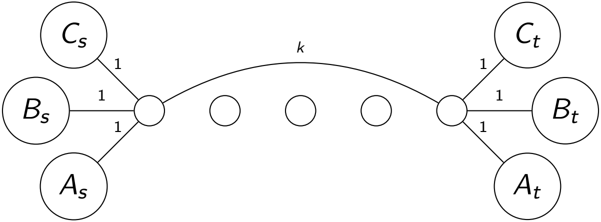

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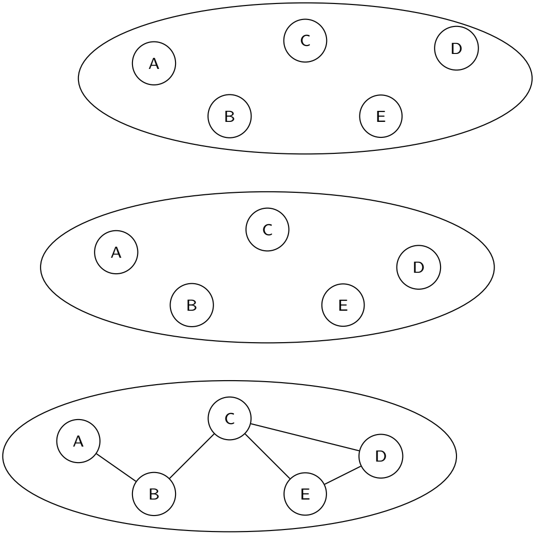

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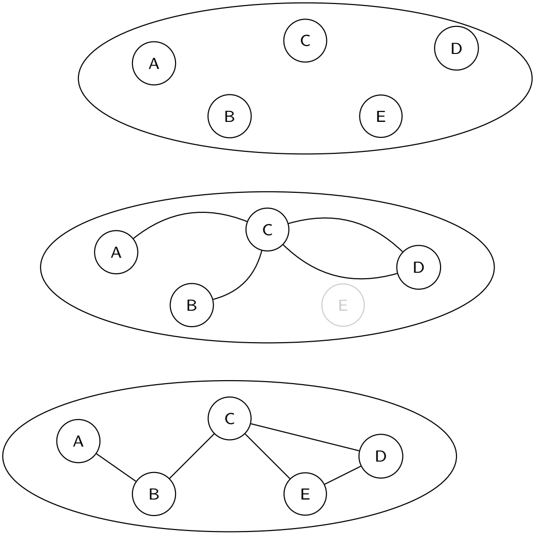
Stacked copies of vertex set on top of original graph



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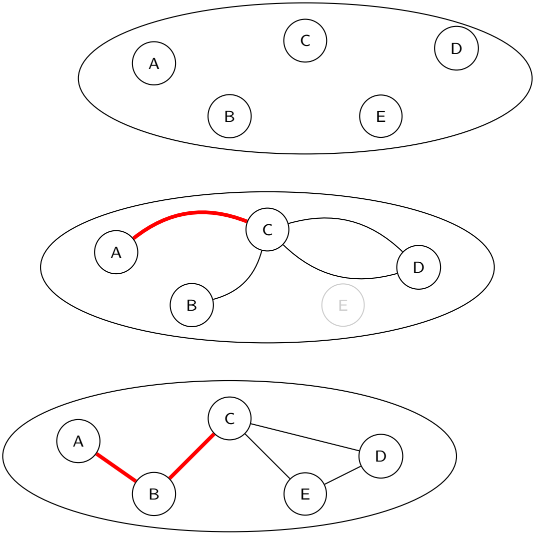

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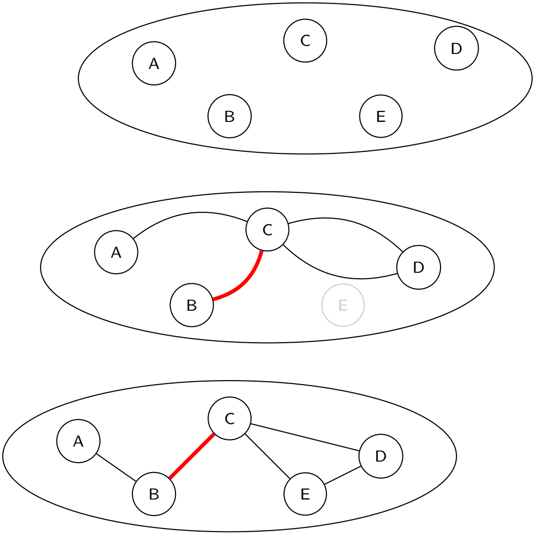

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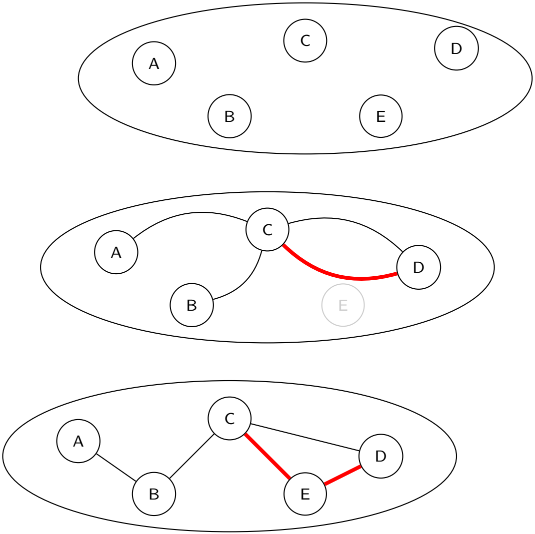

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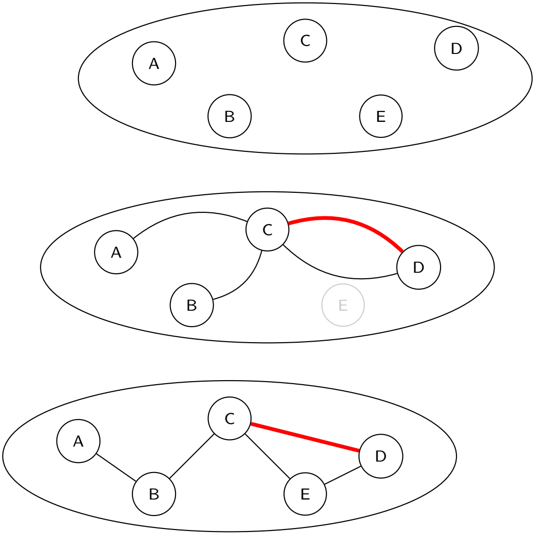

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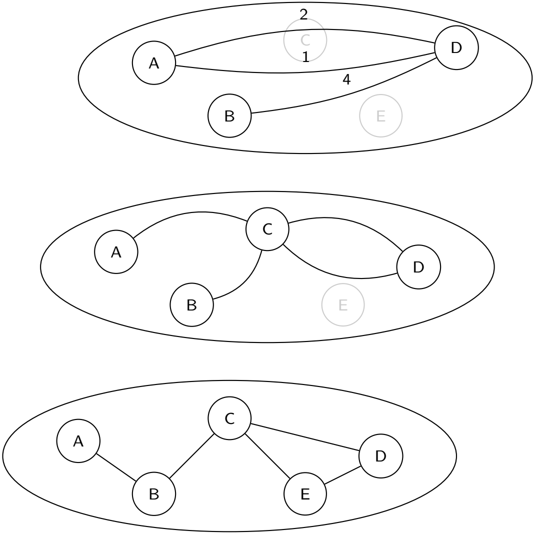

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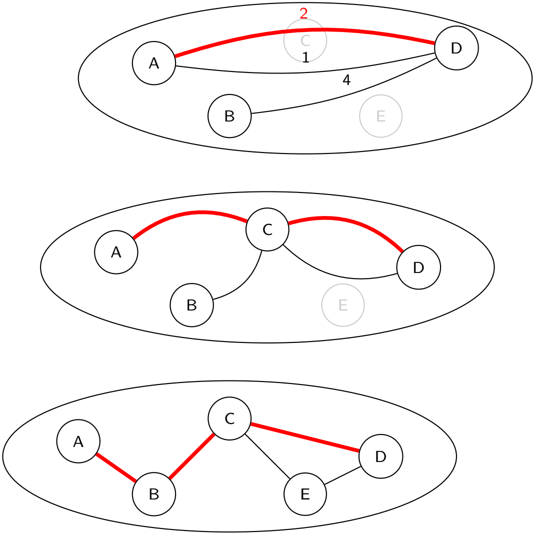

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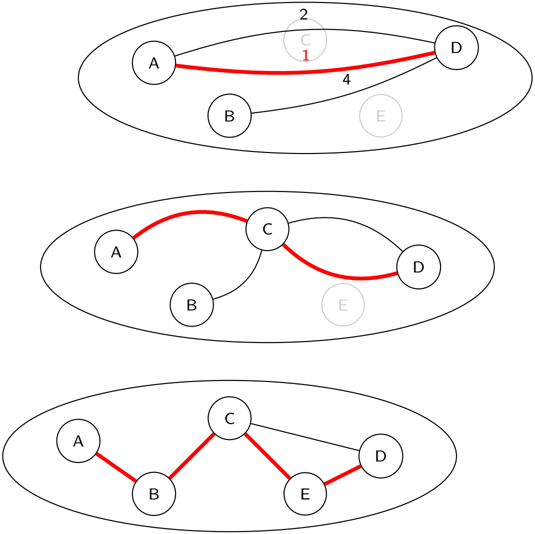

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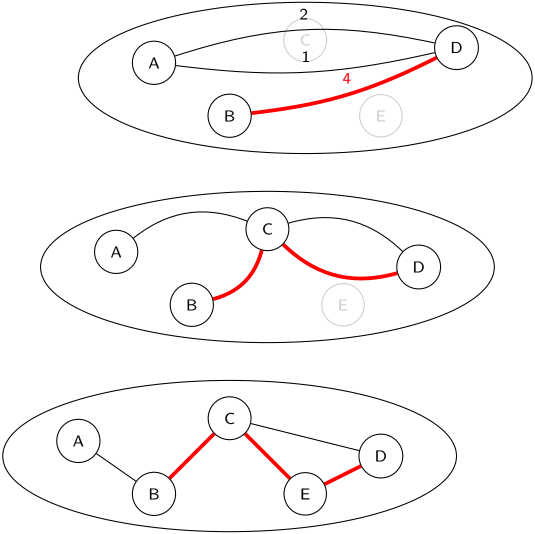

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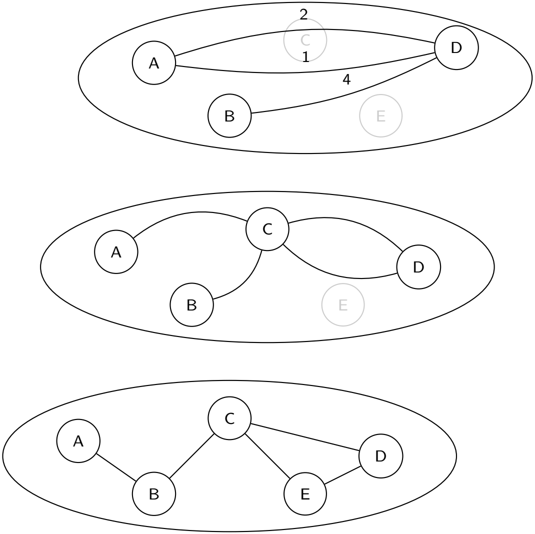

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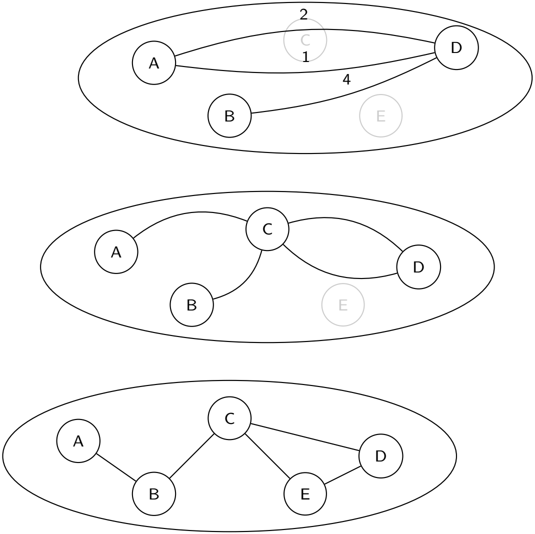

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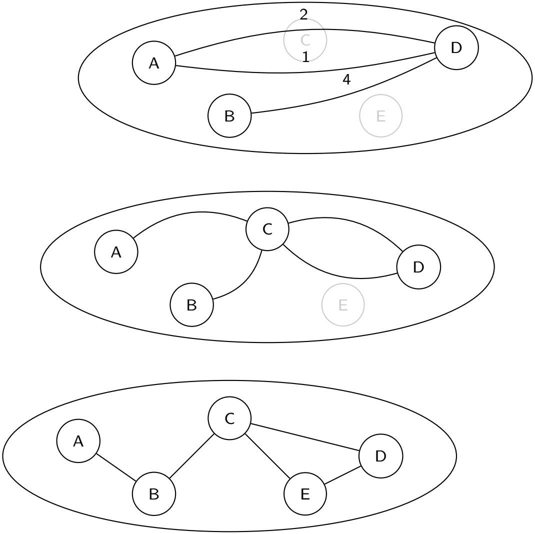

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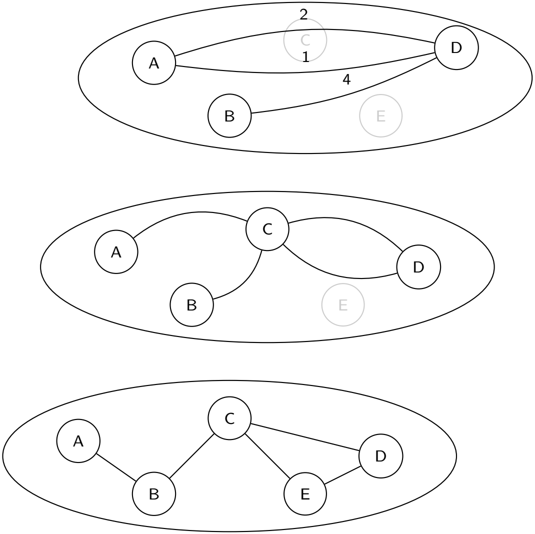

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- Key ingredient: **low-step MCMCF emulators**

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    - i.e., all flow paths have few edges

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- $\mathrm{totlen}(F') \leq s \cdot \mathrm{totlen}(F^*)$
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Low-Step Flow Emulators

A t-**step emulator** of a graph G is a

- graph H on same vertex set
- with mapping Π from *edges* in H to *paths* in G,

Such that

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Parameters: congestion slack κ, length slack s

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- We also give an existential result with a tighter tradeoff

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   - Built on length-constrained expander routing

Length-Constrained Expander Workshop
- **Here at STOC!**

Tuesday 25th - Introduction and overview

- Length-constrained expanders
- **Low-step emulators**

Wednesday 26th - Fast algorithms

- $\mathcal{O}(1)$-**approx min cost multicommodity flow** via emulators
- Algorithmics of LC expander decomposition

Thursday 27th - Dynamic algorithms

- Dynamic emulators $\rightarrow$ $\mathcal{O}(1)$-approx fully dynamic distance oracles!
- Open directions of research

End of Talk

Questions?

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